STABILITY OF OPPOSING-WAVE REGIME IN A RING GAS LASER

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An expression is obtained for the polarization vector of a system with account taken of terms of second order in the ratio of the natural line width to the Doppler width, and also second-order terms with respect to the intensity of the oscillations. The corresponding simplified equations are derived for the opposing wave amplitudes and phase differences. An expression is obtained for the width of the region of the two-wave regime instability. It is shown that the instability region decreases with increasing field amplitude. The existence of a two-wave regime is investigated for the case when the Q factors of the resonator are different for the two waves. It is shown that there exists a detuning region which overlaps the instability partially and in which the two-wave regime is impossible. Under the given conditions, the upper limit of the region greatly exceeds the limit of the instability region.

The effect of atomic collisions on the nature of the wave regime in a laser is considered. It is shown that the widths of the instability region and the region of non-existence of a two-wave regime increase with increasing pressure. New effects appear if the collisions are taken into account. Thus, the dependence of the laser power on the detuning becomes asymmetric relative to the center of the Doppler line. Asymmetry is observed also for the regions of instability and non-existence of the two-wave regime. The effect of the isotopic composition on the non-existence of the two-wave regime is investigated. The calculation shows that a small amount of a second isotope is sufficient for the two-wave regime to exist throughout the entire detuning region in which the self-excitation conditions are satisfied.

The influence of wave coupling on the nature of the wave regime is investigated. It is shown that the boundary of the instability region of the two-wave regime depends weakly on the magnitude of the coupling coefficient and does not depend on it at all if the coefficients are equal. However, the picture is entirely different inside the instability region. Thus, a stationary one-wave regime occurs when the coupling is weak, but no such stationary regime exists if the coupling is strong enough and energy is periodically pumped from one wave to the other occurs. It is shown that the existence of such a nonstationary regime depends on the excess over the excitation threshold.

IN a gas ring laser operating in the one-mode regime it is possible, in general, to excite two waves traveling in the ring in opposite directions. For solid-state lasers such a two-wave regime is as a rule unstable[1], owing to the competition of the opposing waves. In the case of gas lasers, a strong competition of the opposing waves takes place only when the frequency deviations of these waves relative to the center of the Doppler emission line are small, and the two waves interact simultaneously with one and the same group of atoms. As a result of the competition, the two-wave regime is unstable; either one wave suppresses the other completely (stationary one-wave regime), or energy is periodically pumped from one wave to the other (self-oscillation regime). Such self-oscillating regimes in solid-state lasers are described in[2].

The instability of the regime of two opposing waves in a gas ring laser was investigated in[3-5]. Aronovitz[3] found that the width of the instability region is proportional to the speed of rotation of the ring laser and vanishes in a laser at standstill. This result is attributed to the fact that in the immobile laser the width of the instability region is determined by terms of higher order of smallness than those taken into account in Lamb's theory[6], which was used in[3]. Similar results, except that account was taken of the coupling between the opposing waves due to the back scattering, were reported by Zehlnov et al.[4] Zeĭger and Fradkin[5] took into account the terms of second order of smallness in the ratio of the natural line width to the Doppler width, for inhomogeneous broadening, and obtained the width of the instability region of the two-wave regime in a stationary laser. However, they did not take into account the comparable terms of second order of smallness in the oscillation intensity, and therefore obtained no dependence of the width of the instability region on the amplitude. This dependence can be obtained if fifth-approximation terms are taken additionally into account, rather than retaining only third-approximation terms in the amplitude, as was done in all the cited investigations.

In the first part of this paper we obtain the width of the instability region of the two-wave regime as a function of the laser parameters and the oscillation amplitude. It is shown that the instability region decreases with increasing amplitude. We investigate also the question of the possible existence of a solution corresponding to the two-wave mode in the case when the resonator Q-factors are slightly different for the two waves. It turns out that there exists a detuning region, which overlaps slightly the instability region and in which there exists no solution corresponding to the two-wave regime, i.e., the two wave regime is impossible. The upper limit of this region can greatly exceed the limit of the instability region.

In the second part we generalize the results obtained in the first, so as to be able to take into account the effect of atomic collisions. We consider the widths of the instability region, and of the region where the two-wave regime cannot exist, as functions of the laser-gas pressure. We show that the widths of these regions increase with increasing pressure. When collisions are taken into account, certain new effects appear: The dependence of the laser power on the detuning becomes asymmetrical relative to the center of the Doppler line, and an analogous symmetry is observed also for the instability region and the region where the two-wave regime cannot exist.

In the third part of the paper we estimate the effect of the adding a second isotope of the active gas on the width of the regions of instability and non-existence of opposing waves in the laser. It turns out that at a certain small percentage of second-isotope admixture the two-wave regime becomes possible in the entire detuning range in which the self-excitation condition is satisfied.

The fourth part of the paper is devoted to an analysis of the influence of the coupling produced between the wave by the back-scattering on the possible existence of the two-wave regime. We find that the limit of the instability region of this regime depends little on the coupling coefficient, whereas inside the instability region the picture depends essentially on the magnitude of this coefficient: when the coupling is small, a stationary single-wave regime is observed (the amplitude of one of the waves is much smaller than that of the other); at sufficiently large coupling, no such stationary state exists, and energy is periodically pumped from one wave to the other.

1. DETERMINATION OF THE REGION OF EXISTENCE AND STABILITY OF THE TWO-WAVE REGIME

The investigation is carried out for the one-mode regime, so that the electric field in the laser can be represented in the form of a sum of two opposing waves:

$$E = {}^1/_2 \sum_{1,2} (E_{1,2}\, e^{i(\omega t \mp kx)} + \text{c.c.}) = {}^1/_2 \sum_{1,2} \tilde{E}_{1,2} + \text{c.c.} \quad (1.1)$$

Each of these fields should satisfy the wave equation

$$\frac{\partial^2 \tilde{E}_{1,2}}{\partial t^2} + \frac{\omega}{Q_{1,2}} \frac{\partial \tilde{E}_{1,2}}{\partial t} - c^2 \frac{\partial^2 \tilde{E}_{1,2}}{\partial x^2} = -4\pi \frac{\partial^2 P_{1,2}}{\partial t^2}. \quad (1.2)$$

