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KINETIC THEORY OF PROPAGATION OF ULTRASOUND IN MOLECULAR GASES IN AN EXTERNAL STATIONARY FIELD

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On the basis of the kinetic equation, formulas are obtained which describe the dispersion of sound in a molecular gas. The influence of electric and magnetic fields on the ultrasonic absorption is investigated.

THE basic aim of the present research was the determination of the dependence of the propagation velocity and the ultrasonic absorption coefficient on an external (electric or magnetic) field. As a result of the research, we found dispersion of the sound in a gas of molecules which possess rotational degrees of freedom (RDF). These formulas reproduce the results of the general theory of Leontovich-Mandel'shtam for a specific case.[1,2]

The small parameters of our problem will be, first, the ratio of the reciprocals of the relaxation times for RDF and the translational degrees of freedom (TDF),

$$\varepsilon = \Omega_{\text{RDF}} / \Omega_{\text{TDF}} \ll 1$$

and, second,

$$\omega/\Omega_{\text{TDF}} \ll 1,$$

ω is the sound frequency.

The first condition follows from the assumption that, in collisions between the molecules, the change in the rotational states of the molecules is small in comparison with the change in the translational motion of the colliding molecules. The second condition is necessary for the possibility of ultrasonic propagation, because its violation leads to damping at distances of the order of a wavelength. The mathematical problem which arises in the latter case has been frequently discussed in the kinetic theory of monatomic gases (see, for example,[3,4]).

Actually, the first condition is poorly satisfied (for oxygen, $\epsilon_{O_2}^{-1} = 4.1 \pm 0.4$, for nitrogen, $\epsilon_{N_2}^{-1} = 4.7 \pm 0.4$). However, we shall use it, inasmuch as it allows us to explain, with maximum clarity, the laws associated with the freezing out of the RDF for $\omega \gtrsim \Omega_{\text{RDF}}$. This condition is evidently entirely violated only for molecules for which the vibrational degrees of freedom are easily excited (for example, CO or N_2O), which are not considered in our research. If the reinforcement of the inequality is not satisfied, then our conclusions relative to the region of frequencies $\Omega_{\text{RDF}} < \omega < \Omega_{\text{TDF}}$ bear only a qualitative character. It should also be noted that we formally use one small parameter ϵ both for the description of Ω_{RDF} and for the characteristics of the non-spherical nature of the molecules, which govern the dependence of the kinetic phenomena on the external field. As the Senftleben effect shows (see, for example,[5]), the latter characteristic is always small ($\lesssim 0.1$), and all our conclusions that refer to the effect of the external field on the propagation of sound are unchanged if $\Omega_{\text{RDF}} \sim \Omega_{\text{TDF}}$ (but, as before, $\omega/\Omega_{\text{TDF}} \ll 1$).

1. THE HYDRODYNAMIC THEORY OF SOUND ABSORPTION IN AN EXTERNAL FIELD

Inasmuch as the transport coefficients of molecular gases depend on the external magnetic or electric field,[5-10] it must be expected that the sound absorption in the presence of an external field will also change. This effect has been detected experimentally for oxygen in a magnetic field by Itterbeek and Thys.[11] At low frequencies, the theoretical explanation of the effect is trivial. The hydrodynamic sound absorption coefficient is equal to[12]

$$\gamma_0 = \frac{\omega^2}{2\rho v^3}\left[\frac{4}{3}\eta + \zeta + \varkappa\left(\frac{1}{c_v} - \frac{1}{c_p}\right)\right], \quad v = \sqrt{c_p T / c_v m}. \quad (1.1)$$

In an external field, the viscosity coefficients and the thermal conductivity take on a tensor character,[9] and we easily find, for a wave propagating along the x axis (by repeating the calculations of[12]),

$$\gamma = \frac{\omega^2}{2\rho v^3}\left[2\eta_{xxxx} + \varkappa_{xx}\left(\frac{1}{c_v} - \frac{1}{c_p}\right)\right]. \tag{1.2}$$

