SCREENING IN THE ATOMIC PHOTOEFFECT

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The effect of screening on the photoeffect from the K shell is taken into account in the first perturbation-theory approximation. Expansion of the relativistic Coulomb functions in powers of αZ is used as the basis. It is shown that the influence of screening decreases the cross section by several per cent.

1. IN an earlier paper (henceforth cited as I)[1] we investigated the effect of screening in the photoeffect from the K shell in first approximation of perturbation theory in terms of the difference between the screened and Coulomb potentials. We took into account in I only the effect of screening on the wave function of the electron of the continuous spectrum, and this led to an increase of the cross section in the small-angle region. It was assumed that the shift of the K level and the change of the K-shell wave function under the influence of the screening are negligible. This assumption is incorrect. In the present paper we take complete account of screening in first-order perturbation theory and show that its influence is small and leads to an insignificant decrease in the cross section at all angles.

Just as in I, we choose the additional potential connected with the screening, following Moliere,[2] in the form of a sum of Yukawa potentials:[1)]

$$U = -\alpha Z\sum_{l=1}^{4} a_l V_{i\lambda_l} \equiv -\alpha Z\Sigma_l V_{i\lambda_l} \qquad V_{i\lambda_l}(r) = r^{-1}e^{-\lambda_l r}, \quad (1)$$

where

$$\lambda_l = b_l \nu, \qquad \nu = {}^1/_{121} mZ^{1/3}, \quad (1a)$$

m is the electron mass, and the values of the coefficients a and b are

$$a_1 = 0.10, \quad a_2 = 0.55, \quad a_3 = 0.35, \quad a_4 = -1,$$
$$b_1 = 6.0, \quad b_2 = 1.20, \quad b_3 = 0.30, \quad b_4 = 0. \quad (1b)$$

The action of the summation operator Σ_l on a function that does not depend on λ_l yields zero, by virtue of (1b):

$$\Sigma_l \cdot 1 = \sum_{l=1} a_l = 0. \quad (2)$$

The values of a_l, b_l, and λ_l are taken from Moliere's paper,[2] and λ_l^{-1} has the dimensions of length and coincides in order of magnitude with the Thomas-Fermi radius. When λ_l tends to zero, U tends to zero linearly with λ_l, by virtue of (2). Therefore, expansion in powers of U at small values of λ_l is equivalent to expansion in powers of λ_l. The dimensionless expansion parameters can be obtained by dividing λ_l by the momenta that enter in the problem, viz., p—the momentum of the outgoing electron, k—the γ-quantum momentum, q—the momentum transferred to the nucleus, and $\eta = m\alpha Z$—the average electron momentum on the K-shell. We shall assume that we are far from threshold, so that $p \sim k > m$. We note that in this case $q \sim m$ for all angles. In this case the small expansion parameter will be λ_l/η, which is the ratio of the Bohr radius η^{-1} to the Thomas-Fermi radius λ_l^{-1}.

In the expansion in powers of λ_l, the linear term of the expansion corresponds to a constant potential, and this should affect none of the physical consequences. Indeed, it is easy to verify that the terms linear in λ_l drop out from the wave functions of both the discrete and the continuous spectrum when the energy conservation law is taken into account.[2)] Therefore the first nonvanishing terms of the expansion are those proportional to λ_l^2. For the same reason, the second approximation in the potential U contains terms proportional to λ_l^4.

[1)]It is possible to use in lieu of the sum an integral spectral expansion, which does not affect the character of the calculations.

[2)]The K-level shift, and with it the energy conservation law, were not taken into account in I. This led to the appearance of a term linear in λ_l in the correction for the screening, giving a small contribution and increasing the cross section.

For the Coulomb wave functions and the Green's function we shall use an expansion in powers of αZ, discarding terms of order $\alpha^2 Z^2$; this is equivalent to discarding terms of order $\lambda_l^2 \alpha^2 Z^2/\eta^2 = \lambda_l^2/m^2$ in the correction for the screening. Thus, when p, k, $q \sim m$ the inclusion of terms of order λ_l^2/q^2, λ_l^2/p^2, and $\lambda_l^2/k^2 \sim \lambda_l^2/m^2$ would go beyond the accuracy limits, and they will be discarded. We need thus to observe only the terms proportional to λ_l^2/η^2.

