QUASILINEAR THEORY OF EXCITATION OF ELECTROMAGNETIC WAVES IN A SEMICONDUCTOR BY AN ELECTRON BEAM

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Excitation of longitudinal electromagnetic waves in ionic semiconductors by an electron beam is investigated in the quasilinear approximation. An equation is derived for the "background" distribution function of the resonant beam particles. It resembles the diffusion equation in velocity space. Expressions for the frequency and increments of unstable electromagnetic waves are obtained by taking into account the polarization of the semiconductor and the oscillation damping due to carrier collisions. It is shown that the height of the distribution function "plateau" and hence the energy of the oscillation depends significantly on the beam temperature and on the carrier collision frequency. The energy lost by the beam as a result of excitation of the oscillations is determined.

1. INTRODUCTION

THE excitation of longitudinal electromagnetic oscillations in a plasma by an electron beam has been investigated in [1,2], where it was shown that in the quasilinear stage the reaction of the waves on the resonant particles of the beam leads to the formation of a "plateau" in the distribution function and to the saturation of the oscillations. Excitation of oscillations by an electron beam can also occur in solids. In the linear approximation, this problem has been examined in [3,4].

In the present paper we investigate the buildup of one-dimensional longitudinal oscillations in ionic semiconductors during the quasilinear stage. The beam-semiconductor system is assumed here to be spatially homogeneous. An essential step in solving this problem is the necessity to take into account the oscillation damping due to the collisions of the carrier particles with impurities and with one another (other collisions are usually negligible) and to consider the polarization of the semiconductor.

2. BASIC EQUATIONS

As a starting point we use the Vlasov kinetic equations with self-consistent electric field E

$$\frac{\partial f^{(\alpha)}}{\partial t}+v\frac{\partial f^{(\alpha)}}{\partial x}+\frac{e_\alpha}{m_\alpha}E\frac{\partial f^{(\alpha)}}{\partial v}=\mathrm{St}^{(\alpha)}\{f^{(\alpha)}\}, \qquad (2.1)$$

Maxwell's equation

$$\frac{\partial}{\partial t}(E+4\pi P)=-4\pi\sum_\alpha e_\alpha n_\alpha\int_{-\infty} vf^{(\alpha)}dv \qquad (2.2)$$

and the equation for the polarization vector P of the semiconductor without allowance for spatial dispersion[5,6]

$$\partial^2P/\partial t^2+\omega_0{}^2P=\gamma E, \qquad (2.3)$$

where $f^{(\alpha)}$ is the distribution function of the particles of type α (the superscript $\alpha = 1$) corresponds to the electrons of the beam, $\alpha = 2$ to the carriers) with charge e_α, mass m_α, and density n_α, ω_0 is the frequency of the exciton absorption, and γ is a constant characterizing the structure of the exciton bands. Because of the comparatively low energy of the oscillations excited in the beam-semiconductor system, anharmonicity of optical oscillations of the lattice may be neglected.

We represent the distribution function $f^{(\alpha)}$ as the sum of two terms:

$$f^{(\alpha)}=f_0^{(\alpha)}+F^{(\alpha)}. \qquad (2.4)$$

where $f_0^{(\alpha)}\equiv\langle f^{(\alpha)}\rangle$, $\langle F^{(\alpha)}\rangle=\langle E\rangle=\langle P\rangle=0$, and the averaging is carried out over distances much greater than the wavelengths and over times much longer than the periods of the oscillations, or else over a set of special phases. We expand $F^{(\alpha)}$, E and P in Fourier series

$$F^{(\alpha)}=\sum_k F^{(\alpha)}(v,t)e^{i(kx-\omega_kt)},\quad E=\sum_k E_k(t)e^{i(kx-\omega_kt)},$$

$$P=\sum_k P_k(t)e^{i(kx-\omega_kt)},$$

where

$$\frac{\partial}{\partial t}\ln F_k^{(\alpha)}\ll\omega_k,\quad \frac{\partial}{\partial t}\ln E_k\equiv\frac{\partial}{\partial t}\ln P_k\equiv\gamma_k\ll\omega_k.$$

Then we get from (2.1),

$$\frac{\partial f_0^{(\alpha)}}{\partial t}=\frac{e_\alpha}{m_\alpha}\frac{\partial}{\partial v}\sum_k E_k{}^*F_k^{(\alpha)}+\mathrm{St}^{(\alpha)}\{f_0^{(\alpha)}\}. \qquad (2.5)$$

$$F_k^{(\alpha)}=-i\frac{e_\alpha}{m_\alpha}\frac{E_k\partial f_0^{(\alpha)}/\partial v}{\omega-kv}, \qquad (2.6)$$

where

$$\omega\equiv\omega_k+i[\gamma_k+\nu_k^{(\alpha)}(v)],$$

$$\nu_k^{(\alpha)}(v)\equiv-\frac{\mathrm{St}^{(\alpha)}\{f_0^{(\alpha)}+F_k^{(\alpha)}\}-\mathrm{St}^{(\alpha)}\{f_0^{\prime(\alpha)}\}}{F_k^{(\alpha)}}\ll\omega_k$$

