PLASMA–PHONON MAGNETOHYDRODYNAMIC WAVES IN SEMIMETALS

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The effect of mutual drag of carriers and phonons in semimetals on the propagation of magnetohydrodynamic waves at temperatures $T \gg p_F s$ (s is the speed of sound) is considered. It is shown that under conditions of strong mutual drag there can be propagated in semimetals, besides magnetoplasma waves, also two weakly decaying plasma-phonon magnetohydrodynamic waves (Alfven and magnetosonic) in which joint oscillations of the electron-hole plasma and of the phonons interacting with the electrons and holes occur. The frequency of the plasma-phonon waves is lower than the electron, hole, and phonon relaxation frequencies, and their propagation velocity is smaller than that of the magnetoplasma waves by approximately $[T/s^2(m^+ + m^-)]^{1/2}$ times ($m^\pm$ are the hole and electron masses).

1. It was shown in [1,2] that mutual dragging of carriers and phonons exerts a strong influence on the kinetic properties of semimetals. Bass [3] called attention to the role of the mutual dragging of electrons and phonons in the propagation of electromagnetic waves in semimetals. He has shown that, in a semimetal with one type of carrier, a new cyclotron resonance is produced at a frequency $\Omega \approx eHs^2/Tc$, which is obtained from the cyclotron frequency of the electrons by replacing the electron mass by the phonon "mass" T/s^2 (T is the temperature in energy units and s the speed of sound). The resonance takes place at low frequencies, much lower than the electron-phonon and phonon-electron collision frequencies. The resonance takes place at low frequencies, much lower than the electron-phonon and phonon-electron collision frequencies.

In a real case of a semimetal with two types of carrier, the situation becomes more complicated. If the electrons and holes are scattered primarily by phonons, and the phonons primarily by electrons and holes, then an indirect interaction occurs between the electrons and the holes, consisting in the fact that carriers of one polarity influence the drift velocity of the carriers of the other polarity via the phonons dragged by them [2]. As a result, as shown in [2], in a weak magnetic field when the electrons and holes move in opposite directions, there is no mutual dragging in semimetals with equal densities of the electrons (n^-) and holes (n^+) and with isotropic constant-energy surfaces.

It can be shown that under the same conditions, there is likewise no low-frequency cyclotron resonance, owing to the indirect interaction in the semimetals.

2. We consider in this paper the influence of mutual dragging on the propagation of magnetohydrodynamic waves in semimetals with $n^+ = n^-$. Since the electrons and holes oscillate in such a wave in phase, the indirect interaction does not prevent mutual dragging.

Ordinary magnetohydrodynamic waves in semimetals (we shall henceforth call them magnetoplasma waves) are attenuated weakly when their frequency is $\omega \gg \nu$, where ν is the carrier collision frequency. The mutual dragging makes it possible for additional weakly-damped magnetohydrodynamic waves to propagate in semimetals at a frequency $\omega \ll \nu$ and a velocity much lower than that of the magnetoplasma waves by a factor, for a given magnetic field H, of $[T/s^2(m^+ + m^-)]^{1/2}$ (m^+ and m^- are the hole and electron masses). In these waves, simultaneous oscillations take place of the electron-hole plasma and of the phonons interacting with the electrons and holes. We shall henceforth call them plasma-phonon waves.

3. The propagation of electromagnetic waves in a conductor at frequencies $\omega \ll \sigma$ (σ = conductivity) is determined by Maxwell's equations

$$\mathrm{rot}\,\mathbf{H} = \frac{4\pi}{c}\hat{\sigma}\mathbf{E}, \quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \tag{1}$$

For a plane monochromatic wave $\mathbf{E} \sim \exp\{i(\mathbf{k}\cdot\mathbf{r} - \omega t)\}$, propagating in an unbounded conductor, we get therefore

$$k^2\mathbf{E} - \mathbf{k}(\mathbf{kE}) = 4\pi i\omega c^{-2}\hat{\sigma}(\omega)\mathbf{E}. \tag{2}$$

We neglect the spatial dispersion of the tensor $\hat{\sigma}$, a procedure that will be shown to be valid under the conditions considered here.