The explicit form of the tensors η and κ can be taken from[9]. Inasmuch as processes for colliding molecules with the participation of the rotational degrees of freedom (RDF) of the molecules are comparatively complicated in comparison with processes in which only the translational degrees of freedom (TDF) of the molecules take part, then, in general, the coefficient of second viscosity, which characterizes the relaxation processes between RDF and TDF, is of the same order or is large in comparison with the coefficients of first viscosity and thermal conductivity. For this same reason, the change for the second viscosity in the external field is comparatively large. Therefore, it suffices, for an estimate of the effect, to retain in (1.1) and (1.2) the terms connected with the second viscosity, which leads to the simple relation

$$\frac{\gamma - \gamma_0}{\gamma_0} = \frac{\Delta\zeta}{\zeta}, \tag{1.3}$$

where $\Delta\zeta$ is the change in the coefficient of second viscosity in the external field. Equation (1.3) satisfactorily explains the order of magnitude of the effect (~30%) found by Itterbeek and Thys.

It is also easy to predict the value of the effect in the frequency region for which the RDF can be regarded as frozen, while the TDF is still in a state of local equilibrium. In this case, as before, we use Eqs. (1.1) and (1.2), in which we discard terms connected with the second viscosity, and take into account the change in the specific heat.

Inasmuch as the change in the first viscosity and in the thermal conductivity in the external field is of the order of 1%, it is obvious that the effect of the external field on the absorption coefficient in the frequency range under study becomes weaker, but on the other hand the effect of the anisotropy becomes sharper, since now it is not smothered by the isotropic effect on the second viscosity.

2. KINETIC THEORY OF SOUND PROPAGATION

For a description of the effect in the region of frequencies where dispersion of the sound velocity takes place, it is necessary to deduce the dispersion equation from the linearized kinetic equation

$$\frac{\partial\chi}{\partial t} + \sqrt{\frac{2T}{m}}\mathbf{u}\nabla\chi + \gamma[\mathbf{MB}]\frac{\partial\chi}{\partial\mathbf{M}} = -\hat{\Omega}\chi. \tag{2.1}$$ *

Here **u** and **M** are the velocity and the rotational moment of the molecules in dimensionless units, **B** is the external field. The form of γ, which depends on the nature of the gas and the external field **B** (magnetic or electric), and also the remaining notation, can be found in[9]. We use here the following collision integral (which has the dimensions of frequency):

$$\hat{\Omega}\chi = n\hat{I}\chi = n\iiint f_{01}[(\chi+\chi_1)W$$
$$-(\chi'+\chi_1')W']\,d\Gamma_1\,d\Gamma'\,d\Gamma_1',$$
$$f_0 = \text{const}\cdot\exp(-u^2 - E_{\text{rot}}), \tag{2.2}$$

TE_{rot} is the rotational energy of the molecule.

Let the sound wave be described by the nonequilibrium part of the distribution function of the form

$$\chi = \chi(\mathbf{u},\mathbf{M})\exp\left(-i\omega t + i\sqrt{\frac{m}{2T}}\mathbf{qr}\right). \tag{2.3}$$

Then Eq. (2.1) takes the form

$$\left(\hat{\Omega} - i\omega + i\mathbf{qu} + \gamma[\mathbf{MB}]\frac{\partial}{\partial\mathbf{M}}\right)\chi = 0. \tag{2.4}$$