Since the amplitude of the photoeffect without allowance for screening was obtained in I with a relative accuracy on the order of $\alpha^2 Z^2$ (the terms proportional to $\alpha^2 Z^2$ are represented in the forms of integrals that can be readily evaluated for the case when the electron is emitted in the direction of the incident γ quantum), it follows that the correction for screening need be taken into account only in that region of Z where it has the same order of smallness. For the parameter $\Sigma_l \lambda_l^2/\eta^2$ (all the functions of γ_l are acted upon by the summation operator defined in (1) and (1b), we have the following estimates:

$$\begin{aligned} &\Sigma_l\lambda_l^2/\eta^2 \leqslant \alpha Z && \text{for} \quad Z \geqslant 18,\\ &\Sigma_l\lambda_l^2/\eta^2 \leqslant \alpha^2 Z^2 && \text{for} \quad Z \geqslant 32,\\ &\Sigma_l\lambda_l^2/\eta^2 \leqslant \alpha^3 Z^3 && \text{for} \quad Z \geqslant 45.\end{aligned}$$

We shall consider regions of Z such that

$$\alpha^3 Z^3 < \Sigma_l \lambda_l^2/\eta^2 \leqslant \alpha Z.$$

As follows from (1), the second approximation in the potential U has a relative order of smallness $\alpha Z(\Sigma_l \lambda_l^2/\eta^2)^2$, i.e., it exceeds the accuracy of the calculations in the region of Z under consideration.

The amplitude of the photoeffect is given by formula (4) of I. The wave functions of the continuous and discrete spectra, with allowance for screening, can be written in first approximation in the form

$$|\psi_p\rangle = \frac{N_p}{2\pi^2}\left(|\varphi_p\rangle + G_c^{E_0} U|\varphi_p\rangle + \ldots\right) u_p; \tag{3}$$

$$\begin{aligned}|\psi_b\rangle &= G_c^{\varepsilon} U|\psi_b\rangle = |\varphi_b\rangle \\ &+ \left(G_c^{\varepsilon} - \frac{|\varphi_b\rangle\langle\varphi_b|}{\varepsilon-\varepsilon_0}\right)\Big|_{\varepsilon\to\varepsilon_0} U|\varphi_b\rangle + \ldots,\\ |\varphi_b\rangle &= (2\pi)^{3/2} N_b |\varphi_0\rangle u_0,\end{aligned} \tag{4}$$

where $|\varphi_p\rangle$ and $|\varphi_b\rangle$ are the relativistic Coulomb functions of the electrons of the continuous and discrete spectra, G_c is the relativistic Coulomb Green's function, E, ϵ and E_0, ϵ_0 are the energies of the electrons of the continuous spectrum and of the K shell, respectively, with and without allowance for screening:

$$E = k + \varepsilon, \quad E_0 = k + \varepsilon_0, \quad \varepsilon_0 = \sqrt{m^2 - \eta^2}.$$

The K-level shift calculated in first perturbation-theory approximation is

$$\begin{aligned}\varepsilon - \varepsilon_0 &= E - E_0 = (2\pi)^3 N_b^2 \langle\varphi_0|U|\varphi_0\rangle \\ &= -\frac{\alpha Z\eta}{\gamma}\Sigma_l\left(1+\frac{\lambda_l}{2\eta}\right)^{-2\gamma} \approx \frac{1}{2m}\Sigma_l\left(\eta\lambda_l - \frac{3}{2}\lambda_l^2\right),\end{aligned} \tag{5}$$

$\gamma = \sqrt{1-\alpha^2 Z^2}$. Here and below, all the undefined symbols are the same as in I.

