THEORY OF SCATTERING OF ELECTROMAGNETIC WAVES IN FERROMAGNETIC SUBSTANCES

I. A. AKHIEZER and Yu. L. BOLOTIN

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor December 16, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 52, 1393–1405 (May, 1967)

The scattering of electromagnetic waves in ferromagnetic substances is investigated with account of coupling between elastic waves and magnetic-moment oscillations. It is shown that three pairs of lines appear in the scattered-radiation spectrum far from ferroacoustic resonance; these are the longitudinal and transverse sound satellites produced by scattering of an electromagnetic wave by elastic oscillations and the magnon satellites due to scattering of the wave by spin waves. In the range of not very high frequencies, the interaction between the electromagnetic wave and magnetic moment fluctuations yield the main contribution to the magnon satellite intensity. In the high-frequency range, the main contribution to the magnon satellite intensity is due to the interaction between the electromagnetic wave and the oscillations of elastic quantities accompanying the spin wave. In the case of a crystal with a large magnetoelastic constant, an important role is played in the scattering of electromagnetic waves by sound in the low frequency range by the interaction of the waves with the magnetic moment fluctuations accompanying the sound wave. In this case, the intensity of the sound satellites in the ferromagnetic substance differs appreciably from that of the sound satellite in the ordinary (nonferromagnetic) crystal. Scattering of electromagnetic waves is studied in the vicinity of the ferroacoustic resonance point. It is shown that, in addition to the pair of weak lines, an additional pair of bright lines appears in the resonance region of the scattered radiation spectrum. The total intensity of the scattered radiation remains unchanged.

INTRODUCTION

As is well known, if weakly attenuating oscillations can propagate in any medium, then, upon scattering of electromagnetic waves in such a medium, sharp maxima appear in the scattered radiation spectrum, brought about by the excitation (or absorption) of the electromagnetic wave by these oscillations. In particular, sound waves travelling in the medium lead to the appearance of maxima in the scattered-radiation spectrum at frequencies which differ from the frequency of the incident radiation by the sound frequency—the so-called sound satellites.

In ferromagnetic crystals, there is another branch of characteristic oscillations besides the elastic waves—the spin waves (magnons). Therefore, upon scattering of electromagnetic waves in such crystals, magnon satellites should appear in addition to the sound satellites. The magnon satellites should be separated from the fundamental line by the frequency of the spin wave. The possibility of magnon satellites was noted by Bass and Kaganov,[1] who considered the scattering of electromagnetic waves by magnetic-moment fluctuations. The problem of the scattering of electromagnetic waves in ferromagnetic substances has also been treated by Shen and Bloembergen.[2]

The present research is concerned with the scattering of electromagnetic waves in ferromagnetic substances, with account of the coupling between the elastic waves and the magnetic-moment fluctuations. Because of this coupling, the spin wave in the ferromagnetic material is always accompanied by oscillations of the elastic quantities; therefore, it can be expected that the intensity of the magnon satellite is determined in some cases by the scattering of the electromagnetic wave by the elastic-quantity fluctuations accompanying the spin wave (and not by the fluctuations of the magnetic moment themselves). In fact, we shall show that such is the situation in the high-frequency region ($\omega > \omega_0 \zeta^{-1}$, where ω_0 is the frequency of homogeneous ferromagnetic resonance and ζ is a small parameter characterizing the coupling between the elastic and spin waves and is equal to $\sim 10^{-4} - 10^{-6}$ in order of magnitude), while the scattering cross section in this region may turn out to be several orders larger than that calculated by the formulas of Bass and Kaganov.[1]

Similarly, the sound wave in the ferromagnetic material is always accompanied by oscillations in the magnetic-moment density which, as we shall show, make an important contribution to the intensity of the sound satellites in the case of a crystal with a large value of the magnetoelastic constant ($f \gg 1$) in the low frequency region ($\omega \leq f\omega_0$).

The elastic and spin branches of oscillations of the ferromagnetic substance are especially strongly coupled, as is well known, close to the point of ferroacoustic resonance (A. Akhiezer, Bar'yakhtar, and Peletminskiĭ[3]). For this reason, as is shown in the present work, the character of the distribution of scattered radiation changes materially approaching the resonance point. That is, in the case of longitudinal resonance, in addition to the two bright lines (sound satellites) and two weak lines (magnon satellites) four bright lines appear in the scattered radiation spectrum; these lines are due to the scattering of the electromagnetic waves by the coupled magnetoelastic oscillations. Similarly, in the case of transverse resonance, in addition to each of the two weak lines, additional bright lines appear in the scattered radiation spectrum. In both cases, however, the distribution of scattered radiation changes, but the total intensity does not.

1. GENERAL EXPRESSION FOR THE SCATTERING CROSS SECTION

First, we shall obtain the general expression for the scattering cross section of electromagnetic waves in magnetically ordered crystals, taking into account both the density oscillations of the crystal and the magnetic-moment oscillations. The oscillations of the electromagnetic quantities in the scattered wave satisfy the Maxwell equations:

$$\operatorname{rot}\mathbf{E}' = -\frac{1}{c}\frac{\partial}{\partial t}(\mathbf{H}' + 4\pi\mathbf{M}'), \quad \operatorname{rot}\mathbf{H}' = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{D}',$$
$$\operatorname{div}(\mathbf{H}' + 4\pi\mathbf{M}') = 0, \quad \operatorname{div}\mathbf{D}' = 0, \tag{1}$$

where $\mathbf{E}'$ and $\mathbf{D}'$ are the electric field and the electric displacement, $\mathbf{H}'$ is the magnetic field, and $\mathbf{M}'$ the magnetic moment density (the prime denotes quantities referring to the scattered wave). We are interested in scattering in the high frequency region, $\omega \gg \omega_q$, $\omega \gg sq$, where ω is the frequency of the incident wave and ω_q and sq are the frequencies of the spin or sound waves taking part in the scattering. Considering that the frequency change in the scattering is small in this case, we can connect the oscillations of the electric field and the displacement in the scattered wave with the electric field of the incident wave $\mathbf{E}^0$ (see, for example, the monograph of Landau and Lifshitz:[4]):

$$D_i' = \varepsilon E_i' + \delta\varepsilon_{ij}E_j^0, \tag{2}$$

where ϵ is the dielectric constant of the crystal and $\delta\epsilon_{ij}$ are the fluctuations of the dielectric tensor (for simplicity, we shall consider the quantity ϵ to be a scalar, not taking into account the anisotropy of the dielectric properties of the crystal). Noting that the magnetic susceptibility of the ferromagnetic material is small when $\omega \gg \omega_q$, it is easy to connect the oscillation of the magnetic-moment density in the scattered wave with the magnetic field of the incident wave $\mathbf{H}^0$:

$$\partial\mathbf{M}'/\partial t = g[\delta\mathbf{M}, \mathbf{H}^0], \tag{3}$$

where $\delta\mathbf{M}$ is the fluctuation of the magnetic-moment density and g is the gyromagnetic ratio.