Following[9], we divide the collision operator into two parts:

$$\hat{\Omega} = \hat{\Omega}^{(0)} + \varepsilon\hat{\Omega}^{(1)}, \quad \varepsilon \ll 1,$$

where the second, small part describes the correlation between the internal and external degrees of freedom. Inasmuch as such a division is not unique, it can always be arranged that the operator $\hat{\Omega}^{(0)}$ possesses eigenfunctions identical with the eigenfunctions of a collision operator of the Maxwell type, the probability of collision of which depends only on the scattering angle and does not depend on the value of the relative velocity and the rotational momenta, which do not change upon collision:[9,10]

$$\hat{\Omega}^{(0)}\psi_n = \Omega_n\psi_n,$$
$$\Omega_n = \langle\psi_n\hat{\Omega}\psi_n\rangle = \int f_0 d\Gamma\psi_n{}^*\hat{\Omega}\psi_n,$$
$$\psi_n = \sum_{m_1+m_2=m} C^{lm}_{l_1m_1;\,l_2m_2}Y_{l_1m_1}(\mathbf{u})Y_{l_2m_2}(\mathbf{M})L_{r_1}^{l_1+1/2}$$
$$\times(u^2)L_{r_2}^{l_2+\alpha}(E_{\text{rot}})\varphi_s(\sigma), \tag{2.5}$$

n = (lm; l_1l_2, r_1r_2, s). Here Y_{lm} is a spherical harmonic, L_r is the normalized Laguerre polynomial; the polynomials φ_s form an orthonormal basis relative to the auxiliary variables σ (the projection of the spin on the rotational moment in the case of paramagnetic molecules, the projection of the dipole moment in the case of a polar molecule); α is equal to zero if the moment of a (linear) diatomic molecule and is equal to ½ if it is monatomic.

In accord with the determination of Ω_n, the diagonal matrix elements of the operator $\hat{\Omega}^{(1)}$ are equal

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*[MB] ≡ M × B.

to zero. Instead of the functions with n = (00; 00, 10, 0) and n = (00; 00, 01, 0), we shall use their linear combinations ψ_3 and ψ_4 (see (2.10) and (2.11)).

Substituting the expansions

$$\chi = \chi^{(0)} + \varepsilon\chi^{(1)} + \varepsilon^2\chi^{(2)} + \dots,$$
$$\omega = \omega^{(0)} + \varepsilon\omega^{(1)} + \varepsilon^2\omega^{(2)} + \dots \tag{2.6}$$

in (2.4), and collecting terms of like order of smallness in ϵ, we find

$$\begin{aligned}&\hat{K}^{-1}\chi^{(0)} = 0, \qquad \langle\chi^{(0)*}, \chi^{(0)}\rangle = 1,\\ &\hat{K}^{-1}\chi^{(1)} + (\hat{\Omega}^{(1)} - i\omega^{(1)})\chi^{(0)} = 0,\\ &\hat{K}^{-1}\chi^{(2)} + (\hat{\Omega}^{(1)} - i\omega^{(1)})\chi^{(1)} - i\omega^{(2)}\chi^{(0)} = 0,\\ &\hat{K}^{-1} = \hat{\Omega}^{(0)} - i\omega^{(0)} + i\mathbf{qu} + \gamma[\mathbf{MB}]\frac{\partial}{\partial\mathbf{M}}.\end{aligned} \tag{2.7}$$

Substituting the expansion

$$\chi^{(0)} = \sum a_n\psi_n, \tag{2.8}$$

in the first equation of (2.7), we get

$$0 = \sum_{n'} a_{n'}\Big[(\Omega_n - i\omega^{(0)})\delta_{nn'} + iqu_{nn'} + \Big\langle\psi_n, \gamma[\mathbf{MB}]\frac{\partial}{\partial\mathbf{M}}\psi_{n'}\Big\rangle\Big]$$
$$u_{nn'} = \langle\psi_n, \mathbf{eu}\psi_{n'}\rangle, \quad \mathbf{e} = \mathbf{q}/q. \tag{2.9}$$

In what follows, we shall omit the index of zeroth approximation on $\omega^{(0)}$.

