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NONUNIFORM CURRENT DISTRIBUTION IN SEMICONDUCTORS WITH NEGATIVE DIFFERENTIAL CONDUCTIVITY

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We consider the flow of current through a semiconductor with an S-like current-voltage characteristic brought about by the dependence of the momentum and energy relaxation times on the electron temperature. When the load resistance is large, a semiconductor with a uniform density distribution corresponding to the section of the characteristic with negative differential conductivity is unstable against coordinate-dependent disturbances. It is shown that disturbances with a wave vector perpendicular to the direction of the current increase most rapidly and aperiodically. Nonuniform stationary states, in which the semiconductor changes as a result of instability of the uniform state, are investigated. It is found that either narrow or wide layers (domains) of different conductivity, extending in the direction of the current, are formed in the electron plasma of the semiconductor. It is also found that distributions with a minimal number of domains are the most stable ones. Some conclusions regarding the shape of the current-voltage characteristic are made.

In a semiconductor situated in a strong electric field, the current-voltage characteristic j(E) have a section with negative differential conductivity ($\sigma_D < 0$). The characteristic is referred to as S-like or N-like, depending on whether the current density j is a multiply-valued function of the field E or the field is a multiply valued function of the current.

Most mechanisms hitherto considered for the occurrence of an S-like characteristic are based on the growth of the momentum relaxation time τ_p of the electrons with their temperature T, and on the choice of a special temperature dependence of the electron-energy relaxation time τ_e, the electron density remaining constant[1-5]. All these can be called superheat mechanisms, in analogy with the well known superheat instability in a gas plasma[6].

It is known that when $\sigma_d < 0$ the states with uniform distribution of j and E are unstable against small perturbations, as a result of which the semiconductor goes over to a state with an uneven distribution of j and E[7-9]. In semiconductors with an N-like characteristic, a field distribution that is inhomogeneous along the current direction is established. The form and the velocity of motion of the regions (domains) of strong or weak fields have by now been sufficiently thoroughly investigated theoretically[10-13]. If a current stabilized by a large load resistance, such that $\sigma_d < 0$, flows through a semiconductor with an S-like characteristic, then layers (filaments) of different conductivity, extending in the current direction, are produced in the semiconductor. Such a stratification was investigated qualitatively by Ridley[9].

We consider in this paper semiconductors with an S-like characteristic brought about by the superheat mechanism. A hydrodynamic approach is used in this case to investigate the possible states of a semiconductor with inhomogeneous distributions of j and T, and to analyze the stability of such states. Conclusions are drawn concerning the form of the observed current-voltage characteristic; in particular, it is shown that the result obtained in[9] is incorrect.

1. FUNDAMENTAL EQUATIONS. INSTABILITY OF UNIFORM CURRENT DISTRIBUTION

Let us consider an electron plasma in a semiconductor, under conditions when the time of the interelectron collisions is small compared with the time τ_e of scattering of the electron-gas energy by the lattice vibrations, i.e., when a local electron temperature exists. We shall assume processes involving the generation and recombination of the conduction electrons to be insignificant. In addition, we confine ourselves to conditions under which the characteristic scale l_c of the resultant current and temperature inhomogeneities (see below) is much larger than the Debye screening length, and the characteristic frequencies $\omega \lesssim \tau_e^{-1}$ are much smaller than the reciprocal Maxwellian relaxation time $\tau_M \sim \sigma^{-1}$ where σ is the specific electric conductivity. In this case the plasma can be regarded as incompressible.

The continuity equation takes the form

$$\operatorname{div} \mathbf{v} = \frac{1}{ne} \operatorname{div} \mathbf{j} = 0, \tag{1.1}$$

where $\mathbf{j}$ is the current density, $\mathbf{v}$ the drift velocity, and n the constant electron density.

Neglecting inertial terms ($\omega \ll \tau_p^{-1}$) and viscosity ($l_c \gg l_p$, where l_p is the mean free path), the equation of motion reduces to the expression for the current

$$\mathbf{j} = \sigma\mathbf{E} - \sigma\left(\alpha_{st} + \frac{1}{ne}\frac{\partial p}{\partial T}\right)\nabla T. \tag{1.2}$$

Here p is the pressure, α_{st} the part of the differential thermal emf α connected with the electron scattering,

$$\alpha_{st} = \alpha - s/e, \tag{1.3}$$

s is the plasma entropy per particle, and e is the electron charge. In an ideal nondegenerate gas with a power-law dependence of τ_p on the electron energy, $\tau_p \propto \epsilon^r$, we have $\alpha_{st} = r/e$, i.e., it does not depend on the temperature.