4. To obtain $\hat{\sigma}(\omega)$ in the case when the mutual dragging is significant, it is necessary to solve the system of kinetic equations for the distribution functions of the electrons (n_p^-), holes (n_p^+), and phonons (N_q)

$$i\omega(n_p^\pm - n_p^{\pm(0)}) + S_p(n_p^\pm; N_q) - \nu_d^\pm(n_p^\pm - n_p^{\pm(0)}) = \pm e(\mathbf{E} + c^{-1}[\mathbf{v}^\pm\mathbf{H}])\,\partial n_p^\pm/\partial\mathbf{p}, \quad (3)^*$$

$$i\omega(N_q - N_q^{(0)}) + S_q(N_q; n_p^+, n_p^-) - \beta_{fd}(N_q - N_q^{(0)}) = 0.$$

Here $S_p(n_p^\pm; N_q)$ are the electron-phonon and hole-phonon collision integrals, $S_q(N_q; n_p^+, n_p^-)$ is the collision integral for the phonons with the holes and electrons, and $\mathbf{v}^\pm$ are the hole and electron velocities.

The collision frequencies of the holes and of the electrons will be denoted by $\nu^\pm$ with an index describing the scattering mechanism (f - phonons, d - defects, no index - total collision frequencies). Similarly, β with an index denotes the frequency of phonon relaxation (d - by defects, whose roles are assumed principally by the boundaries of the sample, f - by phonons, e - by electrons, h - by holes), β without an index is the total phonon relaxation frequency. The energy spectrum of the electrons, holes, and phonons will be assumed to be isotropic and we shall consider temperatures in the interval $sp_F \ll T \ll \zeta$, where ζ and p_F are the Fermi energy and momentum.

Assuming

$$n_p = n_p^{(0)} - (\mathbf{V}(\varepsilon), \mathbf{p})\frac{\partial n_p^{(0)}}{\partial\varepsilon};$$
$$N_q = N_q^{(0)} - (\mathbf{U}(q), \mathbf{q})\frac{\partial N_q^{(0)}}{\partial(\hbar\omega_q)}, \quad (4)$$

we obtain from (3) a system of equations in terms of the drift velocities of the holes $\mathbf{V}^+(\epsilon)$, electrons $\mathbf{V}^-(\epsilon)$, and phonons $\mathbf{U}(q)$ with momenta $q < 2p_F$, interacting with the electrons and the holes:

$$(-i\omega + \nu^\pm)\mathbf{V}^\pm + \Omega^\pm[\mathbf{h}\mathbf{V}^\pm] - \frac{\nu_f^\pm}{4p^4}\int_0^{2p}\mathbf{U}(q)q^3\,dq = \pm\frac{e\mathbf{E}}{m^\pm}; \quad (5a)$$

$$(-i\omega + \beta)\mathbf{U} + \beta_h\int_{\varepsilon^+(q/2)}^{\infty}\mathbf{V}^+\frac{\partial n_p^{(0)}}{\partial\varepsilon}d\varepsilon + \beta_e\int_{\varepsilon^-(q/2)}^{\infty}\mathbf{V}^-\frac{\partial n_p^{(0)}}{\partial\varepsilon}d\varepsilon = 0 \quad (5b)$$

$^*[\mathbf{v}^\pm\mathbf{H}] \equiv \mathbf{v}^\pm \times \mathbf{H}$.

Here $\Omega^\pm$ are the cyclotron frequencies of the holes and the electrons, and $\mathbf{h} = \mathbf{H}/H$.

The frequencies of the phonon collisions with the holes (electrons) and the frequencies of the hole (electron) collisions with the phonons are related as follows:

$$\beta_h(q) = \frac{m^+s^2}{T}\frac{q}{2p_F}\nu_f^+ \ll \nu_f^+, \quad \beta_e(q) = \frac{m^-s^2}{T}\frac{q}{2p_F}\nu_f^- \ll \nu_f^-. \quad (6)$$

Since $\nu_f^\pm \sim T$, it follows that β_h and β_e do not depend on T. Recognizing that $\partial n_p^{(0)}/\partial\epsilon = -\delta(\epsilon - \zeta)$, we get from (5b):

$$\mathbf{U}(q < 2p_F) = \frac{\beta_h}{\beta - i\omega}\mathbf{V}^+(\zeta) + \frac{\beta_e}{\beta - i\omega}\mathbf{V}^-(\zeta). \quad (7)$$

Substituting (7) in (5a) we obtain a system of algebraic equations with respect to $\mathbf{V}^+$ and $\mathbf{V}^-$:

$$\mathbf{V}^+(1 - \gamma^+(\omega)) + \frac{\Omega^+}{\nu^+ - i\omega}[\mathbf{h}\mathbf{V}^+] - \gamma^+(\omega)\frac{\nu_f^- m^-}{\nu_f^+ m^+}\mathbf{V}^- = \frac{e\mathbf{E}}{m^+(\nu^+ - i\omega)}, \quad (8)$$

$$\mathbf{V}^-(1 - \gamma^-(\omega)) - \frac{\Omega^-}{\nu^- - i\omega}[\mathbf{h}\mathbf{V}^-] - \gamma^-(\omega)\frac{\nu_f^+ m^+}{\nu_f^- m^-}\mathbf{V}^+ = -\frac{e\mathbf{E}}{m^-(\nu^- - i\omega)}.$$

Here and throughout, all the quantities that depend on the energies of the electrons and holes, are referred to the Fermi surface.

