CONTRIBUTION TO THE THEORY OF SPONTANEOUS EMISSION FROM ATOMS IN AN EXTERNAL FIELD

G. E. NOTKIN, S. G. RAUTIAN and A. A. FEOKTISTOV

Institute of Semiconductor Physics, Siberian Section, Academy of Sciences, U.S.S.R.; Moscow Physico-technical Institute

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The quantum theory of the spontaneous emission of atoms in an external electromagnetic field is developed. The external field can be regarded either classically or quantum-mechanically. The general formulas for the probability of spontaneous emission are given in detail for the resonance approximation. The line shape is analyzed for the case of a monochromatic external field and large Doppler broadening. The superposition of the external field leads to deformation of the line contour, which can be interpreted as the consequence (1) of a change in the velocity distribution of the atoms, and (2) of interference effects arising in the mixing of stationary states of the isolated atom by the external field. Under certain conditions, the second factor completely counterbalances the effect of the first.

1. INTRODUCTION

THE problem of the spontaneous emission from an atom in the presence of an external electromagnetic field was first considered in the work of Rautian and Sobel'man,[1] where a calculation was carried out for the simplest case of a fixed atom and a monochromatic field. In a subsequent work,[2] a postulate (in the spirit of the postulates of Klein[3]) was introduced without any foundation. This postulate allows the calculation of the probability of spontaneous emission in the quasiclassical approximation for an arbitrary external field. On the basis of this postulate, an analysis of a number of specific cases was given in [2,4] with account of motion of the atoms.

As is well known, the Doppler broadening, which arises from the thermal motion of the atoms, is much greater than the radiation broadening under normal circumstances, and almost completely masks it. Upon superposition of an external electromagnetic field, a relatively sharp structure appears on the Doppler-broadened line. The characteristic width of this line is determined by the radiative decay. Thus the study of the spectrum of spontaneous emission of atoms in an external field uncovers interesting possibilities for the study of the radiation characteristics of the excited states of atoms.

In the present work a general theory is developed for the spontaneous emission from atoms located in an external field. The postulate of reference [2] is retained in the theory as a special case and the region of its applicability is shown to be limited. In Secs. 3 and 4, a series of special cases is considered. This series demonstrates the general laws and is of interest for the spectroscopic problems mentioned above.

2. GENERAL EXPRESSIONS

1. We shall begin with the expansion of the field in plane waves. The Hamiltonian of the system "atom + field" is described in the following way:

$$\hat{H} = \hat{H}_0 + \hat{M}_\mu', \quad \hat{H}_0 = \hat{H}_a + \hat{H}_f' + \hat{H}'. \qquad (2.1)$$

Here $\hat{H}_a$ and $\hat{H}_f$ are the Hamiltonians of the isolated atom and the free radiation field. From the Hamiltonian of the interaction $\hat{H}'$, we separate the term $\hat{H}_\mu'$ corresponding to the interaction of the atom with waves of type μ. We shall be interested in the emission of a photon $\hbar\omega_\mu$, considering $\hat{H}_\mu'$ in first-approximation perturbation theory and assuming that photons are absent at the moment of excitation of the μ–e atom.

We introduce the following notation. Let $\mathbf{u}$ denote the row of wave functions of the atom and the field without account of their interaction, i.e., the row of solutions of the equation

$$i\hbar\partial u_\alpha / \partial t = (\hat{H}_a + \hat{H}_f) u_\alpha. \qquad (2.2)$$

We shall write the zeroth-approximation wave function Ψ_0, which satisfies the equation

$$i\hbar\frac{\partial}{\partial t}\Psi_0 = \hat{H}_0\Psi_0, \tag{2.3}$$

in the form of an expansion in u_α:

$$\Psi_0 = \mathbf{ua}; \quad \mathbf{a} = \mathbf{sa}_H, \tag{2.4}$$

where $\mathbf{a}_H$ is the column which defines the distribution over the states α at the initial instant of time t_0; $\mathbf{s}$ is a fundamental matrix satisfying the equation

$$i\hbar\dot{\mathbf{s}} = \langle \mathbf{u}^+|\hat{H}'|\mathbf{u}\rangle\mathbf{s}, \quad \mathbf{s}(t_0) = \mathbf{E}, \tag{2.5}$$

$\mathbf{E}$ is a unit matrix. The row elements

$$\Psi_0 = \mathbf{us} \tag{2.6}$$

are linearly independent solutions of Eq. (2.3) corresponding to different initial conditions.

Let Ψ_1 be the correction to the wave function, due to the term $\hat{H}'_\mu$:

$$\Psi = \Psi_0 + \Psi_1. \tag{2.7}$$

The first approximation corresponds to the result that the term $\hat{H}'_\mu\Psi_1$ in the equation for Ψ_1,

$$i\hbar\frac{\partial}{\partial t}\Psi_1 = \hat{H}_0\Psi_1 + \hat{H}_\mu'\Psi_0 + \hat{H}_\mu'\Psi_1, \tag{2.8}$$

is discarded. We write Ψ_1 in a way similar to (2.4):

$$\Psi_1 = \mathbf{u}\sigma; \quad \Psi_1 = \Psi_2\mathbf{a}_H = \mathbf{u}\sigma\mathbf{a}_H. \tag{2.9}$$

From (2.8) we get the following equation for the matrix σ:

$$i\hbar\dot{\sigma} = \langle \mathbf{u}^+|\hat{H}'|\mathbf{u}\rangle\sigma + \langle \mathbf{u}^+|\hat{H}_\mu'|\mathbf{u}\rangle\mathbf{s}, \tag{2.10}$$

with zero initial conditions for σ. Therefore, in the solution of Eq. (2.10), we are interested only in the contribution due to the second term on the right hand side. The homogeneous equation corresponding to (2.10) is identical to (2.5). Therefore, taking the unitarity of $\mathbf{s}$ into account, we can write[5]

$$\sigma(t) = -\frac{i}{\hbar}\mathbf{s}(t)\int_{t_0}^{t}\mathbf{s}^+(t')\langle \mathbf{u}^+|\hat{H}_\mu'|\mathbf{u}\rangle\,\mathbf{s}(t')\,dt'; \tag{2.11}$$

$$\Psi_1(t) = \frac{i}{\hbar}\Psi_0(t)\int_{t_0}^{t}\langle \Psi_0^+(t')\,|\hat{H}_\mu'|\Psi_0(t')\rangle\,dt'. \tag{2.12}$$

Equations (2.11) and (2.12) are essentially the same as the usual expressions of first approximation and are written only for the entire wave function, and not for the corrections to the individual expansion coefficients of Ψ in terms of u_α.

We turn to the computation of the probability of spontaneous emission of a photon of type μ:[6]

$$w_\mu = \frac{1}{\hbar\omega_\mu}\int_{t_0}^{\infty}\langle \Psi^*|\hat{H}_\mu|\Psi\rangle dt. \tag{2.13}$$

The integral in (2.13) is obviously the average increase of energy in the μ-th type of oscillation. The ratio of this energy to the energy of the photon $\hbar\omega_\mu$ is the probability that, after a single act of excitation of the atom, the photon of interest to us will be emitted in a spontaneous transition (the photons $\hbar\omega_\mu$ were absent at the initial time $t = t_0$).