The equation expressing the conservation of the internal energy can be obtained in the same manner as for a gas plasma[14];

$$nc_e dT/dt = \mathbf{R}\mathbf{v} - P - \operatorname{div} \mathbf{w}, \tag{1.4}$$

where P is the power transferred to the electrons by the lattice, c_e is the specific heat of the electrons per particle, and $\mathbf{R}$ is the "friction force," which in accord with (1.2) is equal to

$$\mathbf{R} = ne(\mathbf{j}/\sigma + \alpha_{st}\nabla T). \tag{1.5}$$

The heat-flux density is

$$\mathbf{w} = \alpha_{st}T\mathbf{j} - \varkappa\nabla T, \tag{1.6}$$

where κ is the specific thermal conductivity of the plasma.

If $v \ll v_T$, where v_T is the thermal velocity of the electrons, and if the collisions are weakly inelastic, then the kinetic coefficients and the power P are functions of the electron temperature only. In particular, the power P can be represented in the form

$$P = nc_e(T - T_0)/\tau_e(T), \tag{1.7}$$

where T_0 is the temperature of the semiconductor lattice, which is assumed to be constant.

Substituting (1.5) and (1.6) in (1.4), we get

$$nc_e\frac{dT}{dt} = \nabla(\varkappa\nabla T) + \frac{j^2}{\sigma} - P - T\frac{d\alpha_{st}}{dT}\mathbf{j}\nabla T. \tag{1.8}$$

In the cases in question, the field can be regarded as potential

$$\mathbf{E} = -\nabla\varphi. \tag{1.9}$$

Equations (1.1), (1.2), (1.8), and (1.9) constitute a complete system. The boundary conditions on the side surfaces of the samples are that the normal components of $\mathbf{j}$ and ∇T vanish. The voltage across the sample and the total current are connected by the condition

$$\int_0^{l_x} E_x dx + r\int dy\,dz\,j_x = \mathscr{E}. \tag{1.10}$$

Here E_x and j_x are the components of the field and of the current density in the direction joining the contacts of the sample, l_x is the length of the sample, r is the load resistance, and E the emf of the source.

Let us consider the stability of the homogeneous current distribution. It is stable against perturbations that do not depend on the coordinate if the load resistance is sufficiently large. Indeed, linearizing Eq. (1.8) and the condition (1.10) for perturbations of the form $(\delta T)_0 \exp(-\lambda t)$, we find that $\lambda > 0$ if

$$r > l_x/S|\sigma_d|, \tag{1.11}$$

where S is the sample cross section area, and $\sigma_d = dj/dE_x$ is the differential conductivity, equal to

$$\sigma_d = \sigma\frac{1 + E_x^2(d\sigma/dT)/(dP/dT)}{1 - E_x^2(d\sigma/dT)/(dP/dT)}. \tag{1.12}$$

But even under condition (1.11) the uniform distribution is unstable against inhomogeneous perturbations. Linearizing the initial equations for perturbations of the type

$$\delta T = (\delta T)_0 \exp(i\mathbf{kr} - i(\omega - i\gamma)t),$$

we obtain the dispersion equation

$$i\omega + \gamma = ik_x v\left(1 + \frac{eT}{c_e}\frac{d\alpha_{st}}{dT}\right) + \frac{1}{\tau}\left[1 - \frac{(d\sigma/dT)E_x^2}{dP/dT}\frac{k_\perp^2 - k_x^2}{k_\perp^2 + k_x^2}\right] + \frac{\varkappa}{nc_e}(k_\perp^2 + k_x^2), \tag{1.13}$$

where $k_\perp^2 = k_y^2 + k_z^2$, and

$$\frac{1}{\tau} = \frac{dP/dT}{nc_e} = \frac{1}{\tau_e(T)}\left[1 - \frac{T - T_0}{T}\frac{d\ln(\tau_e/c_e)}{d\ln T}\right].$$

