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WEAKLY INTERACTING PARTICLES IN THE ANISOTROPIC COSMOLOGICAL MODEL

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The possibility of a significant anisotropy in the expansion of the metagalaxy at various stages is currently being discussed in the literature. According to the "hot cosmological model" interaction of weakly interacting particles (neutrinos, gravitons) with other particles and with one another ceases during the expansion. Various momentum components of the freely moving particles are transformed differently during the anisotropic expansion, and the particles begin to move predominantly along a certain direction. It is shown that if such particles are taken into account, then the general pattern of anisotropic expansion and the physics of the processes are greatly altered at an early stage.

1. INTRODUCTION

RECENTLY, homogeneous anisotropic cosmological models have been under extensive discussion.[1-9] There exists a class of anisotropic solutions, which approach the isotropic solution asymptotically during the course of expansion and consequently do not contradict the facts concerning the presently observed isotropy. Interest in such models is due, besides the natural tendency to investigate a broader class of solutions satisfying the observations than the strictly isotropic one, also to the fact that the anisotropic solutions contain an additional parameter, the choice of which leads to essentially different consequences for the chemical composition of free stellar matter.[5,6] It may turn out that it will be necessary to forego a rigorous isotropic solution in order to reconcile the theory of the hot cosmological model with the data on the chemical composition of old stars. Another special cause of interest in anisotropic solutions is the possibility of including in them the magnetic field.[10,7,8]

The purpose of the present article is to show that an analysis of isotropic solution cannot be made without taking into account the role of the weakly-interacting particles (gravitons, neutrinos) which are present in the prestellar matter. Allowance for these particles changes radically the picture of the anisotropic expansion and the physics of the processes during the early stage.[11] The cause of the singular behavior of weakly-interacting particles lies in the fact that different momentum components of the freely moving particles are transformed differently in the anisotropic solution, and the equilibrium spherically-symmetrical distribution of the particle momenta is transformed into preferred motion along one direction (both signs of which are on par). In addition, it is possible that there is no thermodynamic equilibrium at all in the anisotropic solutions at superhigh densities. The consequences ensuing from these phenomena are discussed in the succeeding sections.

We note that if the anisotropic homogeneous solution really existed in the past, then as a result of all the processes it might turn out in particular that the modern average energy of the relict neutrinos is much larger than the energy of the relict quanta corresponding to $T = 3^\circ K$, and relict neutrinos might even be observable by modern means.

2. ANISOTROPIC VACUUM SOLUTION

Let us consider the anisotropic homogeneous plane cosmological solution

$$ds^2 = c^2dt^2 - a_1^2(t)\,dx_1^2 - a_2^2(t)\,dx_2^2 - a_3^2(t)\,dx_3^2. \tag{1}$$

As is well known,[12] such a solution always has near a singularity an asymptotic form

$$\begin{gathered} a_1 = a_{10}t^{p_1}, \quad a_2 = a_{20}t^{p_2}, \quad a_3 = a_{30}t^{p_3}; \\ p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1, \quad p_1 \leqslant p_2 \leqslant p_3. \end{gathered} \tag{2}$$

It follows from (1) that the co-moving volume varies in proportion to $(a_1a_2a_3) \sim t$, the number of the conserved particles per unit volume varies like t^{-1}, and the density of the ultrarelativistic gas with $P = \varepsilon/3$ varies like

$$\rho = \varepsilon/c^2 = Kt^{-4/3}. \tag{3}$$

The character of the expansion during the early stage is determined completely by specifying one exponent, say the smallest one $p_1 = -\alpha$, for which it follows from (2) that $0 \leq \alpha \leq 1/3$. Accordingly, the matter is compressed along the corresponding axis (with the exception of the case $\alpha = 0$). The density of the ultrarelativistic gas is determined by the constant K.

During the early stage, quantities of the form $(\dot{a}/a)^2$ or $\ddot{a}/a$, which enter into the gravitational equations, are of the order t^{-2}, whereas $\varepsilon \sim P \sim t^{-4/3}$, and the terms $4\pi G\rho$ and $4\pi GP/c^2$, which describe the matter, can therefore be neglected.[1)] Thus, the presence of matter does not influence the dynamics of the expansion, as is emphasized by E. Lifshitz and Khalatnikov.[12] This can be called the "vacuum" stage.

