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STATISTICAL THEORY OF INELASTIC PROCESSES IN A PLASMA

I. Kinetic Equations for Coulomb Plasma with Allowance for Inelastic Processes

Yu. L. KLIMONTOVICH

Moscow State University

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A method is developed for describing non-equilibrium processes in a partially ionized plasma by taking into account inelastic processes. The initial set of equations is that for four operator density matrices that describe the states of free and coupled charged-particle pairs (electrons and ions) as well as transitions from bound states to free ones and vice versa. The equations are considered simultaneously with those for the microscopic electromagnetic field strengths. Expressions in which inelastic processes are taken into account are obtained for the field spectral functions, and a set of kinetic equations for the electron, ion, and atom distribution functions is obtained, in which inelastic processes are taken into account. The properties of the collision integrals are such that they ensure conservation of the total number of charged particles, the total momentum, and the energy. The Maxwell-Boltzmann distribution is the equilibrium solution of this set of equations under conditions of chemical (ionization) equilibrium.

The approximation in which the kinetic equations have been derived corresponds to the Born approximation for elastic and inelastic processes, but with atomic motions and polarization of the medium taken into account. The polarization of the medium is due to four processes: motion of free charged particles, variation of the internal state of the atoms, and transitions of the charged particles from the bound state to a free one or vice versa.

A set of equations for the electron, ion, and atom concentrations is derived with aid of the kinetic equations. The ionization coefficient for longitudinal electric field quanta is calculated. The expressions for the equilibrium ionization and recombination coefficients of colliding particles are considered.

INTRODUCTION

IN studies of the statistical theory of non-equilibrium processes in a plasma [1-8], principal attention is paid to the investigation of a fully-ionized plasma. The starting point is either the system of equations for the distribution functions of the coordinates and momenta of all the charged particles [2,8], or the system of equations for the microscopic phase density of each plasma component and the microscopic electromagnetic fields [7]. In the quantum case, the chain of equations for the distribution functions is replaced by a chain of equations for the density matrices of one, two, etc. particles [8], and the equations for the microscopic phase densities are replaced by the corresponding operator equations [9,10]. The initial equations are valid, naturally, for a description of both fully and partly ionized plasma. Limitations arise in the choice of the method of approximately solving the initial equations.

The kinetic equations are derived for a plasma by using the assumption that the correlation functions are small [2,4-8]. In zeroth approximation, that is, when the correlation of the coordinates and momenta of the charged particles is completely neglected, a closed system of equations is obtained for the first distribution functions of the electrons and ions and for the average intensities of the electromagnetic field, namely the Vlasov approximation. This approximation can be called the approximation of completely free charged particles. The term ''free charged particles'' has here a statistical meaning and indicates that the correlation functions are equal to zero.

In the next approximation (in determining the collision integral), the correlation is taken into account but it is assumed to be small. The small parameter is either the ratio of the average potential energy of the interaction to the average kinetic energy to the particles, or else the plasma parameter. Such an approximation is not suitable for the description of the bound states of charged particles in atoms, for in this case the correlation functions are not small. This made it possible, in the derivation of the kinetic equations in [1,2,4-8], to obtain only a collision integral that describes elastic scattering of charge particles—the collision integral in the Landau form.

In the present paper we consider a more general method of approximately solving the exact initial equations; this method makes it possible to describe the processes in a partly ionized plasma with allowance for inelastic interactions between the free charged particles and the atoms and of the inelastic collisions of the atoms with one another.

In many recent papers dealing with ionization, dissociation, and other inelastic processes in gases at sufficiently high temperatures, kinetic equations with allowance for inelastic processes are used to derive hydrodynamic equations. An analysis of these papers and a pertinent bibliography are contained in [11-13]. The kinetic equations themselves, which take inelastic processes into account, are written in analogy with the Boltzmann equation, by introducing appropriate effective cross sections. A consistent statistical derivation of these equations from exact macroscopic equations is one of the main tasks of the present paper.

1. INITIAL MICROSCOPIC EQUATIONS

We consider a partly polarized plasma. In the simplest case, such a system consists of three components: electrons, singly-charged ions, and neutral atoms. The first two components will be designated by indices a and b, and the third by the double index ab. The charged particles of components a and b will be called free, and those of the component ab, that is, the charged particles forming the atoms, will be called bound.

The system on the whole is neutral, so that the total number of negatively charged particles (both free and bound) is equal to the total number of positively charged particles. We denote this number by N. In order not to complicate the problem from the very outset, we confine ourselves here to the case of a Coulomb plasma.

To take into account the bound states in the kinetic equations (within the framework of the classical theory), it is convenient to separate from the very beginning the particle pairs making up the atoms (or, in the more general case, more complicated particle complexes). To this end we can use as the starting point the equation for the microscopic phase density of pairs of charged particles in twelve-dimensional space $\mathbf{r}_a$, $\mathbf{p}_a$, $\mathbf{r}_b$, $\mathbf{p}_b$, that is, for the function

$$N_{ab}(\mathbf{r}_a, \mathbf{p}_a, \mathbf{r}_b, \mathbf{p}_b, t) = \sum_{1\leqslant i\leqslant N} \delta(\mathbf{r}_a - \mathbf{r}_{ia}(t))\,\delta(\mathbf{p}_a - \mathbf{p}_{ia}(t)) \times \delta(\mathbf{r}_b - \mathbf{r}_{ib}(t))\,\delta(\mathbf{p}_b - \mathbf{p}_{ib}(t)),$$

and the equation for the microscopic field [10].

For a consistent description of the inelastic processes it is necessary to use quantum theory, and therefore we shall use in lieu of the classical function the corresponding quantum function—the operator of phase density in the space $\mathbf{r}_a$, $\mathbf{p}_a$, $\mathbf{r}_b$, $\mathbf{p}_b$:

$$N_{ab} = \frac{N}{(2\pi)^6} \int \rho_{ab}\left(\mathbf{r}_a \pm \frac{1}{2}\hbar\boldsymbol{\tau}_a, \mathbf{r}_b \pm \frac{1}{2}\hbar\boldsymbol{\tau}_b \right) \cdot \times \exp\{-i(\boldsymbol{\tau}_a\mathbf{p}_a + \boldsymbol{\tau}_b\mathbf{p}_b)\}\,d\boldsymbol{\tau}_a d\boldsymbol{\tau}_b.$$

Here ρ_{ab} is the corresponding operator density matrix.