The relations (2) and (3) together with the Maxwell equations (1) allow us to express the oscillations of all the electromagnetic quantities in the scattered wave in terms of the amplitude of the incident wave and the fluctuations of the dielectric constant and the magnetic-moment density of the ferromagnetic substance. In particular, we get for $\mathbf{E}'$ (converting to Fourier components):

$$\mathbf{E}' = \frac{4\pi i\omega}{c^2}\left(k'^2 - \frac{\omega'^2}{c^2}\varepsilon\right)^{-1}\mathbf{J}, \tag{4}$$

$$\mathbf{J} = -\frac{i\omega}{4\pi}\hat{\tau}'\delta\hat{\varepsilon}\mathbf{E}^0 + \frac{g\varepsilon}{k^2}\{[\mathbf{k}'\mathbf{E}^0](\mathbf{k}\delta\mathbf{M}) - [\mathbf{k}'\mathbf{k}](\mathbf{E}^0\delta\mathbf{M})\}, \tag{5*}$$

where $\hat{\tau}_{ij} = \delta_{ij} - k'^{-2}k_i'k_j'$, $\mathbf{k}$ being the wave vector of the incident wave, ω' and $\mathbf{k}'$ the frequency and wave vector of the scattered wave, and $\delta\epsilon$ and $\delta\mathbf{M}$ the fluctuations of the dielectric constant and the magnetic moment density with frequency $\Delta\omega = \omega - \omega'$ and wave vector $\mathbf{q} = \mathbf{k} - \mathbf{k}'$. The quantity $\mathbf{J}$, which is bilinear in the amplitude of the incident wave and in the amplitude of the fluctuations, satisfies the role in Eq. (4) of the outside current and can be called the current produced by the scattered wave.

Having expressions (4) and (5) for the field of the scattered wave and the current which produces

*$[\mathbf{k}\mathbf{k}'] \equiv \mathbf{k} \times \mathbf{k}'$.

the scattered wave, we should now compute the intensity of the scattered radiation, i.e., the entrainment of its energy per unit time:

$$Q = \int \mathbf{E}'(\mathbf{r}, t)\mathbf{J}(\mathbf{r}, t)\,d\mathbf{r},$$

and average this quantity over the fluctuations. Taking it into account that the averaged product of the fluctuations of the quantities characterizing the crystal, $\langle \delta f(\mathbf{r}_1, t_1)\, f(\mathbf{r}_2, t_2)\rangle$, depends on the coordinates and on time only in the combinations $\mathbf{r}_1 - \mathbf{r}_2$ and $t_1 - t_2$, it is easy to become convinced that the scattering intensity averaged over the fluctuations, $\langle Q\rangle$, has the form of an integral over the wave vectors of the scattered waves. Dividing the intensity of the scattering in the frequency interval $d\omega'$ and into the element of solid angle do' by the energy flux density of the incident wave $S_0 = c(4\pi)^{-1}|\mathbf{E}^0 \times \mathbf{H}^0|$ and by the value of the scattering volume V, we find the differential scattering cross section

$$d\Sigma = (VS_0)^{-1} d\langle Q\rangle. \tag{6}$$

Noting that the current which produces the scattered wave (and, consequently, the field of the scattered wave) is a sum of two components—a component proportional to $\delta\hat{\varepsilon}$ and a component proportional to δM—it is easy to see that the quantity $d\Sigma$ should be the sum of three quantities:

$$d\Sigma = d\Sigma_e + d\Sigma_m + d\Sigma_{em}, \tag{7}$$

where the quantity $d\Sigma_e$ is bilinear in the amplitude of fluctuations of the dielectric constant, the quantity $d\Sigma_m$ is bilinear in the amplitude of fluctuations of the magnetic moment, and the quantity $d\Sigma_{em}$ is proportional to the product of the fluctuations of the dielectric constant and the fluctuations of the magnetic moment.

In the case of unpolarized radiation, the cross section for scattering of the electromagnetic waves by the fluctuations of the dielectric constant has the form

$$d\Sigma_e = \left(\frac{\omega}{c}\right)^4 \tau_{ij}' \langle \delta\varepsilon_{il}\,\delta\varepsilon_{jm}\rangle_{\mathbf{q},\Delta\omega}\, \tau_{lm} \frac{do'\,d\omega'}{(4\pi)^3}, \tag{8}$$

where $\tau_{lm} = \delta_{lm} - k^{-2} k_l k_m$. The cross section for scattering of the electromagnetic waves by the oscillations of the magnetic moment is determined by the formula

$$d\Sigma_m = g^2 c^{-2} \varepsilon \{\sin^2\vartheta k^2 \langle \delta M^2\rangle_{\mathbf{q},\Delta\omega} + 2\cos\vartheta k_i k_j' \times \operatorname{Re}\langle \delta M_i \delta M_j\rangle_{\mathbf{q},\Delta\omega}\} \frac{do' d\omega'}{4\pi}, \tag{9}$$

where ϑ is the scattering angle (the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$). Finally, the interference component $d\Sigma_{em}$ has the form

$$d\Sigma_{em} = 2g\omega c^{-2} [\mathbf{k}'\mathbf{k}]_i \operatorname{Im}\langle \delta M_j \delta\varepsilon_{ij}\rangle_{\mathbf{q},\Delta\omega} \frac{do' d\omega'}{(4\pi)^2}. \tag{10}$$

Here $\langle \delta\hat{\varepsilon}\delta\hat{\varepsilon}\rangle$, $\langle \delta M_i \delta M_j\rangle$, and $\langle \delta M \delta\hat{\varepsilon}\rangle$ are the Fourier components of the correlators of the fluctuations of the dielectric constant and the magnetic moment density; for example,

$$\langle \delta M_i \delta M_j\rangle_{\mathbf{q},\Delta\omega} = \int \exp\{-i\mathbf{q}\mathbf{r} + i\Delta\omega t\} \langle \delta M_i (\mathbf{r}_1, t_1) \delta M_j (\mathbf{r}_2, t_2)\rangle\, d\mathbf{r}\, dt$$

$$(\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \qquad t = t_1 - t_2).$$

(We shall not cite here the cumbersome formulas for the scattering cross section in the case of polarized radiation.)