It is easy to determine the instant $t = \theta$ of the end of the vacuum solution, when matter already begins to in-

[1)] This conclusion is not valid only for an expanding rigid equation of state [13], which is not considered here. A physical interpretation of the deformation in the anisotropic model is given in [14].

fluence the deformation. To this end, the terms $4\pi G\rho$ and $4\pi GP/c^2$ should be of the order of $\ddot{a}/a \sim t^{-2}$. With the aid of (3) we get

$$\theta^{-2} \approx 4\pi GK\theta^{-4/3}, \quad \theta \approx G^{-3/2}K^{-3/2}. \tag{4}$$

When $t > \theta$, the solution approaches rapidly the isotropic solution:[2)]

$$a_1 \sim a_2 \sim a_3 \sim t^{1/2}, \quad \rho = 3/32\pi Gt^2. \tag{5}$$

Nuclear reactions in the "hot model" under the conditions of such a cosmological solution were considered by Thorne.[6] He showed that the concentration of He^4 in prestellar matter is larger than 30% (by weight) when $1 < \theta < 3 \times 10^{10}$ sec, less than 30% when $\theta > 3 \times 10^{10}$ sec, and can be very small for larger θ. At the same time, for the interval 3×10^9 sec $< \theta < 10^{12}$ sec we obtain a large deuterium content: $C_D \approx 10-25\%$. To satisfy the two conditions $C_{He^4} < 30\%$ and $C_D < 1\%$ we must have $\theta > 10^{13}$ sec. In this case there will be practically no He^4 at all. We note that at large values of θ the only nuclear process is neutron capture by protons. It is easy to show that under these conditions $C_D = \text{const}\ \theta^{-1/2} \ln(\tau_1/\tau_2)$, where τ_1 is the instant when photodissociation stops, $D + \gamma \rightarrow p + n$, and τ_2 is the time of spontaneous decay of the neutron, equal to 1000 sec.

3. FREE PARTICLES IN ANISOTROPIC SOLUTION

We turn to weakly interacting particles—gravitons and neutrinos. For neutrinos, for example, during the early stages of expansion at large temperatures, the scattering, creation, and annihilation precede rapidly enough to maintain complete thermodynamic equilibrium between the neutrinos and the other particles.[3)] In particular, the neutrino density is $\sim T^3$, their average energy ~ 5 kT, and the momentum distribution is isotropic. We shall call this the Pascal stage of expansion (the energy-momentum tensor is isotropic). Starting with a certain instant $t = \tau$, the processes of neutrino interaction become slow compared with the expansion, and the neutrinos are already free particles.

Assume that during the course of further expansion, when $t > \tau$, the free particles interact neither with other particles nor with one another. We shall show in Sec. 5 that this assumption is not always valid for neutrinos, and that the picture is more complicated but the assumption can always be valid for gravitons.

In the isotropic solution, the separation of the neutrinos from the other particles cause no violation of equilibrium, since neutrinos, as well as γ quanta and e^+e^- pairs "cool" in accordance with the same law (with $kT > m_e c^2$). In the anisotropic solution, the density and the momentum of the free particles vary in accordance with the cosmological expansion; each momentum component i_1, i_2, i_3 varies in inverse proportion to a_1, a_2, a_3. The energy $E = c|i|$ is determined by the largest momentum component, so that the average energy is $\overline{E} \sim t^{-p_1} = t^{\alpha}$. The distribution of the particles in momentum space becomes all the more sharply anisotropic.