If the conductivity increases with temperature and $dP/dT > 0$, then $\sigma_d < 0$ under the condition

$$1 - \frac{(d\sigma/dT)E_x^2}{dP/dT} < 0, \tag{1.14}$$

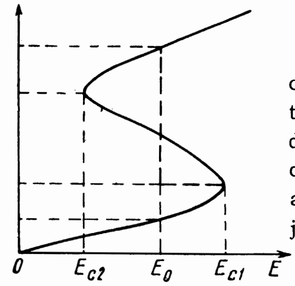

FIG. 1. S-like current-voltage characteristic. Solid line – characteristic for homogeneous current distribution, dashed – when broad domains occur. On the ordinate axis, reading downward: j_3, j_{c2}, j_{c1}, j_1.

which corresponds to an S-like characteristic. The latter is seen from the fact that σ_d becomes infinite at $E_x = E_{c1}$, when the denominator of (1.12) vanishes (see Fig. 1). If $\sigma_d < 0$, then the homogeneous state is unstable ($\gamma < 0$) against perturbations having a sufficiently large ratio $k_\perp^2/k_x^2$. We see that the maximum increment is possessed by perturbations with $k_\perp^2 \gg k_x^2$ (the instability is purely aperiodic when $k_x = 0$), which lead to stratification of the sample in a direction perpendicular to the stationary current. We can propose a simple qualitative explanation for this effect.

A local increase δT in the electron temperature changes the power received by the electrons by an amount

$$\left(\frac{d\sigma}{dT}E_x^2 - \frac{dP}{dT}\right)\delta T \tag{1.15}$$

and the perturbation continues to grow if this change is positive (see (1.14)). If the perturbation δT depends on the coordinate x, it causes a decrease $\delta E_x = -ik_x\varphi$ in the field. In this case it is necessary to add to (1.15) a term connected with the field perturbation and equal to

$$-2\frac{d\sigma}{dT}E_x^2\frac{k_x^2}{k_\perp^2 + k_x^2}\delta T. \tag{1.15a}$$

When $d\sigma/dT > 0$ this term prevents the growth of the perturbation.

As expected, perturbations with sufficiently short wavelengths are attenuated as a result of the thermal conductivity. Consequently, in a sufficiently thin sample, the homogeneous stationary state is stable[15]. From (1.13) we get an expression for the corresponding critical linear dimension

$$l_c = \pi\left(\frac{\varkappa}{E_x^2\, d\sigma/dT - dP/dT}\right)^{1/2}$$
$$= \pi\left(\frac{\varkappa\tau_e}{nc_e}\right)^{1/2}\left(\frac{T-T_0}{T}\frac{d\ln(\sigma\tau_e/c_e)}{d\ln T} - 1\right)^{-1/2}. \tag{1.16}$$

In order of magnitude we have

$$l_c \sim v_T(\tau_p\tau_e)^{1/2}, \tag{1.17}$$

where $v_T = (T/m)^{1/2}$.

Thus, for n-InSb at helium temperatures we have $l_c \sim 10^{-2}$—10^{-3} cm. The data for this material were taken from[16], in which the observed S-like characteristic was apparently connected with the superheat mechanism considered above.

In the case when the conductivity decreases with temperature, $\sigma_d < 0$ if

$$\frac{dP}{dT} + \frac{d\sigma}{dT}E_x^2 < 0,$$

corresponding to an N-like characteristic. It follows from (1.13) that the greatest increment $|\gamma|$ is possessed by perturbations with $k_x^2 \gg k_\perp^2$. The growing fluctuations are uniform over the cross section of the sample.

2. STATIONARY STATES

Let us consider the stationary states, with a current distribution that is not uniform over the cross section, into which the sample can go over as a result of the instability. We shall assume that all quantities are independent of the coordinate x, and that the transverse component of the current $j_\perp$ is equal to zero. Then Eq. (1.8) can be represented in the form

$$\nabla_\perp^2\Theta + dU/d\Theta = 0, \tag{2.1}$$

where

$$\Theta = \int_0^T dT'\varkappa(T'), \tag{2.2}$$

$$U(\Theta) = \int^\Theta d\Theta'[\sigma(\Theta')E_x^2 - P(\Theta')]. \tag{2.3}$$

