ON THE AXIOMS OF QUANTUM FIELD THEORY

D. A. SLAVNOV

Moscow State University

Submitted to JETP editor October 10, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 52, 1224–1232 (May, 1967)

An attempt is made to modify the axiomatic method of quantum field theory in such a way that a nontrivial example may be formulated within the framework of the theory. The relevant axioms are given. The essential point is the use of nonequivalent representations of the canonical commutation relations.

THE so-called axiomatic method of quantum field theory has been developed rather far in recent years (cf., for example,[1-3]). The main difficulty with this method is that up to now it has not yet been possible to formulate a single example with interacting fields within the framework of this method. On the other hand, the Lagrangian formalism (cf., for example,[4,5]), which lies outside the framework of the axiomatic method and has a much less rigorous foundation, allows one to calculate successfully many specific effects in the theory of interacting fields.

In the present paper we make an attempt to modify slightly the axiomatic method, by bringing it closer to the Lagrangian formalism, so as to make it possible to construct a model of interacting fields within its framework. The paper is devoted to the formulation of the relevant axioms.

In quantum field theory one usually assumes that the interaction between the fields plays a double role. First, the characteristics of the particles (charge, mass) arise as a result of the interactions. In the Lagrangian method the bare particles are renormalized. Second, these real particles interact with one another. Still adhering to this point of view, we propose the following method of description of interacting fields: At some instant T the interaction leads to the formation of particles which can approximately be regarded as free particles as far as their mutual interaction is concerned; the further evolution of the system is described in terms of these particles.

Two sorts of fields will be associated with a system of interacting particles: an interacting field, which is needed for the description of the evolution of the system, and a free field, which is necessary for the interpretation of the fields in terms of particles. In the more customary terminology these are the interacting Heisenberg field and, for example, the in field. The description of the interacting fields in terms of particles formed at the instant T will be called the T representation. In order that these particles may be regarded as free, the time T must be sufficiently far away from the time when the reaction takes place. In the limit T may be either $-\infty$ or $+\infty$. In the following we shall take $T \to -\infty$ which corresponds to a description of the system in the in representation. More exactly, we shall regard T as finite but sufficiently large in modulus.

We assume that each state of the interacting fields expressed in terms of the particles formed at the time T is represented by a vector in the Hilbert space $\mathcal{H}(T)$. This type of description is clearly not covariant. However, the covariance of the description can be regained if the time instant T is replaced by a space-like hyperplane $\Sigma(T, n)$, i.e., if the state of the interacting fields is represented by a vector of the Hilbert space $\mathcal{H}(T, n)$. Correspondingly, a state of the system is not to be considered at a fixed time instant t but on some hyperplane $\Sigma(t, n)$. Here we have introduced the following notations. Each hyperplane $\Sigma(t, n)$ is characterized by the unit vector along the normal n $[(n, n) = 1, n_0 > 0]$ and the "distance" t from the origin

$$t = (n, x), \tag{1}$$

where x are the coordinates of a point on the hyperplane.

Before we construct the space $\mathcal{H}(T, n)$, we introduce a special description of the points lying on the hyperplanes. We shall characterize the points of the hyperplane $\Sigma(t, n)$, not by the coordinate x, but by some other quantities which are constructed in the following way. Let $\widetilde{\Lambda}(n)$ be a Lorentz transformation of the form

$$\tilde{\Lambda}_{00} = n_0, \quad \tilde{\Lambda}_{0\alpha} = \tilde{\Lambda}_{\alpha 0} = -n_\alpha; \tag{2}$$
$$\tilde{\Lambda}_{\alpha\beta} = \delta_{\alpha\beta} + (1+n_0)^{-1} n_\alpha n_\beta, \quad \alpha, \beta = 1, 2, 3.$$

We introduce four unit vectors $e_i(0)$ with the projections

$$e_i(0) = \{\delta_{i0}, \delta_{i1}, \delta_{i2}, \delta_{i3}\}, \quad i = 0, 1, 2, 3$$

and construct the vectors

$$e_i(n) = \tilde{\Lambda}^{-1}(n) e_i(0). \tag{3}$$

With the help of the $e_i(n)$ we form the quantities

$$t = (e_0(n), x), \quad z_\alpha = (e_\alpha(n), x); \quad \alpha = 1, 2, 3. \tag{4}$$

