LIMITING CURRENTS AND ELECTRON–ION OSCILLATIONS IN QUASI-NEUTRAL ELECTRON BEAMS

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It is demonstrated experimentally that the restriction (cutoff) phenomenon for a current in a quasi-neutral inhomogeneous electron beam moving along an external magnetic field is in accord with the modern theory of instability of such a beam with respect to buildup of axial asymmetric electron–ion oscillations. In the range of the system parameters in which the oscillations are weakly manifest (such as for small beam electron energies in strong magnetic fields), the restriction (cutoff) of the beam current occurs later and is defined by the well known Pierce instability.[4]

INTRODUCTION

IT is well known[1-3] that the current in a beam of charged particles having the same sign and propagating in an equipotential space has an upper limit (I_0) such that the space charge of the particles leads to the formation of a virtual cathode in the beam. If this space charge is compensated by particles of opposite polarity, then it might appear that the limiting beam current could be greatly increased. However, as shown theoretically by Pierce,[4] an electron beam whose space charge is compensated by (immobile) ions becomes unstable at a current strength I_P which exceeds I_0 by merely 5–6 times. The mechanism of pure electronic instability observed by Pierce is connected with the reaction of the charges induced in the beam-confining walls by any (negative) fluctuation of the beam potential: when $I > I_P$, the electric field of these charges intensifies the initial fluctuation, and the growth of the (negative) sag of the potential continues until a virtual cathode is produced in the beam.

More than twenty years have elapsed since the publication of Pierce's paper. During that time, his result was subjected to numerous theoretical and experimental verifications. However, whereas the theoretical papers agree in the main with Pierce's paper and constitute either further concretization of his work[5] or generalization to a nonlinear case,[6] the conclusions of the experimental papers are quite contradictory (see [2, 3, 7, 8]). Thus, according to the experiments of [7], the limiting current in a compensated electron beam coincides with Pierce's current ($I_L = I_P$), but according to the monographs,[2, 3] whose authors refer to unpublished experiments, this current does not differ practically from the limiting current (I_0) in a beam without ions. In the latest of the experimental papers known to us[8] they investigated only a very particular case, in which a beam (of 1 cm diameter) fills an equipotential tube completely, and the electron energies are limited to the low value 60 eV; in this particular case, the limiting currents were equal to the Pierce currents.

In view of such gaps in the experimental data, we deemed it expedient to carry out the present investigation, the purpose of which was to measure systematically the limiting currents in an electron beam and to investigate the mechanism of their limitation. With respect to this mechanism, we have assumed that it can be connected with a buildup of low-frequency (lf) electron–ion oscillations, which generate strong electric fields that offset the compensation of the beam space charge. Therefore, the purpose of some of the experiments was to study such oscillations and their connection with the limiting currents in an electron beam.

1. EXPERIMENTAL PROCEDURE

The experimental setup is shown in Fig. 1. A beam of primary electrons, emitted by an indirectly-heated tungsten cathode and accelerated in an electric field of the two grids to an energy ($W_1 = eV_1$) of several hundred electrons volts, propagated along the axis of an equipotential (grounded) stainless steel cylinder parallel to an external time-invariant magnetic field. The beam diameter was $2a = 1$ cm, the cylinder diameter $2R_0 = 30$ cm, and the path length L was regulated by moving an anode (15 cm diameter) in a range from 5 to 150 cm. The equipotential volume in which the beam propagated was bounded on the ends by grounded grids with 2 mm mesh, made of tungsten wires 0.2 mm diameter. The distribution of the magnetic field along the beam was homogeneous (accurate to $\sim 3\%$), and the field intensity H was varied from 100 to 8000 Oe.

The space charge of the electron beam was neutralized by means of the ions produced by the beam by the residual gas, the pressure of which was usually (1–2) $\times 10^{-6}$ mm Hg. In order for such neutralization to be possible, two conditions must be satisfied.

First, if the beam is pulsed, then the rate of increase of the beam density must be lower than the rate of formation of ions; otherwise, there will be not enough time to compensate the beam. In our experiments we used initially two beam regimes: continuous and quasi-intermittent—with a current rise time $\tau \approx 2$–3 sec.

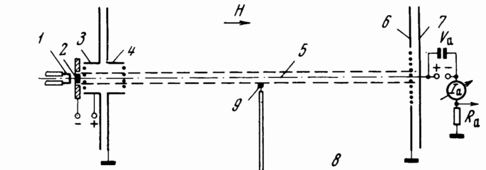

FIG. 1. Experimental setup: 1 – filament, 2— cathode, 3, 4 and 6 – diaphragm, 5 – beam, 7 – anode, 8 – vacuum, 9 – probe.