We consider for simplicity a case when the temperature depends only on one coordinate, and consequently $\Theta = \Theta(y)$. Such an inhomogeneity will be called layered. The equation for $\Theta(y)$,

$$d^2\Theta/dy^2 + dU/d\Theta = 0, \tag{2.4}$$

has the same form as the equation of motion of a particle in a field with potential $U(\Theta)$. Figure 2 shows a plot of $U(\Theta)$ for several values of the field E_x. The condition under which the function $U(\Theta)$ has an extremum,

$$\sigma(\Theta)E_x^2 = P(\Theta), \tag{2.5}$$

has three roots in the field range $E_{c2} < E_x < E_{c1}$ (see Fig. 1); we shall denote them, in increasing

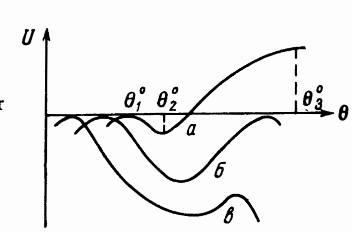

FIG. 2. Potential $U(\Theta)$ for different values of the field E_x: a – $E_0 < E_x < E_{c1}$, b – $E_x = E_0$, c – $E_{c2} < E_x < E_0$.

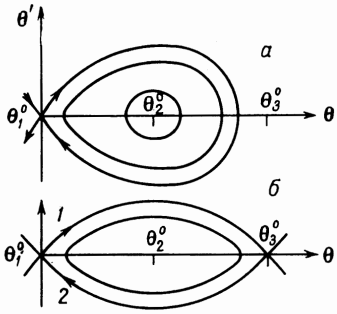

FIG. 3. Phase trajectories of Eq. (2.6): a – $E_0 < E_x < E_{c1}$, b – $E_x = E_0$.

order, by Θ_1^0, Θ_2^0, and Θ_3^0. They correspond to three possible values of the temperature and consequently to three values of the current density for a given E_x and a uniform distribution (Fig. 1):

$$j^{(k)} = \sigma(T_k{}^0)E_x \quad (k = 1, 2, 3).$$

The "energy integral" of Eq. (2.4) is

$$1/2(d\Theta/dy)^2 = U(\Theta_1) - U(\Theta), \tag{2.6}$$

where Θ_1 is the extremal value of the distribution $\Theta(y)$. To analyze the possible solutions of (2.4), it is convenient to draw of the "particle" trajectory (2.6) in the (Θ, $\Theta' \equiv d\Theta/dy$) plane (Fig. 3). For the potential $U(\Theta)$ represented by curve a of Fig. 2 [for which $U(\Theta_3^0) > U(\Theta_1^0)$], the phase trajectories are shown in Fig. 3a. The trajectory passing through the singular point Θ_1^0 (saddle) corresponds to a single narrow hot domain (layer of increased conductivity) with maximum "temperature" $\Theta_1 < \Theta_3^0$ satisfying the condition $U(\Theta_1) = U(\Theta_1^0)$. The trajectories that are close to the one just considered correspond to a series of such domains, and those close to the point Θ_2^0 correspond to weak oscillations of the temperature about T_2^0.

There exists a unique field $E_x = E_0$ (Fig. 1) for which $U(\Theta_1^0) = U(\Theta_3^0)$ (Fig. 2, curve b). Each of the trajectories 1 and 2 (Fig. 3b) passing through Θ_1^0 and Θ_3^0 represents two phases with temperatures T_1^0 and T_3^0 and a smooth transition between them, so that the closed trajectory 1—2 corresponds to a broad domain (of width much greater than the width of the transition layer, and with a flat top).

The potential $U(\Theta)$ for $E_{c2} < E_x < E_0$ is shown in Fig. 2(c). The phase curves are similar to those of Fig. 3a. The difference lies in the fact that the phase curve corresponding to the single domain goes in this case through the singular point Θ_3^0, and the domain is a narrow cold layer.