The sound oscillations are accompanied by oscillations of the density, velocity and temperature (energy density) of the gas. This means that the "states"

$$\psi_1 = 1, \quad \psi_2 = \sqrt{2}\,\mathbf{eu}, \quad \psi_3 = c_v^{-1/2}(u^2 + E_{\text{rot}} - c_v), \tag{2.10}$$

are represented in the expansion (2.8), to which correspond the zero eigenvalues of the collision operator. The matrix elements $u_{nn'}$ connect in (2.9) those states which possess nonzero values of Ω_n, that is, with

$$\begin{aligned}&\psi_4 = \left(\frac{2}{3} - \frac{1}{c_v}\right)^{-1/2}\left[\frac{2}{3}u^2 - \frac{1}{c_v}(u^2 + E_{\text{rot}})\right],\\ &\psi_5 = \sqrt{3}\left[(\mathbf{eu})^2 - \frac{1}{3}u^2\right], \quad \psi_6 = \frac{2}{\sqrt{5}}\mathbf{eu}\left(u^2 - \frac{5}{2}\right),\\ &\psi_7 = \sqrt{2/(c_v - 3/2)}\,\mathbf{eu}(E_{\text{rot}} - c_v + 3/2).\end{aligned} \tag{2.11}$$

For all the states (2.10), (2.11), the matrix element in (2.9), which depends on the external field, is equal to zero, i.e., $\chi^{(0)}$ does not depend on the external field.

Among the states (2.11), there is the state ψ_4 which describes the nonequilibrium distribution of the energy between the TDF and the RDF. If the time for establishment of equilibrium between the TDF and the RDF is large in comparison with the relaxation time inside the TDF, then

$$\Omega_4/\Omega_n \ll 1, \qquad n \geqslant 5. \tag{2.12}$$

In the following, we shall limit ourselves to consideration of a frequency range that is small in comparison with the reciprocal of the TDF relaxation time, i.e., we shall assume that

$$\omega/\Omega_n \ll 1, \qquad n \geqslant 5. \tag{2.13}$$

The relation between ω and Ω_4 we shall assume to be arbitrary. The condition (2.13) leads to the result that only the first seven equations in the set (2.9) are important, in first approximation relative to (2.13). The solution of this set of equations is made easier by the fact that only a small number of the coefficients $u_{nn'}$ in them differ from zero:

$$\begin{aligned}&u_{12} = \frac{1}{\sqrt{2}} \quad u_{23} = \frac{1}{\sqrt{2c_v}}, \quad u_{24} = \sqrt{\left(\frac{2}{3} - \frac{1}{c_v}\right)\frac{1}{2}},\\ &u_{25} = \sqrt{\frac{2}{3}}, \quad u_{36} = \frac{1}{2}\sqrt{\frac{5}{c_v}}, \quad u_{37} = \frac{\sqrt{3}}{2}\left(\frac{2}{3} - \frac{1}{c_v}\right)^{1/2},\\ &u_{46} = \frac{\sqrt{5}}{2}\left(\frac{2}{3} - \frac{1}{c_v}\right)^{1/2}, \quad u_{47} = -\frac{1}{2}\sqrt{\frac{3}{c_v}}, \quad u_{nn'} = u_{n'n}.\end{aligned} \tag{2.14}$$

This allows us to express a_1, a_5, a_6 and a_7 in terms of a_2, a_3, and a_4 and to reduce the system from seven equations to a system of three equations. We find the dispersion equation from the condition of solvability of these latter three equations:

$$\begin{aligned}1 &- x^2\left(u_{12}^2 + u_{23}^2 + \frac{u_{24}^2}{1 + i\Omega_4/\omega}\right) + ix^2u_{25}^2\frac{\omega}{\Omega_5}\\ &+ ix^4\left(u_{23}u_{36} + \frac{u_{24}u_{46}}{1 + i\Omega_4/\omega}\right)^2\frac{\omega}{\Omega_6}\\ &+ ix^4\left(u_{23}u_{37} + \frac{u_{24}u_{47}}{1 + i\Omega_4/\omega}\right)^2\frac{\omega}{\Omega_7} = 0, \quad x = q/\omega.\end{aligned} \tag{2.15}$$