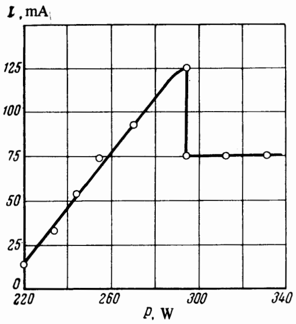

FIG. 2. Dependence of beam current (to the anode) on the cathode heating power P. V_1 = 550 V, H = 1000 Oe, L = 100 cm, p = 2 × 10^{-6} mm Hg, $2R_0$ = 30 cm.

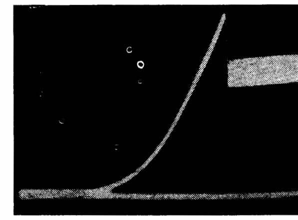

FIG. 3. Oscillogram of electron current to the anode in the quasi-intermittent beam mode. Sweep duration 3 sec, V_1 = 600 V, H = 4000 Oe, L = 100 cm, p = 1 × 10^{-6} mm Hg, $2R_0$ = 30 cm.

Thus, this condition was satisfied with a "margin" of several orders of magnitude, unlike the investigation in [9], where this condition was not satisfied. Since (as will be shown below) our measurements of the limiting current in the electron beam yielded a result that deviated greatly from that of [7], we also measured the limiting currents by the method described in [7], in order to ascertain the cause of this discrepancy. In the latter measurements we used a pulsed beam in which the time of smooth current rise ranged from several microseconds to several tenths of a millisecond, and the electric circuitry of these measurements was precisely that used in [7]. Some of the measurements were made with a setup having the same geometry as in [7], that is, $2R_0$ = 6 cm and L = 10 cm.

Second, the possibility of rapid loss of beam ions at the ends of the system, along the external magnetic field, must be eliminated. In some experiments[3, 9, 10] this condition is not satisfied, and the ions move freely to the cathode. In our experiments this condition was satisfied with the aid of a so-called "ion trap": a time-invariant positive potential V_C = +(50–100) V was applied to the anode (which was located behind a grounded grid) and to a specially introduced "cathode" grid (Fig. 1).

The third condition concerns the neutral-gas pressure. The point is that the only experiments that can be compared with Pierce's theory[4] are those in which the electron beam is not "overcompensated," that is, the ion density does not exceed the beam density; otherwise the beam will contain slow (secondary) electrons, whereas the theory of [4–6] is valid only if there are no such electrons. In order to satisfy this condition, it is necessary to measure the limiting currents of the beam at sufficiently low gas pressure, such that the beam has a small negative potential relative to the walls; this potential retains the ions and pushes out slow electrons. This condition was satisfied in our experiments.

The setup (Fig. 1) included several disc probes (5 mm diameter) which could be moved both along the magnetic field and transversely to it. With the aid of these probes and with the aid of an SCh-8 spectrum analyzer we investigated the frequency characteristics of the lf oscillations and measured the plasma parameters.

The state of the beam in which the virtual cathode was formed was identified by measuring the energies of electrons moving opposite to the beam.[11] The presence of a grounded "anode" grid and the positive potential on the anode has practically eliminated the possible interference to the beam-current measurements on the part of the secondary-emission electrons from the anode.

Before describing the measurements of the limiting currents in the electron beams, we must make the following important remark. As is well known,[12] in an electron beam passing through a rarefied gas, under conditions close to those of our experiments, high-frequency (hf) electronic oscillations may arise and smear out the beam-electron velocity-distribution function and decrease the average beam velocity. It is also known[12] that methods are available for eliminating these oscillations, namely reducing the pressure of the residual gas, attenuating the secondary electron emission from the anode, or decreasing the beam length. With the aid of these measures, we measured the limiting currents in beams under conditions such that there were practically no high frequency oscillations, and the energy spectrum of the beam electrons was close to a δ-function (see [12]).

The limiting current of the electron beam was measured in the following manner. At a fixed beam-electron energy, we plotted the electron current to the anode against the cathode temperature, the latter being regulated by varying the power used to heat it by electron bombardment. A plot of the current measured under constant conditions is shown in Fig. 2. At a certain cathode-heating power, a jumpwise decrease occurred in the electron current to the anode. Since the instant of this jump coincided with the instant of formation of a virtual cathode in the beam (errors determined by the method described in [11]), the magnitude of the anode current directly before the jump was taken to be the limiting current I_l. Measurements in the quasi-intermittent mode, the cathode heating was turned off after a long pause, and the cathode temperature, and with it the beam current, began to increase with a thermal time constant on the order of several seconds. The corresponding oscillogram is shown in Fig. 3. It does not differ in its character from the oscillogram obtained in the pulsed mode, Fig. 4.

The dependence of the maximum beam current (I_{max}) on the front rise time (τ) is shown in Fig. 5. We see that when $\tau \gtrsim 300$ μsec the maximum current does not depend on τ. Therefore the value of I_{max} at $\tau > 300$ μsec was taken to be the limiting current (I_l). Accordingly, $I_{max} = I_0$ when $\tau \approx 5$ μsec.