If the sample thickness l_y is smaller than the half-period of the temperature oscillations near T_2^0, which is equal to $l_c = \pi(U''(\Theta_2^0))^{-1/2}$, then there is no inhomogeneous stationary state under the boundary conditions indicated above. We note that l_c coincides with the crystal wavelength of the perturbations which are attenuated in the inhomogeneous state with $T = T_2^0$ (see (1.13)). When the field E_x approaches the critical field $E_{c1}(E_{c2})$ (Fig. 1), the "temperatures" Θ_1^0 and Θ_2^0 (Θ_2^0 and Θ_3^0) come closer together, so that $U''(\Theta_2^0) \to 0$ and the critical thickness increases.

The width of the narrow domain and the width of the boundary of the broad domain are, naturally, of the order of l_c, i.e., $\sim v_T(\tau_e\tau_p)^{1/2}$.

Equation (2.1) for the stationary distribution $\Theta(y, z)$ is the Euler equation for the functional

$$S = \int dy\, dz \left\{ \frac{1}{2}(\nabla\Theta)^2 - U(\Theta) \right\}, \tag{2.7}$$

i.e., the stationary distribution $\Theta(y, z)$ corresponds to the extremum of the "action" S.

Ridley[9] states that the stationary state corresponds to a minimum of entropy production in the sample. It can be verified quite easily that the latter quantity differs from S (2.7) in the systems under consideration, so that Ridley's statement is incorrect. In[9] he obtained $E_x = E_{c2}$ (in the inhomogeneous stationary state (Fig. 1). However, at this value of the field the points Θ_2^0 and Θ_3^0 coalesce, the potential $U(\Theta)$ has only two equilibrium points, and there exists no solution corresponding to domains.

3. STABILITY OF INHOMOGENEOUS STATIONARY STATES

Let us linearize Eqs. (1.1), (1.2), and (1.8) for perturbations of the type

$$\delta T = \delta T(y)\exp[i(k_x x + k_z z) - \lambda t]. \tag{3.1}$$

For convenience, we represent expression (1.2) for j in the form

$$\mathbf{j} = -\sigma\nabla\psi, \tag{3.2}$$

where

$$\psi = \varphi + \int^{T} dT'\left(\alpha_{st} + \frac{1}{ne}\frac{\partial p}{\partial T'}\right) \tag{3.3}$$

and we put $\delta\Theta = \kappa\delta T$. Then we obtain for $\delta\Theta$ and $\delta\psi$ the system of equations

$$\left[\hat{H}_0 + k_x^2 + k_z^2 + ik_x v\frac{nc_e}{\varkappa}\left(1 + \frac{eT}{c_e}\frac{d\alpha_{st}}{dT}\right) - \lambda\frac{nc_e}{\varkappa}\right]\delta\Theta + 2ik_x\sigma E_x\delta\psi = 0, \tag{3.4}$$

$$\hat{H}_1\delta\psi + ik_x E_x\frac{d\sigma}{d\Theta}\delta\Theta = 0, \tag{3.5}$$

where

$$\hat{H}_0 = -\frac{d^2}{dy^2} - \left.\frac{d^2U}{d\Theta^2}\right|_{\Theta=\Theta(y)}, \tag{3.6}$$

$$\hat{H}_1 = -\frac{d}{dy}\left(\sigma\frac{d}{dy}\right) + (k_x^2 + k_z^2)\sigma. \qquad (3.7)$$

An investigation of the eigenvalues λ of this system is particularly simple in the case when the perturbation does not depend on x, i.e., $k_x = 0$. Then λ is determined from the solution of the following equation (here $k_z^2 \neq 0$, see below):

$$(\hat{H}_0 + k_z^2 - nc_e\lambda/\varkappa)\,\delta\Theta = 0. \qquad (3.8)$$

The operator $\hat{H}_0$ coincides with the energy operator of quantum particle in a field with a potential $-U''(\Theta)$. Figure 4 shows this potential schematically for the case of the boundary of a broad domain.

Let us investigate the eigenvalues of $\hat{H}_0$ for those stationary distributions of Θ(y) whose derivative dΘ/dy satisfies practically the same conditions on the boundaries ($-l_y/2$ and $l_y/2$) as the function Θ(y) itself (boundary of two broad phases, domains located away from the boundary at distances much larger than l_c, distributions that oscillate many times within the length l_y, etc.). In this case the eigenfunction of $\hat{H}_0$ corresponding to the zero eigenvalue is dΘ/dy. To verify this it is sufficient to differentiate Eq. (2.4). If the function dΘ/dy does not vanish inside the sample, then it is the ground state eigenfunction of H_0, and then the smallest eigenvalue of $\hat{H}_0$ is zero[1)], and λ_{min} is a positive quantity when $k_z^2 \neq 0$.