To calculate the polarization vector $P_{1,2}$ we use the equations for the density matrix:

$$\frac{\partial \rho_a}{\partial t} + v \frac{\partial \rho_a}{\partial x} = i \frac{e}{\hbar} (r_{ab}\rho_{ba} - \rho_{ab} r_{ba}) E - \gamma_a (\rho_a - \rho_a^{(0)}),$$

$$\frac{\partial \rho_b}{\partial t} + v \frac{\partial \rho_b}{\partial x} = -i \frac{e}{\hbar} (r_{ab}\rho_{ba} - \rho_{ab} r_{ba}) E - \gamma_b (\rho_b - \rho_b^{(0)}),$$

$$\frac{\partial \rho_{ab}}{\partial t} + v \frac{\partial \rho_{ab}}{\partial x} = i\omega_0 \rho_{ab} + \frac{ie}{\hbar} (\rho_b - \rho_a) r_{ab} E - \gamma_{ab}\rho_{ab}, \quad \rho_{ba} = \rho_{ab}^*. \quad (1.3)$$

Here v is the velocity of the atom, r_{ab} the matrix element of the displacement vector, ω_0 the natural frequency of the transition of the atoms from level b to level a; γ_a and γ_b are quantities characterizing the times of relaxation of ρ_a and ρ_b to specified levels $\rho_a^{(0)}$ and $\rho_b^{(0)}$ in the absence of a field, and γ_{ab} is the spontaneous-emission line width.

We seek the solution of (13) in the form

$$\rho_{a,b} = \rho_{a,b0} + {}^1/_2 (\rho_{a,b1} e^{-2ikx} + \text{c.c.}),$$

$$\rho_{ab} = \sum_{1,2} \rho_{ab}^{(1,2)} e^{i(\omega t \mp kx)}. \quad (1.4)$$

A solution in this form takes into account the modulation of the level populations along the gas-discharge tube. The second time harmonic is not taken into account, since more detailed calculations have shown that it makes a small contribution to the final result (terms of the order of γ/ω).

We seek the solution of (13) in the form of an expansion in terms of the field, confining ourselves to terms of fourth power in the field. As a result we get:

$$\rho_{ab}^{(1,2)} = \frac{er_{ab}D^{(0)}}{2\hbar(\omega-\omega_0 \mp kv - i\gamma_{ab})}[1 - A_{1,2}|E_{1,2}|^2$$
$$-(A_{2,1}-B_{1,2})|E_{2,1}|^2 + A_{1,2}^2|E_{1,2}|^4 + (A_{2,1}^2 - A_{2,1}B_{1,2}$$
$$-B_{1,2}C_{2,1})|E_{2,1}|^4 + (2A_1A_2 - B - B_{1,2}A_{1,2} + B_{1,2}C_{1,2}^*)$$
$$\times |E_1|^2|E_2|^2]E_{1,2},$$
$$D^{(0)} = \rho_b^{(0)} - \rho_a^{(0)}. \tag{1.5}$$

Here

$$A_{1,2} = \frac{e^2|r_{ab}|^2\gamma_{ab}(\gamma_a+\gamma_b)}{2\hbar^2\gamma_a\gamma_b[(\omega-\omega_0\mp kv)^2+\gamma_{ab}^2]}$$

$$B_{1,2} = \frac{e^2|r_{ab}|^2(4kv \pm i(\gamma_a+\gamma_b))}{4\hbar^2(2kv\pm i\gamma_a)(2kv\pm i\gamma_b)}$$
$$\times\left(\frac{1}{\omega-\omega_0+kv\pm i\gamma_{ab}} - \frac{1}{\omega-\omega_0-kv\mp i\gamma_{ab}}\right),$$

$$C_{1,2} = \frac{4kv\mp i(\gamma_a+\gamma_b)}{(2kv\mp i\gamma_a)(2kv\mp i\gamma_b)}\,\frac{e^2|r_{ab}|^2}{4\hbar^2(\omega-\omega_0\pm kv - i\gamma_{ab})},$$

$$B = \frac{e^4|r_{ab}|^4(\gamma_a-\gamma_b)}{8\hbar^4\gamma_a\gamma_b}\,\mathrm{Im}\left\{\frac{4kv - i(\gamma_a+\gamma_b)}{(2kv-i\gamma_a)(2kv-i\gamma_b)}\right.$$
$$\left.\times\left(\frac{1}{\omega-\omega_0-kv+i\gamma_{ab}} - \frac{1}{\omega-\omega_0+kv-i\gamma_{ab}}\right)\right\}.$$

The polarization vector P is represented in the form

$$P = \sum_{1,2} P_{1,2}\, e^{i(\omega t \mp kx)}.$$

Then

$$P_{1,2} = enr_{ba}\int_{-\infty}^{\infty}\rho_{ab}^{(1,2)}(v)\,dv.$$

We average over the velocities under the assumption that the distribution is Maxwellian. In the case of inhomogeneous broadening, when γ_a, γ_b, $\gamma_{ab} \ll ku$, we obtain after averaging, taking into account terms of second order of smallness in $\gamma_{a,b,ab}/ku$:

$$P_{1,2} = \frac{b\pi}{2ku}\left\{\sqrt{\frac{2}{\pi}}\frac{\mu}{ku} - \frac{\mu\gamma_{ab}}{2(ku)^2}a|E_{1,2}|^2 - \left(\frac{\mu\gamma_{ab}}{2(\mu^2+\gamma_{ab}^2)}\right.\right.$$
$$\left.-\frac{\mu\gamma_{ab}}{4(ku)^2}\right)a|E_{2,1}|^2 + i\left[1 - \frac{\mu^2}{2(ku)^2}\right.$$
$$-\left(\frac{1}{2} - \frac{\mu^2+\gamma_{ab}^2}{4(ku)^2}\right)a|E_{1,2}|^2 - \left(\frac{\gamma_{ab}^2}{2(\mu^2+\gamma_{ab}^2)}\right.$$
$$\left.+\frac{\gamma^2}{2(ku)^2} - \frac{\gamma_{ab}^2}{4(ku)^2}\right)a|E_{2,1}|^2 + \frac{3}{8}a^2|E_{1,2}|^4$$
$$+\left(\frac{3}{8} + \frac{\gamma^2}{4\gamma_{ab}^2}\left(\frac{\gamma_{ab}}{\gamma_a+2\gamma_{ab}} + \frac{\gamma_{ab}}{\gamma_b+2\gamma_{ab}}\right)\right)a^2|E_{2,1}|^4$$
$$+\left(\frac{3}{4} - \frac{\gamma^2}{4\gamma_{ab}^2}\left(\frac{\gamma_a-\gamma_{ab}}{2\gamma_{ab}+\gamma_a} + \frac{\gamma_b-\gamma_{ab}}{2\gamma_{ab}+\gamma_b}\right)\right.$$
$$\left.\left.\times\; a^2|E_1|^2|E_2|^2\right\}E_{1,2}. \tag{1.6}$$

Here

$$a = \frac{e^2|r_{ab}|^2}{2\hbar^2\gamma^2} \qquad b = \frac{e^2n|r_{ab}|^2D^0}{\sqrt{2\pi}\,\hbar}$$
$$\gamma^2 = \frac{\gamma_a\gamma_b\gamma_{ab}}{\gamma_a+\gamma_b} \qquad \mu = \omega - \omega_0.$$

Expression (1.6) for the polarization differs from that given by Lamb[6] in that terms of second order in γ/ku and aE^2 are taken into account.