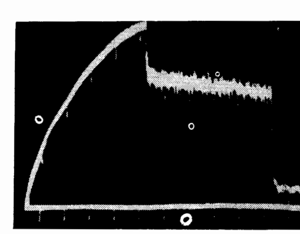

FIG. 4. Oscillogram of electron current to the anode in the pulsed beam mode. Sweep duration 10 msec, V_1 = 600 V, H = 4000 Oe, L = 100 cm, p = 1 × 10^{-6} mm Hg, I_l = 180 mA. The arrow denotes the instant of cutoff of the beam pulse. $2R_0$ = 30 cm.

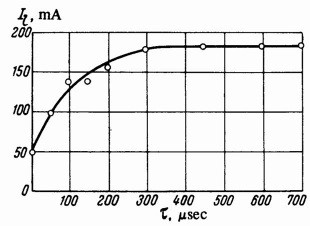

FIG. 5. Maximum beam current I_{max} vs. current rise time (τ). H = 4000 Oe, V_1 = 600 V, L = 100 cm, p = 1 × 10^{-6} mm Hg. $I_{max} = I_0$ when $\tau \to 0$; $I_{max} = I_l$ when $\tau \gtrsim 300$ μsec. $2R_0$ = 30 cm.

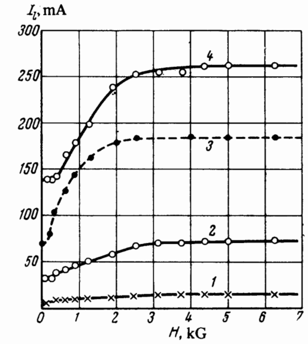

FIG. 7. Limiting current vs. magnetic field for different electron energies. L = 100 cm, $2R_0$ = 30 cm. 1, 2, 4 – Continuous regime, 3 – pulsed regime (τ = 600 μsec); 1 – W_1 = 100 eV, 2 – W_1 = 300 eV, 3 – W_1 = 600 eV, 4 – W_1 = 900 eV.

2. EXPERIMENTAL DATA

1. Limiting Currents

Solid curves 2 and 3 of Fig. 6 show the dependence of the limiting current, measured in the continuous mode on the beam-electron energy in different magnetic fields, from 100 Oe (curve 2) to 6500 Oe (curve 3). The limiting currents at 100 Oe < H < 6500 Oe depend on the energy of the electrons in perfect analogy, and lie in the shaded region between curves 2 and 3; an increase of H in excess of 6500 Oe has practically no effect on the limiting current (saturation). The theoretical curve 1 of Fig. 6 gives the limiting currents (I_0) in a beam without ions; for our case ($L > 2R_0 \gg 2a$), the Smith-Hartman formula applies:[1]

$$I_0 \approx \frac{25 \cdot 10^{-6} V_1^{3/2}}{1 + 2\ln(R_0/a)}, \tag{1}$$

where I_0 is in amperes, and $V_1 = W_1/e$ is energy of the beam electrons in volts. Curve 4 of Fig. 6 corresponds to Pierce's theory,[4] extended in [5] to the case of the geometry under consideration:

$$I_P \approx \frac{150 \cdot 10^{-6} V_1^{3/2}}{1 + 2\ln(R_0/a)} \tag{2}$$

(I_P is in amperes and V_1 is in volts). The dashed curves 2′ and 3′ were taken in the pulse mode (τ = 600 μsec) and pertain to H equal to 100 and 6500 Oe. The dashed curve 1′ is obtained in the pulsed mode at τ = 5 μsec.

We see that in the range of electron energies that are not too small ($W_1 \gtrsim 200$ eV), the limiting beam current (I_l) is much smaller than the Pierce current (I_P). The relation between I_l and I_P greatly depends on the magnetic field intensity. Thus, in weak magnetic fields the current I_l is smaller than I_P by a factor 3–4 and differs relatively little from the current I_0 in the ionless beam. In the case of strong magnetic fields (corresponding to saturation of the I_l(H) dependence) I_l exceeds I_0 by approximately 3.5 times and approaches I_P, remaining nevertheless smaller than I_P by an approximate factor 1.2–2 times.

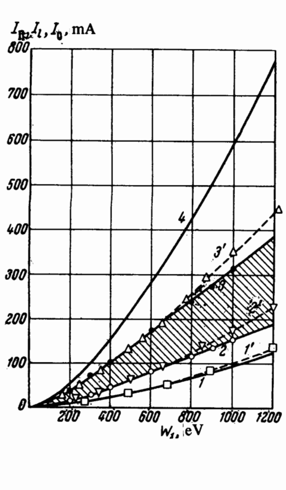

FIG. 6. Limiting current vs. beam-electron energy at different values of the magnetic field, $2R_0$ = 30 cm, L = 100 cm. 1 – Beam without ions, theory [1]; 1′ – experiment in the pulsed regime (τ = 5 μsec), H = 4000 Oe; 2 and 3 – experiment in the continuous regime, H = 100 and 6500 Oe; 2′ and 3′ – experiment in pulse regime (τ = 600 μsec), H = 100 and 6500 Oe; 4 – Pierce's theory [4,5].