Assume that at the instant of separation $t = \tau$ the particle energy density ε^* was a fraction β of the total energy density. During the course of expansion, in the vacuum stage, we have

$$n^* \sim t^{-1}, \quad E^* \sim t^{\alpha}, \quad \varepsilon^* \sim t^{\alpha-1} \tag{6}$$

(n^* is the particle density). Consequently, when $t > \tau$ we have

$$\rho^* = \beta K\tau^{-4/3}(t/\tau)^{\alpha-1}. \tag{7}$$

Here ρ^* becomes the principal term in the total density, and the energy-momentum tensor is quite anisotropic:

$$-T_0^0 = \varepsilon \approx c^2\rho^* \approx T_1^1 \gg T_2^2, T_3^3. \tag{8}$$

The presence of free particles, whose energy density decreases more slowly than that of the interacting particles, reduces appreciably the period of the applicability of the vacuum solution. Preceding in analogy with the determination of θ in Sec. 2, let us obtain this instant $t = q$ (using (7)):

$$q = \theta(\tau/\theta)^{(7-3\alpha)/(9-3\alpha)}\beta^{1/(\alpha-3)}. \tag{9}$$

The ratio of the free-particle energy density ε^* to the energy density of all other particles ε_1 at the instant of termination of the vacuum solution is

$$\varepsilon^*/\varepsilon_1 \approx \frac{\beta}{1-\beta}\left(\frac{q}{\tau}\right)^{\alpha+1/3} = \frac{\beta^{2(4-3\alpha)/3(3-\alpha)}}{1-\beta}\left(\frac{\tau}{\theta}\right)^{2(3\alpha+1)/9(\alpha-3)}. \tag{10}$$

Consequently, inasmuch as β is not particularly small, the effects under consideration shorten the duration of the vacuum solution and bring about $\varepsilon^* \gg \varepsilon_1$ at the end of the vacuum stage.

4. COSMOLOGICAL SOLUTION WITH ANISOTROPIC ENERGY-MOMENTUM TENSOR

In the model without free particles, the vacuum stage was followed by a rapid isotropization of the solution at $t > \theta$. In the model under consideration here, the vacuum stage is followed when $t > q$ by a stage in which the dominating influence in the dynamics is played by the free particles which have a highly anisotropic energy-momentum tensor: $-T_0^0 \sim T_1^1 \gg T_2^2, T_3^3$

The cosmological solution with such an energy momentum tensor is of the form

$$\begin{gathered} -T_0^0 = T_1^1 = \varepsilon, \quad T_2^2 = T_3^3 = 0; \\ a_1 = a_{10}\, y^{P^2-1/4} e^y, \quad a_2 = a_{20}\, y^{1/2-P}, \quad a_3 = a_{30}\, y^{1/2+P}, \\ t = \tau_0 \int y^{P^2-1/4} e^y dy, \quad \varkappa\varepsilon = \tau_0^{-2} y^{-1/2-2P^2} e^{-2y}, \\ \varkappa = 8\pi G/c^2, \quad 0 \leqslant P \leqslant 1/2. \end{gathered} \tag{11}$$

The asymptotics of this solution as $y \rightarrow 0$ and $\varepsilon \rightarrow \infty$ are described by the vacuum solution (2), and the connection between α and P^2 is given by the expression $\alpha = (1 - 4P^2)/(3 + 4P^2)$. The instant of termination of the vacuum solution q corresponds to $y_0 = 1/4 - P^2$. When $y > y_0$, the gravitation of the particles comes into play: compression along the x axis is replaced by expansion, and on the other hand the expansion along x_2 and x_3 slows down. When $y \gg 1$ we have

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[2)] For an exact solution with asymptotic (2) at $t < \theta$ and (5) at $t > \theta$ see [8].

[3)] In anisotropic models, unlike the isotropic ones, conditions are possible when there is no thermodynamic equilibrium at all (for details see the end of Sect. 5). We assume here that during the earlier stages of the expansion there is thermodynamic equilibrium.

$$a_1 \sim t, \quad a_2 \sim (\ln t)^{1/2+P}, \quad a_3 \sim (\ln t)^{1/2-P};$$
$$\varepsilon^* \sim t^{-2}(\ln t)^{-1/2-2P^2}, \quad n \sim t^{-1}(\ln t)^{-P^2-3/4}, \quad E^* \sim t^{-1}(\ln t)^{1/4-P^2}. \quad (12)$$

Thus, the free particles whose energy E* increased during the vacuum stage upon compression along x_1, give rise when $t > q$ to such a realignment of the solution, as to cause their energy to begin to drop rapidly.[4)]