Substituting now (1.1) and (1.6) in (1.2) and separating individual harmonics, we obtain equations for the two opposing fields E_1 and E_2. We introduce the amplitudes $E_{1,2}$ and the phases $\Phi_{1,2}$, and write for them the simplified equations

$$\dot{E}_{1,2} = \frac{\omega d}{2}\left[\eta_{1,2} - \frac{\mu^2}{2(ku)^2} - \alpha a E_{1,2}^2 - \beta a E_{2,1}^2 + \frac{3}{8}a^2E_{1,2}^4\right.$$
$$\left.+\frac{3}{8}a^2(1+\theta_1)E_{2,1}^4 + \frac{3}{4}(1+\theta_2)a^2E_1^2E_2^2\right]E_{1,2},$$
$$\dot{\Phi}_{1,2} = \Delta_0 - \frac{\omega d}{2}\left(\sqrt{\frac{2}{\pi}}\frac{\mu}{ku} - \frac{\mu\gamma_{ab}}{2(\mu^2+\gamma_{ab}^2)}aE_{2,1}^2\right). \tag{1.7}$$

Here

$$d = \frac{4\pi^2 b}{ku}, \qquad \eta_{1,2} = 1 - \frac{1}{Q_{1,2}d},$$
$$\Delta_0 = kc - \omega, \qquad \alpha = \frac{1}{2} - \frac{\mu^2+\gamma_{ab}^2}{4(ku)^2},$$
$$\beta = \frac{\gamma_{ab}^2}{2(\mu^2+\gamma_{ab}^2)} + \frac{\gamma^2}{2(ku)^2} - \frac{\gamma_{ab}^2}{4(ku)^2},$$
$$\theta_1 = \frac{2\gamma^2}{3\gamma_{ab}^2}\left(\frac{\gamma_{ab}}{\gamma_a+2\gamma_{ab}} + \frac{\gamma_{ab}}{\gamma_b+2\gamma_{ab}}\right).$$
$$\theta_2 = \frac{\gamma^2}{3\gamma_{ab}^2}\left(\frac{\gamma_{ab}-\gamma_a}{\gamma_a+2\gamma_{ab}} + \frac{\gamma_{ab}-\gamma_b}{\gamma_b+2\gamma_{ab}}\right).$$

In the derivation of (1.7), we calculated the polarization under the assumption that the amplitudes and phases of the fields assume their steady-state values much more slowly than the polarization, i.e., the polarization manages to follow the variation of the field. This is valid under the following conditions:

$$\eta\omega/Q \ll \gamma, \quad \omega\mu/Qku \ll \gamma,$$

which are well satisfied at small excess above threshold $\eta \ll 1$ and at small values of the detuning $(\mu/ku \ll 1)$.

The system (1.7) admits of three stationary solutions:

$$\text{I} \quad E_{20}^2 = 0, \quad E_{10}^2 \approx \frac{\eta_1}{\alpha a} - \frac{\mu^2}{2\alpha a (ku)^2}; \quad (1.8)$$

$$\text{II.} \quad E_{10}^2 = 0, \quad E_{20}^2 \approx \frac{\eta_2}{\alpha a} - \frac{\mu^2}{2\alpha a (ku)^2}; \quad (1.9)$$

$$\text{III.} \quad E_{10}^2 = E_0^2 + \frac{\delta}{(\alpha - \beta + {}^3/_4\,\theta_1 a E_0^2) a}$$

$$E_{20}^2 = E_0^2 - \frac{\delta}{(\alpha - \beta + {}^3/_4\,\theta_1 a E_0^2) a}. \quad (1.10)$$

Here

$$E_0^2 = \frac{1}{2}(E_{10}^2 + E_{20}^2) = \frac{\eta_0 - \mu^2/2(ku)^2}{(\alpha + \beta) a},$$

$$\eta_0 = (\eta_1 + \eta_2)/2, \quad \delta = (\eta_1 - \eta_2)/2. \quad (1.10')$$

Expressions (1.8), (1.9), and (1.10′) were obtained without allowance for the term of second order in aE^2, since these terms make only small contributions here. Allowance for these terms is quite essential in (1.10), since they are comparable with the small quantity $\alpha - \beta$.

Let us investigate now the possible existence and the stability of the solution (1.10) corresponding to the presence of two waves traveling in opposition. We see that the condition for the existence of two opposing waves is $E_{10}^2 \geq 0$ and $E_{20}^0 \geq 0$. It follows from (1.10) that these conditions are not satisfied if

$$-\frac{|\delta|}{aE_0^2} \leqslant \alpha - \beta + \frac{3}{4}\theta_1 a E_0^2 \leqslant \frac{|\delta|}{aE_0^2}. \quad (1.11)$$

This inequality determines the region of values of μ in which the two-wave regime is impossible.

In order to define this region in explicit form, we assume that the excess of the pump level over threshold is sufficiently large, so that the inequality

$$\eta_0 \gg \mu^2/2(ku)^2 \quad (1.12)$$

is satisfied for all values of μ defined by the condition (1.11). Then, substituting in (1.11) the quantities α, β, and E_0^2, we obtain with allowance for (1.12)

$$\frac{\gamma^2}{(ku)^2} - \frac{3}{2}\theta_1\eta_0 - \frac{2|\delta|}{\eta_0} \leqslant \frac{\mu^2}{\gamma_{ab}^2} \leqslant \frac{\gamma^2}{(ku)^2} - \frac{3}{2}\theta_1\eta_0 + \frac{2|\delta|}{\eta_0}. \quad (1.13)$$

We see from this that in the region under consideration the values of μ^2 are not larger than $\gamma_{ab}^2\gamma^2/(ku)^2 + 2|\delta|\gamma_{ab}^2/\eta_0$, i.e., the condition (1.12) is satisfied if

$$\eta_0 \gg \gamma^2_{ab}\gamma^2/2(ku)^4, \quad \eta_0 \gg \sqrt{|\delta|}\,\gamma_{ab}/ku.$$

Let us determine now the condition for the stability of the opposing-wave regime, assuming that the conditions for its existence are satisfied. The equations for the deviations from the stationary value are:

$$\dot{\varepsilon}_1 = \omega d[-\alpha a E_{10}\varepsilon_1 - \beta a E_{20}\varepsilon_2 + {}^3/_4 a^2 E_{10}^3 \varepsilon_1 + {}^3/_4 a^2 E_{20}^3 (1 + \theta_1)\varepsilon_2$$
$$+ {}^3/_4 a^2 E_{10} E_{20}^2 (1 + \theta_2)\varepsilon_1 + {}^3/_4 a^2 E_{10}^2 E_{20} (1 + \theta_2)\varepsilon_2] E_{10}. \quad (1.14)$$

The equation for ϵ_2 is obtained by permuting the corresponding indices for the fields.