In the region of small electron energies, ($W_1 \lesssim 100$ eV) and in strong magnetic fields (H = 4000 Oe), the limiting currents coincide with the Pierce currents. Figure 6 shows that the limiting currents of the beam in the continuous and in the pulsed modes coincide, and the value of I_0, measured in the pulsed mode at $\tau \approx 5$ μsec, corresponds with the theoretical one,[1] determined by formula (1).

Figure 7 shows the dependence of I_l on H at different beam–electron energies. In connection with the presence of an appreciable I_l(H) dependence, it must be noted that the function $I_{max}(\tau)$ shown in Fig. 5, was plotted at H = 4000 Oe, corresponding to saturation of the I_l(H) dependence. Figure 5 shows that the current I_l exceeds I_0 not by 6 times (as should be the case if the limitation of the current in the beam were to be governed by the Pierce instability[4]), but only by 3.5 times (and not more than).

Thus, the experimental data shown in Figs. 5–7 indicate that in the entire range of variation of the system parameters (with the exception of the region of small beam energies, $W_1 \lesssim 100$ eV), the limiting currents of the beam are greatly smaller than the Pierce values[4,5] determined by formula (2).

These results are patently in disagreement with the conclusions of [7], according to which the limiting currents measured at beam-electron energies 500–2500 eV are, first, equal to the Pierce values (that is, they are determined by formula (2) and exceed the values of I_0 by approximately 6 times) and, second, do not depend on H—in the range from 20 to 2000 Oe.

Since the geometry of our setup differed from the geometry given in [7] in having a larger tube diameter, (in our case $2R_0$ = 30 cm, whereas in [7] $2R_0$ = 6 cm), we have carried out a series of additional measurements of limiting currents, for the purpose of understanding the causes of the aforementioned discrepancy, at two other tube diameters, $2R_0$ = 10 cm and $2R_0$ = 6 cm. The results of these measurements reduced to the following.

A. At $2R_0$ = 10 cm, in the range of energies $W_1 \gtrsim 300$ eV, the limiting current practically coincides with the values measured at $2R_0$ = 30 cm and represented in Figs. 5–7. According to formula (2), this denotes that in this case the difference between I_l and I_P becomes even larger than when $2R_0$ = 30 cm. In the range of small beam energies ($W_1 \lesssim 100$ eV) and in

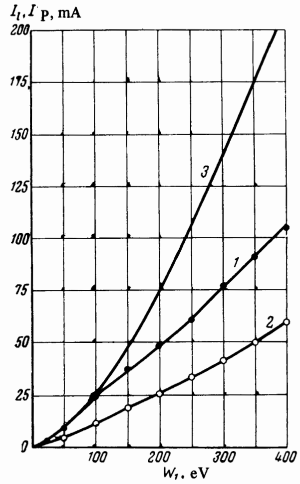

FIG. 8. Dependence of the limiting current on the beam electron energy in the case when $2R_0$ = 10 cm. 1 – H = 4000 Oe, 2 – H = 600 Oe, 3 – Pierce's theory [4,5], L = 100 cm.

strong magnetic fields (H $\gtrsim$4000 Oe), the limiting currents, just as in the case $2R_0$ = 30 cm, coincide with the Pierce currents (that is, in particular, $I_l \propto W_1^{3/2}$). However, when $W_1 \gtrsim$ 150 eV the $I_l(W_1)$ dependence "straightens out" and becomes close to linear. In a weaker magnetic field, the deviation of the limiting currents from the Pierce values begins at lower beam-electron energies. In view of the importance of these facts for the further exposition (see Sec. 3), we present Fig. 8, which differs from the analogous Fig. 7 in showing more details at small beam energies. Figure 8 shows (much more distinctly than Fig. 7) the change in the character of the dependence of $I_l(W_1)$, occurring both when W_1 is increased and when H_1 is decreased.

B. Under conditions of precisely the same geometry as in [7], that is, for $2R_0$ = 6 cm and L = 10 cm, the character of the oscillograms of the beam current and the character of the $I_{max}(\tau)$ and $I_l(H)$ dependences had the same form as in the two preceding cases (Figs. 3, 4, 5, 7). The results of the measurements for the case of $2R_0$ = 6 cm and L = 10 cm are shown in Fig. 9. We see that in moderate magnetic fields (H $\lesssim$ 1200 Oe) the limiting currents are smaller than the Pierce currents by a factor of approximately 2. In sufficiently strong fields, that is, H $\gtrsim$ 4000 Oe (more than double the maximum field of [7]), there is a beam energy range (W $\lesssim$ 500 eV) in which the limiting currents are quite close to the Pierce currents, or even coincide with them. However, at high beam-electron energies the limiting currents again are much smaller than the Pierce currents, owing to the already noted fact of "rectification" of the $I_l(W_1)$ dependence.