Using (2.6), we find from Eqs. (2.2) and (2.3) the dispersion equation

*This is a sample of proposed type substituting for DSJ.

$$\varepsilon'(\omega + i\gamma_k) + \sum_\alpha \frac{\Omega_\alpha^2}{\omega_k + i\gamma_k} \int_{-\infty}^{\infty} \frac{v\,dv}{\omega - kv} \frac{\partial f_0^{(\alpha)}}{\partial v} = 0, \qquad (2.7)$$

where

$$\varepsilon'(\omega_k) = 1 - \frac{4\pi\gamma}{\omega^2 - \omega_0^2}, \quad \Omega_\alpha^2 = \frac{4\pi e_\alpha^2 n_\alpha}{m_\alpha}.$$

It is clear from (2.7) that when $\Omega_1^2 u/\Omega_2^2 v_{T_1} \ll 1$ (u is the mean velocity of the beam, v_{T_1} is its thermal velocity), the expression for the excited frequencies ω_k does not depend on the parameters of the beam. Since quasilinear theory is applicable when $\gamma_k \ll k v_{T_1}$, the dispersion equation (2.7), with due allowance for this inequality, readily reduces to the form

$$\varepsilon'(\omega_k) + i\gamma_k \frac{\partial \varepsilon'(\omega_k)}{\partial \omega_k} - \frac{\Omega_2^2}{\omega_k^2} + i\frac{\Omega_2^2}{\omega_k^3}(2\gamma_k + \nu_k) - \frac{i\pi\Omega_1^2}{k|k|}\frac{\partial f_0^{(1)}}{\partial v} = 0, \qquad (2.8)$$

where the effective carrier collision frequency ν_k is defined by the expression

$$\nu_k = \int_{-\infty}^{\infty} dv\, f_0^{(2)}(v) \frac{\partial}{\partial v}[v\nu_k^{(2)}(v)]. \qquad (2.9)$$

From (2.8), we find the expressions for the excited frequencies and increments

$$\omega_k^2 \equiv \omega_{1,2}^2 = \frac{\omega_0^2 + \Omega_2^2 + 4\pi\gamma}{2} \pm \left[\frac{(\omega_0^2 + \Omega_2^2 + 4\pi\gamma)^2}{4} - \omega_0^2\Omega_2^2\right]^{1/2}, \qquad (2.10)$$

$$\gamma_k \equiv \frac{1}{2E_k}\frac{\partial |E_k|^2}{\partial t} = Z\left\{\frac{\pi}{2}\frac{\Omega_1^2\omega_k^3}{\Omega_2^2 k|k|}\frac{\partial f_0^{(1)}}{\partial v} - \frac{\nu_k}{2}\right\}_{v=\omega_k/k},$$

$$Z = \left[1 + \frac{\omega_k^3}{2\Omega_2^2}\frac{\partial \varepsilon'(\omega_k)}{\partial \omega_k}\right]^{-1}. \qquad (2.11)$$

Since the phase velocity of the wave is much greater than the thermal velocity of the carriers, the reaction of these waves on the carrier distribution may be neglected. Then, substituting $F_k^{(1)}$ from (2.6) into Eq. (2.5) for the beam, and neglecting the Coulomb collisions of the electrons in the beam, we obtain the equation for the diffusion of the beam electrons in velocity space

$$\frac{\partial f_0^{(1)}}{dt} = \frac{\partial}{\partial v} D \frac{\partial f_0^{(1)}}{\partial v}, \qquad (2.12)$$

where

$$D = \frac{\pi e^2}{m_1^2}\sum_k |E_k|^2 \delta(\omega_k - kv).$$

3. QUASILINEAR STAGE

Let us assume that the initial "background" distribution function of the beam particles is Maxwellian:

$$f_0^{(1)}|_{t=0} = f_{0M}^{(1)} \equiv \frac{1}{\sqrt{2\pi}\, v_{T_1}} \exp\left[-\frac{(u - v)^2}{2v_{T_1}^2}\right]. \qquad (3.1)$$

Then for the case $u \gg v_{T_{1,2}}$ and

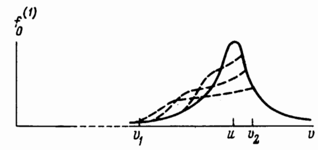
$$\gamma_k \lesssim k v_{T_1} \qquad (3.2)$$

the reactions of the oscillations excited in the velocity region where $\gamma_k > 0$ leads to deformation of the beam-particle "background" distribution function described by (2.12).