The coefficients of the normalized functions $\chi^{(0)}$ are found to be equal to

$$\begin{aligned}&a_1 = \frac{1}{\sqrt{2}}xu_{12}, \quad a_2 = \frac{1}{\sqrt{2}} \quad a_3 = \frac{1}{\sqrt{2}}xu_{23}a_2,\\ &a_4 = \frac{1}{\sqrt{2}}xu_{24}\left(1 + \frac{i\Omega_4}{\omega}\right)^{-1}, \quad a_5 = \frac{1}{\sqrt{2}}xu_{52}\left(-\frac{i\omega}{\Omega_5}\right),\\ &a_6 = \frac{1}{\sqrt{2}}x^2\left(u_{23}u_{36} + \frac{u_{24}u_{46}}{1 + i\Omega_4/\omega}\right)\left(-\frac{i\omega}{\Omega_6}\right),\\ &a_7 = \frac{1}{\sqrt{2}}x^2\left(u_{23}u_{37} + \frac{u_{24}u_{47}}{1 + i\Omega_4/\omega}\right)\left(-\frac{i\omega}{\Omega_7}\right).\end{aligned} \tag{2.16}$$

In zeroth approximation in the parameter (2.13), the dispersion equation (2.15) can be rewritten in the form

$$1-\left(\frac{q}{\omega}\right)^2\frac{v_0^2-(i\omega/\Omega_4)v_\infty^2}{1-i\omega/\Omega_4}=0,\qquad v_0^2=\frac{c_p}{c_v},\qquad v_\infty^2=\frac{5}{3},\tag{2.17}$$

which is completely identical with the dispersion equation of Leontovich-Mandel'shtam.[2]

The dispersion equation (2.15) determines the (phase) velocity of propagation and the sound absorption coefficient:

$$\begin{aligned}v^2&=\frac{T}{m}\left[\frac{c_p}{c_v}+\left(\frac{2}{3}-\frac{1}{c_v}\right)\frac{1}{1+(\Omega_4/\omega)^2}\right],\\ \gamma&=\frac{\omega^2}{2\rho v^3}\left\{\zeta\left[1+\left(\frac{\omega}{\Omega_4}\right)^2\right]^{-1}+\frac{4}{3}\eta+\right.\\ &\quad+\varkappa_6\frac{T}{v^2c_v^2}\operatorname{Re}\left\{\left(1-\frac{i\omega}{\Omega_4}\frac{2}{3}c_v\right)\Big/\left(1-\frac{i\omega}{\Omega_4}\right)\right\}^2\\ &\quad+\varkappa_7\frac{T}{v^2c_v^2}\operatorname{Re}\left(1-\frac{l\omega}{\Omega_4}\right)^{-2}.\end{aligned}\tag{2.18}$$

Here we have introduced the coefficients of first and second viscosity and the heat transfer coefficients into the TDF (κ_6) and the RDF (κ_7), computed in the zeroth approximation in the parametery of non-sphericity ϵ:

$$\zeta=\frac{nT}{\Omega_4}\left(\frac{2}{3}-\frac{1}{c_v}\right),\quad \eta=\frac{nT}{\Omega_5},\quad \varkappa_6=\frac{5}{2}\frac{nT}{m\Omega_6},\quad \varkappa_7=\left(c_v-\frac{3}{2}\right)\frac{nT}{m\Omega_7}.\tag{2.19}$$

As the frequency approaches zero ($\omega \ll \Omega_4$), the expressions (2.18) approach the usual form, which follows from the equations of hydrodynamics (1.1). Naturally, the coefficient a_4 in (2.16) approaches zero in this limit, which means that the TDF and the RDF are found in relative equilibrium.