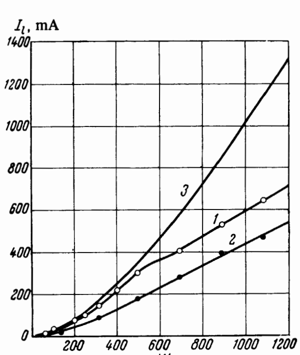

FIG. 9. Dependence of the limiting current on the energy of the beam electrons in the case when $2R_0$ = 6 cm. 1 – H = 5200 Oe, 2 – H = 1200 Oe, 3 – Pierce's theory [4,5]. L = 10 cm.

Thus, when the tube diameter is decreased from $2R_0$ = 30 cm to $2R_0$ = 6 cm, all the phenomena described above (Figs. 3–8) remain valid in principle, the only difference being that the range of small beam energies, within which $I_l \approx I_P$ (in the case of strong magnetic fields) is broadened. Outside this range of the parameters W_1 and H, the limiting currents in the beam remain as before smaller than the Pierce currents and depend essentially on the magnetic field. It appears to us, therefore, that the conclusion drawn in [7], namely that $I_l = I_P$ (accurate to 10–20%) in the entire range of W_1 from 500 to 2500 eV, and furthermore in relatively weak magnetic fields (H = 20–2000 Oe), is in error.

It also follows from the results of experiments with different R_0 that the value of R_0 influences significantly the limiting beam current only when the ratio R_0/a does not exceed several times unity. This is in good agreement with the result of experiment[8] in which the limiting current for $W_1 \lesssim$ 60 eV and R_0/a = 1 turned out to be equal to the Pierce values. This result agrees fully with our data (see, for example, Fig. 8).

Concluding the comparison of experimental data on the limiting currents in beams, we note that in the monographs [2,3], which refer to unpublished papers, it is concluded that $I_l \approx I_0 \ll I_P$; unfortunately, there are no indications in these references to the conditions of the corresponding experiments.

Thus, the aggregate of the available experimental data allows us to state that those cases, in which the limiting currents in the electron beams are equal to the Pierce currents (that is, are determined by formula (2)) pertain to a relatively narrow range of experimental conditions and are more an exception than a general rule. In a much wider range of variation of the system parameters, the limitation of the currents and the beams takes place much earlier than called for by Pierce's theory. The very fact of this limitation (current cutoff) denotes that the compensation of the space charge is upset, in spite of satisfaction of all the necessary preliminary conditions. Inasmuch as in our experiments there were no causes capable of producing statistical decompensation of the beam, it must be assumed (in opposition to the point of view advanced in [13]), that the mechanism whereby the beam compensation is violated is dynamic, that is, it is connected with its instability. Therefore further experiments were aimed at investigating the lf oscillations (in which ions can participate) in the beam.

2. Electron–Ion Oscillations

The experiments reported in this section had as their purpose, first, to identify qualitatively the oscillations responsible for the interruption of the current in an initially quasineutral electron beam. Therefore we paid principal attention to a study of the conditions for the occurrence of these oscillations and their connection with those conditions under which current cutoff takes place in the beam.

The experiments have shown that oscillations with frequencies of hundreds of kHz develop in a compensated electron beam with $I < I_l$. Typical spectra of these oscillations at different beam currents are shown

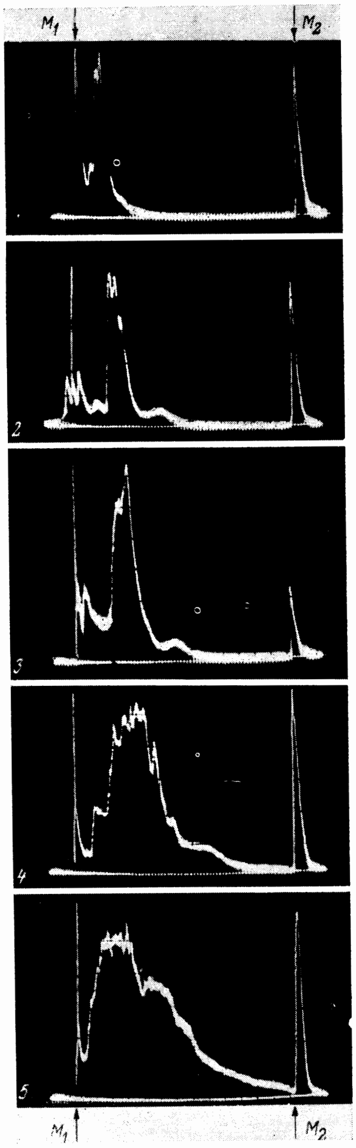

FIG. 10. Spectra of low frequency oscillations – dependence of the amplitude of the probe-current oscillations on the frequency (linear scales on the axis). The arrows indicate the frequency markers M_1 and M_2 ($f_{m1} = 0$), $W_1 = 300$ eV, $p = 2 \times 10^{-6}$ mm Hg, L = 100 cm, H = 2000 Oe, $f_{m2} = 1200$ kHz. 1 – I = 7 mA, 2 – I = 18 mA, 3 – I = 23 mA, 4 – I = 48 mA, 5 – after formation of virtual cathode ($I_1 = 53$ mA). $2R_0 = 30$ cm.

in Fig. 10. We see that with increasing beam current the spectrum of the oscillations broadens appreciably and the frequency corresponding to the maximum of the oscillation amplitude greatly increases (the absolute value of this frequency is close to that of the ion Langmuir frequency for N_2^+ ions).