From (2.11) it follows that the waves with

$$\gamma' \equiv \frac{\pi\Omega_1^2\omega_k^3}{\Omega_2^2 k|k|}\frac{\partial f_0^{(1)}}{\partial v} \gg \nu_k. \qquad (3.3)$$

grow the fastest. Allowance for the reaction of only these oscillations on the distribution function within a time $\tau \leq \nu_k^{-1}$ leads to the establishment of a plateau-like distribution in the velocity region $v_1 \leq v \leq v_2$ (see the diagram). Here $v_2 \approx u + v_{T_1}$, and the boundary v_1



is approximately determined by the equation $\gamma'|_{v = v_1} = \nu_k$, from which we find that

$$v_1 \approx u - \sqrt{2}\, v_{T_1} A, \qquad (3.4)$$

where

$$A = \left(\ln \sqrt{\pi}\frac{\omega_k}{\nu_k}\frac{\Omega_1^2}{\Omega_2^2}\frac{u^2}{v_T^2}\right)^{1/2}.$$

It is interesting to observe that v_1 depends weakly on ν_k.

Assuming for simplicity that at the instant $t = \tau$ a "plateau" has been established in the velocity interval $v_1 < v < v_2$, we find its height from the condition that the number of particles must be conserved

$$f_0^{(1)}|_{t=\tau} = \frac{1}{v_2 - v_1} \approx \frac{1}{v_{T_1}(1 + \sqrt{2}A)}. \qquad (3.5)$$

If the quantity ν_k in (2.11) is neglected in comparison with γ' (which can be done for times $t < \tau$ in the velocity interval $v_1 < v < v_2$), we readily obtain from (2.11) and (2.12) that[1]

$$|E_k|^2 = \frac{4\pi^2 m_1 n_1 \omega_k u^3}{\Omega_2^2} Z \int_{v_1}^{v_2} (f_0^{(1)} - f_{0M}^{(1)})\, dv. \qquad (3.6)$$

At the instant $t = \tau$ the quantity γ' becomes of the order of ν_k in the entire region of instability and the oscillations stop growing. In this case, the maximum intensity in time of the k-th harmonic is determined from (3.6)

$$|E_k|^2|_{t=\tau} = \frac{4\pi^2 m_1 n_1 \omega_k u^3}{\Omega_2^2} Z\left\{\frac{v - u + \sqrt{2}\, v_{T_1} A}{v_{T_1}(1 + \sqrt{2}A)} - \frac{1}{2}\left[1 + \frac{v - u}{|v - u|}\Phi\left(\frac{|v - u|}{v_{T_1}}\right)\right]\right\} \qquad (3.7)$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\, dt.$$

The potential energy of the oscillations W_n at the instant $t = \tau$ is given by the expression

$$W_n = \frac{1}{8\pi}\sum_k \varepsilon' |E_k|^2 = \frac{n_1 m_1 u v_{T_1}}{4}\sum Z(\sqrt{2}A - 1), \qquad (3.8)$$

where the summation is carried over the different modes of the oscillations. From (3.8), we find that the fraction of the initial energy of the beam lost in exciting the oscillations is equal to $(v_{T_1}/u)\sum \Xi(\sqrt{2}A - 1)$.

We now investigate in greater detail the conditions of applicability of the obtained solution. To this end, it suffices to require that inequalities (3.2) and (3.3) be satisfied at the initial instant of time, when the velocity distribution of the beam particles is still Maxwellian. Determining the maximum values of γ' and γ_k in this case, and substituting them into inequalities (3.2) and (3.3), we obtain respectively

$$0.3Z\frac{\Omega_1^2}{\Omega_2^2}\frac{u^3}{v_{T_1}^3}\lesssim 1, \quad (3.9)$$

$$0.3\frac{\Omega_1^2}{\Omega_2^2}\frac{u^2}{v_{T_1}^2}\gg\frac{\nu_k}{\omega_k}. \quad (3.10)$$

For most semiconductors the collision frequency is high, so that for the oscillation mode with $Z \sim 1$ the inequalities (3.9) and (3.10) turn out to be incompatible. In this case, however, for another oscillation mode the quantity Z may be sufficiently small so that inequalities (3.9) and (3.10) are satisfied. If both oscillation modes can build up in the semiconductor, then the mode with the greater Z will build up more rapidly.

In many cases, by changing the carrier density (for example, by illumination), the frequency of one of the oscillation modes (with the greater Z) may increase so much that it enters the region of intrinsic absorption; in this case the branch of oscillations with $Z \ll 1$ will be excited.

In conclusion we make some numerical estimate of the effect of excitation of oscillations in a semiconductor with the parameters $n_2 \cong 10^{15}$ cm^{-3}, $m_2 \cong 0.1m_e$ (m_e = electron mass), $\omega_0 \approx 10^{13}$ sec^{-1}, $4\pi\gamma = 1.1\omega_0^2$, and $\nu_k = 10^{11}$ sec^{-1} (such parameters are characteristic for Ge[7]) by a beam with $n_1 = 10^{12}$ cm^{-3}, $m_1 = m_e$, and $u^2 = 10^3 v_{T_1}^2$. It is not difficult to show that conditions (3.9) and (3.10) are satisfied with the greater increment and frequency $\omega_1 = 1.5 \times 10^{13}$ sec^{-1}, and that 0.2% of the initial energy is lost, according to (3.8), in exciting this branch.

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39