The deviations ϵ_1 and ϵ_2 will grow, i.e., the two-wave regime will be unstable, if

$$[\alpha - {}^3/_4 a^2 E_{10}^2 - {}^3/_4 a^2 E_{20}^2 (1 + \theta_2)]$$
$$\times [\alpha - {}^3/_4 a^2 E_{20}^2 - {}^3/_4 a^2 E_{10}^2 (1 + \theta_2)]$$
$$\leqslant [\beta - {}^3/_4 a^2 E_{20}^2 (1 + \theta_1) - {}^3/_4 a^2 E_{10}^2 (1 + \theta_2)]$$
$$\times [\beta - {}^3/_4 a^2 E_{10}^2 (1 + \theta_1) - {}^3/_4 a^2 E_{20}^2 (1 + \theta_2)]. \quad (1.15)$$

Confining ourselves only to terms of second order in the field, we get

$$\alpha^2 - \beta^2 + {}^3/_4 \alpha a^2 \theta_1 (E_{10}^2 + E_{20}^2) \leqslant 0, \quad (1.16)$$

or, taking (1.10′) and (1.12) into account

$$\frac{\mu^2}{\gamma_{ab}^2} \leqslant \frac{\gamma^2}{(ku)^2} - \frac{3}{2}\eta_0\theta_1. \quad (1.17)$$

Comparing the inequalities (1.13) and (1.17) we see that the upper limit of the region of non-existence of the two-wave regime is always higher (or, at any rate, not lower) than the limit of the instability region of this regime. Therefore, the region of existence of the two-wave regime is always

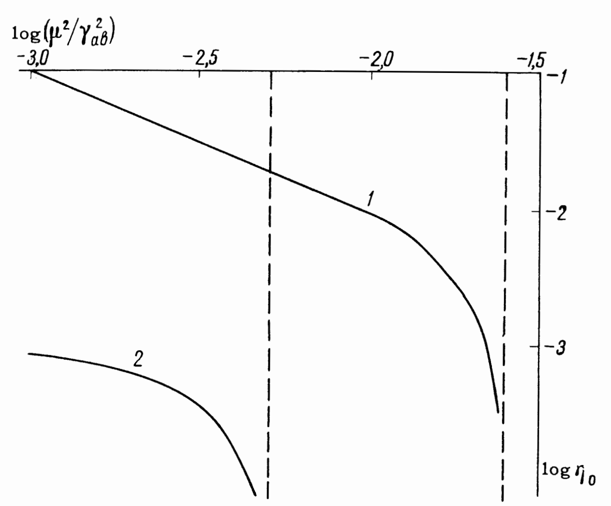

FIG. 1. Limit of the region of existence of the two-wave regime vs. gain at the following values of the parameters: $\gamma_a = \gamma_b = \gamma_{ab}/2$, $\gamma^2/(ku)^2 = 10^{-3}$; 1) $2|\delta| = 10^{-4}$, 2) $|\delta| = 0$.

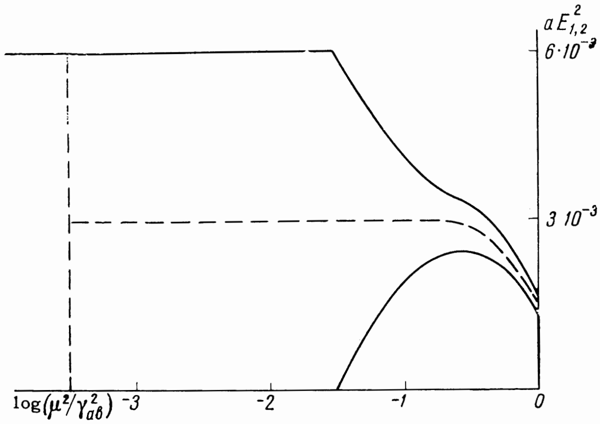

FIG. 2. Amplitudes of the opposing waves vs. detuning. $\eta_0 = 3.16 \times 10^{-3}$, $\gamma_a = \gamma_b = \gamma_{ab}/2$, $\gamma^2/(ku)^2 = 10^{-3}$, and $2|\delta| = 10^{-4}$. Dashed curve – $\delta = 0$.

bounded by the right side of the inequality (1.13), i.e., the two-wave regime exists if

$$\frac{\mu^2}{\gamma_{ab}{}^2} \geqslant \frac{\gamma^2}{(ku)^2} - \frac{3}{2}\theta_1\eta_0 + \frac{2|\delta|}{\eta_0}. \qquad (1.18)$$

The intensities of the opposing waves are determined in this region by the expressions (1.10).

We note that the regions of non-existence and instability of the two-wave regime become narrower when η_0 increases, i.e., when the oscillation amplitude increases (Fig. 1). Figure 2 shows the intensities of the opposing waves as functions of the detuning μ at different values of $|\delta|$.

2. DEPENDENCE OF THE REGION OF EXISTENCE OF THE TWO-WAVE REGIME ON THE PRESSURE

The effect of collisions was investigated theoretically and experimentally for standing-wave gas lasers[7-10]. It was shown that allowance for the collisions reduces in part to a redetermination of the parameters, viz., a change in the level lifetimes and a shift of the transition frequency. In addition, collisions give rise to a new effect—an asymmetric dependence of the laser power on the detuning. In this section we clarify the influence of collisions on the width of the region of existence of the two-wave regime and calculate the intensities of both waves as a function of the detuning.