The change in the energy density of particles whose momentum is directed essentially along the j axis is determined by the relation $\varepsilon_j^* \sim n i_j \sim n a_j^{-1}$. The change in the density of the interacting particles is determined by the relation $\varepsilon_1 \sim n^{4/3}$. Using these relations and (2), (9), (10), and (12) we can find that particles whose momenta are directed along the x_1 axis will play a dominating role up to the instant $t = t_1$:

$$t_1 \approx \theta\beta^{P-3/2}\delta^{2/3P}[\ln(\beta^{-1}\delta^{-2/3})]^{1/2-P}, \quad \delta = \tau/\theta. \quad (13)$$

By the instant t_1, the energy density of the particles whose momenta are directed primarily along x_1 and x_2 become equalized, and greatly exceed the energy density of all other particles: $\varepsilon^*_{x_1} \approx \varepsilon^*_{x_2} \gg \varepsilon^*_{x_3}, \varepsilon_1$.

After the time t_1, the principal role is played by particles with momenta along x_2: $\varepsilon^*_{x_2} \gg \varepsilon^*_{x_1}, \varepsilon^*_{x_3}, \varepsilon_1$, and the solution (11) is again valid, except that a_1 is along the x_2 axis and other values of the parameters are used. Now the expansion proceeds along the earlier axis x_2, and the expansion in the x_1 and x_3 directions is weak. Such a realignment of the solution causes, after a certain time, a rapid drop in $\varepsilon^*_{x_2}$ and an instant when $\varepsilon^*_{x_2} \approx \varepsilon^*_{x_1}$ again is reached. Thus, the two axes will interchange roles and the anisotropy will be "vibrational" with decreasing amplitude and frequency, until all the energy densities become the same order of magnitude: $\varepsilon^*_{x_1} \sim \varepsilon^*_{x_2} \sim \varepsilon^*_{x_3} \sim \varepsilon_1$ (we assume β to be of the order of unity), after which isotropization of the solution begins.

To estimate the isotropization time t_2, let us consider two limiting cases.

1. Assume that the axes x_2 and x_3 in the solution (1) are equivalent, that is, $P = 0$. Then already at the end of the first period of the "oscillations," when $t = t_1$, we get $\varepsilon^* \approx \varepsilon_1$, after which rapid isotropization begins. Consequently, for α close to $1/3$, the isotropization time t_2 can be obtained from (13) with $P = 0$

$$t_2 \approx \theta\beta^{-3/2}[\ln(\beta^{-1}\delta^{-2/3})]^{1/2}. \quad (14)$$

2. Another limiting case corresponds to $\alpha = 0$. In this case we have in the vacuum stage

$$\varepsilon_1^* \approx \varepsilon_2^* = \text{const}, \quad \varepsilon_3^* \sim t^{-2}, \quad \varepsilon_1 \sim t^{-4/3}. \quad (15)$$

After the vacuum stage, the dominating role is played by particles with momentum not along any one axis (as in (11)), but on the x_1, x_2 plane with equal probability along any direction in this plane. The energy-momentum tensor of the particles is in this case

$$-T_0^0 = \varepsilon, \quad T_1^1 = T_2^2 = \varepsilon/2 \geqslant T_3^3. \quad (16)$$

Neglecting the energy of all the other particles, we obtain the exact solution

$$a_1 = a_2 = a_{10}(1 + t/t_0)^{2/3}, \quad a_3 = a_{30}[1 - (1 + t/t_0)^{-1/3}];$$
$$\varkappa\varepsilon = {}^4/_9\, t_0^{-2}(1 + t/t_0)^{-5/3}[(1 + t/t_0)^{1/3} - 1]^{-1}. \quad (17)$$

The effective instant of the essential change in the vacuum solution is t_0. It follows from (15) that by the instant $t_0 \approx \beta^{-1}\theta\delta^{1/3}$ we shall have $\varepsilon_1^* = \varepsilon_2^* \gg \varepsilon_3^*, \varepsilon_1$.

When $t > t_0$ the expansion, in analogy with (11), again proceeds in such a way that the energy density of particles whose momenta lie in the plane decreases more rapidly than $\varepsilon^*_{x_3}$ and ε_1. By the instant

$$t_1 \approx \beta^{-5/2}\theta\delta^{-2/3} \quad (18)$$

we have $\varepsilon_1^* = \varepsilon_2^* \approx \varepsilon_3^* \approx \varepsilon_1$ (β is on the order of unity), and isotropization sets in.