The definitions of t according to (1) and (4) are identical. If we invert (4) we have

$$x = te_0(n) - \sum z_\alpha e_\alpha(n). \tag{5}$$

The vectors x of the form (5) satisfy identically the equation of the hyperplane (1), so that the quantities z_α are independent. Thus the quantities t and n give the position of the hyperplane, and the quantities z_α give the position of the points on this hyperplane. The quantity t will be called the "reduced time" of the point x, and the quantities z_α will be called "reduced coordinates."

Let us now consider the proper orthochronous Poincaré transformation $\mathscr{P} = (\Lambda, a)$. Under this transformation,

$$x \to x' = \Lambda x + a, \tag{6}$$

the hyperplane $\Sigma(t, n)$ goes over into the hyperplane $\Sigma(t', n')$, where

$$n' = \Lambda n, \quad t' = (n', x') = t + (n', a). \tag{7}$$

The reduced coordinates and the reduced time transform like $t \to t'$, $z_\alpha \to z'_\alpha$. The quantities t' and z'_α are defined by (4), where n and x are replaced by n' and x'. One easily finds that

$$t \to t' = t + \tau, \quad z_\alpha \to z_\alpha' = \sum r_{\alpha\beta} z_\beta + c_\alpha, \tag{8}$$

where

$$\tau = (e_0(n'), a), \quad c_\alpha = (e_\alpha(n'), a),$$
$$\sum r_{\alpha\beta} r_{\gamma\beta} = \sum r_{\beta\alpha} r_{\beta\gamma} = \delta_{\alpha\gamma}. \tag{9}$$

It is seen that $r_{\alpha\beta}$ is the three-dimensional matrix of rotation. The explicit form of $r_{\alpha\beta}$ can be obtained without difficulty, but it has a rather complicated form. Thus, under Poincaré transformations the reduced time undergoes a translation, and the reduced coordinates are rotated in three-dimensional space and translated. We emphasize that the quantities determining these transformations (τ, c_α, $r_{\alpha\beta}$) depend not only on the parameters of the Poincaré transformation, but also on n.

For points lying on the same or parallel hyperplanes, the distance between two points in the reduced coordinates $|\mathbf{z} - \mathbf{z}'|$ and the reduced time interval $|t - t'|$ are invariants under Poincaré transformations. We note that the concept of a state at a given instant of the reduced time is relativistically covariant, in contrast to the concept of a state at an instant of ordinary time. The use of the reduced coordinates and the reduced time therefore enables us to introduce in a relativistically covariant manner not only the Heisenberg picture, but also the Schrödinger picture. The latter could not have been done using the ordinary coordinates and time.

Let us now describe the mathematical form of the Hilbert space $\mathscr{H}(T, n)$ whose vectors describe the state of the system. In order not to burden the discussion with complications which are not essential at the present time, we consider only the case of a neutral scalar self-interacting field.

For a realization of the Hilbert space $\mathscr{H}(T, n)$ we take recourse to the Fock space (cf., for example, the book of Schweber[5]). That is, $\mathscr{H}(T, n)$ is the closure in the norm of the manifold $D(T, n)$ whose elements are the finite sequences of functions

$$\{f(\mathbf{z}); T, n\} = \{f_0, f_1(\mathbf{z}_1), \ldots, f_i(\mathbf{z}_1, \ldots, \mathbf{z}_i)\}. \tag{10}$$

Here T and n indicate that this sequence is a vector in $\mathscr{H}(T, n)$. The functions $f_i(\mathbf{z}_1, \ldots, \mathbf{z}_i)$ are symmetric in their arguments and belong to the Schwartz space $S(R_{3i})$;[6] $\mathbf{z}_1, \ldots, \mathbf{z}_i$ are the reduced coordinates of the hyperplane orthogonal to n. If in the usual way (as in the nonrelativistic theory) we introduce in $D(T, n)$ creation and annihilation operators $\psi^\pm(\mathbf{z})$ and the vacuum vector

$$|\Omega; T, n\rangle = \{1, 0, 0, \ldots\},$$

then the vectors of $D(T, n)$ can be written in the form

$$|f; T, n\rangle = f_0 |\Omega; T, n\rangle + \sum \int (d\mathbf{z}) f_i(\mathbf{z}_1, \ldots, \mathbf{z}_i) \psi^+(\mathbf{z}_1) \ldots \psi^+(\mathbf{z}_i) |\Omega; T, n\rangle. \tag{11}$$