Following expansion of the wave function of the continuous spectrum in λ_l, including the expansion (5) of the energy, the linear term cancels out. The quadratic term of (5) contains the parameter λ_l^2/m^2 and can, as already stated, be discarded compared with $E_0/m \gtrsim 1$. The quadratic term in the second member proportional to U in (3) is of the form λ_l^2/p^2, λ_l^2/k^2, or λ_l^2/q^2 and should also be discarded. Therefore the function (3) should be replaced in our approximation by $(2\pi^2)^{-1} N_{p_0} |\varphi_{p_0}\rangle$, where $p_0^2 = E_0^2 - m^2$. We shall henceforth omit the zero index of p and E throughout.

The expression for the photoeffect amplitude now takes the form

$$\begin{aligned} Q &= \langle\psi_p|\hat{A}|\psi_b\rangle = \sqrt{\frac{2}{\pi}} N_p N_b \bar{u}_p (Q^c + Q^s) u_0,\\ Q^c &= \langle\varphi_p|\hat{A}|\varphi_0\rangle,\\ Q^s &= \langle\varphi_p|\hat{A} G_c^{\varepsilon} U|\varphi_0\rangle - \langle\varphi_p|\hat{A}|\varphi_0\rangle\langle\varphi_b|U|\varphi_b\rangle/(\varepsilon-\varepsilon_0)|_{\varepsilon_0-\varepsilon\to 0}.\end{aligned} \tag{6}$$

Here Q^c is the Coulomb amplitude, which contains no dependence on U, and Q^s is the screening correction, which is to be calculated.

2. We shall first calculate the matrix element $\langle s|G_c^{\epsilon} U|\varphi_0\rangle$, where **s** is an arbitrary vector in momentum space. Using (1) and the expression for $|\varphi_0\rangle$ in I:

$$|\varphi_0\rangle = V_{i\eta}|\mathbf{r}\rangle(\overleftarrow{\Gamma}_\eta + O(\alpha^2 Z^2))|_{r\to 0}, \quad \Gamma_\eta = -\frac{\partial}{\partial\eta} + \frac{\alpha Z}{2}\tilde{\nabla}_r, \tag{7}$$

(the arrow over Γ_η indicates that the operator acts on the left) with $\tilde{\nabla}_r = \boldsymbol{\alpha}\nabla_r$, $\boldsymbol{\alpha}$ is a Dirac Matrix, and V is defined by

$$\langle s_1|V_\varkappa|s_2\rangle = \frac{1}{2\pi^2}\frac{1}{(s_1-s_2)^2 - \varkappa^2},$$

we obtain

$$S \equiv \langle s|G_c^{\varepsilon} U|\varphi_0\rangle = -\alpha Z \Sigma_l \langle s|G_c^{\varepsilon} V_{i\lambda_l} V_{i\eta}|\mathbf{r}\rangle\overleftarrow{\Gamma}_\eta|_{r\to 0}. \tag{8}$$

With the aid of the following identities for the Yukawa potentials (see I)

$$V_{\alpha_1+i\varepsilon_1} V_{\alpha_2+i\varepsilon_2} = \int\limits_{\varepsilon_1+\varepsilon_2}^{\infty} d\lambda\, V_{\alpha_1+\alpha_2+i\lambda}, \quad V_{i\varepsilon_1} V_{i\varepsilon_2} = V_0 V_{i(\varepsilon_1+\varepsilon_2)} \tag{9}$$

we represent (8) in the form

$$\begin{aligned} -\Sigma_l \alpha Z G_c^{\varepsilon} V_{i\lambda_l} V_{i\eta}|\mathbf{r}\rangle &= -\alpha Z \Sigma_l G_c^{\varepsilon} V_0 V_{i\mu_l}|\mathbf{r}\rangle \\ &= (\varphi - 1) V_{i\mu_l}|\mathbf{r}\rangle,\end{aligned} \tag{10}$$

where $\mu_l = \lambda_l + \eta$,

$$\varphi = 1 - \alpha Z G_c^\varepsilon V_0 = 1 - \alpha Z G^+ V_0 \varphi. \quad (11)$$