If dΘ/dy vanishes inside the sample (oscillating temperature distribution or domains inside the sample) then the zero eigenvalue of $\hat{H}_0$ corresponding to the function dΘ/dy is not the minimal value. Thus, for oscillating distributions, $\hat{H}_0$ has an entire spectrum of negative eigenvalues λ < 0 for small k_z^2, and such distributions are unstable. For broad domains, however, the negative eigenvalue of $\hat{H}_0$ is exponentially small and when $k_z \neq 0$ we have for them λ > 0.

Indeed, let us consider a broad domain which is not adjacent to the lateral surface of the sample. The potential $-U''(\Theta)$ has in this case the form of two potential wells of width $\sim l_c$, separated by a barrier whose dimension is equal to the width of the domain. The zero eigenvalue of $\hat{H}_0$ corresponds to an antisymmetrical eigenfunction dΘ/dy (such a perturbation represents a small shift of the domain along the y axis). It is known[17] that the lower level (to which the symmetrical function corresponds) differs from the former by an exponentially small quantity, which in our case makes a negative contribution of the order of $\tau_e^{-1}\exp(-l|U''(\Theta_3^0)|^{1/2})$

[1)]Such a method of investigating stability was used earlier in [13].

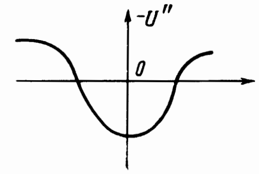

FIG. 4. Potential $-U''[\Theta(y)]$ in the case of a broad-domain boundary.

to λ (l is the width of the "hot" domain, and the characteristic length is $|U''(\Theta_3^0)|^{-1/2} \sim l_c$). Since the minimal value $|k_z| \neq 0$ is of the order of l_z^{-1}, the positive addition to λ, which is proportional to k_z^2, exceeds the exponential one when $l \gtrsim l_c \ln(l_z/l_c)$, thus ensuring the instability of such a domain. Perturbations with $k_z = 0$ change the total resistance of the sample and consequently the voltage across it. Then it is necessary to add to the left side of (3.8) the term

$$-2\sigma E_x\delta E_x = 2\sigma E_x^2\left[\frac{l_x}{l_z r} + \int_{-l_y/2}^{l_y/2} dy\,\sigma(y)\right]^{-1}\int_{-l_y/2}^{l_y/2} dy\,\frac{d\sigma}{d\Theta}\delta\Theta. \qquad (3.9)$$

Let us consider the stability of the boundary between "phases," and also the stability of the broad domain. In this case, as will be shown, the correction to λ is small. It follows from (3.8), with allowance for (3.9), that at small values of |λ| outside the transition layer, δΘ does not depend on the coordinates and its sign is the opposite of the sign of the last integral in (3.9). Consequently, the signs of δΘ inside and outside the transition layer are opposite. Using the asymptotic form of (3.8) with allowance for (3.9), let us express the values of δΘ on both sides of the transition layer in terms of

$$J = \int_{-l_c}^{l_c} dy\,\frac{d\sigma}{d\Theta}\delta\Theta,$$

where the integration is carried out only over the transition layer. Substituting these values in (3.9) and the resultant expression in (3.8), we find that it constitutes a small perturbation, proportional to the quantity

$$J\left(l_x/l_z r + \int_{-l_y/2}^{l_y/2} dy\,\sigma_d\right)^{-1}.$$

the correction to λ for large values of r is positive, of the order of $\tau_e^{-1}(l_c/l)$, where l is the width of the phase that determines the conductivity of the sample (l can be of the order of l_y). Consequently, the broad domain is stable against perturbations with $k_z = 0$.

Perturbations with $k_x \neq 0$, corresponding to small |λ|, are surface waves whose amplitude at small $|k_x|$ decreases far from the phase boundary like $\exp(-\gamma|y|)$, where γ is equal to $(\sigma_d/\sigma)^{1/2}|k_x|$

when $k_z = 0$. Perturbations with $k_x \neq 0$ will not be considered in detail separately.