In place of the variables $\mathbf{r}'_a$, $\mathbf{r}''_a$, $\mathbf{r}'_b$, $\mathbf{r}''_b$, it will be more convenient to use the variables

$$\mathbf{R} = (m_a\mathbf{r}_a + m_b\mathbf{r}_b)/(m_a + m_b), \quad \mathbf{r} = \mathbf{r}_a - \mathbf{r}_b. \tag{1.1}$$

In terms of these variables, the equation for the operator density matrix takes the form

$$i\hbar\frac{\partial\rho_{ab}}{\partial t} = -\frac{\hbar^2}{2M}(\Delta_{\mathbf{R}'} - \Delta_{\mathbf{R}''})\rho_{ab} - \frac{\hbar^2}{2\mu}(\Delta_{\mathbf{r}'} - \Delta_{\mathbf{r}''})\rho_{ab} + \{[\Phi_{ab}(|\mathbf{r}'|) + U_{ab}(\mathbf{R}', \mathbf{r}')] - [\Phi_{ab}(|\mathbf{r}''|) + U_{ab}(\mathbf{R}'', \mathbf{r}'')]\}\rho_{ab}. \tag{1.2}$$

Here

$$M = m_a + m_b, \qquad \mu = m_a m_b/(m_a + m_b),$$

$$U_{ab}(\mathbf{R}, \mathbf{r}) = e_a U^M(\mathbf{r}_a) + e_b U^M(\mathbf{r}_b). \tag{1.3}$$

The equation for the microscopic potential U^M can be expressed in the terms of these variables in the form

$$\Delta U^M(\mathbf{q}, t) = -4\pi\int\left[e_a\delta\left(\mathbf{q} - \left(\mathbf{R} + \frac{m_b}{m_a + m_b}\mathbf{r}\right)\right) + e_b\delta\left(\mathbf{q} - \left(\mathbf{R} - \frac{m_a}{m_a + m_b}\mathbf{r}\right)\right)\right]\rho_{ab}(\mathbf{R}, \mathbf{R}, \mathbf{r}, \mathbf{r}, t)\,d\mathbf{R}\,d\mathbf{r}. \tag{1.4}$$

Equations (1.2) for the function ρ_{ab} describes the distribution of the particle pairs, both free and bound. In order to separate the free and the bound states and to describe the transitions between them, we proceed as follows.

We use the eigenfunctions of the energy operator of an individual atom. They are represented by the equations

$$-\frac{\hbar^2}{2\mu}[\Delta_r + \Phi_{ab}(|\mathbf{r}|)]\Psi_\alpha(\mathbf{r}) = E_\alpha\Psi_\alpha,$$

$$-\frac{\hbar^2}{2M}\Delta_{\mathbf{R}}\Psi_{\mathbf{P}}(\mathbf{R}) = E_{\mathbf{P}}\Psi_{\mathbf{P}}. \tag{1.5}$$

The orthonormalization conditions for the eigenfunctions is written in the form

$$\int \Psi_{\mathbf{P}'}(\mathbf{R}) \Psi^{*}_{\mathbf{P}''}(\mathbf{R})\, d\mathbf{R} = \frac{(2\pi\hbar)^3}{V} \delta(\mathbf{P}' - \mathbf{P}''),$$

$$\Psi_{\mathbf{P}}(\mathbf{R}) = \frac{1}{\sqrt{V}} \exp\left\{\frac{i\mathbf{P}\mathbf{R}}{\hbar}\right\},$$

$$\int \Psi_{\alpha}(\mathbf{r})\, \Psi^{*}_{\beta}(\mathbf{r})\, d\mathbf{r} = \delta_{\alpha\beta} = \begin{cases} \delta_{nm} & \text{for } \alpha = n,\ \beta = m \\ 0 & \text{for } \alpha = \mathbf{p}',\ \beta = m;\ \alpha = n,\ \beta = \mathbf{p}'' \\ (2\pi\hbar)^3 V^{-1} \delta(\mathbf{p}' - \mathbf{p}'') & \text{for } \alpha = \mathbf{p}',\ \beta = \mathbf{p}'' \end{cases} \tag{1.6}$$

Thus, the values $\alpha = n$ and $\beta = m$ pertain to the discrete spectrum, while $\alpha = \mathbf{p}'$ and $\beta = \mathbf{p}''$ pertain to the continuous spectrum.

Using the eigenfunction expansion, we represent the operator density matrix in the form

$$\rho_{ab}(\mathbf{R}', \mathbf{R}'', \mathbf{r}', \mathbf{r}'', t) = \frac{V^2}{(2\pi\hbar)^6} \sum_{\alpha\beta} \int \rho_{\alpha\beta}(\mathbf{P}', \mathbf{P}'', t)\, \Psi_{\alpha}(\mathbf{r}') \Psi^{*}_{\beta}(\mathbf{r}'') \Psi_{\mathbf{P}'}(\mathbf{R}') \times \Psi^{*}_{\mathbf{P}''}(\mathbf{R}'')\, d\mathbf{P}'\, d\mathbf{P}''. \tag{1.7}$$

From (1.2) and (1.7) we obtain a system of equations for the functions $\rho_{ab}(\mathbf{P}', \mathbf{P}'', t)$:

$$i\hbar \frac{\partial \rho_{\alpha\beta}(\mathbf{P}', \mathbf{P}'', t)}{\partial t} = (E_{\alpha} + E_{\mathbf{P}'} - E_{\beta} - E_{\mathbf{P}''}) \rho_{\alpha\beta}(\mathbf{P}', \mathbf{P}'') + \frac{V}{(2\pi\hbar)^3} \sum_{\gamma} \int [U_{\alpha\gamma}(\mathbf{P}', \mathbf{P}_1) \rho_{\gamma\beta}(\mathbf{P}_1, \mathbf{P}'') - \rho_{\alpha\gamma}(\mathbf{P}', \mathbf{P}_1) U_{\gamma\beta}(\mathbf{P}_1, \mathbf{P}'')]\, d\mathbf{P}_1,$$

$$\alpha = n,\ \mathbf{p}';\ \beta = m, \mathbf{p}''. \tag{1.8}$$

Here

$$U_{\alpha\beta}(\mathbf{P}', \mathbf{P}'', t) = \int \Psi^{*}_{\alpha}(\mathbf{r}) \Psi^{*}_{\mathbf{P}'}(\mathbf{R})\, U_{ab}(\mathbf{R}, \mathbf{r}, t) \Psi_{\mathbf{P}''}(\mathbf{R}) \Psi_{\beta}(\mathbf{r})\, d\mathbf{r}\, d\mathbf{R}, \tag{1.9}$$

$$\sum_{\gamma} \cdots = \sum_{n} \cdots + \frac{V}{(2\pi\hbar)^3} \int \ldots d\mathbf{p}_1. \tag{1.10}$$