In order to obtain equations for the density-matrix elements, with allowance for the pressure effects, we proceed in the following manner: We introduce in the right sides of (1.3) additional terms describing the collisions of the atoms. We denote them by J_a, J_b, and J_{ab}. The functions J_a and J_b are represented in the form

$$J_a = -\tilde{\gamma}_a(\rho_a - \rho_a^{(0)}), \quad J_b = -\tilde{\gamma}_b(\rho_b - \rho_b^{(0)}). \quad (2.1)$$

In this approximation, allowance for the collisions in the equations for the diagonal elements reduces to replacement of γ_a and γ_b by $\gamma_a + \tilde{\gamma}_a$ and $\gamma_b + \tilde{\gamma}_b$, respectively. We shall henceforth take γ_a and γ_b to mean their values with collisions taken into account.

We represent the additional term J_{ab} in the equation for the rapidly-varying off-diagonal element of the density matrix ρ_{ab} in the form

$$J_{ab} = -(\tilde{\gamma}_{ab} - i\tilde{\omega}_0)\rho_{ab} + \Delta\frac{er_{ab}}{\hbar}D^{(0)}E. \qquad (2.2)$$

Here $\tilde{\gamma}_{ab}$, $\tilde{\omega}_0$, and Δ are functions of the pressure. In the derivation of (2.2) it was assumed that the collision integral J_{ab} is determined by the rapidly-varying functions ρ_{ab} and E. In view of the smallness of the collision effects, we have retained in (2.2) only the terms linear in ρ_{ab} and E. The quantities $\tilde{\gamma}_{ab}$ and $\tilde{\omega}_0$ will henceforth be included in γ_{ab} and ω_0.

As a result, the equations for ρ_a and ρ_b remain unchanged in form, and the equation for ρ_{ab} takes the form

$$\frac{\partial\rho_{ab}}{\partial t} + v\frac{\partial\rho_{ab}}{\partial x} = i\omega_0\rho_{ab} + \frac{ier_{ab}}{\hbar}E(D - i\Delta D^{(0)}) - \gamma_{ab}\rho_{ab}. \qquad (2.3)$$

It follows from this equation that allowance for the collisions leads to an additional phase shift between the field and the polarization vector. A similar effect was taken into account by Fork and Pollak[7]. In practice Δ can be identified with the parameter c introduced in[7].

Calculating the polarization of the medium with allowance for collisions, we obtain the following expression for the complex polarizability:

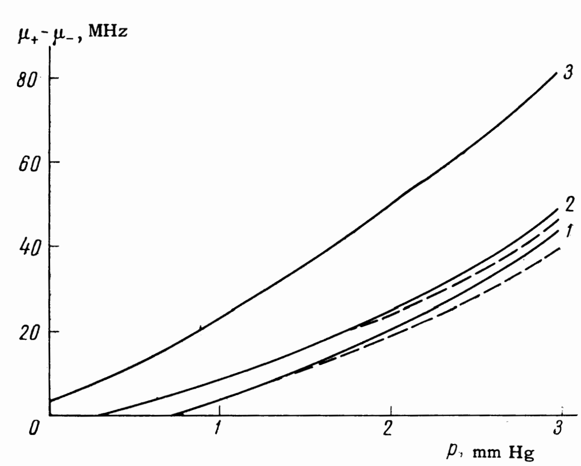

FIG. 3. Dependence of the widths of the instability region and of the region of non-existence of the two-wave regime on the pressure of He³ at $\eta = 0.01$. Curves 1, 2, and 3 correspond to values $\delta = 0$, 10^{-5}, and 10^{-4}. The dashed curves were calculated for $\Delta = 0$.

$$\varkappa_{1,2}(\Delta)=\varkappa_{1,2}(0)\,(1-i\Delta), \qquad (2.4)$$

where $\kappa_{1,2}(0)$ is the polarizability calculated on the basis of (1.3) with the parameters γ_a, γ_b, γ_{ab}, and ω_0 redefined as above.

When $\Delta = 0$ the polarizability $\kappa_{1,2}(0)$ is a symmetrical function of $\mu = \omega - \omega_0$, which is the frequency detuning relative to the center of the Doppler emission line (the imaginary part of $\kappa(0)$ is an even function of μ, and the real part is odd). When collisions are taken into account, asymmetry occurs when $\Delta \neq 0$.

It follows from (2.4) that

$$\operatorname{Im}\varkappa_{1,2}(\Delta)=\operatorname{Im}\varkappa_{1,2}(0)-\Delta\operatorname{Re}\varkappa_{1,2}(0)$$

$$=\frac{b\pi}{2ku}\Big[1-\frac{\mu^2}{2(ku)^2}-\sqrt{\frac{2}{\pi}}\frac{\mu\Delta}{ku}-aaE_{1,2}$$

$$-\Big(\beta-\frac{\mu\gamma_{ab}\Delta}{2(\mu^2+\gamma_{ab}^2)}\Big)aE_{2,1}^2+O(aE^2)\Big]. \qquad (2.5)$$

When $\Delta = 0$ the maximum gain takes place at $\mu = 0$, i.e., when $\omega = \omega_0$. If $\Delta \neq 0$, then the maximum gain shifts to the point $\mu_0 = -\sqrt{2/\pi}\,\Delta ku$, and the function $\operatorname{Im}\kappa_{1,2}(\Delta)$ becomes an asymmetric function of both $\mu - \mu_0$.

The region of instability of the two-wave regime is determined at $\Delta \neq 0$ by the condition

$$\Big(\frac{\mu}{\gamma_{ab}}+\frac{\Delta}{2}\Big)^2\leqslant\frac{\Delta^2}{4}+\frac{\gamma^2}{(ku)^2}-\frac{3}{2}\theta_1\eta_0. \qquad (2.6)$$

It follows therefore that the center of the instability region shifts to the point $\mu_1 = -\Delta\gamma_{ab}/2$, and the width of the region is increased somewhat as the result of Δ. The instability region is essentially asymmetrical with respect to the point μ_0 corresponding to the maximum gain.

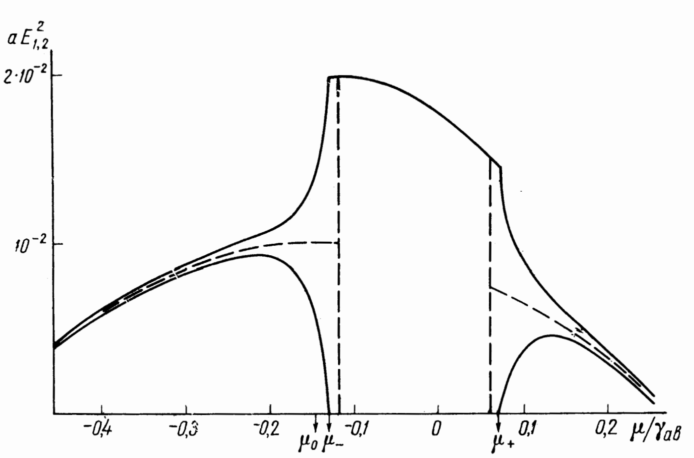

FIG. 4. Dependence of the intensities of the opposing waves on the relative frequency detuning $\mu/\gamma_{ab} = \omega - \omega_0/\gamma_{ab}$ at He^3 pressure 3 mm Hg, $\eta_0 = 0.01$, $|\,\delta\,| = 10^{-5}$. $\delta = 0$ for the dashed curves.