Here and in the following $(d\mathbf{z}) = d\mathbf{z}_1 \ldots d\mathbf{z}_i$. Strictly speaking, the operators $\psi^\pm$ must be provided with indices T, n since they act in different spaces, but since the way they act is the same, we shall omit the indices T, n in the operators. The scalar product is defined as

$$\langle g | f \rangle = g_0^* f_0 + \sum \int (d\mathbf{z}) g_i^*(\mathbf{z}_1, \ldots, \mathbf{z}_i) f_i(\mathbf{z}_1, \ldots, \mathbf{z}_i). \tag{12}$$

From the operators ψ^+ and ψ^- we can construct symmetric operators φ_{+1} and φ_{-1} which satisfy the free field equations

$$\varphi_\sigma(t, \mathbf{z}) = \frac{1}{(2\pi)^3} \sqrt{\frac{\sigma}{2}} \int d\mathbf{u} d\mathbf{k} e^{i\mathbf{k}(\mathbf{z}-\mathbf{u})} \omega^{-\sigma} \times [e^{i\omega t}\psi^+(\mathbf{u}) + \sigma e^{-i\omega t}\varphi^-(\mathbf{u})], \tag{13}$$

where $\omega = \sqrt{\mathbf{k}^2 + m^2}$, $m^2 > 0$, $\sigma = \pm 1$ ($\sigma = 1$ for the field operator, $\sigma = -1$ for the operator of the canonical momentum).

We give the following physical interpretation of the vectors of $\mathscr{H}(T, n)$. Using the operators (13), we rewrite (11) in the form

$$|f; T, n\rangle = f_0|\Omega; T, n\rangle + \sum \int (d\mathbf{z}) f_i'(\mathbf{z}_1, \ldots, \mathbf{z}_i) \varphi_+^+(0, \mathbf{z}_1) \ldots \varphi_+^+(0, \mathbf{z}_i) |\Omega; T, n\rangle. \tag{14}$$

Here φ_+^+ is the positive frequency part of φ_+. The quantity f' is expressed through f in an elementary way, and we shall omit the prime in the following. We assume that the vector (14) would describe a state of our system on the hyperplane $\Sigma(T, n)$ in the Schrödinger picture if the system had developed from a state without interaction at $t = T$. In other words, we assume that on the hyperplane $\Sigma(T, n)$ the state of our system can be approximated by a state of non-interacting particles described by the field φ. In the following we shall use the Heisenberg picture, i.e., we assume that the vector (14) describes a state of our system of interacting particles at an arbitrary time instant, where all the time dependence of the system is incorporated in the operators of the interacting fields. We put this in the form of an axion:

Axiom I. A state of a system of interacting fields in the representation corresponding to the hyperplane $\Sigma(T, n)$ is described by a normalized vector of the Hilbert space $\mathscr{H}(T, n)$.

Let us now turn to the construction of a representation of the Poincaré group. A transformation of the coordinate system can lead to two kinds of changes in the vectors describing the state of the system. First, the coordinate transformation may induce a transformation of the vectors within $\mathscr{H}(T, n)$. Second, if the transformation of the coordinates changes the hyperplane $\Sigma(T, n)$, then the interpretation of the state vectors must also change. The system of interacting fields will no longer be interpreted in terms of particles corresponding to the hyperplane $\Sigma(T, n)$ but in terms of particles corresponding to some other hyperplane $\Sigma(T', n')$. Thus the state of the system will no longer be described by a vector of the space $\Sigma(T, n)$ but by a vector of the space $\Sigma(T', n')$. More definitely, we introduce the following axiom:

Axiom II. To each vector $|f; T, n\rangle$ [of the form (14)] of $D(T, n)$ and to an arbitrary proper orthochronous Poincaré transformation $\mathscr{P} = (\Lambda, a)$ there corresponds a vector $|f\{\Lambda, a\}; \Lambda n, T+\tau\rangle$ of $D(T+\tau, \Lambda n)$:

$$|f_{\{\Lambda,a\}}; \Lambda n, T+\tau\rangle = f_0|\Omega; T+\tau, \Lambda n\rangle + \sum \int (d\mathbf{z}) f_i(\mathbf{z}_1, \ldots, \mathbf{z}_i) \varphi_+^+(\tau, r\mathbf{z}_1 + \mathbf{c}) \ldots \varphi_+^+(\tau, r\mathbf{z}_i + \mathbf{c}) \times |\Omega; T+\tau, \Lambda n\rangle \equiv f_0|\Omega; T+\tau, \Lambda n\rangle + \sum \int (d\mathbf{z}) f_i(-\tau; r^{-1}(\mathbf{z}_1 - \mathbf{c}), \ldots, r^{-1}(\mathbf{z}_i - \mathbf{c})) \times \varphi_+^+(0, \mathbf{z}_1) \ldots \varphi_+^+(0, \mathbf{z}_i) |\Omega; T+\tau, \Lambda n\rangle, \tag{15}$$

where r, c, and τ are given by (8) and (9).

Formula (15) also serves as a definition of the functions $f_i(-\tau; \ldots)$. If we introduce the operator $U(\Lambda, a)$ by

$$U(\Lambda, a) |f_{\{\Lambda, a\}}, \Lambda n, T+\tau\rangle = |f; n, T\rangle, \tag{16}$$

it will evidently be isometric. We emphasize that in general, $U(\Lambda, a)$ takes a vector from one Hilbert space to another. Thus it is impossible to introduce an operator of the energy corresponding to the generator $U(1, a)$.

It is seen from (15) that the Poincaré transformation $\mathscr{P}$ sets the sequence of functions $\{f(\mathbf{z})\}$, describing some configuration of fields corresponding to particles formed at the instant T, in correspondence with the sequence $\{f(-\tau; r^{-1}(\mathbf{z} - \mathbf{c}))\}$, describing some other configuration of fields corresponding to particles formed at the instant $T + \tau$. It may be assumed that for sufficiently large negative times and sufficiently small τ the particles formed at the time $T + \tau$ will be the same as those formed at the time T. We recall that we regard the particles as free for such times T. Then the Poincaré transformation reduces to a change in the configuration of the fields describing the same particles.

Correspondingly, we shall regard the sequences $\{f(\mathbf{z})\}$ and $\{f(-\tau; r^{-1}(\mathbf{z} - \mathbf{c}))\}$ as elements of the same Hilbert space (the definition of the norm is the old one). It is easy to see that they are connected by a unitary operator $U_0(\Lambda, a)$, which is constructed in an elementary fashion. The operator $U_0(\Lambda, a)$ has seven generators which can be given a physical meaning (reduced energy, momentum, and angular momentum). In particular, we take the generator $U_0(1, \tau e_0(n))$ as the operator of the "reduced energy" E, and the generators $U_0(1, c_\alpha e_\alpha(n))$ as the components of the "reduced momentum" Q_α. It is easy to see that, if the operator of energy-momentum P is defined as the generator $U_0(1, a)$, then

$$P = E e_0(n) - \sum Q_\alpha e_\alpha(n). \tag{17}$$

It is verified by an elementary calculation that P has the correct transformation properties, the vacuum is an eigenvector of P with vanishing eigenvalue, and the spectrum of the operator P lies in the forward light cone. Thus the spectral condition is fulfilled. The angular momentum operator can be treated analogously. We call attention to the fact that in our approach there are no generators corresponding to quantities with no direct physical interpretation.