We substitute (2.7) in (2.13) and discard the term $\langle \Psi_1^*|\hat{H}_\mu|\Psi_1\rangle$. We also note that

$$\langle \Psi^*|\hat{H}_\mu|\Psi\rangle = -i\omega_\mu\langle \Psi^*|\hat{H}_\mu'|\Psi\rangle. \tag{2.14}$$

Formula (2.13) then becomes

$$w_\mu = -\frac{i}{\hbar}\int_{t_0}^{\infty}\langle \Psi_1^*|\hat{H}_\mu'|\Psi_0\rangle dt - \text{к. с.}$$

$$= -\frac{i}{\hbar}\int_{t_0}^{\infty}\mathbf{a}_H^+\sigma^+\langle \mathbf{u}^+|\hat{H}_\mu'|\mathbf{u}\rangle\mathbf{sa}_H dt - \text{к. с.} \tag{2.15}$$

In what follows, two other forms of writing w_μ will be convenient. If the function Ψ_1 in (2.15) is expressed in terms of Ψ_0 with the help of (2.12), we can obtain

$$w_\mu = \frac{1}{\hbar^2}\int_{t_0}^{\infty}\langle \Psi_0^*|\hat{H}_\mu'|\Psi_0\rangle\,dt\int_{t_0}^{\infty}\langle \Psi_0^+|\hat{H}_\mu'|\Psi_0\rangle\,dt'. \tag{2.16}$$

The general structure of the expression (2.16) is identical with that for the case in which the external field is absent—the probability w_μ is the sum of the squares of the matrix elements of the perturbation, computed with the help of the wave function of the zeroth approximation. The difference is that Ψ_0 in (2.16) takes into account the interaction of the atom with the strong field which "mixes" the states of the isolated atom. It is then clear that the spectral composition and the integrated intensity of the spontaneous emission depend on the external field.

Another representation of w_μ corresponds to the elimination of the zeroth-approximation function from (2.15); this is easily done with the help of Eq. (2.10):

$$w_\mu = [\mathbf{c}^+\mathbf{c}]_{t\to\infty}, \quad \mathbf{c} = \sigma\mathbf{a}_H. \tag{2.17}$$

Equation (2.17) contains a summation over all states whose probability differs from zero as $t \to \infty$. In this relation, it is convenient that only one (ground) state of the atom is included. We note that Eq. (2.17) is essentially a generalization to the case of an external field of the well-known equations of the Weisskopf-Wigner theory (see, for example, [7], Eq. (18.13)).

In the derivation of Eqs. (2.15)–(2.17), two assumptions were made. First, in the use of the first approximation, the appearance of only a single photon of frequency ω_μ is possible. States with two and more photons of the same type do not appear. The error connected with this assumption is negli-

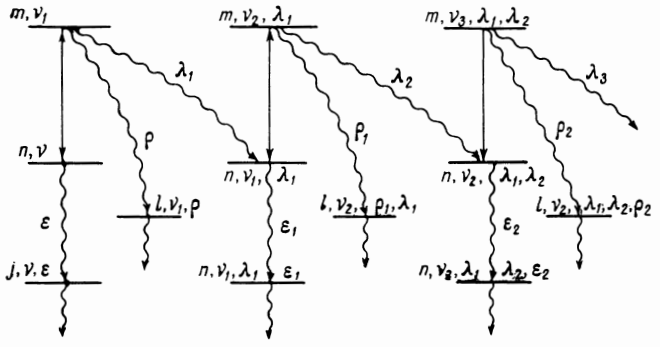

FIG. 1. Scheme of zeroth-approximation states. The straight arrows indicate transitions induced by the external field. The wavy arrows indicate spontaneous transitions.

gible. Second, we have limited ourselves to the analysis of the spontaneous emission for types of oscillations where photons are absent at the initial instant of time. Therefore Eqs. (2.15)–(2.17) are inapplicable, in particular, at those frequencies where there is an external field. Otherwise, the expressions obtained are completely general and exact.

2. Equations (2.15)–(2.17) are too complicated for specific calculations, since they contain a summation over a large number of field states. This summation can be carried out partially in explicit form in the so-called resonance approximation (the frequencies of the external field are close to the transition frequency, the distance between the levels being much greater than the width of the levels and the energy of interaction of the atom with the field). Here, however, it is necessary to consider separately the spontaneous emission via the different transitions.

We shall assume that the spectrum of the external field contains a comparatively small number of harmonics, the frequencies of which are concentrated near a single transition $m \to n$. We first consider w_μ just for this transition. Figure 1 shows the diagram of the zeroth-approximation states that appear after excitation of the atom to the level m or n. The external field (we shall also call it "strong" and number its states by the indices ν_1) produces the transitions $m \to n$, which are drawn in Fig. 1 as straight-line arrows. For sufficient intensity of the strong field, the probabilities of finding the atom in the levels m and n are equal in order of magnitude, independently of what level of the atom was excited. The indices λ_i, ϵ_i, and ρ_i denote the spontaneous emission of the photons $\hbar\omega_{\lambda_i}$, $\hbar\omega_{\epsilon_i}$, and $\hbar\omega_{\rho_i}$, respectively, by the transitions $m \to n$, $m \to l$, and $n \to j$. The levels l and j can in turn decay or be ground levels—that is immaterial.

In contrast with the transitions $m \to l$ and $n \to j$, at which the strong field is absent, for the transition $m \to n$, it is necessary to consider the possibility of the spontaneous emission of a large number of photons. Actually, after the excitation of the atom, for example, to the level m, and the spontaneous emission of the photon λ_i, the state n, ν_1, λ_1 appears; the strong field induces the transition of the atom to the level m (state m, ν_2, λ_1) and it is necessary to consider the state n, ν_2, λ_1, λ_2, etc. Thus there arises a sequence of groups of states, and each group is characterized by the presence of k photons $\lambda_1, \lambda_2, \ldots, \lambda_k$.

To begin with, we complete the summation in (2.17) over all the spontaneous photons λ_i, ϵ_i, ρ_i. For the states of the k-th group, the submatrices of the amplitudes of the probabilities $\sigma_i^{(k)}$ (the atom in the levels $i = m, n, l, j$; the elements of the submatrices $\sigma_i^{(k)}$ are enumerated by the indices of the strong field) are connected, in agreement with (2.10), by the equations

$$i\hbar\dot{\sigma}_l^{(k)} = H'_{\rho_k}\sigma_m^{(k)}, \quad i\hbar\dot{\sigma}_j^{(k)} = H'_{\varepsilon_k}\sigma_n^{(k)}, \qquad (2.18)$$

which should be solved for the zero initial conditions. Integrating (2.18) and substituting the results in (2.17), we obtain

$$w_\mu = \sum_k \Big\{ \sum_{l\rho_k} \frac{|H'_{\rho_k}|^2}{\hbar^2} \int_{t_0}^{\infty} dt \int_{t_0}^{t} dt' \, \mathbf{A}_H^+ \sigma_m^{(k)+}(t)\, \sigma_m^{(k)}(t')\, \mathbf{A}_H e^{-i\Omega_{\rho_k}(t-t')} + \sum_{j\varepsilon_k} \frac{|H'_{\varepsilon_k}|^2}{\hbar^2} \int_{t_0}^{\infty} dt \int_{t_0}^{t} dt' \, \mathbf{A}_H^+ \sigma_n^{(k)+}(t)\, \sigma_n^{(k)}(t')\, \mathbf{A}_H e^{-i\Omega_{\varepsilon_k}(t-t')} \Big\};$$

$$\Omega_{\rho_k} = \omega_{\rho_k} - \omega_{ml}, \qquad \Omega_{\varepsilon_k} = \omega_{\varepsilon_k} - \omega_{nj}.$$