We have thus reduced the initial equation (1.2) to a system of equations for four operator density matrices:

$$\rho_{nm},\ \rho_{\mathbf{p}'\mathbf{p}''},\ \rho_{n\mathbf{p}''},\ \rho_{\mathbf{p}'m}(\mathbf{P}', \mathbf{P}'', t)$$

The first two describe the bound and free states of the charged particles, respectively, and the last two described transitions between the free and bound states.

2. KINETIC EQUATIONS FOR THE DISTRIBUTION FUNCTIONS OF THE ELECTRONS, IONS, AND ATOMS

We present the derivation of the kinetic equations in two stages. We first obtain the kinetic equations for the distribution functions of pairs of charged particles, and from them we obtain the sought-for equations for the distribution functions of the electrons, ions, and atoms.

We confine ourselves here to the case of a spatially homogeneous plasma. Under this condition we have

$$\bar{\rho}_{\alpha\beta}(\mathbf{P}', \mathbf{P}'', t) = \delta_{\alpha\beta} \frac{(2\pi\hbar)^3}{V} \delta(\mathbf{P}' - \mathbf{P}'') f_{\alpha}(\mathbf{P}', t),$$

$$U \equiv \overline{U^M} = 0. \tag{2.1}$$

Here $f_{\alpha}(\mathbf{P}', t)$ is the distribution function of pairs of free ($\alpha = \mathbf{p}'$) and bound ($\alpha = n$) charged particles.

After averaging the system (1.8), we obtain a system of two equations for the function $f_{\alpha}(\mathbf{P}', t)$. We write it in the form

$$\frac{\partial f_{\alpha}(\mathbf{P}', t)}{\partial t} = -\frac{2V}{(2\pi\hbar)^3} \times \sum_{\beta} \int \operatorname{Im} \overline{[\delta\rho_{\alpha\beta}(\mathbf{P}', \mathbf{P}'', t)\, \delta U_{\alpha\beta}(\mathbf{P}', \mathbf{P}'', t)]}\, d\mathbf{P}'' \equiv J_{\alpha}(\mathbf{P}', t). \tag{2.2}$$

Here J_{α} are the corresponding collision integrals. They are determined by the correlation function of the random collisions $\delta\rho_{\alpha\beta} = \rho_{\alpha\beta} - \bar{\rho}_{\alpha\beta}$; $\delta U_{\alpha\beta} = U_{\alpha\beta}$, since $U^M = 0$.

The next step in obtaining the kinetic equations is perfectly analogous to that given in Sec. 14 of the book[7] in the derivation of the kinetic equations without allowance for the inelastic processes. We consequently omit all the intermediate steps and write down directly the final expression for the collision integral in the following form:

$$J_{\alpha}(\mathbf{P}', t) = 4n \frac{V}{(2\pi\hbar)^3} \sum_{\beta\alpha_1\beta_1} \int d\mathbf{P}''\, d\mathbf{P}_1'\, d\mathbf{P}_1''\, d\omega\, d\mathbf{k}\, |P_{\alpha\beta}(\mathbf{k})|^2 \times |P_{\alpha_1\beta_1}(\mathbf{k})|^2 k^{-4} |\varepsilon(\omega, \mathbf{k})|^{-2} \delta(\hbar\mathbf{k} - (\mathbf{P}' - \mathbf{P}'')) \times \delta(\hbar\omega - (E_{\alpha} + E_{\mathbf{P}'} - E_{\beta} - E_{\mathbf{P}''}))\, \delta(\mathbf{P}' + \mathbf{P}_1'' - (\mathbf{P}'' + \mathbf{P}_1'))\, \delta(E_{\alpha} + E_{\mathbf{P}'} + E_{\beta_1} + E_{\mathbf{P}_1''} - (E_{\alpha_1} + E_{\mathbf{P}_1'} + E_{\beta} + E_{\mathbf{P}''}))\, [f_{\alpha_1}(\mathbf{P}_1') f_{\beta}(\mathbf{P}'') - f_{\alpha}(\mathbf{P}') f_{\beta_1}(\mathbf{P}_1'')], \tag{2.3}$$

where $n = N/V$. The matrix elements in (2.3) are determined by the expression

$$P_{\alpha\beta}(\mathbf{k}) = \int \left[e_a \exp\left(i \frac{m_b}{m_a + m_b} \mathbf{k}\mathbf{r}\right) + e_b \exp\left(-i \frac{m_a}{m_a + m_b} \mathbf{k}\mathbf{r}\right)\right] \Psi_{\alpha}^{*}(\mathbf{r}) \Psi_{\beta}(\mathbf{r})\, d\mathbf{r}. \tag{2.4}$$

$\epsilon(\omega, \mathbf{k})$ is the dielectric constant of the partly ionized plasma. It is determined by the formula

$$\varepsilon(\omega, \mathbf{k}) = 1 + \frac{4\pi n}{\mathbf{k}^2}\frac{V}{(2\pi\hbar)^3}$$
$$\times \sum_{\alpha\beta} \int d\mathbf{P}'\, d\mathbf{P}'' \frac{|P_{\alpha\beta}(\mathbf{k})|^2 (f_\alpha(\mathbf{P}') - f_\beta(\mathbf{P}''))\, \delta(\hbar\mathbf{k} - (\mathbf{P}' - \mathbf{P}''))}{\hbar(\omega + i\Delta) - (E_\alpha + E_{\mathbf{P}'} - E_\beta - E_{\mathbf{P}''})}. \tag{2.5}$$

The double sum $\sum\limits_{\alpha\beta}$ has the meaning

$$\sum_{\alpha\beta} \ldots = \sum_{nm} \ldots + \frac{V}{(2\pi\hbar)^3}\left(\sum_n \int \ldots d\mathbf{p}'' + \sum_m \int \ldots d\mathbf{p}'\right)$$
$$+ \frac{V^2}{(2\pi\hbar)^6} \int \ldots d\mathbf{p}' d\mathbf{p}''. \tag{2.6}$$

It follows from (2.5) that the polarizability of a partly ionized plasma consists of four parts: the polarizability produced by the free particles (first term), bound particles (fourth term), and the polarizabilities arising in transitions from the bound to the free state and vice versa. The collision integrals (2.3) take into account the possibility of production of bound states (atoms) from free particles belonging to one pair only. In order to take into account the possibility of formation of atoms from free particles of arbitrary pairs, we proceed as follows.