The experiments have also shown that the beam-current cutoff (formation of the virtual cathode) is always preceded by a sharp growth in the amplitude of the lf oscillations, beginning at a certain "critical" current I_{cr} (Fig. 11). The ratio of this critical current to the limiting beam current (I_l) with changing system parameters (in particular, the values of L, W_1, and H) changes relatively little: under the conditions of our experiments this ratio was in the range from 0.7 to 1; the lower of these limits corresponded to the larger values of L(~ 100 cm) and smaller H (hundreds of Oe), and the upper one to smaller L (tens of cm) and larger H (thousands of Oe). Therefore, in attempting the theoretical interpretation of the experimental data, we shall henceforth, by way of a qualitative approximation, set I_{cr} equal to I_l, neglecting some quantitative differences between them.

3. Discussion of Experimental Data

The investigation of the nature of the lf oscillations described above (for $I > I_{cr}$) was started with an attempt to establish their connection with the hf electronic oscillations, which are also excited under conditions close to those of our experiments,[12] and which have frequencies ranging from several MHz to several tens of MHz. It has turned out that these two types of oscillations are not directly connected: there are conditions under which the lf oscillations are produced in spite of the absence of hf oscillations. This takes place either in sufficiently small magnetic fields ($H \leq 500$ Oe), or at a small beam length ($L \leq 40$ cm), or at a small gas pressure ($p \leq (1-2) \times 10^{-6}$ mm Hg), and in the absence of a large number of secondary-electron emission from the anode; in the latter case no hf oscillations are produced, owing to the absence of a sufficient number of slow electrons.[12] 1)

In order to attempt to understand the possible mechanism of the buildup of lf oscillations in quasineutral electron beam (with $I > I_{cr}$), we turn to the theory of electron-ion oscillations in electron beams (see [13-21]). We take a system of charged particles with a radius limited to a, made up of fast (beam) electrons, ions, and slow (plasma) electrons, and situated in a longitudinal magnetic field. The direction of the magnetic field coincides with the direction of motion of the beam (z axis). We consider the possibility of spontaneous buildup, in this system, of oscillations such that the wavelength along z is much larger than the radius ($\lambda_z = 2\pi/k_z \gg a$) and the particle oscillates in the following manner: the electrons along the magnetic field, and the ions almost transversely to the magnetic field (such a system is close to the model of the quasineutral electron beam under the conditions of our experiments). In this case we take immediately account of the circumstance that there are facts which do not fit within the framework of the representations of the two-stream instability of the homogeneous plasma. First, there is the very pronounced fact that the critical current (I_{cr}), corresponding to a sharp increase in the observed lf oscillations (Fig. 11), and the limiting current of the beam (I_l) decrease greatly with decreasing magnetic field (Fig. 7), whereas according to the theory of the homogeneous ("beam") plasma[13-18, 21] the stability of the beam in the range of conditions under consideration should not depend on the magnetic field (see below). Second, in the experiment, at sufficiently small H and large W_1, the limiting (critical) current of the beam is more readily proportional to the square of the beam velocity (u) than to its cube (as would follow from the theory of a homogeneous plasma[12]). Therefore, in order to obtain a more consistent comparison of theory with experiment, we have turned directly to the theory of two-stream instability of an inhomogeneous plasma,[19, 20] in which effects connected with the radial

1)For a more complete identification of the nature of the low-frequency oscillations that precede the current cutoff in a quasi-neutral electron beam, we are presently investigating their spatial and spectral characteristics.

inhomogeneity of the beam are taken into account.