The column $\mathbf{A}_H$ gives the initial photon distribution of the strong field. We replace the summation over ρ_k, ϵ_k by integration over Ω_{ρ_k}, Ω_{ϵ_k} and by summation over the remaining parameters of the oscillators of the field, as usual. After integration over the frequencies, $\delta(t - t')$ appears, which allows us to carry out the integration over t'. Summation now gives the first Einstein coefficients $2\gamma_{ml}$ and $2\gamma_{nj}$ for the transitions $m \to l$ and $n \to j$:

$$w_\mu = \int_{t_0}^{\infty} \mathbf{A}_H^+ \Big\{ 2\gamma_m \sum_k \sigma_m^{(k)+}\sigma_m^{(k)} + 2\gamma_n \sum_k \sigma_n^{(k)+}\sigma_n^{(k)} \Big\} \mathbf{A}_H \, dt;$$

$$\gamma_m = \sum_l \gamma_{ml}, \qquad \gamma_n = \sum_j \gamma_{nj}. \qquad (2.19)$$

Formula (2.19) has a simple interpretation: $\sum_k \sigma_m^{(k)+}\sigma_m^{(k)}$ and $\sum_k \sigma_n^{(k)+}\sigma_n^{(k)}$ are the total probabilities that the atom is found in the levels m and n, while a photon of type μ appears in the field; the probabilities of decay of these states in the time dt are $2\gamma_m\,dt$ and $2\gamma_n\,dt$. Consequently, the inte-

grand in (2.19) is the rate of spontaneous emission, and the integral is the probability of emission of the photon within the relaxation time.

In Eq. (2.19), there is left only the summation over the groups of states. We shall show that all the $\sigma_m^{(k)}$ and $\sigma_n^{(k)}$ can be expressed in terms of the same matrix. With this aim, we write the equations for $\sigma_m^{(k)}$ and $\sigma_n^{(k)}$:

$$i\hbar\,[\dot{\sigma}_m^{(k)} + \gamma_m \sigma_m^{(k)}] = \mathbf{H}''_{mn}\sigma_n^{(k)};$$
$$i\hbar\,[\dot{\sigma}_n^{(k)} + \gamma_n \sigma_n^{(k)}] = \mathbf{H}''_{nm}\sigma_m^{(k)} + H_\mu' s_m^{(k)};$$
$$i\hbar\,[\dot{s}_m^{(k)} + \gamma_m s_m^{(k)}] = \mathbf{H}''_{mn} s_n^{(k)};$$
$$i\hbar\,[\dot{s}_n^{(k)} + \gamma_n s_n^{(k)}] = \mathbf{H}''_{nm} s_m^{(k)} + H'_{\lambda_k} s_m^{(k-1)};$$
$$k = 0, 1, 2, \ldots; \quad s_m^{(-1)} \equiv 0; \quad H_\mu' = \langle \psi_n^* u_1^* | \hat{H}_\mu' | u_0 \psi_m \rangle;$$
$$\mathbf{H}''_{nm} = \langle \psi_n^* \mathbf{U}^+ | \hat{H}'' | \mathbf{U}\psi_m \rangle. \tag{2.20}$$

Here $\mathbf{U}$ is the row of eigenfunctions of the strong field, u_1 and u_0 are the eigenfunctions of the μ-th type of oscillation for the numbers of photons 1 and 0, and ψ_m and ψ_n are the eigenfunctions of the isolated atom with the strong field. The constants γ_m and γ_n arise as in (2.19) as the result of summation over all the spontaneous transitions from the levels m and n.

In the presence of the strong field, the spontaneous emission will take place for excitation of the atom both to level m and to level n. In order to include both these cases, we introduce the matrices

$$\sigma^{(k)} = \begin{pmatrix} \sigma_{mm}^{(k)} & \sigma_{mn}^{(k)} \\ \sigma_{nm}^{(k)} & \sigma_{nn}^{(k)} \end{pmatrix}, \quad \mathbf{S}^{(k)} = \begin{pmatrix} s_{mm}^{(k)} & s_{mn}^{(k)} \\ s_{nm}^{(k)} & s_{nn}^{(k)} \end{pmatrix}, \tag{2.21}$$

where the second index indicates to what level the atom is excited. The equations for $\sigma^{(k)}$ and $\mathbf{S}^{(k)}$, according to (2.20), have the form

$$i\hbar\,[\dot{\sigma}^{(k)} + \boldsymbol{\gamma}\sigma^{(k)}] - \mathbf{H}''\sigma^{(k)} = H_\mu'\boldsymbol{\beta}^+\mathbf{S}^{(k)}; \tag{2.22}$$

$$i\hbar\,[\dot{\mathbf{S}}^{(k)} + \boldsymbol{\gamma}\mathbf{S}^{(k)}] - \mathbf{H}''\mathbf{S}^{(k)} = H_{\lambda_k}'\boldsymbol{\beta}^+\mathbf{S}^{(k-1)}; \quad k = 1, 2, \ldots; \tag{2.23}$$

$$i\hbar\,[\dot{\mathbf{S}} + \boldsymbol{\gamma}\mathbf{S}] - \mathbf{H}''\mathbf{S} = 0; \quad \mathbf{S} \equiv \mathbf{S}^{(0)}; \quad \mathbf{S}(t_0) = \begin{pmatrix} \mathbf{E} & 0 \\ 0 & \mathbf{E} \end{pmatrix}, \tag{2.24}$$

$$\boldsymbol{\beta} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad \boldsymbol{\gamma} = \begin{pmatrix} \gamma_m & 0 \\ 0 & \gamma_n \end{pmatrix}; \quad \mathbf{H}'' = \begin{pmatrix} 0 & \mathbf{H}_{mn}'' \\ \mathbf{H}_{nm}'' & 0 \end{pmatrix}. \tag{2.25}$$

The probability w_μ, averaged over the excitations of the atom, can be written in the following fashion in place of (2.19), with the help of $\sigma^{(k)}$:

$$w_\mu = \mathrm{Sp}\left\{\boldsymbol{\rho}\int_{t_0}^{\infty}\sum_k 2\sigma^{(k)+}\boldsymbol{\gamma}\sigma^{(k)}dt\right\}, \quad \boldsymbol{\rho} = \begin{pmatrix} \rho_m & 0 \\ 0 & \rho_n \end{pmatrix},$$
$$\rho_i = \frac{Q_i}{Q_m + Q_n}\mathbf{A}_H, \tag{2.26}$$

where Q_m and Q_n are the numbers of acts of excitation of the levels m and n per unit time.