For the description of the evolution of the interacting system we must introduce the operators of the interacting fields. It is easy to see that the operators $\varphi_\sigma(t, \mathbf{z})$ form an irreducible representation of the canonical commutation relations for each fixed value of t, where all these representations are unitarily equivalent. Let us introduce the operators $A_\sigma(t, \mathbf{z}; T)$, which also form irreducible representations of the canonical commutation relations but not necessarily equivalent ones. The operators $A_\alpha(t, \mathbf{z}; T)$ will be treated like operators of the interacting fields in the T representation. The properties of the operators A_σ, will be formulated in a set of axioms.

Axiom III. For fixed values of t and T the quantities $A_\sigma(t, \mathbf{z}; T)$ $(\sigma = \pm 1)$ are generalized operator functions on the Schwartz space $S(R_3)$.

The essential point in Axiom III is the fact that the field operators are defined for a fixed value of t. Usually it is assumed that the fields are averaged over t. But when the fields are averaged over t it becomes unclear how the chronological ordering of the fields can be introduced. It is known what important role this ordering process plays in the construction of the theory in the Lagrangian approach. The fact that up to now no one has succeeded in constructing a single model of interacting fields in the axiomatic method, is largely explained by the circumstance that in the axiomatic method there is no equivalent of the chronological ordering process. In the adopted version of Axiom III this difficulty naturally does not arise.

Axiom IV. The region of definition of $A_\sigma(t, \mathbf{z}; T)$ is $D(T, n)$, the region of values lies in $D(T, n)$.

Again, strictly speaking the operators A_σ must be provided with an additional index n, but we assume that the action of the operators A_σ in the corresponding Hilbert space is independent of n, and hence omit the index n.

Axiom V. The operators $A_\sigma(t, \mathbf{z}; T)$ satisfy the canonical commutation relations

$$[A_\sigma(t, \mathbf{z}; T), \quad A_\rho(t, \mathbf{u}; T)] = {}^1\!/_2 i(\sigma - \rho)\delta(\mathbf{z} - \mathbf{u}), \quad \sigma, \rho = \pm 1. \tag{18}$$

The fact that the field operators must satisfy the canonical commutation relations is a reflection of the principle of correspondence with nonrelativistic quantum mechanics and is assumed in many formulations of quantum field theory. From this circumstance conclusions are drawn regarding the observable quantities. However, we must keep in mind that if the fields are regarded as generalized operator functions which must be averaged over the coordinates as well as over the time, it is meaningless to talk about equal-time commutation relations, let alone, to draw any conclusions from them. Nevertheless, this is what is usually done. In our approach such contradictions do not arise. Because of Axiom III our canonical commutation relations have a completely clear mathematical meaning.

Axiom VI. The operators $A_\sigma(t, \mathbf{z}; T)$ are symmetric (Hermitean).

This axiom is not very essential. It reflects the fact that we work with neutral fields.

All observable quantities must be expressed in terms of the operators of the interacting fields A_σ. To this end, the system of these operators must be complete in some sense. In order to give these words a clear mathematical meaning, we form the ring $\mathfrak{A}$ of operators $\psi^\pm$, i.e., the set of all possible finite sums and products of these operators. The ring of operators $\varphi_\sigma(t, \mathbf{z})$ evidently coincides with $\mathfrak{A}$. We have then:

Axiom VII. For fixed t and T the ring of operators $A_\sigma(t, \mathbf{z}; T)$ coincides with $\mathfrak{A}$, where

$$A_\sigma(t, \mathbf{z}; T) = \varphi_\sigma(t, \mathbf{z}) \quad \text{for} \quad t = T. \tag{19}$$

It follows from this axiom that the operators ψ^+ can be written in the form of certain polynomials in $A_\sigma(t, \mathbf{z}; T)$. This immediately implies that the $A_\sigma(t, \mathbf{z}; T)$ form an irreducible representation of (18). Since any vector of $D(T, n)$ can be obtained from the vacuum by acting on it with some polynomial of the operators ψ^+ and therefore, of $A_\sigma(t, \mathbf{z}; T)$, the vacuum will be a cyclic vector for the fields $A_\sigma(t, \mathbf{z}; T)$. Formula (19) expresses the fact that the fields φ_σ correspond to particles formed at the instant T.