2. SCATTERING OF ELECTROMAGNETIC WAVES BY SPIN WAVES

The relations (7)—(10) express the scattering cross section of the electromagnetic waves in a magnetically ordered crystal in terms of the correlators of the fluctuations of the dielectric constant of the crystal and the magnetic-moment density. The correlators of the fluctuations of quantities which characterize the ferromagnetic substance, with account of the coupling between the elastic oscillations and oscillations of the magnetic moment, were obtained previously by the author.[5] It was shown there that the correlators have sharp maxima close to the characteristic oscillation frequencies of the ferromagnetic material. The longitudinal and transverse sound waves are such characteristic oscillations, far from the ferroacoustic resonance point, with a dispersion law $\omega = s_l q$ and $\omega = s_t q$, as well as a spin wave with dispersion law $\omega = \omega_q$, where $s_l(s_t)$ is the velocity of longitudinal (transverse) sound and ω_q is the frequency of the spin wave, equal to

$$\omega_q = (\Omega\Omega_1)^{1/2}, \quad \Omega_1(q) = gM_0(\beta + H_0/M_0 + \alpha q^2), \quad \Omega(\mathbf{q}) = gM_0(\beta + H_0/M_0 + \alpha q^2 + 4\pi\sin^2\chi), \tag{11}$$

M_0 is the equilibrium value of the magnetic moment density, H_0 the external magnetic field (directed along the axis of easy magnetization), α the constant of exchange interaction, β a constant characterizing the anisotropy of the magnetic properties of the crystal, and χ the angle between the wave vector of the spin wave $\mathbf{q}$ and the axis of anisotropy (the z axis). Because of the presence of the sharp maxima in the expressions for the correlation functions, sharp maxima also appear in the expression for the scattering cross section of electromagnetic waves at the frequencies $\omega' = \omega \pm s_l q$ and $\omega' = \omega \pm s_t q$ (longitudinal and transverse sound satellites) and at the frequencies $\omega' = \omega \pm \omega_q$ (magnon satellites), where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$

is the change in the wave vector in the scattering.

We first estimate the relative contribution to the intensity of the magnon satellites of each of the three components in Eq. (7) for the scattering cross section. For this, we note that $\delta\hat{\epsilon} \sim \delta u_{ik}$ in order of magnitude, where δu_{ik} is the fluctuation of the deformation tensor. As was shown in [5], the fluctuations of the elastic quantities in the spin wave are smaller in order of magnitude by a factor of $f_0\zeta^{-1}$ than the fluctuations of the magnetic-moment density, $\delta u_{ik} \sim f_0^{-1}\zeta\delta M/M_0$, where $\zeta = f_0^2 M_0^2(\rho_0 s^2)^{-1}$ is a small parameter characterizing the coupling between the spin and elastic waves (ρ_0 is the equilibrium value of the density of the crystal, s the sound velocity, $f_0 = \max\{f, 1\}$, and f the magnetoelastic constant; in order of magnitude, $\zeta \sim 10^{-4} - 10^{-6}$). Taking it into account that the quantities $d\Sigma_m$ and $d\Sigma_{em}$ contain (in comparison with the quantity $d\Sigma_e$) additional small parameters $(\Delta\omega/\omega)^2$ and $\Delta\omega/\omega$, it is easy to see that the contribution of these quantities to the intensity of the magnon satellites is proportional respectively to $(f_0\Delta\omega/\omega\zeta)^2$ and $f_0\Delta\omega/\omega\zeta$. Thus, in the region of not very high frequencies ($gM_0 \ll \omega \ll f_0\zeta^{-1}gM_0$) the principal contribution to the scattering cross section of the electromagnetic waves is made by their interaction with the magnetic-moment oscillations, described by Eq. (9). In the high frequency region ($\omega > f_0\zeta^{-1}gM_0$), the principal role in the scattering is played by the elastic fluctuations; the scattering cross section in this case is determined by Eq. (8). Finally, in the intermediate region ($\omega \sim f_0\zeta^{-1}gM_0$) the contributions to the scattering cross section of each of the three components in (7) are of equal order of magnitude.

We shall first consider the scattering of electromagnetic waves by spin waves in the region of not very high frequencies ($gM_0 \ll \omega \ll f_0\zeta^{-1}gM_0$). In this region, as has already been shown, the principal role is played by the interaction of the electromagnetic wave with the magnetic-moment oscillations.[1)] Therefore, substituting the known expression for the correlator of the magnetic moment density in Eq. (9)

$$\langle\delta M_i\delta M_j\rangle_{\mathbf{q},\omega} = 2\pi\hbar|N_\omega + 1|gM_0\Omega_{ij}\delta(\omega^2 - \omega_{\mathbf{q}}^2), \quad (12)$$

$$\Omega_{xx} = \Omega_1(q), \quad \Omega_{xy} = \Omega_{yx}^* = i\omega, \quad \Omega_{yy} = \Omega(\mathbf{q}),$$
$$\Omega_{xz} = \Omega_{zx} = \Omega_{yz} = \Omega_{zy} = \Omega_{zz} = 0 \quad (12')$$