We consider potential oscillations with frequencies lying in the range

$$\omega_{Hi} < \omega < k_z u \tag{3}$$

(if the ions of the residual gas have a mass on the order of that of the nitrogen molecule, then those inequalities in (3) correspond to the conditions of our experiments). In this case the dispersion equation of the oscillations (without account of the thermal motion of the particles) takes the form[19]

$$\left[\frac{\omega_1^2}{(\omega-k_z u)^2}\frac{k_z^2}{k^2}-\frac{2s\omega_1^2}{k^2a^2\omega_{He}(\omega-k_z u)}\right] + \left[\frac{k_z^2}{k^2}\frac{\omega_2^2}{\omega^2}-\frac{2s\omega_2^2}{k^2R^2\omega_{He}\omega}\right]+\frac{\omega_+^2}{\omega^2}=1, \tag{4}$$

where ω_1, ω_2, and ω_+ are the Langmuir frequencies of the beam electrons, plasma electrons, and ions, ω_{He} and ω_{Hi} are the Larmor frequencies of the electrons and ions, k_z is the projection of the wave vector of the oscillations k under direction of the electron oscillations (we can put $k_z \approx 2\pi/L$), a is the "radius" of the beam, s is the number of the azimuthal mode (the number of azimuthal wavelengths spanned by the perimeter of the beam), and R is the "radius" of the plasma (in [19] we consider an example of quadratic distribution of the density over the radius of the beam and the plasma).

The left side of (4) contains three terms (square brackets) which take into account, respectively, oscillations of three components of the system: beam electrons, plasma electrons, and plasma ions. Each of the electronic terms consists of two parts, of which the first takes into account the longitudinal oscillations, and the second the transverse (drift) motion in the crossed fields: perturbed electric field (E_φ) and the main magnetic field (H). The third ("ionic") term consists of one part, since it is possible in our approximation ($\omega > \omega_{Hi}$) to neglect the "drift" motion of the electrons in first approximation.

From (4) we can see that since $\omega < k_z u$, the drift (or convective) terms have opposite signs for the electrons of the beam and for the electrons of the plasma; the convective beam term contributes to the buildup of oscillations, while the plasma term contributes to their damping. Therefore the buildup of axially asymmetrical oscillations ($E_\varphi \neq 0$, that is, $s \neq 0$) is possible only if the plasma convective term is smaller than the beam term, that is, if

$$\alpha \equiv \frac{n_2}{n_1} < \frac{\omega}{k_z u}\frac{R^2}{a^2}, \tag{5}$$

where n_1 and n_2 are the densities of the beam and plasma electrons. Since $\omega < k_z u$, the condition (5) denotes that when $R \approx a$ we have $\alpha < 1$. We assume below that condition (5) is satisfied, and therefore we neglect in the dispersion equation (4) the second member of the second term compared with the second member of the first term. We then get in lieu of (4)

$$\frac{\omega_1^2}{(\omega-k_z u)^2}\frac{k_z^2}{k^2}-\frac{2s\omega_1^2}{k^2a^2\omega_{He}(\omega-k_z u)}+\frac{\omega_+^2}{\omega^2}\left[1+\frac{\alpha}{1+\alpha}\frac{M}{m}\frac{k_z^2}{k^2}\right]=1, \tag{4'}$$

where m/M is the ratio of the electron mass to the ion mass.

From (4′) we can determine the frequency of the oscillations (in general—complex: $\omega = \omega_r + i\gamma$) and the critical current of their excitation I_{cr} (by definition I_{cr} is the beam current such that $\gamma > 0$ when $I > I_{cr}$). Neglecting in (4′) the value of ω compared with $k_z u$, we get

$$\omega^2 \approx \omega_+^2\left[1+\frac{\alpha}{1+\alpha}\frac{M}{m}\frac{k_z^2}{k^2}\right]\Big/\left\{1-\frac{\omega_1^2}{k^2u^2}\left[1+\frac{2su}{a^2\omega_{He}k_z}\right]\right\}, \tag{6}$$

$$I_{cr} \equiv \frac{m}{4e}a^2u\omega_{1cr} \approx \frac{ma^2}{4e}k^2u^3\left(1+\frac{2su}{a^2\omega_{He}k_z}\right)^{-1} \tag{7}$$

(ω_{1cr} is the Langmuir frequency of the beam when its current is I_{cr}) and the instability increment for $I \gg I_{cr}$:

$$\gamma \sim (k_z u\omega_{Hi}/s)^{1/2}. \tag{6'}$$

The accuracy of expressions (6) and (7) is determined by the ratio of the terms in the dispersion equation (4′). It is easy to show that if the convection term of the beam is large compared with the "ordinary" beam term, then expressions (6) and (7) are sufficiently accurate. In such a case (which should be realized, for example, in sufficiently weak magnetic fields, the oscillations are almost aperiodic ($\gamma \gg \omega_r$). In the opposite case we get from (6)

$$I_{cr} \approx \frac{ma^2}{4e}k^2u^3. \tag{7'}$$

The value of I_{cr} determined by (7′) coincides with the Pierce current[4, 5] and with the almost equivalent current of the buildup of the so-called Buneman oscillation,[15] which were considered for the first time in [14]. The instability increment in this case (when $I \gg I_{cr}$) is

$$\gamma \approx k_z u\left[\alpha+(1+\alpha)\frac{m}{M}\frac{k^2}{k_z^2}\right]^{1/2}. \tag{6''}$$

From (7) and (7′) we see that the convective effects ($s \neq 0$) decrease the critical excitation current of the oscillations.

We now proceed to a comparison of the developed theory with experiment.