We shall use the expression obtained in [3] (formulas (6) and (19) of that paper) for the Moller operator φ:

$$\varphi = \varphi^0 + \alpha Z G^- T_0 + (\alpha Z)^2 \varphi^0 G^+ V_0 G^- T_0 + \ldots \quad (12a)$$

$$= 1 + \alpha Z G^+ T_0 + (\alpha Z)^2 G^+ V_0 G^- T_0 + O(\alpha^3 Z^3), \quad (12b)$$

where

$$T_0 = T_{i\lambda}|_{\lambda=0}, \qquad T_{i\lambda} = V_{i\lambda}\varphi^0;$$

φ^0 is the nonrelativistic Moller operator, defined with the aid of a relation analogous to (11):

$$\varphi^0 = 1 - \alpha Z G^0 V_0 \varphi^0. \quad (11a)$$

The operators G and T are defined as follows:

$$\langle \mathbf{f} | G^j | \mathbf{v} \rangle = G^j(\mathbf{f})\delta(\mathbf{f} - \mathbf{v}), \quad j = 0, +, -, \quad (13)$$

with

$$G^0(\mathbf{f}) = \frac{-2\varepsilon}{f^2 - \tau^2 - ih}, \qquad G^\pm(\mathbf{f}) = \frac{\hat{f} \pm \varepsilon + \beta m}{f^2 - \tau^2 - ih},$$

$$\tau^2 = \varepsilon^2 - m^2, \quad h \to 0, \quad \hat{f} = \alpha \mathbf{f}, \quad (13a)$$

α and β are Dirac matrices;

$$\langle \mathbf{r} | T_{i\lambda} | \mathbf{s} \rangle = \int_0^1 (\exp) \frac{\partial}{\partial x} (x \langle \mathbf{r}x | V_{\tau\Lambda} | \mathbf{s} \rangle) dx, \quad (14)$$

where

$$(\exp) = \exp\left\{ i \frac{\alpha Z \varepsilon}{\tau} \int_x^1 \frac{dx'}{x'\Lambda'} \right\} = \left(\frac{1 - i\lambda/\tau}{1 + i\lambda/\tau} \frac{1 + \Lambda}{1 - \Lambda} \right)^{i\alpha Z\varepsilon/\tau}; \quad (14a)$$

$$\Lambda = \Lambda(x) = [(1 - r^2 x/\tau^2)(1 - x) - \lambda^2 x/\tau^2]^{1/2}, \qquad \Lambda' = \Lambda(x'). \quad (14b)$$

The first two terms of (12b) were rewritten with the aid of definition (11a) for φ^0, and in the third term φ^0 was replaced by unity, which is equivalent to discarding the terms of highest order in αZ.[3)] All the terms (12b) are meaningful for all momenta p, down to $p \to 0$.[3]

Substituting (12b) in (10) and using (9), we get

$$S = \alpha Z \langle \mathbf{s} | G^+ (1 + V_0 G^-) T_0 V_{i\mu_l} | \mathbf{r} \rangle \overleftarrow{\Gamma}_\eta |_{r \to 0}. \quad (15)$$

With the aid of the identities (9) and the definition (14), we can easily obtain the relations

$$\langle \mathbf{s} | T_0 V_{i\mu_l} | \mathbf{r} \rangle = \int_{\mu_l}^\infty \langle \mathbf{r} | T_{i\lambda} | \mathbf{s} \rangle d\lambda, \quad (16a)$$

$$G^\pm(\mathbf{s}) \langle \mathbf{r} | T_{i\lambda} | \mathbf{s} \rangle = \int_0^1 (\exp) \left(\frac{m\beta \pm \varepsilon}{2\tau\Lambda} \frac{\partial}{\partial(\tau\Lambda)} + \frac{\tilde{\nabla}_{rx}}{2} \right) \langle \mathbf{s} | V_{\tau\Lambda} | \mathbf{r}x \rangle. \quad (16b)$$

It is necessary to put r = 0 in (15) after taking the gradient, and consequently the terms quadratic in r can be discarded prior to taking the gradient.[4)] The expression (14b) for Λ then simplifies to $\Lambda = [1 - x - (\lambda/\tau)^2 x]^{1/2}$.