The free charged particles remain most of the time at distances such, that the eigenfunctions of the continuous spectrum can be replaced by plane waves, that is, at the chosen normalization,

$$\Psi_{\mathbf{p}}(\mathbf{r}) \rightarrow \frac{1}{\sqrt{V}} \exp\left\{\frac{i\mathbf{pr}}{\hbar}\right\}. \tag{2.7}$$

In this approximation, the matrix element $|P_{p'p''}(\mathbf{k})|^2$ of the free particles is given by

$$|P_{\mathbf{p}'\mathbf{p}''}(\mathbf{k})|^2 = \frac{(2\pi\hbar)^3}{V}\left[e_a^2\delta\left(\mathbf{p}' - \mathbf{p}'' - \frac{m_b}{m_a + m_b}\hbar\mathbf{k}\right)\right.$$
$$\left. + e_b^2\delta\left(\mathbf{p}' - \mathbf{p}'' + \frac{m_a}{m_a + m_b}\hbar\mathbf{k}\right)\right]. \tag{2.8}$$

To describe the motion of free particles it is more convenient to use the variables

$$\mathbf{P} = \mathbf{p}_a + \mathbf{p}_b, \quad \mathbf{p} = (m_b\mathbf{p}_a - m_a\mathbf{p}_b)/(m_a + m_b). \tag{2.9}$$

Here p_a and p_b are the momenta of the free electrons and ions. Following such a change of variables we have

$$f_{\mathbf{p}}(\mathbf{P}, t) \rightarrow f(\mathbf{p}_a, \mathbf{p}_b, t). \tag{2.10}$$

We note that the function

$$Nf_{\mathbf{p}}(\mathbf{P}, t) = Nf(\mathbf{p}_a, \mathbf{p}_b, t) \tag{2.11}$$

determines the mean value of pairs of free charged particles with momenta p_a and p_b. In order to take into account the possibility of formation of atoms from free particles of arbitrary pairs, we make the following substitution in the terms the collision integral (2.3) which describe the transitions from the free states to the bound states and vice versa:

$$Nf(\mathbf{p}_a, \mathbf{p}_b, t) \rightarrow Nf(\mathbf{p}_a, t) Nf(\mathbf{p}_b, t). \tag{2.12}$$

A similar substitution is made in expression (2.5) for the dielectric constant.

We can now write the sought kinetic equations for the distribution functions of the free electrons $f(p_a, t)$, the free ions $f(p_b, t)$, and the bound charged particles (atoms) $f_n(\mathbf{P}, t)$. The last function determines the probability of an atom of momentum $\mathbf{P}$ in the state n.

Taking account of the fact that

$$f(\mathbf{p}_a) = \frac{V}{(2\pi\hbar)^3}\int f(\mathbf{p}_a, \mathbf{p}_b)\, d\mathbf{p}_b,$$
$$f(\mathbf{p}_b) = \frac{V}{(2\pi\hbar)^3}\int f(\mathbf{p}_a, \mathbf{p}_b)\, d\mathbf{p}_a, \tag{2.13}$$

we obtain from (2.2) and (2.3) the sought-for system of kinetic equations.

$$\frac{\partial f(\mathbf{p}_a, t)}{\partial t} = \frac{V}{(2\pi\hbar)^3}\int J(\mathbf{p}_a, \mathbf{p}_b)\, d\mathbf{p}_b \equiv J(\mathbf{p}_a, t), \tag{2.14}$$

$$\frac{\partial f(\mathbf{p}_b, t)}{\partial t} = \frac{V}{(2\pi\hbar)^3}\int J(\mathbf{p}_a, \mathbf{p}_b)\, d\mathbf{p}_a \equiv J(\mathbf{p}_b, t),$$
$$\partial f_n(\mathbf{P}, t)/\partial t = J_n(\mathbf{P}, t). \tag{2.15}$$

The distribution functions of the electrons, ions, and atoms are normalized in the following fashion:

$$\frac{V}{(2\pi\hbar)^3}\int f(\mathbf{p}_a, t)\, d\mathbf{p}_a = \frac{N_a}{N} \equiv c_a,$$
$$\frac{V}{(2\pi\hbar)^3}\int f(\mathbf{p}_b, t)\, d\mathbf{p}_b = \frac{N_b}{N} \equiv c_b; \tag{2.16}$$

$$\frac{V}{(2\pi\hbar)^3}\sum_n \int f_n(\mathbf{P}, t)\, d\mathbf{P} = \frac{N_{ab}}{N} \equiv c_{ab}; \tag{2.17}$$

Here $c_a = c_b$ and $c_a + c_{ab} = 1$; c_a, c_b, and c_{ab} are the electron, ion, and atom concentrations.

In the equilibrium state, the distribution functions are given

$$Nf_n(\mathbf{P}) = \exp\left\{\frac{\mu_{ab} - E_n - E_{\mathbf{P}}}{\varkappa T}\right\},$$
$$Nf(\mathbf{p}_a) = \exp\left\{\frac{\mu_a - E_{\mathbf{p}_a}}{\varkappa T}\right\}, \tag{2.18}$$

where

$$E_{\mathbf{P}} = \mathbf{P}^2/2M, \; E_{\mathbf{p}_a} = \mathbf{p}_a^2/2m_a \text{ and}$$

$$\mu_{ab} = \varkappa T \ln\left[\frac{N_{ab}}{V}\left(\frac{2\pi\hbar^2}{M\varkappa T}\right)^{3/2}\frac{1}{Z}\right],$$

$$\mu_a = \varkappa T \ln\left[\frac{N_a}{V}\left(\frac{2\pi\hbar^2}{m_a \varkappa T}\right)^{3/2}\right] \tag{2.19}$$

(Z is the partition function). The expressions for the functions $f(p_b)$ and μ_b are obtained from (2.8) and (2.19) by the substitution a → b.