(the y axis is chosen along the normal to the plane $(\mathbf{q}, \mathbf{n})$; $\mathbf{n}$ is a unit vector along the z axis), we get

$$d\Sigma = |N_{\Delta\omega} + 1|(gk/c)^2\varepsilon\hbar(gM_0)^2\nu(\mathbf{k}, \mathbf{k}')\delta(\Delta\omega^2 - \omega_{\mathbf{q}}^2)do'd\omega', \quad (13)$$

$$\nu(\mathbf{k}, \mathbf{k}') = (1 - \cos\vartheta\cos\theta\cos\theta')\left(\beta + \frac{H_0}{M_0} + 4\alpha k^2\sin^2\frac{\vartheta}{2}\right) + \frac{\pi}{2}\sin^{-2}\frac{\vartheta}{2}\Big\{4\sin^2\vartheta\sin^2\frac{\vartheta}{2} - \sin^2\vartheta(\cos\theta - \cos\theta')^2 + 2\cos\vartheta k^{-4}(\mathbf{n}[\mathbf{k}, \mathbf{k}'])^2\Big\}, \quad (13')$$

where θ (θ') is the angle between the vector $\mathbf{k}$ ($\mathbf{k}'$) and the z axis, and the frequency of the spin wave $\omega_{\mathbf{q}}$ is connected with the wave vector $\mathbf{k}$ of the incident wave and the angles θ, θ', and ϑ by the relation

$$\omega_{\mathbf{q}} = gM_0\left(\beta + \frac{H_0}{M_0} + 4\alpha k^2\sin^2\frac{\vartheta}{2}\right)^{1/2}\Big\{\beta + 4\pi + \frac{H_0}{M_0} + 4\alpha k^2\sin^2\frac{\vartheta}{2} - \pi\sin^{-2}\frac{\vartheta}{2}(\cos\theta - \cos\theta')^2\Big\}^{1/2}. \quad (14)$$

The expression for the scattering cross section is greatly simplified if either the incident or the scattered wave is propagated along the anisotropy axis. In this case,

$$\nu(\mathbf{k}, \mathbf{k}') = \sin^2\vartheta\left(\beta + \frac{H_0}{M_0} + 4\alpha k^2\sin^2\frac{\vartheta}{2} + 2\pi\cos^2\frac{\vartheta}{2}\right),$$

$$\omega_{\mathbf{q}} = gM_0\left(\beta + \frac{H_0}{M_0} + 4\alpha k^2\sin^2\frac{\vartheta}{2}\right)^{1/2}\left(\beta + \frac{H_0}{M_0} + 4\pi\cos^2\frac{\vartheta}{2} + 4\alpha k^2\sin^2\frac{\vartheta}{2}\right)^{1/2}.$$

We now proceed to the investigation of the scattering of electromagnetic waves by spin waves in the high-frequency region $\omega > f_0\zeta^{-1}gM_0$. As has already been shown, in this region, the principal contribution to the scattering cross section is made by the component $d\Sigma_e$ which characterizes the interaction of the electromagnetic waves with the fluctuations of the dielectric constant. The fluctuations of the dielectric constant are connected with fluctuations in the deformation tensor (see [4]) by the following relation:

$$\delta\varepsilon_{ij} = \sigma_1\delta u_{ij} + \sigma_2\delta_{ij}\delta u_{nn}, \quad (15)$$

where σ_1 and σ_2 are constants of order of magnitude $\epsilon - 1$ (as before, we do not take into account the anisotropy of the dielectric properties of the crystal). Using the equation of continuity and the connection between the tensor u_{ij} and the vector

[1)] The scattering of electromagnetic waves by the magnetic moment oscillations was studied by Bass and Kaganov; [1] the formulas they obtained give the correct order of magnitude for the intensity of the magnon satellites for $\omega < f_0\zeta^{-1}gM_0$; however, they incorrectly describe the angular dependence of the intensity. This is connected with the fact that incorrect expressions were used by the authors for the correlation functions of the magnetic moment density.

displacement u, this relation is conveniently represented in the form

$$\delta\varepsilon_{ij}=\frac{i}{2}\sigma_1(q_i\delta u_j^t+q_j\delta u_i^t)-\left(\sigma_1\frac{q_iq_j}{q^2}+\sigma_2\delta_{ij}\right)\frac{\delta\rho}{\rho_0},\quad(15')$$

where $\delta\rho$ is the density fluctuation and $\delta\mathbf{u}^t=\delta\mathbf{u}-q^{-2}\mathbf{q}(\mathbf{q}\cdot\mathbf{d})$ is the fluctuation of the shear vector.

Substituting (15′) in (8), we can now express the scattering cross section of electromagnetic waves in the high-frequency region in terms of the fluctuations of the shear vector and the crystal density. It turns out here that (in the case of small change in the frequency in the scattering process, which is the case of interest to us) the scattering cross section $d\Sigma_e$ is decomposed into a sum of two components:

$$d\Sigma_e=d\Sigma_c+d\Sigma_s,\quad(16)$$

where the quantity $d\Sigma_c$, which is proportional to the correlator of the density fluctuations, describes the scattering of the electromagnetic waves by the compressional deformations accompanying the spin wave, and the quantity $d\Sigma_s$, which is proportional to the correlator of the fluctuations of the vector $\mathbf{u}^t$, describes the scattering of the electromagnetic waves by the shear deformations accompanying the spin wave. After simple transformations, we get for the quantities $d\Sigma_c$ and $d\Sigma_s$

$$d\Sigma_c=\left(\frac{\omega}{c}\right)^4\left\{\left[\frac{\sigma_1}{2}(1+\cos\vartheta)+\sigma_2\cos\vartheta\right]^2+\sigma_2^2\right\}\times\rho_0^{-2}\langle\delta\rho^2\rangle_{\mathbf{q},\Delta\omega}\frac{do'd\omega'}{(4\pi)^3},$$

$$d\Sigma_s=\left(\frac{\omega}{c}\right)^4\frac{\sigma_1^2}{4}\{2k^2\sin^2\vartheta\langle\delta u^{t2}\rangle_{\mathbf{q},\Delta\omega}-(1-\cos\vartheta)(k_i+k_i')(k_j+k_j')\langle\delta u_i^t\delta u_j^t\rangle_{\mathbf{q},\Delta\omega}\}\frac{do'd\omega'}{(4\pi)^3}.\quad(17)$$