At high frequencies ($\omega \gg \Omega_4$, but, as before, $\omega \ll \Omega_m$, $n \geq 5$), Eqs. (2.18) are almost identical with the formulas which follow from the hydrodynamics of a monatomic gas:

$$v^2=\frac{5}{3}\frac{T}{m},\quad \gamma=\frac{\omega^2}{2\rho v^3}\left\{\zeta\left(\frac{\Omega_4}{\omega}\right)^2+\frac{4}{3}\eta+\frac{4}{15}\varkappa_6 m\right\}.\tag{2.20}$$

The relations (2.18) and (2.20) dispel the misunderstanding (which frequently arises) associated with the fact that the first viscosity coefficient, according to (2.19), becomes infinite as the probability of exchange of energy between the TDF and the RDF approaches zero (i.e., as the quantity Ω_4 approaches zero). It is obvious that as $\Omega_4 \to 0$, the frequency range for which the hydrodynamics is valid also approaches zero.

3. DISPERSION OF SOUND IN AN EXTERNAL FIELD

For the determination of the dependence of the dispersion law and the sound absorption on the external field, one must turn to Eqs. (2.7) of higher order in ϵ. We multiply the second and third equations in (2.7) on the left by $\chi^{(0)}$ and carry out a statistical averaging. Inasmuch as

$$\int d\Gamma\chi^{(0)}\hat{K}^{-1}F=\int d\Gamma(\hat{K}^{-1}\chi^{(0)})F=0,$$

for an arbitrary function F, we get

$$\begin{aligned}\omega^{(1)}&=i\langle\chi^{(0)*}\hat{\Omega}^{(1)}\chi^{(0)}\rangle,\\ \omega^{(2)}&=i\langle\chi^{(0)*}(\hat{\Omega}^{(1)}-i\omega^{(1)})\hat{K}(\hat{\Omega}^{(1)}-i\omega^{(1)})\chi^{(0)}\rangle.\end{aligned}\tag{3.1}$$

The change in the dispersion law (2.6) in an external field is determined by the difference

$$\begin{aligned}\Delta\omega&=\omega(\mathbf{B})-\omega(0)=i\varepsilon^2\langle\chi^{(0)*}\hat{\Omega}^{(1)}\Delta\hat{K}\hat{\Omega}^{(1)}\chi^{(0)}\rangle,\\ \Delta\hat{K}&=K(\mathbf{B})-K(0).\end{aligned}\tag{3.2}$$

Substituting the expression (2.8) here, we find

$$\Delta\omega=-\frac{i}{n}\sum_{kk'}a_ka_{k'}\Omega_{kk'}c_{kk'},\tag{3.3}$$

$$c_{kk'}=(\varepsilon^2n/\Omega_k\Omega_{k'})\langle\psi_k,\hat{\Omega}^{(1)}\Delta\hat{K}\hat{\Omega}^{(1)}\psi_{k'}\rangle.\tag{3.4}$$

Following[9], we limit ourselves to the class of models of collisions between molecules in which the transitions to states depending only on M and σ are made very difficult. Then, in the expansion

$$\hat{\Omega}^{(1)}\psi_k=\sum_n\langle\psi_n,\hat{\Omega}^{(1)}\psi_k\rangle\psi_n$$

only functions with large values of $\Omega_n \sim \Omega_{TDF}$ appear, and the term $(-i\omega+i\mathbf{q}\cdot\mathbf{u})$ in the operator $\hat{K}$ relative to them can be neglected (see (2.7)). Then Eq. (3.3) does not depend on the frequency and reduces to a quantity characterizing the change in the kinetic coefficient.[9] For example,

$$\begin{aligned}c_{55}&=\frac{\varepsilon^2n}{\Omega_5^2}\langle\psi_5,\hat{\Omega}^{(1)}K\hat{\Omega}^{(1)}\psi_5\rangle=\frac{4}{15}\sum_m|Y_{2m}(\mathbf{e})|^2\Delta\operatorname{Re}c^{(2)}_{2m;2m},\\ c^{(2)}_{2m;2m}&=\varepsilon^2n\langle Y_{2m}(\mathbf{u}),\hat{K}\hat{\Omega}^{(1)}\hat{K}\hat{\Omega}^{(1)}\hat{K}Y_{2m}(\mathbf{u})\rangle.\end{aligned}\tag{3.5}$$

These expressions describe the effect of the external field on the velocity and the ultrasonic absorption:

$$\Delta v=(v/\omega)\operatorname{Re}\Delta\omega,\qquad \Delta\gamma=-v^{-1}\operatorname{Im}\omega.\tag{3.6}$$

The behavior of quantities of the type (3.5) in the external field was investigated in[9]. Digressing from the crossover effects in an electric field (see Sec. 4), one can say that the correction to the dispersion law is an even function in which the static transfer coefficients are close to saturation, i.e., when the precession frequency of the molecule exceeds Ω_{TDF}.