The quantities

$$\gamma^\pm(\omega) = \frac{1}{4p^4}\frac{\nu_f^\pm}{\nu^\pm - i\omega}\int_0^{2p}\frac{\beta_{h,e}}{\beta - i\omega}q^3\,dq \quad (9)$$

characterize the degree of mutual dragging of the carriers and the phonons. As $\gamma^\pm \to 0$, the system of equations (8) breaks up into two independent equations which determine $\mathbf{V}^+$ and $\mathbf{V}^-$ in the absence of dragging. The mutual dragging is appreciable if $|\gamma^+|, |\gamma^-| \approx 1$. This means, first, that $\nu^\pm = \nu_f^\pm$ (the electrons and holes are scattered primarily by phonons) and $\beta = \beta_h + \beta_e$ (the phonons are scattered primarily by holes and electrons). With this, $\gamma^+(0) + \gamma^-(0) = 1$. Second, the frequency ω should be much lower than the collision frequencies ν and β, i.e., in order that the carriers and phonons drag each other, they must collide many times within a time equal to the oscillation period.

5. Solving the system (8) and calculating the current $j = ne(\mathbf{V}^+ - \mathbf{V}^-)$, we obtain the following expressions for the ohmic and Hall conductivities $\sigma_\parallel$ and $\sigma_\perp$:

$$\sigma_\parallel(\omega) = \frac{ne^2}{\Delta}\left[\frac{1 - \gamma^-(\omega)(1 + \nu_f^+m^+/\nu_f^-m^-)}{m^+(\nu^+ - i\omega)}\right.$$

$$+\frac{1-\gamma^+(\omega)(1+\nu_f^-m^-/\nu_f^+m^+)}{m^-(\nu^- - i\omega)}\Bigg]$$

$$\times\left[1-\gamma^+(\omega)-\gamma^-(\omega)+\frac{\Omega^+\Omega^-}{(\nu^+-i\omega)(\nu^--i\omega)}\right] \quad (10)$$

$$\sigma_\perp(\omega)=\frac{ne^2}{\Delta}\Bigg[\frac{1-\gamma^-(\omega)(1+\nu_f^+m^+/\nu_f^-m^-)}{m^+(\nu^+-i\omega)}$$

$$+\frac{1-\gamma^+(\omega)(1+\nu_f^-m^-/\nu_f^+m^+)}{m^-(\nu^--i\omega)}\Bigg]$$

$$\times\left[\frac{\Omega^+(1-\gamma^-(\omega))}{\nu^+-i\omega}-\frac{\Omega^-(1-\gamma^+(\omega))}{\nu^--i\omega}\right] \quad (11)$$

Here Δ is the determinant of the system (8):

$$\Delta=\left[1-\gamma^+(\omega)-\gamma^-(\omega)+\frac{\Omega^+\Omega^-}{(\nu^+-i\omega)(\nu^--i\omega)}\right]^2 + \left[\frac{\Omega^+(1-\gamma^-(\omega))}{\nu^+-i\omega}-\frac{\Omega^-(1-\gamma^+(\omega))}{\nu^--i\omega}\right]^2 \quad (12)$$

6. The time-dependent dispersion of the conductivity is due to both the frequency dependence of the electron and hole distribution function (a contribution on the order of $\omega/\nu^{\pm}$), and to the frequency dependence of the phonon distribution function (contribution on the order of ω/β). Since $\beta/\nu \approx ms^2/T \ll 1$, the main contribution to the frequency dependence of σ is made by the terms of order ω/β connected with the dragging, while the terms of order ω/ν, which cause temporal dispersion in the absence of dragging, can be neglected.