The fundamental matrix of Eqs. (2.22)–(2.24) is the same. It satisfies Eq. (2.24) for unity initial conditions, i.e., it is identical with $\mathbf{S}$. The functions $\sigma^{(k)}$ and $\mathbf{S}^{(k)}$ have zero initial conditions, and one must consider only the solutions due to the right-hand sides. It is seen from Eqs. (2.23) that the states of the (k – 1)-st group are the "source of excitation" of the states of the k-th group, which is graphically illustrated in the scheme of Fig. 1. If the fundamental matrix $\mathbf{S}$ is known, then $\mathbf{S}^{(k)}$ and $\sigma^{(k)}$ can be found by quadratures:

$$\mathbf{S}^{(k)} = -\frac{i}{\hbar}\mathbf{S}(t)\int_{t_0}^{t}\mathbf{S}^{-1}(t')H_{\lambda_k}'\boldsymbol{\beta}^+\mathbf{S}^{(k-1)}(t')\,dt';$$
$$\sigma^{(k)} = -\frac{i}{\hbar}\mathbf{S}(t)\int_{t_0}^{t}\mathbf{S}^{-1}(t')H_\mu'\boldsymbol{\beta}^+\mathbf{S}^{(k)}(t')\,dt'. \tag{2.27}$$

Thus all the $\mathbf{S}^{(k)}$ and $\sigma^{(k)}$ can be expressed in terms of the fundamental matrix $\mathbf{S}$ of Eq. (2.24).

We now express w_μ in terms of $\mathbf{S}$. It follows from Eq. (2.22) and its conjugate that

$$2\sigma^{(k)+}\boldsymbol{\gamma}\sigma^{(k)} = -\frac{d}{dt}[\sigma^{(k)+}\sigma^{(k)}]$$
$$-\frac{i}{\hbar}\{\sigma^{(k)+}H_\mu'\boldsymbol{\beta}^+\mathbf{S}^{(k)} - \mathbf{S}^{(k)+}\boldsymbol{\beta}H_\mu'^*\sigma^{(k)}\}.$$

Inasmuch as $\sigma^{(k)} = 0$ for $t = t_0$ and $t \to \infty$, then it follows from (2.26) and (2.27) that

$$w_\mu = 2\,\mathrm{Re}\sum_k \mathrm{Sp}\left\{\rho\frac{i}{\hbar}\int_{t_0}^{\infty}\mathbf{S}^{(k)+}\boldsymbol{\beta}H_\mu'^*\sigma^{(k)}dt\right\}$$
$$= \sum_k 2\,\mathrm{Re}\,\mathrm{Sp}\left\{\rho\frac{1}{\hbar^2}\int_{t_0}^{\infty}dt\int_{t_0}^{t}dt'\,H_\mu'^*(t)H_\mu'(t')\mathbf{S}^{(k)+}(t)\right.$$
$$\left.\times\ \boldsymbol{\beta}\mathbf{S}(t)\mathbf{S}^{-1}(t')\boldsymbol{\beta}^+\mathbf{S}^{(k)}(t')\right\}. \tag{2.28}$$

The first row in (2.28) is similar to (2.15), the second, to (2.16). Equation (2.28) differs from this in that summation has been carried out in it over all states arising in the spontaneous transitions $m \to l$ and $n \to j$. There remains only the summation over $\lambda_1, \lambda_2, \ldots, \lambda_k$ and over the groups of states of Fig. 1. Systematic use of (2.27) permits us to represent all the $\mathbf{S}^{(k)}$ in terms of $\mathbf{S}$. After this, summation is carried out over the λ_k. As a result, we obtain the following final formula for w_μ:

$$w_\mu = 2\frac{|H_\mu'|^2}{\hbar^2}\mathrm{Sp}\left(\rho\,\mathrm{Re}\left\{\int_{t_0}^{\infty}dt\int_{t_0}^{t}dt'\,e^{-i\Omega_\mu(t-t')}\right.\right.$$
$$\times\left[\mathbf{S}^+(t)\boldsymbol{\beta}\mathbf{S}(t)\mathbf{S}^{-1}(t')\boldsymbol{\beta}^+\mathbf{S}(t')\right.$$
$$\left.\left.+\sum_{k=1}^{\infty}(2\gamma_{mn})^k\int_{t_0}^{t'}dt_1\ldots\int_{t_0}^{t_{k-1}}dt_k\,\mathbf{A}^+(t_1,\ldots,t_k)\mathbf{S}^+(t)\right.\right.$$

$$\times\ \beta S(t)S^{-1}(t')\beta^{+}S(t')A(t_1,\ldots,t_k)\Big]\Big\}\Big);$$

$$A(t_1,\ldots,t_k)=S^{-1}(t_1)\beta^{+}S(t_1)\ldots S^{-1}(t_k)\beta^{+}S(t_k),$$

$$\Omega_\mu=\omega_\mu-\omega_{mn}. \qquad (2.29)$$

The first row in (2.29) is determined by the left-hand states of the scheme of Fig. 1, while the contribution of the k-th group to w_μ is given by the k-th term of the series. In the limiting case of very weak external field, $H''=0$, the matrix S is diagonal, as a consequence of which all the terms of the series are identically equal to zero. The first term in (2.29) gives the usual expression for w_μ:

$$w_\mu=\frac{|H_\mu'|^2}{\hbar^2}\frac{\pi}{2\gamma_m}\frac{(\gamma_m+\gamma_n)/\pi}{\Omega_\mu^2+(\gamma_m+\gamma_n)^2}. \qquad (2.30)$$

In the presence of an external field, the spectrum will naturally be different.

In the limiting case of a very strong field, the matrix S attenuates like $\exp\{-(\gamma_m+\gamma_n)(t-t_0)\}$, and the k-th term of the series will have the factor $[\gamma_{mn}/(\gamma_m+\gamma_n)]^k$, i.e., (2.29) can be regarded as the expansion of w_μ in powers of the parameter $\gamma_{mn}/(\gamma_m+\gamma_n)$. Physically, this is completely clear from Fig. 1, inasmuch as γ_{mn} is the integrated rate of excitation of states of the k-th group from the (k − 1)-st, while $\gamma_m+\gamma_n$ is the damping rate. If the inequality

$$\gamma_{mn}\ll\gamma_m+\gamma_n, \qquad (2.31)$$

is satisfied, which holds in many cases, then one can discard the series and limit oneself to the first row in (2.29):

$$w_\mu\cong\frac{|H_\mu'|^2}{\hbar^2}2\mathrm{Sp}\left\{\rho\,\mathrm{Re}\int_{t_0}^{\infty}dt\int_{t_0}^{t}dt'\exp\{-i\Omega_\mu(t-t')\}\right.$$
$$\left.\times\ S^{+}(t)\beta S(t)S^{-1}(t')\beta^{+}S(t')\right\}. \qquad (2.29')$$

We note that the approximation corresponds to the situation in which in the Hamiltonian $\hat{H}$ in (2.1), we discard not $\hat{H}'_\mu$ but all the interaction terms with zero oscillations that are in resonance with the transition frequency ω_{mn}.