Using (7), (9)–(12), and (16a) we get

$$S = \alpha Z (S_1 + S_2 + S_3), \quad (17)$$

$$S_1 = \Sigma_l G^+(\mathbf{s}) \langle 0 | T_{i\mu_l} | \mathbf{s} \rangle = \Sigma_{\mu_l} \int_0^1 (\exp) dx \left(\frac{m}{\tau\Lambda_l} \frac{\partial}{\partial(\tau\Lambda_l)} + \frac{\tilde{\nabla}}{2} \right) \langle \mathbf{s} | V_{\tau\Lambda_l} | \mathbf{r} \rangle |_{r \to 0}, \quad (17a)$$

$$S_2 = \frac{\alpha Z}{2} \Sigma_l G^+(\mathbf{s}) \tilde{\nabla}_r \int_{\mu_l}^\infty d\lambda \langle \mathbf{r} | T_{i\lambda} | \mathbf{s} \rangle = \frac{\alpha Z \eta}{8} \Sigma_l \int_0^1 (\exp) x\, dx \int_{\mu_l/\eta}^1 d\sigma \left(\frac{3}{\eta\Lambda} + \frac{\partial}{\partial(\eta\Lambda)} \right) \times \frac{\partial}{\partial(\eta\Lambda)} \langle \mathbf{s} | V_{i\eta\Lambda} | 0 \rangle + O(\alpha^4 Z^4), \quad (17b)$$

$$S_3 = \alpha Z \Sigma_l G^+(\mathbf{s}) \int_0^1 (\exp) \frac{\partial}{\partial x} (x \langle \mathbf{s} | V_0 G^- V_{\tau\Lambda_l} | 0 \rangle) dx = \alpha Z \Sigma_l G^+(\mathbf{s}) \frac{\tilde{\nabla}_r}{2} \int_0^1 (\exp) dx \int_{-i\tau\Lambda_l}^\infty d\lambda \langle \mathbf{s} | V_{i\lambda} | \mathbf{r} \rangle + O(\alpha^2 Z^2). \quad (17c)$$

In (17a) and (17c) $\Lambda_l = [1 - x - (\mu_l/\tau)^2 x]^{1/2}$. In the derivation of (17b) we used the property of the operator Σ_l (Eq. (2)).

$$\Sigma_l \int_{\mu_l}^\infty = \Sigma_l \left(\int_{\mu_l}^\eta + \int_\eta^\infty \right) = \Sigma_l \int_{\mu_l}^\eta,$$

since the second integral does not depend on λ_l. In (17b) we replaced τ by $\tau_0 = \sqrt{\epsilon_0^2 - m^2} = i\eta$, since here, unlike (17a) and (17c), the integrand has no singularity at x = 0. In (17a) we replaced $(m + \epsilon)/2\tau\Lambda_l$ by $m/\tau\Lambda_l$, which is equivalent to discarding terms of order $\alpha^2 Z^2$.

With the aid of (17a)–(17c) we get:

$$\langle \varphi_p | \hat{A} G_c^\varepsilon U | \varphi_0 \rangle = \alpha Z (Q_1 + Q_2 + Q_3), \quad (18)$$

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3)By substituting (12b) in (10) and (6) we can replace the function $\langle \varphi_p |$ in (6) by the Born term $\langle \mathbf{p} |$, after which the investigated term in (12) takes the form of the corresponding term in (8) with $\mathbf{s} = -\mathbf{q} = \mathbf{p} - \mathbf{k}$. It is easy to verify from (11a) or (14) that when $q \approx m$ the replacement of φ_0 in (12b) by unity actually corresponds to discarding terms of order of αZ, in spite of the fact that the expansion of φ_0 is carried out in terms of the parameter $i\zeta = i\alpha Z\varepsilon / \sqrt{\varepsilon^2 - m^2} \sim 1$.