Thus, when α is close to zero, when the more rapid expansion occurs alternately along the x_1 and x_2 axes, but $\varepsilon_1^*, \varepsilon_2^* \gg \varepsilon_3^*, \varepsilon_1$, we can use (18) to estimate the isotropization time. Consequently, in this case

$$t_2 \approx \theta\delta^{-2/3}. \quad (19)$$

In the general case, the isotropization time lies between the values (14) and (19). The isotropization of the solution has in the general case a vibrational character, and when the anisotropy is already small, the solution is written in the form

$$a_j \sim a_{j0}t^{1/2}\left\{1 + b_j t^{-1/4}\sin\left[\frac{\sqrt{3}}{2}\ln(c_j t)\right] + \ldots\right\}. \quad (20)$$

This means that the isotropic expansion of the free particles is stable.

We note finally that for gravitons the "separation" occurs under conditions when quantum gravitation effects are significant.[15] Therefore the gravitons may not be in equilibrium with the other particles at the instant of "separation."

In the presence of equilibrium $\beta \sim 10^{-2}$, but if there is no equilibrium, then β can differ even more from unity. If $\beta \ll 1$, then we should have after isotropization at any rate $\varepsilon^* \approx \varepsilon_1$, that is, a contemporary graviton energy density on the order of the quantum energy density, amounting to approximately 10^{-12} erg/cm^3 or $\sim 10^{-33}$ g/cm^3. This means that when $\beta \ll 1$ the distribution of the graviton momenta is anisotropic, and if at the instant of "separation" their average energy was of the order of the energy of the other particles, but their density was small, $n^* \ll n_1$, then at the present time we also have $n^* \ll n_1$, and $\bar{E}^* \gg E_\gamma$.

So far we have disregarded the possibility of instability of the described processes. The anisotropic vacuum solution itself is exponentially unstable.[12] If the initial perturbations are small, then this instability may not have time to develop. But isotropization of a strictly directional particle beam can occur as the result of collective interaction. We shall not consider this here. We note only that if such isotropization occurs during the stage $E^* \gg E$, then the graviton energy is larger than the γ-quantum energy even at present.

As to the results of Thorne on the chemical composition of prestellar matter,[6] they are valid without stipulation only in the case when all the processes occur during the vacuum stage. For other values of the anisotropic parameter it is necessary to take into ac-

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4)An analogous re-arrangement of the solution with the magnetic field under anisotropic axially-symmetrical compression leads to limitation of the growth of the field, as noted in [6] and [8]. This phenomenon is connected not with the pressure gradient (the medium is homogeneous), but with the relativistic effect of the gravitational influence of the anisotropic pressure.

count the fact that there are periods when the volume density during the phase of the nuclear reactions does not vary in the same manner as in Thorne's calculations. Thus, for example, in the model (17) and in the time interval $t_0 = \theta\delta^{1/3} < t < \theta\delta^{-2/3}$, the volume density varies like $n \sim t^{-4/3}$.

5. NEUTRINOS IN THE ANISOTROPIC SOLUTION

The behavior of the neutrinos differs from that considered above in that in the case of anisotropy that is accompanied by compression along one of the axes during the early stage, a fraction of the neutrinos and the antineutrinos traveling opposite to them acquire an energy such that under certain conditions the probability of their irreversible transformation into electrons and positrons again becomes noticeable. This does not take place for gravitons, since the graviton interaction cross sections do not depend on the energy.[16]

We shall assume at first that there are no gravitons at all,[5] and consider for concreteness the behavior of electronic neutrinos. The combined behavior of the neutrinos and the gravitons will be considered later.

The instant of "separation" of the neutrinos τ will be determined from the condition that the relaxation and hydrodynamic times be equal, which yields

$$f = \sigma n c \tau = 1. \tag{21}$$

Here σ is the cross section of the interaction $e^+ + e^- \rightleftarrows \nu_e + \bar{\nu}_e$; $\sigma \sim E^2$, and if $E < 300$ GeV we shall assume this condition to be satisfied (the case $E > 300$ GeV will be discussed at the end of this section); n is the concentration of the particles and c is the velocity of light.