We must substitute in these formulas the expressions for the density-fluctuation correlator and the shear vector at the spin wave:[5]

$$\langle\delta\rho^2\rangle_{\mathbf{q},\omega}=2\pi\hbar|N_\omega+1|q^2\rho_0D\delta(\omega^2-\omega_{\mathbf{q}}^2),$$
$$\langle\delta u_i^t\delta u_j^t\rangle_{\mathbf{q},\omega}=\frac{\nu_i\nu_j^*}{|\mathbf{v}|^2}2\pi\hbar|N_\omega+1|\rho_0^{-1}U\delta(\omega^2-\omega_{\mathbf{q}}^2),\quad(18)$$

where

$$D={}^1/_4q^2gM_0\Omega_1(q)M_0^2(4\pi-\beta-2f)^2\rho_0^{-1}(\omega_q^2-s_l^2q^2)^{-2}\sin^22\chi,$$

$$U=q^2gM_0\Omega_1(q)M_0^2\left\{\frac{\omega_q^2}{\Omega_1^2(q)}f^2\cos^2\chi+[(\beta+f)\sin^2\chi-f\cos^2\chi]^2\right\}\rho_0^{-1}(\omega_q^2-s_t^2q^2)^{-2},$$

$$\mathbf{v}=(\mathbf{q}\cos\chi-\mathbf{n}q)[f\cos^2\chi-(\beta+f)\sin^2\chi]-i[\mathbf{nq}]\times\omega_{\mathbf{q}}\Omega_1^{-1}(q)f\cos\chi,$$

f is the magnetoelastic constant and χ is the angle between the vector $\mathbf{q}$ and the anisotropy axis. As a result, we get

$$d\Sigma_c=|N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6\varepsilon\hbar\rho_0^{-1}R_cD\delta(\Delta\omega^2-\omega_{\mathbf{q}}^2)\frac{do'd\omega'}{(4\pi)^2},$$
$$d\Sigma_s=|N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6\varepsilon\hbar\rho_0^{-1}R_sU\delta(\Delta\omega^2-\omega_{\mathbf{q}}^2)\frac{do'd\omega'}{(4\pi)^2}\quad(19)$$

where the frequency of the spin wave ω_q is connected with the wave vector of the incident wave k and the angles θ, θ', and ϑ by Eq. (14):

$$R_c=(1-\cos\vartheta)\{[{}^1/_2\sigma_1(1+\cos\vartheta)+\sigma_2\cos\vartheta]^2+\sigma_2^2\},$$

$$R_s=\frac{\sigma_1^2}{4}\sin^2\vartheta-\frac{\sigma_1^2}{4}\sin^2\frac{\vartheta}{2}\sin^{-2}\chi\left\{[(\beta+f)\sin^2\chi-f\cos^2\chi]^2\times(\cos\theta+\cos\theta')^2+k^{-4}(\mathbf{n}[\mathbf{kk}'])^2f^2\left(\frac{\Delta\omega}{\Omega_1(q)}\right)^2\cos^2\chi\sin^{-2}\frac{\vartheta}{2}\right\}\times\left\{[(\beta+f)\sin^2\chi-f\cos^2\chi]^2+\left(\frac{\Delta\omega}{\Omega_1(q)}\right)^2f^2\cos^2\chi\right\}^{-1}$$

and we have substituted the following quantities for q and χ:

$$q=2k\sin\frac{\vartheta}{2},\quad\chi=\arccos\frac{\cos\theta-\cos\theta'}{2\sin(\vartheta/2)}.\quad(20)$$

We now consider the scattering of electromagnetic waves in the intermediate frequency range $\omega\sim f_0\zeta^{-1}gM_0$. In this region, the scattering cross section is determined by the general formula (7), while all three components entering into this formula have the same order of magnitude. The quantities $d\Sigma_e$ and $d\Sigma_m$, which describe the scattering of electromagnetic waves by the elastic fluctuations and by the fluctuations of the magnetic moment, are determined by Eqs. (13), (16), and (19); the interference component $d\Sigma_{em}$ can be reduced, by using (15′), to the form

$$d\Sigma_{em}=g\omega c^{-2}\left\{\sigma_1q_i[\mathbf{kk}']_j\,\mathrm{Re}\,\langle\delta M_i\delta u_j^t\rangle_{\mathbf{q},\Delta\omega}+2\sigma_2\rho_0^{-1}[\mathbf{kk}']\,\mathrm{Im}\,\langle\delta\mathbf{M}\delta\rho\rangle_{\mathbf{q},\Delta\omega}\right\}\frac{do'd\omega'}{(4\pi)^2}.\quad(21)$$

Without writing out here the expressions for the correlation functions entering into this relation (these correlation functions are not difficult to calculate by using the method of [5]), we immediately write down the final formula for $d\Sigma_{em}$:

$$d\Sigma_{em}=|N_{\Delta\omega}+1|\frac{\Delta\omega}{\omega}\left(\frac{\omega}{c}\right)^4\varepsilon\hbar(gM_0)^2\rho_0^{-1}G\delta\times(\Delta\omega^2-\omega_{\mathbf{q}}^2)\frac{do'\,d\omega'}{4\pi},$$
$$G={}^1/_8\,qk^{-2}\sin2\chi[\mathbf{qn}][\mathbf{kk}']\{f\sigma_1(\omega_{\mathbf{q}}^2-s_t^2q^2)^{-1}+2\sigma_2(\beta+2f-4\pi)(\omega_{\mathbf{q}}^2-s_l^2q^2)^{-1}\}.\quad(22)$$

We shall now pause briefly on the subject of the relation between the frequency and the direction of propagation of the scattered wave. The δ function which is contained in Eqs. (13), (19), and (22) allows us to find the frequency of the radiation scattered in a particular direction if the frequency and the direction of propagation of the incident wave are known. Using (14), we have for the frequency ω':

$$\omega' = \omega \pm |\Delta\omega|,$$
$$|\Delta\omega| = gM_0\left(\beta + \frac{H_0}{M_0} + 4\alpha k^2 \sin^2\frac{\vartheta}{2}\right)^{1/2}\left\{\beta + \frac{H_0}{M_0} + 4\pi + 4\alpha k^2 \sin^2\frac{\vartheta}{2} - \pi \sin^{-2}\frac{\vartheta}{2}(\cos\theta - \cos\theta')^2\right\}^{1/2}. \quad (23)$$

Taking it into account that the expression for the scattering cross section contains the factor $|N_{\Delta\omega} + 1|$, it is easy to establish that only scattering with a decrease in frequency can take place at very low temperatures ($T \ll \hbar|\Delta\omega|$); if $T \geq \hbar|\Delta\omega|$, then the intensities of both magnon satellites are identical in order of magnitude.

In conclusion, we note that the expressions derived above for the scattering cross section (and also for the correlation functions) do not take into account the damping of the spin waves. In order to consider the spin-wave damping, it suffices to make in these expressions the substitution

$$\delta(\Delta\omega^2 - \omega_{\mathbf{q}}^2) \to \pi^{-1} 2\gamma\omega_{\mathbf{q}}\{(\Delta\omega^2 - \omega_{\mathbf{q}}^2)^2 + (2\gamma\omega_{\mathbf{q}})^2\}^{-1},$$

where γ is the damping decrement of spin waves.