3. SPECTRAL FUNCTION OF THE FIELD AND POLARIZABILITY OF PARTLY IONIZED PLASMA

We return to expression (2.5) for the dielectric constant of a partly ionized plasma. We represent it in the form

$$\varepsilon(\omega, \mathbf{k}) = 1 + 4\pi(\alpha_{ff} + \alpha_{bf} + \alpha_{fb} + \alpha_{bb}). \tag{3.1}$$

The indices f and b denote free and bound states.

For the polarizability α_{ff} we obtain from (2.5), with allowance for (2.7) and (2.8), the well known expression [14,15]

$$\alpha_{ff}(\omega, \mathbf{k}) = n\frac{V}{(2\pi\hbar)^3} \times \sum_a \frac{e_a^2}{\hbar k^2}\int d\mathbf{p}\,\frac{f_a(\mathbf{p} + {}^1/_2\hbar\mathbf{k}) - f_a(\mathbf{p} - {}^1/_2\hbar\mathbf{k})}{\omega + i\Delta - \mathbf{kv}}, \tag{3.2}$$

where a = e, i.

The polarizabilities α_{bf} and α_{fb}, which characterize the transitions between the bound and free states, are defined by the expressions

$$\alpha_{bf}(\omega, \mathbf{k}) = \alpha_{fb}{}^*(-\omega^*, -\mathbf{k})$$
$$= n\frac{V^2}{(2\pi\hbar)^6}\sum_m \int d\mathbf{p}_a\, d\mathbf{p}_b d\mathbf{P}\,\frac{|P_{\mathbf{p}m}(\mathbf{k})|^2}{k^2} \times \frac{\delta(\hbar\mathbf{k} - (\mathbf{p}_a + \mathbf{p}_b - \mathbf{P}))[Nf(\mathbf{p}_a)f(\mathbf{p}_b) - f_m(\mathbf{P})]}{\hbar(\omega + i\Delta) - (\mathbf{p}_a{}^2/2m_a + \mathbf{p}_b{}^2/2m_b - E_m - E_{\mathbf{P}})}; \tag{3.3}$$

where $\mathbf{p} = (m_b\mathbf{p}_a - m_a\mathbf{p}_b)/(m_a + m_b)$. We took the condition (2.12) into account here.

The expression for α_{bb} follows directly from (2.5) with $\alpha = n$ and $\beta = m$.

We now consider the expression for the spectral field-intensity function, which will be used in Sec. 4. In the case of a partly ionized plasma, this function is defined by the formula

$$(\delta\mathbf{E}\,\delta\mathbf{E})_{\omega,k} = \frac{2nV}{\hbar^2}\sum_{\alpha\beta}\int d\mathbf{P}'\,d\mathbf{P}''\,[f_\alpha(\mathbf{P}') - f_\beta(\mathbf{P}'')] \times \frac{|\mathbf{P}_{\alpha\beta}(\mathbf{k})|^2}{\mathbf{k}^2|\varepsilon(\omega, \mathbf{k})|^2}\,\delta(\hbar\mathbf{k} - (\mathbf{P}' - \mathbf{P}'')) \times \delta(\hbar\omega - (E_\alpha + E_{\mathbf{P}'} - E_\beta - E_{\mathbf{P}''})). \tag{3.4}$$

The double sum has here the meaning of (2.6), and therefore the spectral function (3.4), like the dielectric constant, consists of four terms. In the terms due to the transitions from the free state to the bound state and back it is necessary to take the condition (2.12) into account. In the equilibrium case, the spectral field function is defined by the usual expression [15,7]. For that part of the spectral function which is determined by transitions between free states of charged particles, we obtain from the general formula (2.4), with account taken of (2.7) and (2.8), a simpler expression which coincides with the expression for the spectral function of a fully ionized plasma.

Let us consider certain results that follow from the foregoing general formulas.

Using (3.4), we can determine the spatial spectral function of the field. In the equilibrium case, we obtain for this function

$$(\delta\mathbf{E}\delta\mathbf{E})_k = 4\pi\varkappa T\left[\left(1 - \frac{1}{\varepsilon(0, \mathbf{k})}\right) + 2\sum_{l=1}^{\infty}\operatorname{Re}\left(1 - \frac{1}{\varepsilon(2i\pi l\varkappa T/\hbar, \mathbf{k})}\right)\right]. \tag{3.5}$$

In the classical approximation, when $\hbar = 0$, only the first term remains in this formula. We consider the two extreme cases when the degree of ionization is close to unity and zero, respectively.

In the fully ionized plasma, the classical formula can be used at values $k < k_{max} \sim \sqrt{\mu\kappa T}/\hbar$. This condition can be obtained from (3.2) and (3.5), but it is simpler to derive it from the uncertainty relation. It follows from this estimate that $r_{min} \sim 1/k_{max} \sim \hbar/\sqrt{\mu\kappa T}$. For a low-temperature plasma we have $r_{min} > a$, where a is the Bohr radius. This enables us to use the classical form, assuming the spectral function to be equal to zero when $r < r_{min}$ $(k > k_{max})$. In the case of a fully ionized plasma in the classical approximation [7,15]

$$\varepsilon(0, k) = 1 + 1/r_d{}^2k^2, \tag{3.6}$$

where r_d is the Debye radius. From (3.5) and (3.6) we get an expression for the mean square deviation of the field

$$\overline{(\delta\mathbf{E})^2} = \frac{1}{(2\pi)^3}\int(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}}d\mathbf{k} \sim \frac{4\pi\varkappa T}{(2\pi)^3}\int_{k<k_{max}}\frac{d\mathbf{k}}{r_d{}^2k^2 + 1} \approx \frac{8}{\hbar}\sum_a e_a{}^2 n_a\sqrt{\frac{\mu}{\varkappa T}}\varkappa T. \tag{3.7}$$

We used here the expression $r_d^2 = \kappa T/\Sigma_\alpha 4\pi e_a^2 n_a$, and took account of the fact that $k_{max} \gg 1/r_d$.