Simple estimates show that the strong field will significantly affect w_μ if the number of its photons is much greater than unity. This means that the strong field can be considered quasiclassically. Here the eigenfunctions U are absent in the expansion of Ψ_0 and the elements of the matrix S will not be matrices but simple functions. But then (2.29′) is identical, apart from the sign, with the expression obtained in [2,4]. Thus the postulate of [2] holds only in the quasiclassical approximation and upon satisfaction of the condition (2.31).

3. We now turn to transitions between one of the levels m, n and any third level (see Fig. 2). Here the spectrum of spontaneous emission will obviously undergo changes in comparison with the case of an isolated atom.

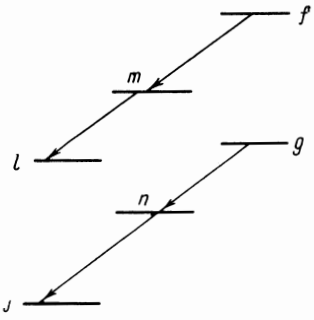

FIG. 2. Scheme of states of an isolated atom and spontaneous transitions between them.

Calculation of w_μ for the indicated transition differs little from the case m → n. Therefore, we shall not dwell on the details of the calculation, and only write down the final result. For transitions from m and n to a lower level, Eq. (2.29) remains in force if $\beta S(t)S^{-1}(t')\beta^{+}$ is replaced by $\beta\exp\{-\gamma_l(t-t')\}\beta^{+}$ for the transition m → l, and by $\beta^{+}\exp\{-\gamma_j(t-t')\}\beta$ for the transition n → j (Fig. 2). For transitions from higher levels to m and n (i.e., for f → m and g → n, Fig. 2) the factors in (2.29) to the left of β and to the right of β^{+} are replaced; furthermore, the necessity of summation over the processes of excitation of the levels m and n disappears, since only the excitation of the initial levels f and g are essential in the case considered. For example, for g → n, one can find

$$w_\mu=2\frac{|H_\mu'|^2}{\hbar^2}\mathrm{Re}\Big\{\alpha_n^{+}\int_{t_0}^{\infty}dt\int_{t_0}^{t}dt'$$
$$\times\ \exp\{-i\Omega_\mu(t-t')-\gamma_g(t+t'-2t_0)\}$$
$$\times\Big[S(t)S^{-1}(t')+\sum_{k=1}^{\infty}(2\gamma_{mn})^k\int_{t_0}^{t'}dt_1\ldots\int_{t_0}^{t_{k-1}}dt_k\,A^{+}(t_1,\ldots,t_k)$$
$$\times\ S(t)S^{-1}(t')A(t_1,\ldots,t_k)\Big]\alpha_n\Big\};\quad \alpha_n=\begin{pmatrix}0\\A_H\end{pmatrix}. \qquad (2.32)$$

The probability w_μ for the transition f → m is obtained from (2.32) by replacing γ_g by γ_f and the matrix α_n by

$$\alpha_m=\begin{pmatrix}A_H\\0\end{pmatrix}.$$

4. We consider the case in which the elements of the fundamental matrix are linear combinations of exponential functions of the time. This case is of interest from the view of applications. Such a form of S exists if, for example, H″ in (2.24) is a periodic function of the time.[5] We limit ourselves,

for simplicity, to the quasiclassical approximation and set

$$S(t)=\begin{pmatrix}\sum_s A_{ms}e^{-\alpha_s t} & \sum_p A_{np}e^{-\alpha_p t}\\ \sum_r B_{mr}e^{-\beta_r t} & \sum_q B_{nq}e^{-\beta_q t}\end{pmatrix}. \quad (2.33)$$

Then the integration in (2.29) leads to the following result:

$$w_\mu=\rho_m \mathrm{Re}\Big\{\sum_{sr}\frac{C_{sr}}{\alpha_s'+\beta_r'+i(\Omega_\mu-\alpha_s''+\beta_r'')}+\sum_{sq}\frac{C_{sq}}{\alpha_s'+\beta_q'+i(\Omega_\mu-\alpha_s''+\beta_q')}\Big\}$$
$$+\rho_n \mathrm{Re}\Big\{\sum_{pr}\frac{D_{pr}}{\alpha_p'+\beta_r+i(\Omega_\mu-\alpha_p''+\beta_r'')}+\sum_{pq}\frac{D_{pq}}{\alpha_p'+\beta_q'+i(\Omega_\mu-\alpha_p''+\beta_q'')}\Big\}, \quad (2.34)$$
$$\alpha=\alpha'+i\alpha'', \quad \beta=\beta'+i\beta'',$$

while the coefficients C_{sr} etc. do not depend on the frequency Ω_μ. According to (2.34), the line of spontaneous emission consists of components of Lorentzian shape; the location of their maxima is determined by the energy difference of the quasistationary states $E_m-E_n+\hbar(\alpha''-\beta'')$, while their width is determined by the sum of the damping constants $\alpha'+\beta'$. In other words, in regard to the width and location of the maxima, the situation is as if a transition took place from each "state" (m, s) and (m, p) to the "state" (n, r) and (n, q). Of course, each exponential function is not a solution of the Schrödinger equation, but their combination (2.33) is, and real physical meaning attaches to the entire set of Lorentzian components in (2.34) and not to each one individually. Nevertheless, in many problems, the form (2.34) is very convenient for the qualitative analysis of the phenomena. In particular, the integrated probability of spontaneous emission is equal to the sum of the integrated probabilities of the individual components. Therefore, if the contours corresponding to the different terms in (2.34) do not intersect, then the picture of the transitions between (m, s), (m, p) and (n, r), (n, q) characterizes sufficiently completely the intensity distribution in the line. If the overlap of components is appreciable, then interference effects are present and the resulting line contour can have nothing in common with the sum of the Lorentzian components.

In the following sections, general formulas will be put down for the case in which the external field is a plane traveling monochromatic wave with frequency ω. Here the fundamental matrix has the form (in the quasiclassical approximation)

$$S(t)=\begin{pmatrix}A_1e^{\alpha_1 t}+A_2e^{\alpha_2 t} & B(e^{\alpha_1 t}-e^{\alpha_2 t})\\ B(e^{\alpha_1 t}-e^{\alpha_2 t})e^{i\Omega' t} & (A_1e^{\alpha_1 t}+A_1e^{\alpha_2 t})e^{i\Omega' t}\end{pmatrix}; \quad (2.35)$$

$$\alpha_{1,2}=-\frac{i\Omega'+\Gamma}{2}\pm\sqrt{\left(\frac{i\Omega'+\gamma}{2}\right)^2-G^2},$$
$$A_1=1-A_2=-\frac{\alpha_2+\gamma_m}{\alpha_1-\alpha_2}, \quad B=-\frac{iG}{\alpha_1-\alpha_2}, \quad (2.36)$$
$$\Omega'=\omega-\omega_{mn}-\mathbf{kv}, \quad \Gamma=\gamma_m+\gamma_{n_2} \quad \gamma=\gamma_n-\gamma_m,$$
$$G=p_{mn}E/2\hbar,$$