3. SCATTERING OF ELECTROMAGNETIC WAVES BY SOUND OSCILLATIONS

Let us now consider the scattering of electromagnetic waves by sound oscillations. In the case of magnetically ordered crystals, contributions to the cross section of this process are made not only by the interaction of the electromagnetic waves with the density fluctuations and the deformation tensor, but also by the interaction of this wave with the fluctuations of the magnetic moment accompanying the sound wave.

We first estimate the relative contribution to the intensity of the sound satellites of each of the three components in Eq. (7) for the scattering cross section of the electromagnetic wave. For this purpose, we note that, as was shown in [5], the fluctuations of the magnetic moment in the sound wave are equal in magnitude to $\delta M \sim f_0 M_0 \delta u_{ik}$. Taking it into account that the quantities $d\Sigma_m$ and $d\Sigma_{em}$ contain (in comparison with the quantity $d\Sigma_e$) the additional small parameters $(gM_0/\omega)^2$ and gM_0/ω, it is easy to see that the contribution of these quantities to the intensity of the sound satellites is proportional respectively to $(f_0 gM_0/\omega)^2$ and $f_0 gM_0/\omega$. Thus, in the case of a ferromagnetic substance with a small value of the magnetoelastic constant ($f \ll 1$), the scattering of the electromagnetic wave by the sound oscillations is determined principally by its interaction with the fluctuations of elastic quantities, and is characterized by the scattering cross section (8). In the case of the same ferromagnetic substance with a large value of the magnetoelastic constant ($f \gg 1$) the principal contribution to the intensity of the sound satellites in the low-frequency region ($gM_0 \ll \omega \leq fgM_0$) can be made by the interaction of the electromagnetic wave with the magnetic moment fluctuations accompanying the sound wave, an interaction which is characterized by the scattering cross sections (9) and (10).

Obviously, for $\omega \gg f_0 gM_0$, the cross section for scattering of electromagnetic waves by sound oscillations is determined in the case of a ferromagnetic crystal by the same relations as in the case of an ordinary (not magnetically ordered) crystal. This scattering cross section can be found by substituting in (17) the well-known expressions for the correlators of the fluctuations of the density and the shear vector at the sound waves:

$$\langle\delta\rho^2\rangle_{\mathbf{q},\omega} = 2\pi\hbar|N_\omega + 1|q^2\rho_0\delta(\omega^2 - s_l^2q^2), \quad (24)$$
$$\langle\delta u_i^t\,\delta u_j^t\rangle_{\mathbf{q},\omega} = 2\pi\hbar\,|N_\omega + 1|\,\rho_0^{-1}\left(\delta_{ij} - \frac{q_iq_j}{q^2}\right)\delta(\omega^2 - s_t^2q^2).$$

As a result, we get

$$d\Sigma = |N_{\Delta\omega} + 1|\left(\frac{\omega}{c}\right)^6 \varepsilon\hbar\rho_0^{-1}\{G_c\delta(\Delta\omega^2 - s_l^2q^2) + G_s\delta(\Delta\omega^2 - s_t^2q^2)\}\frac{do'd\omega'}{(4\pi)^2},$$
$$G_c = (1 - \cos\vartheta)\left\{\left[\frac{\sigma_1}{2}(1 + \cos\vartheta) + \sigma_2\cos\vartheta\right]^2 + \sigma_2^2\right\},$$
$$G_s = {}^1/_4\,\sigma_1^2\sin^2\vartheta. \quad (25)$$

In the case of a crystal with a large magnetoelastic constant ($f \gg 1$), there is, as has already been pointed out, a region of frequencies ($gM_0 \ll \omega \leq fgM_0$) in which an important contribution to the intensity of the sound satellites is made by the interaction of the electromagnetic wave with the magnetic-moment fluctuations accompanying the sound wave. Therefore, in the calculation of the scattering cross section of the electromagnetic waves in this frequency region, allowance for all three components in Eq. (7) may turn out to be necessary. The first of these components—the quantity $d\Sigma_e$—is determined as before by Eq. (25). In order to find two other components, one must substitute the correlation functions $\langle\delta M_i\delta M_j\rangle$, $\langle\delta M_i\delta u_j^t\rangle$ and $\langle\delta M_i\delta\rho\rangle$ in Eqs. (17) and (21). These functions can easily be determined by using the relations of [5].

Noting that in the low frequency case of interest to us $k \leq c^{-1}fgM_0$ and, consequently, $\omega_k \gg sk$ and $\alpha k^2 \ll 1$, and setting $f \gg 1$, we get for the components of the tensor $\langle \delta M_i \delta M_j \rangle$:

$$\langle \delta M_x^2 \rangle_{\mathbf{q},\omega} = 2\pi\hbar\, |N_\omega + 1|\, f^2 M_0^2 q^2 \rho_0^{-1} \left(\beta + \frac{H_0}{M} + 4\pi \sin^2\chi\right)^{-2}$$
$$\times \{\delta(\omega^2 - s_l^2 q^2)\sin^2 2\chi + \delta(\omega^2 - s_t^2 q^2)\cos^2 2\chi\},$$
$$\langle \delta M_y^2 \rangle_{\mathbf{q},\omega} = 2\pi\hbar\, |N_\omega + 1|\, f^2 M_0^2 q^2 \rho_0^{-1} \left(\beta + \frac{H_0}{M_0}\right)^{-2}$$
$$\times\ \delta(\omega^2 - s_t^2 q^2)\cos^2\chi, \tag{26}$$

where χ is the angle between the vector $\mathbf{q}$ and the axis of anisotropy, and the y axis is chosen perpendicular to the $(\mathbf{q}, \mathbf{n})$ plane (the remaining components of the tensor $\langle \delta M_i \delta M_j \rangle$ are small in this case). Substituting (26) in (9) and assuming, for simplicity, that either the incident or the scattered wave is propagated along the axis of anisotropy, we find for the scattering cross section $d\Sigma_m$:

$$d\Sigma_m = |N_{\Delta\omega} + 1|\, g^2 k^4 c^{-2} \varepsilon\hbar f^2 M_0^2 \rho_0^{-1}$$
$$\times \{G_l \delta(\Delta\omega^2 - s_l^2 q^2) + G_t \delta(\Delta\omega^2 - s_t^2 q^2)\}\, do'\, d\omega',$$
$$G_t = 2\sin^2\frac{\vartheta}{2}\sin^4\vartheta \left\{\left(\beta + \frac{H_0}{M_0} + 4\pi\cos^2\frac{\vartheta}{2}\right)^{-2}\cos^2\vartheta\right.$$
$$\left. + \left(\beta + \frac{H_0}{M_0}\right)^{-2}\sin^2\frac{\vartheta}{2}\right\},$$
$$G_l = 2\sin^2\frac{\vartheta}{2}\sin^4\vartheta \left(\beta + \frac{H_0}{M_0} + 4\pi\cos^2\frac{\vartheta}{2}\right)^{-2}. \tag{27}$$

