DISCONTINUITIES OF SHOCK ADIABATS AND THE NON-UNIQUENESS OF SOME SHOCK COMPRESSIONS

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It is shown that a shock adiabat can in principle be discontinuous or possess such a shape that the shock compression problem (decay of discontinuity) may yield non-unique solutions. A condition of non-uniqueness is presented which is similar to that suggested by Bancroft et al. for doubling of waves.

SHOCK compression of matter can give rise to a phase transition. At the point where the compression is turned on, the shock adiabat experiences a kink, and this can lead to a singularity in the compression. Thus, Bancroft et al.[1] observed that in the case of a strong kink of p(v) to the left (more accurately, if the continuation of the shock adiabat goes upward to the left of the Michelson ray AB_1 on Fig. 1a), two waves, one following the other, are produced in lieu of one. Urlin and Ivanov[2] have shown that a kink to the right is also possible, and obtained an expression for the slope of the shock adiabat at the place of inclusion of the phase transition. It is seen from their formula that the shock adiabat can experience here not only a kink, but even a discontinuity (more accurately, a jump). Let us discuss this in somewhat greater detail, after which we shall show that such anomalies can lead to a qualitatively new phenomenon, namely non-uniqueness of the shock compression.

Let OK be the shock adiabat, which reaches at the point K the boundary separating the α and β phases in the plane (pT) (Fig. 2).

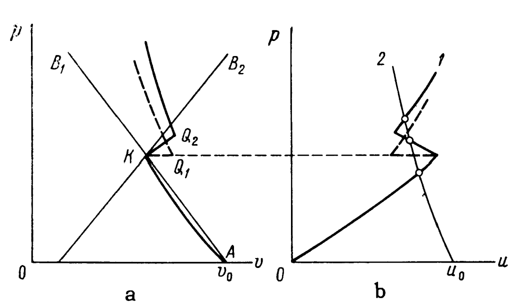

FIG. 1. Anomalous shock adiabats: one having a discontinuity (dashed), and one emerging beyond the limits of the angle between the Michelson ray and its mirror reflection (solid). In case b we show also the non-unique regimes which arise when plates having adiabats 1 and 2 collide (three intersections).

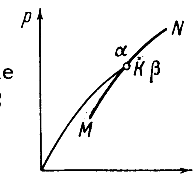

FIG. 2. Shock adiabat OK, reaching the limit MN of the boundary between α and β phases.

If a fraction x of material has gone over under the shock compression from α to β, then $v = (1 - x)v_\alpha + xv_\beta$ (v—specific volume), i.e., $x = (v_\alpha - v)/(v_\alpha - v_\beta)$ and $\epsilon = (1 - x)\epsilon_\alpha + x\epsilon_\beta$ (ϵ—internal energy).

But $\epsilon_\beta = \epsilon_\alpha + p(v_\alpha - v_\beta) + q$, where q is the heat of the transition. According to the Clapeyron–Clausius formula

$$q = -p(v_\alpha - v_\beta)\frac{d\ln T}{d\ln p},$$

where the derivative is taken over the phase boundary. Denoting it by n, we obtain

$$\varepsilon_\beta = \varepsilon_\alpha + p(v_\alpha - v_\beta)(1-n),\ \varepsilon = \varepsilon_\alpha + p(v_\alpha - v)(1-n).$$

On the shock wave, $\epsilon = {}^1\!/_2\, p(v_0 - v)$ (we assume that $p_0 = 0$ and $\epsilon_0 = 0$), i.e., $\epsilon_\alpha + p(v_\alpha - v)(1 - n) = {}^1\!/_2\, p(v_0 - v)$, hence

$$p = \frac{\varepsilon_\alpha}{{}^1\!/_2(v_0 - v) - (1-n)(v_\alpha - v)}.$$

In particular, when $n = {}^1\!/_2$ we get $p = 2\epsilon_\alpha/(v_0 - v_\alpha) = \text{const}$, i.e., the section of the shock adiabat p(v) is horizontal. The upper part of the curve is shifted to the right, since in our case $v_\beta > v_\alpha$. Thus, when p increases smoothly v can decrease, and when $n = {}^1\!/_2$ this change is jumplike. In Fig. 1a this is shown by the dashed line KQ_1. It is clear that intermediate cases with a less abrupt kink, as shown by the line KQ_2, are also possible.[1)]

[1)]The case of a stronger kink ($n < 1/2$), when not only p(v) but also v(p) is non-unique, calls for further study.

The possible existence of kinks and even jumps of shock adiabats should be kept in mind when experimental data are approximated by means of smooth curves or by means of formulas.

In the case of strong kink to the right, the dependence p(u) for shock compression (u—mass velocity) can also become non-unique. The curve p(u) proceeds from K to the left, if u decreases with increasing p (Fig. 1b). It is seen from the formula $u = \sqrt{p(v_0 - v)}$ that this will take place if p increases more slowly than $1/(v_0 - v)$, i.e., when the shock adiabat goes to the right of the mirror-reflection KB_2 of the Michelson ray (Fig. 1a). The case $n = 1/2$ is shown dashed in Fig. 1b.

Thus, the conditions under which the shock compression is anomalous are quite symmetrical. The compression is anomalous if the shock adiabat goes beyond the limits of the angle between the Michelson ray and its mirror reflection, i.e., when $|dp/dv| < |\Delta p/\Delta v|$. On emerging to the left, the wave splits (the Bancroft case), and on emerging to the right the non-uniqueness sets in.

Let us consider, for example, the collision between plates made of substances having anomalous and ordinary shock adiabats (curves 1 and 2 of Fig. 1b). Their points of intersection correspond to the states after impact with velocity u_0. In our case there are three of them, i.e., the problem of the decay of the discontinuity has three solutions; the question is, which of them is realized in experiment.

Physical uniqueness would be restored if two out of the three regimes turn out to be unstable and decayed into the third. However, each of them can be caused by a fixed pressure (for example, by action exerted on the surface of the material by a light gas whose pressure does not depend on the velocity u); in this case the regime is unique and cannot be replaced by another stationary regime. The lower and upper regimes are apparently stable also to non-one-dimensional perturbations (bending of the front), since the local dependences of p on v and on u have for them the usual character. For the central regime, this character is unusual and the question of its stability remains open.

Thus, we encounter a physical non-uniqueness: several possible regimes remain, and the choice of one of them may be determined by conditions which usually do not play any role. For example, in a collision of plates, the presence of a thin liner between them, having a different rigidity and influencing the character of the establishment of the pressure (growth or decrease in this process), can become important. Usually such an influence is forgotten, but here it can change the entire phenomenon strongly.

In conclusion we note that the feasibility of the described cases still does not mean that they of necessity exist; there may be no substances with such phase transitions, but it is of interest to search for them experimentally. It is possible to investigate many substances and many of their initial states (p_0, v_0).

[1] D. Bancroft, E. Peterson, and S. Minshall, J. Appl. Phys. **27**, 291 (1956).

[2] B. D. Urlin and A. A. Ivanov, DAN SSSR **149**, 1303 (1963), Soviet Phys. Doklady **8**, 380 (1963).

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