The result (3.7) agrees with formula (1.12) of [16], where an expression was considered for the quantum correlation function of a fully ionized plasma.

We now consider the other limiting case. In the dipole approximation, that is, when $ka \ll 1$ (a—Bohr radius) we get from the formula for the function $a_{bb}(\omega, k)$, under the condition $m_e \ll m_i$, the expression

$$\alpha_{bb}(i\Delta,\mathbf{k})=\frac{2e^2n_{ab}}{3\hbar}\sum_{nm}\frac{|\mathbf{r}_{nm}|^2\omega_{mn}\rho_n}{\omega_{mn}^2+\Delta^2}. \tag{3.8}$$

When ka ≫ 1, the matrix element $|P_{nm}(\mathbf{k})|^2$ vanishes, and therefore we can use for the estimate the formula (3.8), putting $\alpha_{bb}(i\Delta, \mathbf{k}) = 0$ for ka > 1. In this approximation, the spatial spectral function is

$$(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k}}\sim 4\pi\varkappa T\frac{e^2a^2n_{ab}}{I}\quad\text{for } ka<1 \tag{3.9}$$

and vanishes when ka > 1; I is the ionization potential. It follows therefore that the mean square deviation is

$$\overline{(\delta\mathbf{E})^2}\sim 4\pi\varkappa T\frac{e^2}{aI}n_{ab},$$

that is,

$$\overline{(\delta\mathbf{E})^2}/4\pi\varkappa Tn_{ab}\sim 1. \tag{3.10}$$

The mean square deviation determined by formulas (3.10) is larger than or of the same order as the corresponding quantity in the case of a fully ionized plasma.

A more detailed analysis of the functions $\epsilon(\omega, \mathbf{k})$ and $(\delta\mathbf{E}\cdot\delta\mathbf{E})_{\omega\mathbf{k}}$ in the case of a partly ionized plasma is a separate problem.

4. CLASSIFICATION OF COLLISION PROCESSES IN A PARTLY IONIZED PLASMA. RELAXATION TIMES

Using expressions (2.5) and (3.4) for the dielectric constant and the spectral function of the field, we can write the collision integral (2.3) in the form

$$J_\alpha(\mathbf{P}',t)=\frac{1}{(2\pi)^3\hbar}\sum_\beta\int dP''\,d\omega\,d\mathbf{k}\frac{|P_{\alpha\beta}(k)|^2}{k^2}\delta(\hbar\mathbf{k}-(\mathbf{P}'-\mathbf{P}''))$$
$$\times\delta(\hbar\omega-(E_\alpha+E_{\mathbf{P}'}-E_\beta-E_{\mathbf{P}''}))\Big\{(\delta\mathbf{E}\,\delta\mathbf{E})_{\omega\mathbf{k}}[f_\beta(\mathbf{P}'',t)$$
$$-f_\alpha(\mathbf{P}',t)]-\frac{4\pi\hbar\varepsilon''(\omega,\mathbf{k})}{|\varepsilon(\omega,\mathbf{k})|^2}[f_\beta(\mathbf{P}'',t)+f_\alpha(\mathbf{P}',t)]\Big\}. \tag{4.1}$$

If the time of relaxation of the function $(\delta\mathbf{E}\cdot\delta\mathbf{E})_{\omega\mathbf{k}}$ to its stationary value is much shorter than the relaxation time of the distribution function, then expressions (2.3) and (4.1) are equivalent. In the opposite case it is necessary to use for the spectral function an additional equation—the equation for plasmons. The collision integral in the form (4.1) is convenient for a classification of the collision processes.

We write down the system (2.14) for the distribution functions of the electrons and ions in the form

$$\partial f_a(\mathbf{p}_a',t)/\partial t=(J_a(\mathbf{p}_a',t))_1+(J_a(\mathbf{p}_a',t))_2. \tag{4.2}$$

Here and below a = e and i for the electrons and ions, respectively.

The first term in the right side of (4.2) describes processes in which the number of free charged particles is conserved. In order to obtain for it an explicit expression, we put in (2.3) $\alpha = \mathrm{p}'$ and $\beta = \mathrm{p}''$, make the change of variable (2.9), use expression (2.8), and integrate with respect to p_b. As a result we obtain

$$(J_a(\mathbf{p}_a',t))_1=\frac{e^2}{(2\pi)^3\hbar}\int dp_a''d\omega\,d\mathbf{k}\frac{1}{k^2}\delta(\hbar\mathbf{k}-(\mathbf{p}_a'-\mathbf{p}_a''))$$
$$\times\delta\Big(\hbar\omega-\Big(\frac{\mathbf{p}_a'^2}{2m_a}-\frac{\mathbf{p}_a''^2}{2m_a}\Big)\Big)\Big\{(\delta\mathbf{E}\delta\mathbf{E})_{\omega\mathbf{k}}[f_a(\mathbf{p}_a'',t)$$
$$-f_a(\mathbf{p}_a',t)]-\frac{4\pi\hbar\varepsilon''(\omega,\mathbf{k})}{|\varepsilon(\omega,\mathbf{k})|^2}[f_a(\mathbf{p}_a'',t)+f_a(\mathbf{p}_a',t)]\Big\}. \tag{4.3}$$

When ħ = 0 this expression coincides in form with the Landau collision integral in which additional account is taken of the polarization[4,5,7]. The difference lies in the fact that the functions $(\delta\mathbf{E}\cdot\delta\mathbf{E})_{\omega\mathbf{k}}$ and $\epsilon(\omega, \mathbf{k})$ in (4.3) take into account both the elastic and inelastic processes.