4)The product of two gradients $\tilde{\nabla}_r \tilde{\nabla}_r$ can be expressed in terms of the derivative $\partial/\partial(\tau\Lambda)$ and the gradient $\tilde{\nabla}_r$.

where

$$Q_1 = \Sigma_l \int_0^1 (\exp)\,dx\, \langle \varphi_p | V_{\tau\Lambda_l} | \mathbf{k}\rangle\, \hat{e}\left(\frac{m}{\tau\Lambda_l}\frac{\partial}{\partial(\tau\Lambda_l)} + \frac{\overleftarrow{\tilde{\nabla}}_k}{2}\right) + O(\alpha^2 Z^2), \quad (18a)$$

$\hat{e} = \gamma\mathbf{e}$, $\mathbf{e}$ is the photon polarization vector, and $\gamma = -i\beta\boldsymbol{\alpha}$. The operator $\partial/\partial(\tau\Delta_l)$ acts only on $\langle \varphi_p | \Lambda_{\tau\Lambda} | \mathbf{k}\rangle$;

$$Q_2 = \frac{\alpha Z\eta}{8}\Sigma_l \int_0^1 (\exp)\,x\,dx \int_{\mu_l/\eta}^{1} d\sigma\left(\frac{3}{\eta\Lambda} + \frac{\partial}{\partial(\eta\Lambda)}\right)\frac{\partial}{\partial(\eta\Lambda)} \times \langle \mathbf{p} | V_{i\eta\Lambda} | \mathbf{k}\rangle\, \hat{e} + O(\alpha^4 Z^4), \quad (18b)$$

$$Q_3 = \alpha Z \Sigma_l \hat{e} G^+(-\mathbf{q})\, \frac{\tilde{\nabla}_k}{2}\int_0^1 (\exp)\,dx \int_{-i\tau\Lambda_l}^{\infty} d\lambda\, \langle \mathbf{p} | V_{i\lambda} | \mathbf{k}\rangle + O(\alpha^2 Z^2), \quad (18c)$$

$$\mathbf{q} = \mathbf{k} - \mathbf{p}.$$

We call attention to the fact that the expression

$$\langle \varphi_p | V_{\tau\Lambda_l} | \mathbf{k}\rangle\, \hat{e}\left(\frac{\eta}{\tau\Lambda_l}\frac{\partial}{\partial(\tau\Lambda_l)} + \alpha Z\frac{\tilde{\nabla}_k}{2}\right)$$

in (18a) coincides in construction with the expression

$$\langle \varphi_p | V_{i\eta} | \mathbf{k}\rangle\, \hat{e}\left(-\frac{\partial}{\partial\eta} + \alpha Z\frac{\tilde{\nabla}_k}{2}\right),$$

which we had in I for the amplitude of the photoeffect without allowance for screening. After expanding in powers of $\tau\Lambda_l$ ($\eta \le -i\tau\Lambda_l \le \mu_l$), which is actually in powers of $\tau\Lambda_l/q$ and $\tau\Lambda_l p/mk$ and corresponds to expansion of Q^C in powers of η (η/q and $\eta p/mk$) in I, this expression takes the form

$$A + \frac{\alpha Z}{\tau\Lambda_l}B + O\left(\alpha^2 Z^2, \alpha Z\frac{\tau\Lambda_l}{m}, \frac{\tau^2\Lambda_l^2}{m^2}\right);$$

A and B do not depend on $\tau\Lambda_l$ and consequently on x. The corresponding expansion of Q^C leads to the expression

$$A + \frac{\alpha Z}{i\eta}B + O(\alpha^2 Z^2)$$

with the same values of A and B.

Thus,[5)]

$$Q_1 = \Sigma_l \int_0^1 (\exp)\,dx\left(A + \frac{\alpha Z}{\tau\Lambda_l}B\right) + O\left(\alpha^2 Z^2 \Sigma_l \frac{\lambda_l^2}{\eta^2}\right) = \Sigma_l (A J_1 + \alpha Z B J_2), \quad (19)$$

[5)]An estimate for the discarded terms in (19) is obtained after expanding them in powers of $\lambda_l/2\eta$.

$$J_1 = \int_0^1 (\exp)\,dx = 4\int_0^1 t^{-i\zeta}\frac{a - bt}{(a + bt)^3}dt, \quad (19a)$$

$$J_2 = \int_0^1 (\exp)\frac{dx}{\tau\Lambda_l} = 4\frac{1}{\tau}\int_0^1 t^{-i\zeta}\frac{dt}{(a + bt)^2}, \quad (19b)$$

where

$$a = 1 + i\mu_l/\tau, \quad b = 1 - i\mu_l/\tau, \quad \zeta = \alpha Z\varepsilon/\tau,$$

$$(\exp) = \exp\left\{i\zeta\int_x^1 \frac{dx'}{x'\Lambda_l'}\right\}, \quad \Lambda_l' = \Lambda_l(x').$$