The region of non-existence of the two-wave solution also becomes essentially asymmetrical relative to the point $\mu_0 = -\sqrt{2/\pi}\,\Delta ku$. This region is determined by the inequality

$$\Big|\Big(\frac{\mu}{\gamma_{ab}}+\frac{\Delta}{2}\Big)^2-\frac{\Delta^2}{4}-\frac{\gamma^2}{(ku)^2}+\frac{3}{2}\theta_1\eta_0\Big|\leqslant\frac{2|\delta|}{\eta_0}. \qquad (2.7)$$

Since the lower limits of the non-existence regions lie inside the instability region, the total width of the region in which there is no two-wave regime is determined by the position of the upper limits of the non-existence region. They are designated μ_- and μ_+ in Fig. 4.

To illustrate the effects connected with the collisions, we used the pressure dependences obtained in[7] for the parameters γ_a, γ_b, γ_{ab}, and Δ of a neon-helium laser. Figure 3 shows the dependences of the widths of the instability region and the region of non-existence of the two-wave solution on the partial pressure of the He^3 at a pump level excess above pressure $\eta_0 = 0.01$ and at relative Q differences $|\delta|$ equal to 10^{-4} and 10^{-5}. The same figure shows for comparison also the curves for $\Delta = 0$. Allowance for Δ has negligible effect on the widths of the regions under consideration.

Figure 4 shows the dependences of the wave intensities $E^2_{1,2}$ on the frequency detuning μ/γ_{ab} at He^3 pressure 3 mm Hg. Owing to the asymmetry of the region of non-existence of the two-wave regime relative to the maximum gain μ_0, the amplitudes of the opposing waves depend essentially on the sign of the detuning $\omega - \omega_0$.

In concluding this section, we present numerical estimates of the parameters that determine the stability of the regime of two opposing waves in a neon-helium laser at a helium pressure 2.5 mm Hg and at $\sqrt{2}ku = 1000$ MHz: $\gamma_{ab} = 200$ MHz, $\gamma_b = 60$ MHz, $\gamma_a = 15$ MHz, $\frac{3}{2}\theta_1 \approx \frac{1}{16}$, and $\gamma = 50$ MHz. The width of the region of instability at $\eta_0 \to 0$ is equal to $\mu_+ - \mu_- = 2\gamma_{ab}\gamma/ku \approx 30$ MHz. At $\eta_0 = 8 \times 10^{-2}$ the instability vanishes and all that remains is the region of non-existence of the two-wave regime, the width of which depends on the magnitude of the relative difference δ between the Q factors.

3. INFLUENCE OF ADMIXTURE OF A SECOND ISOTOPE ON THE REGION OF EXISTENCE OF THE TWO WAVE REGIME

All the conclusions drawn above are valid for gas lasers containing only a pure gas isotope (for example, neon in a neon-helium laser). On the other hand, a small admixture of another isotope can greatly alter the entire picture.

We denote the relative concentration of the main isotope by N_1, and that of the impurity isotope by N_2, with $N_1 + N_2 = 1$. Let us assume that $N_2 \ll N_1$. Then the polarizability $\kappa_{1,2}$ can be represented in the form

$$\varkappa_{1,2} = N_1\varkappa_{1,2}^{(1)} + N_2\varkappa_{1,2}^{(2)}, \tag{3.1}$$

where $\kappa_{1,2}^{(1)}$ is defined by an expression given in the first section (see (1.6)), provided one substitutes there $\mu = \mu_1 = (\omega - \omega_{01})$ (ω_{01} is the average transition frequency of the atoms of the main isotope). The expression for $\kappa_{1,2}^{(2)}$ is somewhat different, since it is impossible to assume in its calculation that $\mu_2 = \omega - \omega_{02}$ is much smaller than ku, as was done in the calculation of $\kappa_{1,2}^{(1)}$. For the impurity isotope, μ_2 may turn out to be comparable with ku. If this circumstance is taken into account, then the expression for $\kappa_{1,2}^{(2)}$ takes approximately the form

$$\varkappa_{1,2}^{(2)} = \frac{b\pi}{ku}\exp\left\{-\frac{\mu_2^2}{2(ku)^2}\right\}\left\{\Phi\left(\frac{\mu_2}{\sqrt{2}\,ku}\right) - \frac{\mu_2\gamma_{ab}}{2(ku)^2}a|E_{1,2}|^2 - \frac{\gamma_{ab}}{2\mu_2}a|E_{2,1}|^2 + i\left[1 - \frac{1}{2}a|E_{1,2}|^2 - \left(\frac{\gamma_{ab}^2}{2\mu_2^2} + \frac{\gamma^2}{2(ku)^2}\right)a|E_{2,1}|^2\right]\right\}. \tag{3.2}$$

Substituting now (1.6) and (3.2) in (3.1), we get an expression for $\kappa_{1,2}$. The imaginary part of this expression coincides in form with the imaginary part of the polarizability for one isotope, if we introduce the notation:

$$N_2' = N_2\exp\{-\mu_2^2/2(ku)^2\},$$

$$\alpha' = \frac{N_1}{2}\left(1 + \frac{N_2'}{N_1} - \frac{\mu_1^2 + \gamma_{ab}^2}{2(ku)^2}\right),$$

$$\beta' = \frac{N_1}{2}\left[\frac{\gamma_{ab}^2}{\mu_1^2 + \gamma_{ab}^2} + \frac{N_2'}{N_1}\frac{\gamma_{ab}^2}{\mu_2^2} + \frac{\gamma^2}{(ku)^2} - \frac{\gamma_{ab}^2}{2(ku)^2}\right].$$