We have considered above only layered distributions of the temperature and of the current. Obviously, the condition $E_x = E_0$ determines the existence of broad domains, not necessarily plane ones, provided the radius of curvature of the domain boundary is much larger than the width of the transition layer. An analysis of Eq. (2.1) for a distribution with cylindrical symmetry shows that, just as in the case of a layered distribution, narrow hot domains exist when $E_x > E_0$, and narrow cold domains when $E_x < E_0$.

As verified above, a narrow plane domain inside a sample is unstable against perturbations that do not change the total resistance of the sample (they have the form of jumpers). Let us show that cylindrical domains are stable against such perturbations. In the case of a cylindrically-symmetrical stationary distribution, the equation for a perturbation in the form $\delta\Theta(\rho)\exp(im\ \varphi - \lambda t)$ is as follows for $m \neq 0$ (these are precisely the perturbations which do not change the total resistance of the sample):

$$\left(H^{(m)} - \frac{nc_e}{\varkappa}\lambda\right)\delta\Theta(\rho) = 0, \qquad (3.10)$$

where

$$H^{(m)} = -\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d}{d\rho}\right) - U''(\Theta) + \frac{m^2}{\rho^2}. \qquad (3.11)$$

The derivative of the stationary distribution $d\Theta/d\rho$, as can be readily verified, is the eigenfunction of $\hat{H}^{(1)}$ corresponding to a zero eigenvalue. Therefore, if the temperature varies monotonically from the axis towards the boundary of the sample ($d\Theta/d\rho$ has no nodes), then the smallest value of λ at $m = 1$ is equal to zero. In the opposite case the distribution is unstable ($\lambda < 0$).

4. CONCLUSIONS

We have established above that if the total current in the sample is specified and its density $\bar{j}$ averaged over the cross section exceeds j_{c1} (Fig. 1), then the resultant instability causes the sample to break up into domains that have different conductivities and are parallel to the current. The form of the domain is described by Eq. (2.1). The most stable distributions are those with the smallest number of domains.

The observed current-voltage characteristic, i.e., the $j(E_x)$ dependence, does not coincide with the $j(E_x)$ characteristic for the uniform distribution shown in Fig. 1. Let us see what determines the field E_x (which does not depend on the coordinates) after the sample has become stratified into domains. If the $j(E_x)$ dependence is such that the current densities j_1 and j_3 in the cold and hot stable "phases" are of the same order of magnitude, then the condition that the average density be equal to a specified value of $\bar{j}$ (larger than j_{c1}) can be satisfied only in the presence of a broad domain (domains). Indeed, when $E_x < E_{c1}$ and $\bar{j} > j_{c1}$, the increase of the current due to the appearance of a hot but narrow ($l_c \gg l_y$) domain cannot offset the decrease of the current in the cold phase. In this case, when $\bar{j} > j_{c1}$, the field drops to E_0—the field corresponding to the broad domains (see Sec. 2). With increasing $\bar{j}$, the voltage across the sample remains unchanged (the width of the domain increases), and the observed characteristic has the form shown dashed in Fig. 1. We note that when $\bar{j}$ approaches j_2, the characteristic should deviate from a vertical line, since the width of the cold phase is decreased thereby to a value $l \lesssim l_c$. The vertical section of the current-voltage characteristic in a semiconductor (n-InSb) in which negative σ_d is produced by the superheat mechanism was apparently observed in[16].

If j_1 and j_3 differ greatly, then in some interval of $\bar{j} > j_{c1}$ the condition for the total current can be satisfied only in the presence of narrow domains. With increasing $\bar{j}$, the voltage in the sample should change, i.e., the observed characteristic is not vertical.

When the current $\bar{j}$ decreases from values close to j_3, stratification into domains occurs when $\bar{j} = j_{c2}$, so that hysteresis takes place. We see that at a given value of the total current in the sample (for example, in the interval of $\bar{j}$ from j_{c2} to j_3, Fig. 1) the system can be in two stationary states—homogeneous or inhomogeneous—with different values of E_x. Both states are stable against small perturbations. The sample can go over from one such state into another under the influence of the large perturbations required for the purpose, provided such perturbations exist in the system. In principle, the sample can be stratified into broad domains when $\bar{j} < j_{c1}$.

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