where $\mathbf{k}$ and E are the wave vector and the amplitude of the external field, $\mathbf{v}$ the velocity of the atom, p_{mn} the matrix elements of the dipole moment for $m\to n$. We note that (2.35) is a special case of Eq. (2.33). We shall assume that the condition (2.31) is satisfied, and shall use Eq. (2.29′) for the calculation of w_μ. As has already been recalled above, interest attaches to the case of a gas, where the motion of the atoms must be taken into account. Here the quantity Ω_μ in Eqs. (2.29) and (2.29′) must be replaced by

$$\Omega_\mu'=\Omega_\mu-\mathbf{k}_\mu\mathbf{v}=\omega_\mu-\omega_{ij}-\mathbf{k}_\mu\mathbf{v}, \quad (2.37)$$

where ω_{ij} is the frequency of the transition under study, and $\mathbf{k}_\mu$ the wave vector for spontaneous emission. Averaging over the velocities is completed for the Maxwellian distribution

$$(\sqrt{\pi}\bar v)^{-3}\exp\{-\mathbf{v}^2/\bar v^2\}, \quad \bar v=\sqrt{2kT/m}.$$

3. THE TRANSITIONS f → m and g → n

We begin with consideration of the transitions $f\to m$ and $g\to n$ (Fig. 2), inasmuch as it is convenient to trace the role of the excitation of the levels of the atom in an external field by means of this simple case, and to make clear the specific interference effects.

For the transition $g\to n$, substitution of (2.35) in (2.32) gives the following formula for the angular and spectral densities of spontaneous emission:

$$\langle w_\mu\rangle=\frac{\gamma_{gn}}{4\pi\gamma_g}\frac{1}{\pi} \quad (3.1)$$
$$\times\mathrm{Re}\left\langle\frac{A_2}{\gamma_g-\alpha_1+i(\Omega_\mu'-\Omega')}+\frac{A_1}{\gamma_g-\alpha_2+i(\Omega_\mu'-\Omega')}\right\rangle.$$

The angle brackets denote averaging over the velocities. The structure of Eq. (3.1) will be clear if we note that, in accord with (2.35), the lowest level n of the transition is "split" by the field into two "sublevels," with respective quasienergies $E_n-\alpha_1''-\Omega'$ and $E_n-\alpha_2''-i\Omega'$, widths $(-\alpha_1')$ and $(-\alpha_2')$, and probability amplitudes A_2 and A_1. Therefore, the two terms in (3.1) correspond to the situation wherein the upper level g combines with each of the sublevels of the state n.

Inasmuch as $A_1+A_2=1$, the integrated probability of spontaneous emission does not depend on the external field

$$W=\int_{-\infty}^{\infty} w_\mu d\omega_\mu = \frac{1}{4\pi}\frac{\gamma_{gn}}{\gamma_g}. \qquad (3.2)$$

This should be expected, since the atoms located in the upper level g do not interact with the external field. Thus all the changes of w_μ concern only the line shape, i.e., they have a purely "interference" origin.

In the case of very high fields,

$$\alpha_{1,2}\cong -\frac{i\Omega'+\Gamma}{2}\pm iG-; \qquad A_{1,2}\cong\frac{1}{2}, \quad G\gg k\bar v$$

and in place of (3.1) we have

$$\langle w_\mu\rangle=\frac{W}{2\pi}\mathrm{Re}\Big\langle\Big(\gamma_g+\frac{\Gamma}{2}+i\Big[\Omega_\mu-\frac{\Omega}{2}-G -\Big(\mathbf{k}_\mu-\frac{\mathbf{k}}{2}\Big)\mathbf{v}\Big]^{-1} +\Big(\gamma_g+\frac{\Gamma}{2}+i\Big[\Omega_\mu-\frac{\Omega}{2}+G-\Big(\mathbf{k}_\mu-\frac{\mathbf{k}}{2}\Big)\mathbf{v}\Big]^{-1}\Big\rangle. \qquad (3.3)$$

In this case, two well delineated lines are obtained, with maxima at the frequencies

$$\Omega_\mu=\omega_\mu-\omega_{gn}=\Omega/2\pm G.$$

The line widths depend on the relation $\gamma_g+\Gamma/2=\gamma_g+(\gamma_m+\gamma_n)/2$ and the "Doppler" width $|\mathbf{k}_\mu-\mathbf{k}/2|\bar v$; this depends in turn on the observation direction $\mathbf{k}_\mu/k_\mu$ and the ratio of the frequencies of the spontaneous emission and of the external field. The interference effects are maximum if $\mathbf{k}_\mu=\mathbf{k}/2$, i.e., the direction of observation is identical with $\mathbf{k}$ and $\omega_\mu=\omega/2$. In this case, the motion of the atom does not affect the width; it is determined by the sum $\gamma_g+(\gamma_m+\gamma_n)/2$. If the observation is made in the direction $-\mathbf{k}$ for the same ratio of frequencies, i.e., $\mathbf{k}_\mu=-\mathbf{k}/2$, then the atomic motion is more significant.

We shall not analyze (3.3) in further detail, since the fields that are attainable at the present time are insufficient for realization of the conditions of applicability of this formula; we return to the case of not very strong fields, when the two components in (3.1) are not resolved. Here, it is natural to reduce the terms in (3.1) to a common denominator, and to consider them together:

$$\langle w_\mu\rangle=\frac{W}{\pi}\mathrm{Re}\Big\langle\Big[\gamma_g+\gamma_n+i\Omega_\mu'+\frac{G^2}{\gamma_m+\gamma_g+i(\Omega_\mu'-\Omega')}\Big]^{-1}\Big\rangle. \qquad (3.4)$$

For $G=0$, Eq. (3.4) transforms directly into the ordinary expression for w_μ. It is seen from (3.4) that the external field leads to a change in the line width and in the position of its maximum. The changes here depend, in particular, on

$$\Omega_\mu'-\Omega'=(\omega_\mu-\omega_{gn})-(\omega-\omega_{mn})-(\mathbf{k}_\mu-\mathbf{k})\mathbf{v}. \qquad (3.5)$$

First, we shall consider the case $\mathbf{k}_\mu=\mathbf{k}$, where the effect of the external field is a maximum. Then (3.4) can be expressed in terms of the probability integral of complex argument:[8]

$$\langle w_\mu\rangle=\frac{W}{\sqrt{\pi}\,k_\mu\bar v}\mathrm{Re}\{e^{z^2}[1-\Phi(z)]\};\quad \Phi(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt;$$

$$z=\frac{1}{k_\mu\bar v}\Big\{\gamma_g+\gamma_n+i\Omega_\mu+\frac{G^2}{[\gamma_g+\gamma_m+i(\Omega_\mu-\Omega)]}\Big\}. \qquad (3.6)$$