We recall that the functions $(\delta\mathbf{E}\cdot\delta\mathbf{E})_{\omega\mathbf{k}}$ and $\epsilon(\omega, \mathbf{k})$ each consist of four parts. As a result, the integral (4.3) consists in turn of four parts which describe four processes:

1) $\mathbf{p}_a'+\mathbf{p}_{1b}''\leftrightarrow\mathbf{p}_a''+\mathbf{p}_{1b}'$, $\mathbf{p}_a'+\mathbf{p}_{1a}''\leftrightarrow\mathbf{p}_a''+\mathbf{p}_{1a}'$;
2) $\mathbf{p}_a'+m_1\mathbf{P}_1''\leftrightarrow\mathbf{p}_a''+\mathbf{p}_{1a}'+\mathbf{p}_{1b}'$;

3) $\mathbf{p}_a'+\mathbf{p}_{1a}''+\mathbf{p}_{1b}''\leftrightarrow\mathbf{p}_a''+n\mathbf{P}''$;
4) $\mathbf{p}_a'+m_1\mathbf{P}_1''\leftrightarrow\mathbf{p}_a''+n_1\mathbf{P}_1'$.

Here 1) is the elastic-scattering process, 2) is the direct process (from left to right)—ionization of an atom by collision with an electron ($\mathrm{p}_a' = \mathrm{p}_e'$) or an ion ($\mathrm{p}_a' = \mathrm{p}_i'$), the inverse process—recombination upon collision of three particles; 3) is the process inverse to 2), and 4) is the inelastic-scattering process. In all these processes, the number of particles with momentum p_a remains unchanged, all that takes place is a change in momentum ($\mathrm{p}_a' \rightleftarrows \mathrm{p}_a''$).

In order to obtain an expression for the second term on the right side of (4.2), it is necessary to put in (2.3) $\alpha = \mathrm{p}_a'$ and $\beta = \mathrm{m}$, make the substitution (2.9), and integrate with respect to p_b'. As a result we obtain

$$(J_a(\mathbf{p}_a',t))_2=\frac{1}{(2\pi)^3\hbar}\frac{V}{(2\pi\hbar)^3}\sum_m\int d\mathbf{p}_b'\,d\mathbf{P}''\,d\omega\,d\mathbf{k}\frac{1}{k^2}|P_{\bar{\mathbf{p}},m}|^2$$
$$\times\delta(\hbar\mathbf{k}-(\mathbf{p}_a'+\mathbf{p}_b'-\mathbf{P}''))$$
$$\times\delta\Big(\hbar\omega-\Big(\frac{\mathbf{p}_a'^2}{2m_a}+\frac{\mathbf{p}_b'^2}{2m_b}-E_m-E_{\mathbf{P}''}\Big)\Big)$$
$$\times\Big\{(\delta\mathbf{E}\,\delta\mathbf{E})_{\omega\mathbf{k}}[f_m(\mathbf{P}'')-Nf(\mathbf{p}_a')f(\mathbf{p}_b')]$$
$$-\frac{4\pi\hbar\varepsilon''(\omega,\mathbf{k})}{|\varepsilon(\omega,\mathbf{k})|^2}[f_m(\mathbf{P}'')+Nf(\mathbf{p}_a')f(\mathbf{p}_b')]\Big\}, \tag{4.4}$$

where we put $\bar{p} = (m_b p_a' - m_a p_b')/(m_a + m_b)$. After substituting the functions $(\delta\mathbf{E}\cdot\delta\mathbf{E})_{\omega k}$ and $\epsilon(\omega, \mathbf{k})$, this integral breaks up in turn into four parts, which describe the processes:

5) $\mathbf{p}_\alpha' + \mathbf{p}_b' + \mathbf{p}_{1a}'' \leftrightarrow m\mathbf{P}'' + \mathbf{p}_{1a}'$,
6) $\mathbf{p}_a' + \mathbf{p}_b' + m_1\mathbf{P}_1'' \longleftrightarrow m\mathbf{P}'' + \mathbf{p}_{1a}' + \mathbf{p}_{1b}'$,
7) $\mathbf{p}_a' + \mathbf{p}_b' + \mathbf{p}_a'' + \mathbf{p}_b'' \longleftrightarrow m\mathbf{P}'' + m_1\mathbf{P}_1''$,
8) $\mathbf{p}_a' + \mathbf{p}_b' + m_1\mathbf{P}_1'' \leftrightarrow m\mathbf{P}'' + m_1\mathbf{P}_1''$.

Here 5), 7), and 8) are recombination and ionization processes which change the number of the free charged particles, and 6) is an inelastic-scattering process accompanied by particle exchange.

We can also consider in similar fashion processes described by the collision integral in (2.15) for the atom distribution function.

It is important to note here the following. In the case of a zero degree of ionization, there remains in the collision integral $J_n(\mathbf{P}', t)$ only one term, describing the process

$$n\mathbf{P}' + m_1\mathbf{P}_1'' \longleftrightarrow m\mathbf{P}'' + n_1\mathbf{P}_1'.$$

It represents elastic and inelastic scattering of the atoms. In order to take into account the possibility of formation of bound states for atoms (molecules) it is necessary to take into account the possibility of production of bound complexes of four charged particles.

It is known (see, for example,[11,17]) that the equilibrium state is established in three distinct stages. Equilibrium if first established with respect to the translational degrees of freedom—a Maxwellian distribution; this if followed by equilibrium with respect to the internal degree of freedom—Boltzmann distribution—and finally by a state of ionization equilibrium. The actual picture of establishment of equilibrium in an electron-ion plasma is more complicated. This is due, in particular, to the large difference between the electron and ion masses.

5. EQUATIONS FOR THE CONCENTRATIONS OF FREE AND BOUND CHARGED PARTICLES. IONIZATION AND RECOMBINATION COEFFICIENTS

Let us consider the state of a plasma in which the nonequilibrium character is due only to the fact that the concentrations n_a, n_b, and n_{ab} do not satisfy the ionization-equilibrium condition

$$\frac{n_a n_b}{n_{ab}} = \left(\frac{\mu\varkappa T}{2\pi\hbar^2}\right)^{3/2}\frac{1}{Z} \tag{5.1}$$

and are consequently functions of the time.

Let us multiply (2.14) by $nV(2\pi\hbar)^3$ and integrate with respect to p_a'. Using the Maxwell-Boltzmann distribution for the functions $f(p_a)$ and $f(p_b)$, and recognizing that the following condition

$$\int J_1(\mathbf{p}_a', t)\,d\mathbf{p}_a' = 0 \tag{5.2}$$

is satisfied for the collision integral (4.1), we obtain an equation for the concentration n_a. To obtain the equations for the concentrations n_a and n_{ab} it is necessary to take account of the fact that $n_a = n_b$ and $n_a + n_{ab} = n$. If the relaxation time of the spectral function of the field is of the order of or longer than the relaxation times of the functions n_a, n_b, and n_{ab}, then it is necessary to add to this system also an equation for the function $(\delta\mathbf{E}\cdot\delta\mathbf{E})_{\omega k}$. In the opposite limiting case the spectral function is given by (3.4), in which it is necessary to substitute the Maxwell-Boltzmann distribution for the functions $f_n(\mathbf{P})$, f_a, and f_b.