The integrals (19a) and (19b) exist when $\mathrm{Re}\, i\zeta < 1$. The analytic continuation of J_1 to the case $\mathrm{Re}\, i\zeta \ge 1$ is of the form

$$J_1 = \frac{4}{e^{-2i\pi\delta} - 1}\int_1^{0^+} t^{-i\zeta}\frac{a - bt}{(a + bt)^3}dt, \quad \delta = i\zeta - 1. \quad (20)$$

A cut along the real axis, from 0 to ∞, is made in the t-plane. The integration contour begins at the point t = 1 on the real axis, surrounds the point t = 0 along a circle of small radius ρ, and returns to t = 1 along the lower edge of the cut. Carrying the foregoing integration, we get

$$J_1 = -\frac{4}{(1 + i\mu_l/\tau)^2}\left[\frac{1}{\delta\rho^\delta} + \int_\rho^1 \frac{dt}{t}\frac{1 - bt/a}{(1 + bt/a)^3}\right]. \quad (20a)$$

We have set $i\zeta$ equal to unity in the integrand, since the lower limit of integration differs from zero. Regarding δ as a small quantity

$$\delta = (\gamma - 1) + \frac{\Delta}{\alpha^2 Z^2} + \frac{3}{2}\gamma\frac{\Delta^2}{\alpha^4 Z^4} + O\left(\frac{\Delta^3}{\alpha^6 Z^6}\right)$$

(where $\Delta = (\epsilon - \epsilon_0)/m$ and γ is defined in (5)), we expand the pole term of (20a) in terms of δ:

$$J_1 = -\frac{4}{(1 + i\mu_l/\tau)^2} \times \left[\frac{1}{\delta} + 1 - i\frac{\mu_l}{\tau} + \frac{1 + (\mu_l/\tau)^2}{4} - \ln\frac{1 + i\mu_l/\tau}{2}\right]. \quad (20b)$$

Proceeding analogously with J_2, we get

$$J_2 = -\frac{4}{\tau(1 + i\mu_l/\tau)^2}\left[\frac{1}{\delta} + \frac{1 - i\mu_l/\tau}{2} - \ln\frac{1 + i\mu_l/\tau}{2}\right]. \quad (21)$$

The pole terms in (20b) and (21) are of the form $1/\delta$ and not $1/\Delta$ (the Green's function G_c^ϵ has a pole $1/\Delta$). The shift of the pole position is due to the fact that we have used in the calculations an approximate expression for the relativistic Coulomb Green's function, confining ourselves to terms of relative accuracy ~ αZ. In view of this, we should expand the pole term $1/\delta$ in powers of αZ, regarding, in purely formal fashion, the quantity $1 - \gamma = \alpha^2 Z^2/2 + O(\alpha^4 Z^4)$ as small compared with $\Delta/\alpha^2 Z^2$, and only then allowing Δ to go to zero (cf. (6)). To the required degree of accuracy, we have

$$\frac{1}{\delta}=\frac{\alpha^2Z^2}{\Delta}-\frac{3}{2}+O\left(\frac{\alpha^4Z^4}{\Delta},\alpha^2Z^2\right). \quad (22)$$

Recognizing that $\tau = i\eta\,(1 - \Delta/\alpha^2Z^2)$ (see (13a) and (5)) and using the property of the sum Σ_l (2), we readily obtain

$$\Sigma_l J_1=\Sigma_l i\eta J_2=M'(Z)\equiv\Sigma_l\frac{1}{(1+\lambda_l/2\eta)^2} \quad (23)$$
$$\times\left[-\frac{\alpha^2Z^2}{\Delta}+\frac{5}{2}+\frac{\lambda_l}{2\eta}-\frac{1}{1+\lambda_l/2\eta}+\ln\left(1+\frac{\lambda_l}{2\eta}\right)\right],$$

$$\alpha ZQ_1=M'(Z)\left(A+\frac{\alpha Z}{i\eta}B\right)$$
$$=M'(Z)\langle\varphi_p|V_{i\eta}|\mathbf{k}\rangle\,\hat{e}\overleftarrow{\Gamma}_\eta+O\left(\alpha^2Z^2\Sigma_l\frac{\lambda_l^2}{\eta^2}\right). \quad (24)$$