For a large Doppler width,

$$k_\mu\bar v\gg\gamma_g+\gamma_n+G/(\gamma_g+\gamma_m), \qquad (3.7)$$

one can write (3.6) in the form

$$\langle w_\mu\rangle\cong\frac{W}{\sqrt{\pi}\,k_\mu\bar v}\exp\{-\Omega_\mu^2/(k_\mu\bar v)^2\}\Big\{1-\frac{2}{\sqrt{\pi}}\frac{\gamma_g+\gamma_n}{k_\mu\bar v} -\frac{2}{\sqrt{\pi}}\frac{G^2}{(\gamma_g+\gamma_m)k_\mu\bar v}\frac{(\gamma_m+\gamma_g)^2}{(\Omega_\mu-\Omega)^2+(\gamma_m+\gamma_g)^2}\Big\}. \qquad (3.8)$$

It is seen from (3.8) that a "valley" of Lorentzian shape appears against the "background" of the Doppler broadened line; the valley width is determined by the parameter $G^2/[k_\mu\bar v(\gamma_g+\gamma_m]$, which is smaller than the socalled saturation parameter $G^2/\gamma_m\gamma_n$ by a factor $k_\mu\bar v(\gamma_m+\gamma_n)/\gamma_m\gamma_n$. It is of interest that the valley width $\gamma_g+\gamma_m$ is determined by the damping rate γ_m of level m, which does not participate directly in the transition $g\to n$. This is obviously connected with the fact that the external field "mixes" the states m and n of the isolated atom.

We note that for these effects to appear strict satisfaction of the condition $\mathbf{k}_\mu=\mathbf{k}$ is not necessary. From (3.5) and (3.4), it is easy to see that the satisfaction of the inequalities

$$|\mathbf{k}_\mu-\mathbf{k}|/k_\mu,\quad \vartheta\ll(\gamma_g+\gamma_m)/k_\mu\bar v, \qquad (3.9)$$

is sufficient, where ϑ is the angle between $\mathbf{k}_\mu$ and $\mathbf{k}$. For atoms with an atomic weight of about 20, $T\sim400\,^\circ$K, $\gamma_g+\gamma_m\sim10^8$ sec^{-1} and $\gamma=1\mu$, the conditions (3.9) mean that $\vartheta\sim1^\circ$ and $|\lambda_\mu-\lambda|\sim100$ Å are entirely admissible.

It is not difficult to see that for a significant difference between $\mathbf{k}_\mu$ and $\mathbf{k}$, the effect of the field will be much smaller than in the case under discussion. Therefore, we shall not analyze the situation in detail for $\mathbf{k}_\mu\neq\mathbf{k}$.

We now turn to the transition $f\to m$, i.e., the transition to the upper of two levels which are connected by the external field (Fig. 2). Here we have

$$\langle w_\mu\rangle=\frac{W}{\pi}\mathrm{Re}\Big\langle\frac{A_1}{\gamma_f-\alpha_1+i\Omega_\mu'}+\frac{A_2}{\gamma_f-\alpha_2+i\Omega_\mu'}\Big\rangle.$$

$$= W\pi^{-1}\,\mathrm{Re}\langle\{\gamma_f+\gamma_m+i\Omega_\mu' + G^2/[\gamma_f+\gamma_n+i(\Omega_\mu'+\Omega')]\}^{-1}\rangle. \quad (3.10)$$

Formula (3.10) is obtained from (3.4) by the substitution of γ_f for γ_g and $-\Omega'$ for Ω'. Here the interference effects are very intense for observation along the direction opposite to $\mathbf{k}$. This can offer a certain advantage from the experimental viewpoint.

4. THE TRANSITIONS m → *l* and n → j

In the case of the transitions m → *l* and n → j (Fig. 2), it is necessary to use Eq. (2.29′) with those changes which were mentioned in Sec. 2. For the transition m → *l*, simple calculations give

$$\langle w_\mu\rangle = \frac{\gamma_{ml}}{4\pi\gamma_m}\frac{1}{\pi} \times \mathrm{Re}\left\langle\left[\gamma_l+\gamma_m+i\Omega_\mu'+\frac{G^2}{\gamma_l+\gamma_n+i(\Omega_\mu'-\Omega')}\right]^{-1} \times\left\{\rho_m-\left(\frac{\rho_m}{\gamma_m}-\frac{\rho_n}{\gamma_n}\right)\frac{G^2}{\Omega'^2+\Gamma^2(1+G^2/\gamma_m\gamma_n)} \times\left[\Gamma+\gamma_m\frac{\Gamma+i\Omega'}{\gamma_l+\gamma_n+i(\Omega_\mu'-\Omega')}\right]\right\}\right\rangle. \quad (4.1)$$

The factor in the first row of Eq. (4.1) has the same form as in (3.10). If $\rho_m/\gamma_m = \rho_n/\gamma_n$, i.e., if the populations of levels m and n interacting with the field are equal, then the line shape will be the same as in the transition f → m (with accuracy to within the replacement of γ_f by γ_l and $\mathbf{k}_\mu$ by $-\mathbf{k}_\mu$), and the integrated probability of emission does not depend on the external field. In the opposite case,

$$\int_{-\infty}^{\infty}\langle w_\mu\rangle d\omega_\mu = \frac{\gamma_{ml}}{4\pi\gamma_m}\left\langle\rho_m-\left(\frac{\rho_m}{\gamma_m}-\frac{\rho_n}{\gamma_n}\right)\frac{\Gamma G^2}{\Omega'^2+\Gamma^2(1+G^2/\gamma_m\gamma_n)}\right\rangle. \quad (4.2)$$

As is well known,[1] the factor

$$\frac{\Gamma}{\gamma_m}G^2\frac{1}{\Omega'^2+\Gamma^2(1+G^2/\gamma_m\gamma_n)}$$

is the probability of stimulated emission of a photon by the external field. Therefore, the second term in (4.2) and the term in (4.1) corresponding to it express the change in the population of level m as the result of transitions induced by the external field. As a consequence of the selectivity of the interaction of the atom with the external field, the demonstrated change in the population depends on the velocity of the atom $\mathbf{v}$ (the "Bennet gap"), which is an additional reason for the change in the line shape. Furthermore, in Sec. 3 it was shown that the effect of the field on the factor in the second row of (4.1) is important for $G^2 \sim (\gamma_l+\gamma_n)k_\mu\bar{v}$; the change in the population is determined by the value of the parameter $G^2/\gamma_m\Gamma$. Consequently, for a large value of the Doppler width $k_\mu\bar{v}$, the basic changes in the line shape of spontaneous emission will be connected with the second and not the first row in (4.1).