In the hydrodynamic limit ($\omega \ll \Omega_4$) the correction (3.3) is easily expressed in terms of the correction to the static transfer coefficient and Eq. (1.2) can be obtained.

For frequencies $\omega < \Omega_4$ and $\omega \sim \Omega_4$, the most important role is played by the second viscosity. Therefore, the frequency dependence of the effect in the external field can be studied if the most important term left in (3.3) is

$$\varepsilon^2\omega^{(2)} \approx \frac{i}{n} a_4{}^2\Omega_4{}^2 c_{44} = -i\frac{\omega^2\Delta\zeta}{2\rho v^2}\left(1-\frac{i\omega}{\Omega_4}\right)^2,$$

$$\Delta\zeta = \frac{4T}{9}\Delta c^{(2)}_{00;00}. \tag{3.7}$$

Substituting this expression in (3.5), we find

$$\frac{\Delta v}{v} \approx \left(\frac{2}{3}-\frac{1}{c_v}\right)\frac{\Delta\zeta}{\zeta}\left[\frac{\omega/\Omega_4}{1+(\omega/\Omega_4)^2}\right]^2, \tag{3.8}$$

$$\frac{\Delta\gamma}{\gamma} \approx \frac{\Delta\zeta}{\zeta}\frac{1-(\omega/\Omega_4)^2}{1+(\omega/\Omega_4)^2} \qquad \gamma \approx \frac{\omega^2\zeta}{2\rho v^3}\left(1+\left(\frac{\omega}{\Omega_4}\right)^2\right)^{-1}. \tag{3.9}$$

It follows from these formulas that the effect of the external field on the velocity of sound is most important for $\omega \sim \Omega_4$ (the dispersion region). The change in the sound absorption in the external field is greatest at low frequencies. Upon further increase in the frequency $\omega \gg \Omega_4$, but $\omega \ll \Omega_{TDF}$) terms connected with the second viscosity cease to pay a determining role and the anisotropic terms, which are connected with the first viscosity and the thermal conductivity, become important. In this limit, the change in the velocity in the external field is very small, while the effect of the external field on the absorption coefficient is described by the equation

$$\Delta\gamma = \frac{\omega^2}{2\rho v^3}\left\{-\left(\frac{\Omega_4}{\omega}\right)^2\Delta\zeta + 2\Delta\eta'_{xxxx} + \frac{4m}{15}\Delta\varkappa'_{xx}\right\}.$$

$$\Delta\eta'_{xxxx} = \frac{2T}{15}\left\{3\left(\cos^2\theta-\frac{1}{3}\right)^2\Delta c_{20;20}\right.$$

$$\left. + 4\cos^2\theta\sin^2\theta\Delta\,\mathrm{Re}\,c_{21;21} + \sin^4\theta\Delta\,\mathrm{Re}\,c_{22;22}\right\},$$

$$\Delta\varkappa_{xx}' = \frac{2T}{3m}\{\cos^2\theta\Delta c'_{10;10} + \sin^2\theta\Delta c'_{11;11}\}. \tag{3.10}$$

In the latter formula, in complete analogy with Eq. (2.2), the values of $\Delta c'_{lm;\,lm}$ are obtained from the corresponding static values $\Delta c_{lm;\,lm}$ if in the latter functions $A_{lm} = Y_{lm}(u)\cdot(u^2 + E_{rot} - c_p)$, which describes the total energy flux, is replaced by the function $A_{lm} = Y_{lm}(u)(u^2 - 5/2)$, which describes the flux of translational energy. The RDF are frozen at high frequencies and do not make a contribution to $\Delta\gamma$ just as in γ (see (2.20)). Equation (3.10) agrees with the qualitative results of Sec. 1.