An immediate consequence of axiom VII is the formula

$$A_\sigma(t, \mathbf{z}; T) = \sum \int (du) J_i^\sigma(t; T; \mathbf{z}; \ \mathbf{u}_1, \ldots, \mathbf{u}_i) \times \varphi_{\sigma_1}(T, \mathbf{u}_1) \ldots \varphi_{\sigma_i}(T, \mathbf{u}_i), \tag{20}$$

where J_i^σ are certain coefficient functions. In order to guarantee the relativistic invariance of the theory we impose the following restrictions on the dependence of the J_i^σ on their arguments:

Axiom VIII. The functions $J_i^\sigma(t;\ T;\ \mathbf{z};\ u_1, \ldots, u_i)$ depend only on $t - T$, $|u_1 - \mathbf{z}|, \ldots, |u_i - \mathbf{z}|$.

That axiom VIII guarantees the relativistic invariance of the theory can be seen in the following way. Let us go from one coordinate system to another with the help of the Poincaré transformation $\mathcal{P} = (\Lambda, a)$. Then the field operator $A(t, \mathbf{z};\ T)$ goes over into $A(t + \tau, r\mathbf{z} + \mathbf{c};\ T + \tau)$. We call attention to the fact that not only are the coordinates and the time $(t, \mathbf{z})$ changed on which the field operator depends, but also the T representation is altered. Indeed, if in the old coordinate system the state was described in terms of particles corresponding to the hyperplane $\Sigma(T, n)$, then in the new coordinate system the same description must be given in terms of particles corresponding to the hyperplane $\Sigma(T + \tau, \Lambda n)$. It is easy to verify that axiom VIII leads to the formula

$$U^+(\Lambda, a) A(t, \mathbf{z}; T) U(\Lambda, a) = A(t + \tau;\ r\mathbf{z} + \mathbf{c};\ T + \tau). \quad (21)$$

Comparing (21) and (16), we conclude that the theory is invariant under Poincaré transformations.

The local properties of the theory are formulated in the following way.

Axiom IX. The operators $A_\sigma(t, \mathbf{z};\ T)$ satisfy the strong locality condition

$$[A_\sigma(t_1, \mathbf{z}_1; T), A_\rho(t_2, \mathbf{z}_2; T)] = 0 \text{ for } |\mathbf{z}_1 - \mathbf{z}_2| > |t_1 - t_2|. \quad (22)$$

Axiom IX can be weakened somewhat:

Axiom IXa. The operators $A_\sigma(t, \mathbf{z};\ T)$ satisfy the weakened locality condition

$$[A_\sigma(t_1, \mathbf{z}_1; T),\ A_\rho(t_2, \mathbf{z}_2; T)] = 0 \quad \text{for} \quad |\mathbf{z}_1 - \mathbf{z}_2| > |t_1 - t_2| + \eta, \quad (22a)$$

where η is some small positive quantity.

As already noted, the quantities $|\mathbf{z}_1 - \mathbf{z}_2|$ and $|t_1 - t_2|$ are invariant under Poincaré transformations. Therefore the weakened locality condition has a covariant form. It is seen that the weak locality condition is conveniently formulated in terms of the reduced coordinates and the reduced time.

It is known that the introduction of the concept of a state localized at a given moment in some spatial region meets with great difficulties in the theory of quantized fields. With the help of the operators $A_\sigma(t, \mathbf{z};\ T)$ we can construct states which have properties which justify their being called localized in some region of the reduced coordinates at a given instant of reduced time. As already noted, any vector of $D(T, n)$ can be obtained from the vacuum by acting on it with an operator of the type

$$\sum_i \int (d\mathbf{z}) f_i(t; \mathbf{z}_1, \ldots, \mathbf{z}_i) A_{\sigma_1}(t, \mathbf{z}_1; T) \ldots A_{\sigma_i}(t, \mathbf{z}_i; T), \quad (23)$$

which belongs to the ring $\mathfrak{A}_+$. This ring is generated by the operators ψ^+. Consider two vectors $|f\rangle$ and $|g\rangle$ of $D(T, n)$. We write them down with the help of operators of the type (23) (with the same t). With these same vectors we form two sets of functions $\{f_i(t;\ \mathbf{z}_1, \ldots, \mathbf{z}_i)\}$ and $\{g_i(t;\ \mathbf{z}_1, \ldots, \mathbf{z}_i)\}$. Using the canonical commutation relations we can show that the states $|f\rangle$ and $|g\rangle$ are orthogonal if the supports in $\mathbf{z}$ of all f are located in the region G_1 and the supports of all g in the region G_2, and these regions do not intersect.