Under conditions of strong mutual dragging, i.e., when the following inequalities are satisfied:

$$\frac{\nu^\pm-\nu_f^\pm}{\nu^\pm}\ll 1,\quad \frac{\beta-\beta_e-\beta_h}{\beta}\ll 1,\quad \frac{\beta}{\omega}\ll 1, \quad (13)$$

and in strong magnetic fields satisfying the inequalities

$$\frac{\Omega^+\Omega^-}{\nu^+\nu^-}\gg\frac{\omega}{\beta},\quad \frac{\Omega^+\Omega^-}{\nu^+\nu^-}\gg 1-\gamma^+(0)-\gamma^-(0), \quad (14)$$

we get from (10)—(12)

$$\sigma_\| = \frac{nc^2M}{H^2}(\Gamma-i\omega), \quad (15)$$

$$\sigma_\perp = -i\frac{\omega cM}{eH}\frac{\nu^-m^- - \nu^+m^+}{\nu^-m^-+\nu^+m^+}\sigma_\|, \quad (16)$$

where

$$M=\frac{4}{3}\frac{T}{s^2},$$

$$\Gamma=\frac{3}{4}\beta(1-\gamma^+(0)-\gamma^-(0))+\frac{3}{2}\frac{\omega^2}{\beta}\left(1+\frac{\nu^+\nu^-}{\Omega^+\Omega^-}\right). \quad (17)$$

Thus, $\sigma_\|$ and $\sigma_\perp$ have the form customary for conductivity in a strong magnetic field, except that $m^+ + m^-$ is replaced by M, and the effective attenuation is Γ; by virtue of (14) and (6) we have $\sigma\perp \ll \sigma_\|$.

7. Substituting (15) and (16) in (2) we find in the usual manner [4] that in the frequency interval

$$1-\gamma^+(0)-\gamma^-(0)\ll\omega/\beta\ll 1 \quad (18)$$

there can propagate in the semimetal two weakly-damped plasma-phonon waves: a magnetosonic wave

$$\omega = Hk/\sqrt{4\pi nM} \quad (19a)$$

and an Alfven wave

$$\omega = (\mathbf{Hk})/\sqrt{4\pi nM}. \quad (19b)$$

The velocities of these waves are smaller than the velocities of the magneto-plasma waves by a factor $[4T/3s^2(m^+ - m^-)]^{1/2} \gg 1$. Their damping is equal to $\Gamma/2$ and is small if the double inequality (18) is satisfied.

We note that in the stationary case, the mutual dragging has a strong influence on the conductivity in a strong magnetic field if a condition weaker than (18) is satisfied, namely $1-\gamma^+(0)-\gamma^-(0) \ll 1$.

The expression for $1-\gamma^+(0)-\gamma^-(0)$ depends on the scattering mechanism that causes the loss of momentum by the system of electrons, holes, and phonons. If such a mechanism is the scattering of the phonons by the boundaries of a sample with dimension d, as is the case when $\nu_d/\nu_f \ll s/d\beta \ll 1$ or $\nu_d \ll (T/ms^2)s/d \ll \nu_f$, then

$$1-\gamma^+(0)-\gamma^-(0)=\frac{4}{3}\frac{s}{d\beta(2p)}\approx\frac{\beta_d}{\beta(2p)}\ll 1. \quad (20)$$

On the other hand, if $\nu_f \gg \nu_d \gg (T/sm^2)s/d$, then the momentum is lost essentially in scattering of electrons and holes by defects. With this,

$$1-\gamma^+(0)-\gamma^-(0)=\frac{m^+\nu_d^+ + m^-\nu_d^-}{m^+\nu_f^+ + m^-\nu_f^-}\ll 1. \quad (21)$$

Since the frequencies ω under consideration are smaller than the collision frequencies for electrons and holes, it follows that if the electron and hole mean free paths l^- and l^+ are smaller than the Larmor radius r, then the spatial dispersion is negligible if $kl^{\pm} \ll 1$. On the other hand, if $r \ll l^{\pm}$, then the spatial dispersion is negligible if $kr \ll 1$. These conditions are readily satisfied for a magnetosonic wave and for an Alfven wave when the angles between k and H are not too close to $\pi/2$.

8. The anisotropy of the electron and hole spectra, which takes place in most semimetals, leads, as noted in [2], to the appearance of groups of phonons which, by virtue of the energy and mo-

mentum conservation laws, interact only with one type of carrier, so that the indirect interaction of the electrons and the holes becomes weaker. Since, however, the strong mutual dragging of each type of carrier with "its own" and with "foreign" phonons remains in force, the results obtained here do not change significantly.

If the extrema of the energy bands are located near the boundaries of the Brillouin zone, then the scattering of the carriers by the phonons can be accompanied by umklapp processes. The question of the role of such processes in the effects under consideration is worthy of a special study.

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