In estimating the role of the interference component in the scattering cross section $d\Sigma_{em}$, we note that, in accord with (21), this component is determined by the real part of the correlator $\langle \delta M_i \delta u_j^t \rangle$ and the imaginary part of the correlator $\langle \delta M_i \delta\rho \rangle$. On the other hand, it follows from the relations of [5] that when $sq \ll \omega_q$ the components proportional to f^2 in the correlator $\langle \delta M_i \delta u_j^t \rangle$ are purely imaginary, while those in the correlator $\langle \delta M_i \delta\rho \rangle$ are purely real. Thus the quantity $d\Sigma_{em}$ (in contrast with the quantity $d\Sigma_m$) makes no appreciable contribution to the intensity of the sound satellites even in the low frequency region; therefore, we shall not write down the expression for $d\Sigma_{em}$ here.

We pause briefly on the subject of the relation between the frequency and the direction of propagation of the scattered wave. The δ functions contained in Eqs. (25) and (27) permit us to find the frequencies of the waves scattered in a definite direction. It is easy to see that four waves are propagated at any given angle ϑ to the direction of the incident wave, with frequencies

$$\omega' = \omega \pm 2s_l k \sin(\vartheta/2), \quad \omega' = \omega \pm 2s_t k \sin(\vartheta/2). \tag{28}$$

(Account of the anisotropy of the elastic properties of the crystal, which removes the degeneracy between the two transverse sound waves with unlike polarizations, leads to splitting of each of the transverse sound satellites into two.)

Thus, far from the ferroacoustic resonance, three pairs of lines appear in the spectrum of radiation scattered in the ferromagnetic substance: magnon, longitudinal sound, and transverse sound satellites. Comparing Eqs. (25) and (27) with the relations obtained in the previous section for the cross section for scattering of the electromagnetic wave by spin waves, it is easy to see that in the low frequency region $gM_0 \ll \omega < gs\rho_0^{1/2}$, the magnon satellites have a greater intensity than the sound satellites, while in the high frequency region $\omega > gs\rho_0^{1/2}$ the sound satellites are characterized by the greater intensity.

4. SCATTERING OF ELECTROMAGNETIC WAVES IN THE VICINITY OF FERROACOUSTIC RESONANCE

We now proceed to study the scattering of electromagnetic waves near the point of ferroacoustic resonance. We first note that the ferroacoustic resonance can appear in the scattering of electromagnetic waves only in the high frequency region. In fact, from the resonance condition $\omega_q \sim sq$, it follows that $q > s^{-1}gM_0$; therefore, the frequency of the incident wave should satisfy the inequality $\omega > gM_0c/s$. Using Eqs. (7)—(10), and recognizing that, as shown in [5], $\delta M \sim s\rho_0^{1/2}\delta u_{ij}$, close to the ferroacoustic resonance it is easy to see that in the region of frequencies $\omega > gM_0c/s$ the principal contribution to the scattering cross section of the electromagnetic waves is made by the interaction of these waves with the fluctuations of the density and the shear vector, an interaction characterized by the scattering cross section (17).

In the vicinity of the point of longitudinal ferroacoustic resonance ($|\omega_q - s_l q| \leq \zeta^{1/2}\omega_q$) two branches of coupled magnetoacoustic oscillations with frequencies $\omega_\pm$ are propagated, in place of the spin and longitudinal sound waves in the ferromagnetic material; here

$$\omega_\pm^2 = \tfrac{1}{2}(\omega_{\mathbf{q}}^2 + s_l^2 q^2) \pm \tfrac{1}{2}\{(\omega_{\mathbf{q}}^2 - s_l^2 q^2)^2$$
$$+ \Omega_1 g M_0^3 \rho_0^{-1} q^2 \sin^2 2\chi (\beta + 2f - 4\pi)^2\}^{1/2}. \tag{29}$$

There are great fluctuations of the density of the crystal in these waves; the correlation function for these fluctuations has the form [5]

$$\langle \delta\rho^2 \rangle_{\mathbf{q},\omega} = \pi\hbar\, |N_\omega + 1|\, q^2 \rho_0 \{\delta(\omega^2 - \omega_+^2) + \delta(\omega^2 - \omega_-^2)\}. \tag{30}$$

Substituting this expression in the first of Eqs. (17), we get

$$d\Sigma = \frac{1}{2}|N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6 \varepsilon\hbar\rho_0^{-1}G_c\{\delta(\Delta\omega^2-\omega_+^2) + \delta(\Delta\omega^2-\omega_-^2)\}\frac{do'd\omega'}{(4\pi)^2}. \quad (31)$$

Comparing this formula with Eq. (25), we see that each of the longitudinal sound satellites is split, on approaching the point of longitudinal ferroacoustic resonance, into two lines of equal intensity.

In the vicinity of the point of transverse ferroacoustic resonance ($|\omega_q - s_t q| \leq \zeta^{1/2}\omega_q$), three branches of coupled magnetoacoustic oscillations are propagated with frequencies $s_t q$ and $\omega_{1,2}$, in place of the spin and transverse sound waves in the ferromagnetic substance; here

$$\omega_{1,2}^2 = {}^1/_2(\omega_q^2 + s_t^2q^2) \pm {}^1/_2\{(\omega_q^2 - s_t^2q^2)^2 + 4\eta g M_0^3\rho_0^{-1}q^2\}^{1/2},$$
$$\eta = \Omega_1[(\beta+f)\sin^2\chi - f\cos^2\chi]^2 + \Omega f^2\cos^2\chi. \quad (32)$$