Let us now consider the state $|f\rangle$. We express it with the help of operators of the type (23), using at one time the operators $A_\sigma(t, \mathbf{z};\ T)$ and another time the operators $A_\sigma(t', \mathbf{z};\ T)$. This same vector f we set in correspondence with two sets of functions $\{f_i(t;\ \mathbf{z}_1, \ldots, \mathbf{z}_i)\}$ and $\{f_i(t';\ \mathbf{z}_1, \ldots, \mathbf{z}_i)\}$. Let the supports of all functions of the first set be located in the region G. Using the axiom IX we can then show that the supports of the functions of the second set will be located in the region G′ which is defined by the conditions

$$\text{if } \mathbf{z}' \in G, \quad \mathbf{z}' \in G', \text{ then то } |\mathbf{z} - \mathbf{z}'| \leqslant |t - t'|. \quad (24)$$

If axiom IXa is used instead of IX, then the region G will be defined by the inequality $|\mathbf{z} - \mathbf{z}'| \le |t - t'| + \eta$.

The region G may be called the region of localization of the state at a given instant of the reduced time. This appellation is justified, first by the fact that two states which at a given moment are localized in nonintersecting regions are orthogonal to one another; secondly, a state localized at a given instant in some region G, will at later moments be localized in regions lying in the light shadow of G.

Axiom VII does not propose that the operators $A_\sigma(t, \mathbf{z};\ T)$ for different t span equivalent representations (18). Moreover, it can be shown that the operators $A_\sigma(t, \mathbf{z};\ T)$ can only describe noninteracting fields if these operators are unitarily equivalent, where the vacuum is invariant under this unitary transformation and (22) is satisfied (this is a variant of Haag's theorem [2,7]).

In this connection it becomes clear why we had to introduce the set of Hilbert spaces $\mathscr{H}(T, n)$. Indeed, fixing some T_0, we can connect the operators A_σ at different times t with one another using (20). We obtain a certain operator algebra. All these operators act in $\mathscr{H}(T_0, n)$. On the other hand, this algebra can be regarded as abstract without specifying the spaces in which the operators act. In this case the spaces $\mathscr{H}(T, n)$ can be regarded as spaces generated by this algebra, where the different $\mathscr{H}(T, n)$ are characterized by the fact that the operators A_σ have a vacuum in the corresponding space at $t = T$. If all operators were unitarily equivalent, we could restrict ourselves to this space. But if the operators are not equivalent, we must introduce a set of spaces, since the space in which the operators A_σ have a vacuum at some time t, cannot have a vacuum for the operators A_σ at another time t'.

In the beginning of this paper we said that we want to work in the in representation. For this it would be desirable if the operators $A_\sigma(t, \mathbf{z};\ T)$ had in some sense a limit for $T \to -\infty$. More precisely, expressions of the type

$$\lim_{T\to-\infty} \langle g; T | A(t, \mathbf{z}_1; T) \ldots A(t, \mathbf{z}_i; T) | f; T\rangle. \qquad (25)$$

should exist as generalized functions.

In order to describe the evolution of the system we must know the dependence of the operators $A_\sigma(t, \mathbf{z};\ T)$ on t, i.e., we must practically know the form of the functions J_i^σ. Choosing the functions J_i^σ in some way, we can determine a specific model of interacting fields. In other words, the knowledge of the functions J_i^σ plays in our case the same role as the knowledge of the Lagrangian in the Lagrangian approach.

Thus the problem of the construction of a nontrivial example of the theory has been reduced to the problem of constructing functions J_i^σ which satisfy all axioms enumerated above, or at least all equations which they must satisfy. It turns out that these equations can be formulated relatively easily, at least if the locality condition is taken in the form (22a). It can be shown that these equations have a solution. However, all this requires a special consideration.

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Translated by R. Lipperheide
151