4. ODD EFFECT IN THE SOUND ABSORPTION

As was noted in[10], in a gas of polar molecules of the symmetric top type, an unusual cross effect can take place in the presence of an electric field. In this effect, the velocity gradients produce an additional thermal flux; on the other hand, a temperature gradient acts on the viscous stress tensor:

$$q_i = -\varkappa_{ik}(\nabla T)_k - \xi_{ipq}\left(\frac{\partial V_{0q}}{\partial x_p}+\frac{\partial V_{0p}}{\partial x_q}\right),$$

$$\sigma_{ik} = \frac{2}{T}\xi_{pik}(\nabla T)_p + \eta_{ikpq}\left(\frac{pV_{0q}}{\partial x_p}+\frac{\partial V_{0p}}{\partial x_q}\right),$$

$$\xi_{ipq} = T\sqrt{\frac{2T}{m}}\left\{\frac{1}{3\sqrt{3}}h_i\delta_{pq}c_{10;00} + \frac{1}{\sqrt{15}}c_{10;20}h_i\left(h_ph_q-\frac{1}{3}\delta_{pq}\right)\right.$$

$$\left. + \frac{1}{3\sqrt{5}}c_{11;21}[(\delta_{ip}-h_ih_p)h_q + (\delta_{iq}-h_ih_q)h_p]\right\},$$

$$h_i = \frac{E_i}{E}. \tag{4.1}$$

In[10], the coefficient $c_{11;\,21}$ is actually computed for a specific interaction model. The coefficient $c_{10;\,00}$ has the order of smallness of ϵ, while $c_{10;\,20}$ and $c_{11;\,21} \sim \epsilon^2$. The dependence of these coefficients on the magnitude of the electric field is determined by integrals of the form

$$\int d\Gamma f_0\frac{\sigma\xi M}{M^2+\sigma^2\xi^2} \qquad \xi \sim \frac{dE}{\Omega_{TDF}}. \tag{4.2}$$

It follows from ξ_{ipq} that the cross effect changes sign upon change in sign of the field. One therefore expects that a similar effect that is odd in the electric field will be observed in the sound absorption.

The fact that in a polar gas matrix elements $\langle\psi_{n'}, \hat{\Omega}^{(1)}\psi_n\rangle$ can be different from zero for transitions between the states (2.5) with dependence on the velocity of different parity ($l - l'$ is an odd number), leads to the result that in the calculation of the correction to the dispersion law (3.2) one must take into account the terms

$$\Delta(\varepsilon^2\omega^{(2)}) = \frac{2i}{n}\sum_{k=4,5} a_k\Omega_k \sum_{k'=6,7} a_{k'}\Omega_{k'}c_{kk'}. \tag{4.3}$$

This correction has virtually no effect on the sound velocity. In the low frequency limit, $\Delta\epsilon^{(2)}\omega^{(2)}$ can be expressed in terms of the tensor of the cross effect, and the additional term in the absorption coefficient is seen to be equal to

$$\Delta\gamma = \frac{2\omega^2}{\rho v^4 c}\xi_{xxx}. \tag{4.4}$$

The behavior of the cross effect as the frequency increases is entirely similar to the behavior of the basic even effect (see Sec. 3). In particular, for frequencies where the RDF can be regarded as completely frozen ($\omega \gg \Omega_i$) the cross effect takes the form (4.4) (with the replacement of c_V by 3/2 and v_0 by v_∞), if in the expression .1) for ξ_{xxx} we cancel $c_{10;\,00}$ and replace $c_{1m;\,2m}$ by quantities constructed similar to $c'_{1m;\,1m}$ in (3.10). The relative role of this effect here falls off by a factor ϵ.

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