Thus, the region values of μ_1 in which the two-wave regime is impossible can be determined as before from the condition (1.11), provided we substitute α' and β' for α and β. Taking (1.12) into account, the condition (1.11) takes for this case the form

$$\frac{\gamma^2}{(ku)^2} - \frac{N_2'}{N_1} - \frac{3}{2}\theta_1\eta_0 - \frac{2|\delta|}{\eta_0} \leqslant \frac{\mu_1^2}{\gamma_{ab}^2} \leqslant \frac{\gamma^2}{(ku)^2} - \frac{N_2'}{N_1} - \frac{3}{2}\theta_1\eta_0 + \frac{2|\delta}{\eta_0} \tag{3.3}$$

The inequality (3.3) can never be satisfied, i.e., the two-wave regime is always possible up to those values of μ at which the self-excitation conditions are satisfied for one of the waves, provided only

$$N_2 \geqslant \exp\left\{\frac{\mu_2^2}{2(ku)^2}\right\}\left(\frac{\gamma^2}{(ku)^2} + \frac{2|\delta|}{\eta_0} - \frac{3}{2}\theta_1\eta_0\right). \tag{3.4}$$

Let us estimate the order of magnitude of N_2 at which the two-wave regime is always possible. Let $\gamma_a = \gamma_b = \gamma_{ab}/2$, $\gamma^2/(ku)^2 = 10^{-3}$, $2|\delta| = 10^{-4}$, $\eta_0 = 5 \times 10^{-3}$, and $\mu_2 = ku$. Then the minimum value of N_2 is found to be of the order of 3—4%.

4. EFFECT OF COUPLING BETWEEN THE OPPOSING WAVES AS A RESULT OF SCATTERING FROM THE MIRRORS

If we take into account the presence of scattering from the mirrors, then the truncated equations for the amplitudes and phases of the laser oscillations are slightly modified[4,11]:

$$\dot{E}_1 = \frac{\omega d}{2}\left[\eta - \frac{\mu^2}{2(ku)^2} - \alpha a E_1^2 - \beta a E_2^2\right]E_1 - \frac{\omega d}{2}m\sin\Phi\, E_2,$$

$$\dot{E}_2 = \frac{\omega d}{2}\left[\eta - \frac{\mu^2}{2(ku)^2} - \alpha a E_2^2 - \beta a E_1^2\right]E_2 + \frac{\omega d}{2}m\sin\Phi\, E_1,$$

$$\dot{\Phi} = -\frac{\omega d}{2}\left(\frac{\mu\gamma_{ab}}{2(\mu^2 + \gamma_{ab}^2)}a - \frac{m\cos\Phi}{E_1E_2}\right)(E_1^2 - E_2^2). \tag{4.1}$$

To simplify the calculation we consider here a somewhat idealized case, when the Q factors and the reflection coefficients are strictly equal for both opposing waves. We have also omitted from (4.1) terms containing fifth powers of the field. When the Q's are equal ($\delta = 0$), these terms do not play an important role in the calculation of the wave amplitudes. If they are taken into account, then it is necessary to take in all the formulas that follow the substitution

$$\beta - \alpha \rightarrow \beta - \alpha - {}^3/_2\theta_1\eta. \tag{4.2}$$

Equations (4.1) have one of the following stationary solutions:

$$E_{10}^2 = E_{20}^2 = \frac{\eta - \mu^2/2(ku)^2}{(\alpha + \beta)a}, \quad \Phi = 0. \tag{4.3}$$

The solution (4.3) corresponds to the two-wave regime in the laser and coincides with the analogous solution obtained without allowance for the coupling, i.e., with (1.10) when $\delta = 0$. An investigation of the stability of this solution shows that the instability region does not depend on the coupling coefficient and is determined by the condition (1.16). We note only that this result was obtained because the coupling coefficients were chosen here to be identical. In the opposite case, the region of instability of the two-wave regime turns out to depend little on the coupling coefficients (in the case of weak coupling), and in addition, there appears a region in which such a regime is impossible. The width of this region is proportional to the modulus of the difference between the complex coupling coefficients. Therefore this case is to a certain degree analogous to the difference of the Q's.

Let us investigate now the other possibilities of solving Eqs. (4.1) in that detuning region where the solution (4.3) is unstable.

Let the coupling be sufficiently weak, so that the condition

$$\frac{m^2\alpha^2}{\eta^2[(\beta-\alpha)^2+\mu^2/4\gamma_{ab}^2]}\ll 1 \qquad (4.4)$$

is satisfied. We note that the inequality (4.4) is better satisfied near the stability region. Indeed, at $\mu = \mu_{\text{lim}}$, i.e., at the limit of the instability region, we have, as follows from (1.17), $\mu^2/\gamma_{ab}^2 \sim \gamma^2/(ku)^2$. At the same time, $(\beta - \alpha)$ vanishes at $\mu = \mu_{\text{lim}}$. Thus, when $\mu \sim \mu_{\text{lim}}$ the inequality (4.4) takes the form

$$\frac{m^2\alpha^2}{\eta^2(\gamma/ku)^2}\ll 1.$$

As $\mu \to 0$ the value of $(\beta - \alpha)$ is on the order of $\gamma^2/(ku)^2$ and consequently the inequality (4.4) takes the form

$$m^2\alpha^2(ku)^4/\eta^2\gamma^4 \ll 1.$$

If we investigate the satisfaction of this inequality as a function of the excess of the pump level above threshold, then (4.2) must be taken into account. Then the inequality (4.4) becomes somewhat modified at the stability limit

$$\frac{m^2\alpha^2}{\eta^2(\gamma^2/(ku)^2-3\theta_1\eta/2)}\ll 1. \qquad (4.5)$$

It follows from (4.5) that the denominator of the left part of the inequality first increases at small η, reaching a maximum value at

$$\eta=\eta_m=\frac{4}{9\theta_1}\frac{\gamma^2}{(ku)^2}, \qquad (4.6)$$

and then begins to decrease. Therefore inequality (4.4) is satisfied better in the region where η is close to η_m.

When (4.4) is satisfied, the stationary solution of (4.1) can be sought in the form

$$E_{10}=mE_0,\quad E_{20}=lE_0,\quad \Phi=\Phi_0\quad (l\gg m). \qquad (4.7)$$

Substituting (4.7) in (4.1) and retaining the terms of first order of smallness, we get

$$aE_0^2=\frac{\alpha}{\eta[(\beta-\alpha)^2+\mu^2/4\gamma_{ab}^2]},\ l^2=\frac{\eta^2}{\alpha^2}\left[(\beta-\alpha)^2+\frac{\mu^2}{4\gamma_{ab}^2}\right],$$
$$\cos\Phi_0=\mu alE_0^2/2\gamma_{ab}, \qquad (4.8)$$

whence

$$aE_{10}^2=\frac{am^2}{\eta[(\beta-\alpha)^2+\mu^2/4\gamma_{ab}^2]},\quad aE_{20}^2=\frac{\eta}{\alpha}. \qquad (4.9)$$

It is seen from (4.8) that when condition (4.4) is satisfied we actually have $l \gg m$.