There are large fluctuations of the components of the shear vector in these waves; the correlation function for these fluctuations has the form [5]

$$\langle\delta u_i^t\,\delta u_j^t\rangle_{q,\omega} = \frac{\pi}{2}\hbar|N_\omega+1|\rho_0^{-1}\left(\delta_{ij} - \frac{q_iq_j}{q^2}\right)\{\delta(\omega^2-\omega_1^2) + 2\delta(\omega^2 - s_t^2q^2) + \delta(\omega^2-\omega_2^2)\}. \quad (33)$$

Substituting this expression in the second of Eqs. (17), we get

$$d\Sigma = \frac{1}{4}|N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6 \varepsilon\hbar\rho_0^{-1}G_s\{\delta(\Delta\omega^2-\omega_1^2) + 2\delta(\Delta\omega^2 - s_t^2q^2) + \delta(\Delta\omega^2-\omega_2^2)\}\frac{do'd\omega'}{(4\pi)^2}. \quad (34)$$

Comparing this formula with Eq. (25), we see that, in place of a single pair (or two pairs in the presence of anisotropy in the elastic properties of the crystal) of transverse sound satellites, three pairs of satellites appear in the spectrum of the scattered radiation on approaching the point of transverse ferroacoustic resonance. As in the case of transverse resonance, the summary intensity of the satellites is not changed in this case. So far as the brightness of the individual satellites is concerned, the brightnesses of the lines with frequencies $\omega \pm \omega_1$, $\omega \pm s_t q$ and $\omega \pm \omega_2$ are related to one another as $1:2:1$.

If we take into account the damping of the magnetoelastic waves, then the δ-functions in relations (31) and (34) are replaced by the expressions

$$\delta(\Delta\omega^2-\omega_j^2) \to \pi^{-1}2\gamma_j\omega_j\{(\Delta\omega^2-\omega_j^2)^2 + (2\gamma_j\omega_j)^2\}^{-1},$$

where ω_j is one of the frequencies of the magnetoelastic waves ($\omega_{1,2}$, $s_t q$, $\omega_\pm$) and γ_j is the damping decrement of the corresponding wave. Noting that the gap between the magnetoelastic satellites is of the order of $\zeta^{1/2}\omega_q$, it is easy to see that a change takes place in the character of the distribution of scattered radiation as one approaches the point of ferroacoustic resonance in the case of a sufficiently small damping of the magnetoelastic waves, $\gamma_j < \zeta^{1/2}\omega_q$. If $\gamma_j \geq \zeta^{1/2}\omega_q$, then the satellites with frequencies $\omega + \omega_+$ and $\omega + \omega_-$ (or alternatively) satellites with frequencies $\omega - \omega_+$ and $\omega - \omega_-$; $\omega + \omega_1$ and $\omega + \omega_2$, etc.) are superimposed on one another; in this case, the scattering cross section is determined by the same formulas as in the nonresonant region.

We now consider briefly the conditions for the observation of ferroacoustic resonance by means of the angular and spectral distribution of the radiation scattered in the ferromagnetic material. As has already been noted, for scattering of electromagnetic waves by spin waves, the change in frequency $|\Delta\omega|$ is uniquely determined by the direction of propagation of the scattered wave (for a given frequency and direction of propagation of the incident wave); the value of $|\Delta\omega|$ as a function of the angles θ, θ', and ϑ is given in this case by Eq. (23). In the scattering of electromagnetic waves by sound, the change in frequency is connected (according to (28)) with the scattering angle by the relation $|\Delta\omega| = 2ks\sin(\vartheta/2)$, where $s = s_l$ ($s = s_t$) in the case of longitudinal (transverse) sound.

In order that the phenomenon of ferroacoustic resonance appear in the scattering of electromagnetic waves, it is necessary that the conditions (23) and (28) be satisfied simultaneously, or, more exactly, that the relation

$$F \equiv \left(\frac{sq}{\omega_q}\right) = 2(ks)^2(1-\cos\vartheta)(gM_0)^{-2} \times \left(\beta + \frac{H_0}{M_0} + 4\alpha k^2\sin^2\frac{\vartheta}{2}\right)^{-1} \times \left\{\beta + \frac{H_0}{M_0} + 4\pi + 4\alpha k^2\sin^2\frac{\vartheta}{2} - \pi\sin^{-2}\frac{\vartheta}{2}(\cos\theta - \cos\theta')^2\right\}^{-1} \quad (35)$$

be different from unity by a quantity of the order of $\zeta^{1/2}$, $F = 1 + 0(\zeta^{1/2})$.

In particular, if either the incident or the scattered wave is propagated along the axis of anisotropy and $k < s(\alpha g M_0)^{-1}$, then the scattering angle ϑ should be close to the angle $\vartheta_0(k)$, where

$$\vartheta_0(k) = \arccos\frac{(ks)^2 - {}^1/_2 gM_0^2(\beta + H_0/M_0)(\beta + H_0/M_0 + 2\pi)}{(ks)^2 + (gM_0)^2\pi(\beta + H_0/M_0)}$$

In the general case of arbitrary angles θ and θ', for observation of the ferroacoustic resonance it is necessary that the angle of scattering be close to the angle $\vartheta_0(k, \theta, \theta')$. In this case, the angle ϑ_0 is determined from the equation $F = 1$, where the function F is given by Eq. (35). A change in the character of the distribution of scattered radiation takes place in a narrow range of angles $\vartheta = \vartheta_0\{1 + 0(\zeta^{1/2})\}$ and the change in frequency

$$|\Delta\omega| = 2ks \sin\frac{\vartheta_0}{2}\{1 + O(\zeta^{1/2})\},$$

therefore, for the discovery of this effect, a resolution in the scattering angle of $\sim 0.01\,\vartheta_0$ is necessary, as well as a resolution in the frequency of $\sim 0.01|\Delta\omega|$.

In conclusion, the authors express their gratitude to V. G. Bar'yakhtar and V. V. Eremenko for useful discussions.

[1] F. G. Bass and M. I. Kaganov, JETP **37**, 1390 (1959), Soviet Phys. JETP **10**, 986 (1960).

[2] Y. R. Shen and N. Bloembergen, Phys. Rev. **143**, 371 (1966).

[3] A. I. Akhiezer, V. G. Bar'yakhtar and S. V. Peletminskiĭ, JETP **35**, 228 (1958), Soviet Phys. JETP **8**, 157 (1959).

[4] L. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media)(Gostekhizdat, 1957).

[5] I. A. Akhiezer and Yu. L. Bolotin, JETP **52**, 787 (1967), Soviet Phys. JETP **25**, 517 (1967).

Translated by R. T. Beyer
171