For large values of $k_\mu\bar{v}$, the average of (4.1) over the velocity can be obtained comparatively simply. We shall only put down the formulas which pertain to the class of observation along $\mathbf{k}$ and $-\mathbf{k}$, since interesting interference cases appear here:

$$\langle w_\mu\rangle = \frac{\gamma_{ml}}{4\pi\gamma_m}\frac{\exp\{-\Omega_\mu^2/(k_\mu\bar{v})^2\}}{\sqrt{\pi}\,k_\mu\bar{v}} \times\left\{\rho_m-\frac{k_\mu}{k}\left(\frac{\rho_m}{\gamma_m}-\frac{\rho_n}{\gamma_n}\right)\frac{\Gamma_+G^2}{(\Omega_\mu-k_\mu\Omega/k)^2+\Gamma_+^2}\right\}; \quad (4.3)$$

$$\Gamma_+ = \gamma_l+\gamma_n+(k_\mu/k-1)\Gamma; \quad \mathbf{k}_\mu\mathbf{k}>0; \quad k_\mu\geqslant k; \quad G^2\ll\gamma_m\gamma_n;$$

$$\langle w_\mu\rangle = \frac{\gamma_{ml}}{4\pi\gamma_m}\frac{\exp\{-\Omega_\mu^2/(k_\mu\bar{v})^2\}}{\sqrt{\pi}\,k_\mu\bar{v}} \times\left\{\rho_m-\frac{k_\mu}{k}\left(\frac{\rho_m}{\gamma_m}-\frac{\rho_n}{\gamma_n}\right)\frac{\Gamma_-G^2}{(\Omega_\mu-k_\mu\Omega/k)^2+\Gamma_-^2}\right\}; \quad (4.4)$$

$$\Gamma_- = \gamma_l+\gamma_m+k_\mu\Gamma/k, \quad \mathbf{k}_\mu\mathbf{k}<0; \quad G^2\ll\gamma_m\gamma_n.$$

As in the case of Sec. 3, a "valley" appears in the line of spontaneous emission, the width of which is much less than the Doppler width $k_\mu\bar{v}$. For observation along $-\mathbf{k}$ (Eq. (4.4)), the valley width Γ_- is such as could be expected on the basis of representations of the "Bennet gap:" the term $(k_\mu/k)\Gamma$ is the width of the valley in the velocity distribution, converted to the Doppler shift near the frequency ω_μ; to this term is added the width $\gamma_l+\gamma_m$, due to the lifetimes of the combining levels *l* and m. For observation along $\mathbf{k}$ (Eq. (4.3)), the valley width Γ_+ cannot generally depend on the velocity distribution if $k_\mu \approx k$. Only for $k_\mu \gg k$ will $\Gamma_+ \approx \Gamma_-$. Furthermore, γ_m does not enter into Γ_+, but rather γ_n, i.e., the width of the level n, which does not directly take part in the transition m → *l*.

In the analysis of Eqs. (4.1), (4.3), (4.4), we have assumed that the population of the upper level m is greater than the lower level n. In the opposite case, the sign of the difference $\rho_m/\gamma_m - \gamma_n/\rho_n$ is changed and a peak appears in the line, rather than a valley, with the same parameters.

Let us now consider the spectrum of spontaneous emission in the transition n → j (Fig. 2). Here we get

$$\langle w_\mu \rangle = \frac{\gamma_{nj}}{4\pi\gamma_n} \mathrm{Re}\left\langle \left[\gamma_j + \gamma_n + i\Omega_\mu' + \frac{G^2}{\gamma_j + \gamma_m + i(\Omega_\mu' + \Omega')}\right]^{-1} \times \left\{\rho_m + \left(\frac{\rho_m}{\gamma_m} - \frac{\rho_n}{\gamma_n}\right)\frac{G^2}{\Omega'^2 + \Gamma^2(1 + G^2/\gamma_m\gamma_n)} \times \left[\Gamma + \gamma_n \frac{\Gamma - i\Omega'}{\gamma_j + \gamma_m + i(\Omega_\mu' + \Omega')}\right]\right\}\right\rangle . \quad (4.5)$$

Equation (4.5) is obtained from (4.1) by the substitutions $m \leftrightarrow n$, $\gamma_l \to \gamma_j$, $\Omega' \to -\Omega'$, and the most significant differences are the following. The factor $[\rho_m/\gamma_m - \rho_n/\gamma_n]$ appears in (4.5) with a different sign than in (4.1). Therefore a "peak" in the transition $n \to j$ will correspond to a valley in the transition $m \to l$ and vice versa. This circumstance is clearly understood, inasmuch as the changes in the populations of levels m and n, brought about by the field, are opposite in sign. The second difference pertains to the width of the "peaks" (valleys) in observation of spontaneous emission along $\mathbf{k}$ and $-\mathbf{k}$. In the approximation analogous to (4.3) and (4.4), we have

$$\Gamma_+' = \gamma_j + \gamma_n + \frac{k_\mu}{k}\Gamma; \quad \Gamma_-' = \gamma_j + \gamma_m + \frac{k_\mu - k}{k}\Gamma, \quad (4.6)$$

that is, the analog of the transition $n \to j$ and $\mathbf{k}_\mu \cdot \mathbf{k} > 0$ is $\mathbf{k}_\mu \cdot \mathbf{k} < 0$ for the transition $m \to l$, and vice versa.

5. DISCUSSION OF RESULTS

It was shown above that the external field can significantly change the spectrum of spontaneous emission of a quantum system. These changes can be interpreted as the result of a mixing of stationary states of the isolated atom in the external field. Unusual interference phenomena appear here, as a result of which any discussions based on the picture of "splitting" of the levels of an isolated atom can lead to a seriously wrong conclusion. This applied both to the adjacent transitions considered in the present research as well as to the spontaneous emission by the transition $m \to n$, which was considered in [1,2,4].

We note that similar interference effects occur, to a known extent, in those cases when, by selection of processes of excitation the states of the atom at the initial instant of time are "prepared" in the form of a linear combination of stationary states (see, for example, [9-11]). From the viewpoint of formal classification, the case of interest to us is distinguished by the fact that mixing of states takes place after the excitation of the system, while at the initial instant the atom is in some stationary state.

The established features of spontaneous emission are of interest from the spectroscopic point of view, inasmuch as they permit the experimental determination of the radiation broadening of the levels, which is masked by the Doppler broadening under ordinary conditions. For example, let the widths Γ_+, Γ_-, Γ_+', and Γ_-' of the "valleys" be measured in the transitions $m \to l$ and $n \to j$; according to (4.3), (4.4), and (4.6), we have

$$\begin{aligned} \Gamma_+ &= \gamma_l + \gamma_n + \frac{k_\mu - k}{k}(\gamma_m + \gamma_n); \\ \Gamma_- &= \gamma_l + \gamma_m + \frac{k_\mu}{k}(\gamma_m + \gamma_n); \\ \Gamma_+' &= \gamma_j + \gamma_n + \frac{k_\mu'}{k}(\gamma_m + \gamma_n); \\ \Gamma_-' &= \gamma_j + \gamma_m + \left(\frac{k_\mu'}{k} - 1\right)(\gamma_m + \gamma_n). \end{aligned} \quad (5.1)$$

Thus, a set of four linear equations is obtained in the unknowns γ_l, γ_j, γ_m, and γ_n. It is easy to show that the determinant of the system (5.1) is not equal to zero for any k_μ, k_μ', or k (excluding the practically uninteresting case $k_\mu + k_\mu' = k$). Therefore, measurements of $\Gamma_\pm$ and $\Gamma_\pm'$ make it possible to determine the radiation widths of all the levels l, j, m, and n. A similar situation occurs in the investigation of two transitions from one of the levels m or n (for example, $m \to l$ and $m \to l'$).

It is necessary to emphasize that only radiative relaxation has been considered in the present research, while various processes in atomic collisions have been completely ignored. The account of collisions requires an additional analysis, and we hope to turn to this problem in another place.

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