The solution (4.9) is stable under a definite limitation imposed on the coupling coefficient m:

$$\frac{m^2\alpha^3}{\eta^2[(\beta-\alpha)^2+\mu^2/4\gamma_{ab}^2]}\leqslant\beta-\alpha. \qquad (4.10)$$

The inequality (4.10) does not contradict the condition (4.4) and everything said concerning (4.4) can be said concerning (4.10).

Thus, a stationary one-wave regime can exist in the instability region of the two-wave regime only when condition (4.10) is satisfied, i.e., when the coupling is sufficiently weak and when the excess of the pump over threshold lies in a certain region near η_m.

In the opposite case, the stationary single-wave regime is impossible, and a nonstationary regime sets in, wherein the amplitudes of both waves execute periodic oscillations. If the coupling is sufficiently large, these oscillations become sinusoidal and opposite in phase. In the intermediate case, obviously, the oscillations will be of the relaxation type, and the phase shift between the oscillations of the two waves will no longer be equal to π. We consider here only the regime of sinusoidal oscillations with opposite phases.

To obtain this regime, it is convenient to go over in (4.1) to new variables

$$x=E_1^2-E_2^2,\quad y=E_1^2+E_2^2. \qquad (4.11)$$

In terms of these variables, Eqs. (4.1) acquire the following form:

$$\dot{x}=\omega d[\eta x-aaxy-m\sqrt{x^2-y^2}\sin\Phi], \qquad (4.12)$$

$$\dot{y}=\omega d[\eta y-\beta ay^2+{}^1/_2(\beta-\alpha)\quad \alpha(x^2+y^2)], \qquad (4.13)$$

$$\dot{\Phi}=-\frac{\omega d}{2}\left(\frac{\mu a}{2\gamma_{ab}}-\frac{m\cos\Phi}{\sqrt{y^2-x^2}}\right)x. \qquad (4.14)$$

The solution of Eqs. (4.12)—(4.14) can be easily obtained, and in this case we obtain precisely a sinusoidal regime if we assume that the coupling is large enough, i.e.,

$$m\gg\eta(\beta-\alpha)\ \text{and}\ m\gg\mu\eta/\gamma_{ab}. \qquad (4.15)$$

These two inequalities can be replaced by the single inequality

$$\frac{m^2}{\eta^2[(\beta-\alpha)^2+\mu^2/\gamma_{ab}^2]}\gg 1. \qquad (4.16)$$

Since a ~ 1, the inequality (4.16) is the inverse of (4.4).

When the first condition of (4.15) is satisfied, x varies much more rapidly than y. This difference in the character of the variation of x and y is due to the fact that y represents the total wave energy, and if the oscillations of the intensities of the two waves are out of phase, then the total energy remains approximately constant in time. It follows therefore that the first condition of (4.15) is the condition for the oscillations to be in phase opposition. It is always satisfied near the boundary of the instability region, but this does not mean that oscillations will always be produced near the boundary, since the second condition of (4.15) may not be satisfied here.

Owing to the slow variation of y when the first condition of (4.15) is satisfied, we put in (4.13), in the zeroth approximation, $\dot{y} = 0$. Recognizing that in the instability region we have $(\beta - \alpha) \ll \beta$, α and $x^2 \leq y^2$, we obtain the following expression for y:

$$y = \frac{2\eta}{(\alpha+\beta)a} \approx \frac{\eta}{\alpha a} \approx \frac{\eta}{\beta a} \approx 2\frac{\eta}{a} \equiv y_0. \quad (4.17)$$

Let us consider now the equation for the phase Φ and ascertain under which conditions the phase of the oscillations can be regarded as stationary. This is possible if the expression in the brackets of (4.14) is small, i.e.,

$$\cos\Phi - \frac{\mu a}{2\gamma_{ab} m}\sqrt{y^2 - x^2} \approx 0 \quad (4.18)$$

for all instants of time. Since x^2 is smaller than $y^2 \approx y_0^2$ at all instants of time, the condition (4.18) yields

$$\cos\Phi \leqslant \mu a y_0 / 2m\gamma_{ab}. \quad (4.19)$$

Taking (4.17) and the second condition of (4.15) into account, we get

$$\cos\Phi \lesssim \mu\eta / m\gamma_{ab} \ll 1. \quad (4.20)$$

Substituting (4.20) and (4.17) in (4.12) and integrating, we get

$$x = \pm y_0 \sin \omega d\, mt. \quad (4.21)$$

Taking now (4.11) and (4.17) into account, we have

$$E_{1,2}^2 = \frac{\eta}{a}(1 \pm \sin\Omega t), \quad (4.22)$$

where $\Omega = \omega dm$.

Thus, when conditions (4.15) are satisfied, the intensities of both waves execute harmonic oscillations of frequency $\Omega = \omega dm$ and in phase opposition. Let us estimate this frequency. If $\omega \approx 3 \times 10^{15}$, $d \approx 1/Q = 2 \times 10^{-9}$, and $m \approx 10^{-4}$, then $f = \Omega/2\pi \approx 100$ Hz.

If the coupling coefficient does not satisfy even one of the conditions of (4.15), but is larger than the critical value defined by the condition (4.10), then the stable mode will likewise will be an oscillatory mode, but the form of the oscillations will be different.

It was already noted that the condition (4.10), just as condition (4.4), is satisfied all the more rapidly the larger μ, i.e., the closer the oscillation frequency to the boundary of the stability region of the two-wave regime, whereas the conditions (4.15) are more readily satisfied jointly the smaller μ, i.e., the closer the oscillation frequency is to the center of the Doppler line. Therefore, for certain values of the coupling coefficient, a regime is possible in which there are intensity oscillations near the center of the Doppler line, and a stationary one-wave regime exists near the boundary of the instability region. In addition, conditions (4.15) are always satisfied for sufficiently small excesses of the pump level over threshold, η, the satisfaction of these conditions then becomes more difficult as η increases, and if (4.2) is taken into account, it is again facilitated with further increase of η.

We note also that in Eqs. (4.1) we have introduced the reflection coefficient m, which determines the coupling between the waves, multiplied by the coefficient ωd. Namely, this entire coefficient $M = \omega dm$ determines the frequency at which the energy is pumped from one wave to the other ($\Omega = M$).

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