Further, it is easy to verify that

$$Q_2\sim\alpha^2Z^2\Sigma_l\lambda_l^2/\eta^2 \quad (25)$$

and makes no contribution in our approximation. Calculating Q_3 (Eq. (18c)), we get

$$Q_3=\alpha ZM'(Z)\frac{1}{q^3}\frac{\pi}{4}+O\left(\alpha^2Z^2\Sigma_l\frac{\lambda_l^2}{\eta^2}\right),\ q=\sqrt{(\mathbf{k}-\mathbf{p})^2}. \quad (26)$$

Combining (24), (25), and (26) and taking (5) into account, we obtain for Q^S (Eq. (6)) the following expression:

$$Q^s=M(Z)\langle\varphi_p|\hat{A}|\varphi_0\rangle+O(\alpha^3Z^3\Sigma_l\lambda_l^2/\eta^2), \quad (27)$$

$$M(Z)\equiv\Sigma_l\frac{1}{(1+\lambda_l/2\eta)^2}$$
$$\times\left[\frac{5}{2}+\frac{\lambda_l}{2\eta}-\frac{1}{1+\lambda_l/2\eta}+\ln\left(1+\frac{\lambda_l}{2\eta}\right)\right], \quad (27a)$$

Here $\langle\varphi_p|\hat{A}|\varphi_0\rangle$ is the Coulomb amplitude of the photoeffect Q^C (we recall that the expansion of Q^C begins with terms of order αZ), from which we have left out all terms whose relative order of smallness is higher than that of the first.

In the region of values of Z under consideration ($Z \gtrsim 18$), the factor M(Z) can be expanded in powers of $\lambda_l/2\eta$. As a result we get

$$M(Z)=-3\Sigma_l\left(\frac{\lambda_l}{2\eta}\right)^2+O\left(\Sigma_l\left(\frac{\lambda_l}{2\eta}\right)^3\right)=-\frac{3}{4}\sum_{l=1}^{3}a_l\left(\frac{\lambda_l}{\eta}\right)^2, \quad (27b)$$

where a_l and λ_l are defined by (1a) and (1b). The values for M(Z), calculated from (27b) for different Z, are:

Z:	20	40	60
$-M(Z)$:	0.037	0.014	0.009

Thus, the amplitude and the cross section of the photo effect in the screened field of the nucleus can be represented in the form

$$Q=[1+M(Z)]\,Q^c+Q_1{}^c,\qquad d\sigma=[1+2M(Z)]\,d\sigma^c+d\sigma_1{}^c; \quad (28)$$

Q^C and $d\sigma^C$ are the Coulomb amplitude and cross section, combining the terms whose relative accuracy is of order not higher than first in αZ; Q_1^C and $d\sigma_1^C$ are of order α^2Z^2 relative to the principal terms of Q^C and $d\sigma^C$, respectively.

As seen from (28) and (27b), the screening decreases the cross section. This agrees with the results of Matese and Johnson.[4] When the Moliere potential (1) is used, we find that we get $M(Z) < {}^3Z^3$ already for $Z \sim 40$, and allowance for screening is on over-refinement of the accuracy in our approximation.

In conclusion, we note once more that the screening correction obtained in this manner is suitable in the region of medium Z, from $Z \sim 20$ to $Z \sim 40$. At larger values of Z it is so small that it goes beyond the limits of the accuracy used to calculate the Coulomb amplitude and cross section. With increasing Z, the relative contribution of this correction to the amplitude and to the cross section decreases like $Z^{-4/3}$ (the discarded terms include some which increase with increasing Z, but their relative contribution to the amplitude is $\sim \alpha^3Z^3$).

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