1. The beam Langmuir frequency ω_{1P} corresponding to the Pierce current is equal to [4, 5]

$$\omega_{1P}^2 = k^2u^2. \tag{8}$$

Comparison of (8) with (7) and (7′) shows that at not too large values of H the excitation of the low frequency oscillations under consideration should begin at currents much lower than the Pierce currents, and the instability should have an almost aperiodic character (that is, it can lead to an interruption of the beam). For example, for a = 0.5 cm, s = 1, u = 10^9 cm/sec, L = 100 cm, $k_z \approx 2\pi/L \approx 6\times10^{-2}$ cm^{-1}, and H = 1500 Oe, the second term in the denominator (7) amounts to approximately 4, and $I_{cr} \approx I_P/5$. This is in agreement with the experimentally established fact that if H is not large enough the limitation of the current in the beam takes place in currents that are smaller than I_P by an approximate factor of 3.5 (Figs. 6–8).

2. If we neglect in the denominator of (7) the unity term responsible for the instability of the homogeneous beam, we get

$$I_{cr} \propto \omega_{1cr}^2 u \propto u^2 H \propto W_1 H. \tag{7''}$$

We see that I_{cr} is proportional to W_1 and H. Expression (7″) is valid only for sufficiently small H. At large

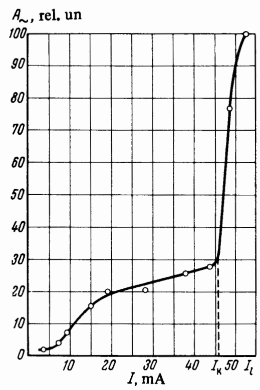

FIG. 11. Dependence of the amplitude of the low frequency oscillations of the probe current (H~) on the beam current. $W_1 = 300$ eV, $H = 2000$ Oe, $L = 100$ cm, $p = 2 \times 10^{-6}$ mm Hg, $2R_3 = 30$ cm.

H (and small u) the ratio of the terms in the denominator of (7) is reversed (the inhomogeneity of the beam ceases to influence the buildup of the oscillations), and I_{cr} ceases to depend on H; then, $I_{cr} \to I_P$. These conclusions of the theory agree with the form of the $I_{cr}(H, W_1)$ plots shown in Figs. 6–9. In particular, it is seen from Figs. 8 and 9 that for sufficiently large values of H and small W_1 the limiting current coincides with the Pierce current, that is, $I_{cr} \propto W_1^{3/2}$; at small values of H and large values of W_1 we get the already-noted transformation of the $I_{cr}(W_1)$ into an almost linear relation in accordance with (7″).

Thus, the theoretically calculated I_{cr} determined by formulas (7) and (7′) can be identified with the experimentally measured critical current at which an abrupt increase of the low frequency oscillations (Fig. 11) begins and eventually (when $I = I_l$) cuts off the beam current. It should be borne in mind (Figs. 6–9) that in the experiments we always have $I_l > I_0$. Therefore the measured values of I_l and the theoretical values of I_{cr} no longer agree under such conditions (for example, when H is too small), whereas from (7) it follows formally that $I_{cr} < I_0$. As regards the oscillations existing in the "pre-critical" regime ($I < I_{cr}$ in Fig. 11), we shall not stop to interpret them in this paper. We note only that similar oscillations were observed in [9, 10, 22], where certain points of view were advanced with respect to their possible mechanisms.

3. The presented theory shows (see condition (5)) that the presence of a certain admixture of plasma electrons in the beam increases the critical current I_{cr}, in that the increase is the larger, the greater the velocity of the beam electrons and the smaller the beam length. In particular, in a short beam of high-energy electrons (for example, as in [7], where L = 10 cm and $W_1 = (0.5-2.5$ keV), the presence in the beam of an even relatively small admixture of plasma electrons can be to a noticeable increase in the limiting beam current. Accordingly, our experiments have shown that in an insufficiently outgassed installation it is possible to obtain larger limiting currents than those shown in Figs. 6–9.

4. The influence of the tube diameter ($2R_0$) on the limiting beam current (Figs. 6–9) is apparently due (besides the cause connected with formula (3)) to the fact that for sufficiently small R_0 it is very difficult to axially asymmetrical oscillations (E_φ, $s \neq 0$) to appear, and effects connected with the inhomogeneity of the beam cease to play an important role.

Thus, the results of our comparison of the experimental data with the theory give serious grounds for assuming that the limitation (cutoff) of the current in a quasineutral electron beam—in a wide range of variation of its parameters—is caused by the axially asymmetrical electron-ion oscillations of a homogeneous "beam" plasma.[2)] The main laws governing the excitation of these oscillations (critical excitation currents and their dependence on the velocity of the beam and the magnetic field intensity) are correctly described by modern theory. For a more complete verification of the theory it is necessary to investigate the spatial structure of the electron-ion oscillations and their dispersion properties. Such an investigation is now being carried out by us.

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