Let us consider the case when the level of the fluctuations of the longitudinal electric field greatly exceeds the equilibrium level of the fluctuations. Under this condition, nonequilibrium ionization takes place, at the expense of the energy of the longitudinal-field quanta. In order to determine the corresponding ionization coefficient (which we shall denote by $\alpha^{(\|)}$), we use the equilibrium relation (5.1) for the concentrations. As a result we obtain the equation

$$dn_a/dt = \alpha^{(\|)} n_a. \tag{5.3}$$

To determine the explicit form of the coefficient $\alpha^{(\|)}$, we take account of the following facts:

In the case of ionization by a longitudinal field, we can consider the limiting case when there is no spatial dispersion ($k = 0$). The ionization coefficient is obtained in the form

$$\alpha^{(\|)} = \frac{e^2}{\hbar}\frac{V}{(2\pi\hbar)^3}\sum_m \int d\mathbf{p}\,d\omega\,\frac{|\mathbf{r}_{\mathbf{p}m}\mathbf{k}|^2}{k^2}$$
$$\times\,\delta\left(\hbar\omega - \left(\frac{\mathbf{p}^2}{2\mu} - E_m\right)\right) e^{-p^2/2\mu\varkappa T}(\delta\mathbf{E}\delta\mathbf{E})$$
$$\times\,{}_{\omega\mathbf{k}}\frac{1}{Z}\left(e^{\hbar\omega/\varkappa T} - 1\right) n_{ab}.$$

We note that the ionization coefficient $\alpha^{(\|)}$ is expressed in terms of the imaginary part of the polarizability:

$$\alpha^{(\|)} = \frac{1}{\pi\hbar}\int \frac{\alpha_{bf}''}{n_{ab}}(\delta\mathbf{E}\delta\mathbf{E})_\omega d\omega. \tag{5.4}$$

We now consider the recombination and ionization processes which occur when plasma particles collide. To obtain equations describing these processes, we eliminate the functions $(\delta\mathbf{E}\cdot\delta\mathbf{E})_{\omega k}$

and $\epsilon(\omega, \mathbf{k})$. The equation obtained in this manner is written in the following fashion:

$$dn_a/dt = (\alpha n_a n_{ab} - \beta n_a^3) + (\alpha_1 n_{ab}^2 - \beta_1 n_a^2 n_{ab}) + (\alpha_2 n_{ab}^2 - \beta_2 n_a^2 n_b^2) + (\alpha_3 n_a^2 n_{ab} - \beta_3 n_a^2 n_{ab}). \tag{5.5}$$

Here α is the impact-ionization coefficient; β is the triple-recombination coefficient (two electrons and an ion and two ions and an electron); β_1 is the coefficient of recombination in triple collision of an electron, ion, and atom; α_1 is the corresponding ionization coefficient upon collision of two atoms; β_2 is the coefficient of recombination in the collision of two pairs of charged particles (resulting in the formation of two atoms); α_3 and β_3 are exchange coefficients.

The coefficient α is given by

$$\alpha = \frac{4V}{(2\pi\hbar)^3} \sum_c e_c^2 \sum_m \int d\mathbf{p}' d\mathbf{P}' d\mathbf{P}'' d\mathbf{p}_c' d\mathbf{p}_c'' d\omega\, d\mathbf{k} \times \frac{|P_{\mathbf{p}'m}|^2}{k^4 |\varepsilon(\omega, \mathbf{k})|^2} \delta(\mathbf{P}' + \mathbf{p}_c'' - (\mathbf{P}'' + \mathbf{p}_c')) \times \delta\left(\frac{\mathbf{p}'^2}{2\mu} + \frac{\mathbf{P}'^2}{2M} + \frac{\mathbf{p}''^2}{2m_c} - \left(E_m + \frac{\mathbf{P}''^2}{2M} + \frac{\mathbf{p}_c'^2}{2m_c}\right)\right) \times \delta(\hbar\mathbf{k} - (\mathbf{P}' - \mathbf{P}''))\, \delta\left(\hbar\omega - \left(\frac{\mathbf{p}'^2}{2\mu} + \frac{\mathbf{P}'^2}{2M} - E_m - \frac{\mathbf{P}''^2}{2M}\right)\right) \times \frac{1}{[2\pi(\mu M)^{1/2} \varkappa T]^3 Z} \exp\left\{-\left(E_m + \frac{\mathbf{P}'^2}{2M} + \frac{\mathbf{p}_c'^2}{2m_c}\right)\frac{1}{\varkappa T}\right\}. \tag{5.6}$$

The coefficients β and α are connected by the relation

$$\beta = (2\pi\hbar^2/\mu\varkappa T)^{3/2} Z\alpha.$$

We perform the calculations in (5.6) under the following assumptions: $\epsilon(\omega, \mathbf{k}) = 1$, that is, polarization is disregarded; the ionization is produced by electrons; the thermal motion of the atoms is disregarded; the ionization proceeds from the ground level. Under these conditions, (5.6) takes the form

$$\alpha = \frac{2^{10}}{3^7} 35 e^{-4/3} \frac{a^3 \mu e^4}{\hbar^3} \frac{\varkappa T}{I} e^{-I/\varkappa T}. \tag{5.7}$$

In deriving this formula, we used the expression given in p. 667 of[18] for the effective ionization cross section.

When the ionization coefficient is determined within the framework of classical theory[17], the obtained formula differs from (5.7) in the factor proceeding the exponent ($\kappa T/I$ is replaced by $(\kappa T/I)^{1/2}$). This difference is equivalent to having the dependence of the effective cross section $\sigma(E)$ on the excess energy near the threshold not linear but of the form $\sigma \approx (E - I)^{3/2}$. We note that the Born approximation is also used for the numerical calculations of the effective cross sections (see, for example,[19]).

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