We shall assume that $\tau < \theta$, for otherwise the neutrinos become separated after isotropization of the solution and no anisotropy effects occur. Up to the instant τ, during the Pascal stage, we have $\sigma \sim E^2 \sim t^{-2/3}$ and $n \sim t^{-1}$. In the isotropic Friedman model $\sigma \sim E^2 \sim t^{-1}$ and $n \sim t^{-3/2}$. Knowing that in the Friedmann model the instant of separation (the separation temperature is ~ 3 MeV) $\tau' \approx 0.1$ sec for[6] ν_e or $\tau' \approx 5 \times 10^{-3}$ sec for ν_μ, and using the relations written out above, we express τ in terms of τ' and θ. We get

$$\tau = (\tau')^{9/4}\theta^{-5/4}. \tag{22}$$

The instant of separation is determined by the energy density and by the rate of volume expansion, and since these quantities do not depend on α, it follows that τ is likewise independent of α.

When $t > \tau$, the average energy of neutrinos with momenta along x_1 increases, $\bar{E}^\nu \sim i_1 \sim t^\alpha$, $0 \le \alpha \le 1/3$. Accordingly, the interaction cross section increases. For the neutrino scattering process

$$\nu_e + e \rightleftarrows \nu_e + e \tag{23}$$

the cross section σ is proportional to $\bar{E}_\nu \bar{E}_e$. From the condition $\sigma n c t < 1$ of free motion of the neutrino and from the relations $n \sim t^{-1}$ and $\bar{E}_e \sim t^{-1/3}$ we find that this process admits of a growth of $\bar{E}_\nu$ not faster than $t^{1/3}$, but since $\alpha \le 1/3$, $\bar{E}_\nu$ cannot increase more rapidly. Thus, after the Pascal stage the neutrinos do not interact with other particles.

However, the opposing streams of neutrinos and antineutrinos along the x_1 axis do not interact. Indeed, for the process

$$\nu_e + \bar{\nu}_e \to e^+ + e^- \tag{24}$$

the cross section $\sigma \sim E_\nu^2 \sim t^{2\alpha}$, and if the number of particles is conserved the condition $\sigma n c t < 1$ will not be satisfied. Consequently, the process (24) will lead during the vacuum stage to an irreversible transformation of the neutrinos into e^+, e^- pairs which are instantaneously thermalized.[7] Let us determine the rate of growth of the average neutrino energy $\bar{E}_\nu$ and the decrease of their concentration n_ν under these conditions.

In momentum space, the neutrino distribution is represented by an ellipsoid with axes i_1, i_2 and i_3. The process (24) at each instant limits the quantity $|i_1|$ to a certain value, and the Liouville theorem is valid for the remaining particles with $|i_1|$ smaller than this value, and therefore $n_\nu \sim |i_1 i_2 i_3|$, and the cross section $\sigma \sim i_1^2$. From the condition $\sigma n c t = 1$ we get

$$E_\nu \sim t^{\alpha/3}, \quad n_\nu \sim t^{-(1+2/3\alpha)}, \quad \varepsilon_\nu \sim t^{-(1+\alpha/3)}. \tag{25}$$

From the energy conservation law we can easily calculate the rate of change of the total energy density of the e^+, e^- pairs and of the quanta $\varepsilon_{\gamma, e\pm}$ (which are in equilibrium), with allowance for the "heating" by the process (24), as well as the rate of growth of the entropy as a result of such heating. We obtain for $t \gg \tau$

$$\frac{\varepsilon_{\gamma,e^\pm}}{\varepsilon_\nu} \approx \frac{4\alpha}{1-\alpha} + \left(\frac{\tau}{t}\right)^{(1-\alpha)/3}, \tag{26}$$

$$\frac{S}{S_0} \approx \left[1 + \frac{4\alpha}{1-\alpha}\left(\frac{t}{\tau}\right)^{(1-\alpha)/3}\right]^{3/4}. \tag{27}$$

The instant of termination of the vacuum stage $t = q$ is obtained in the same manner as in the preceding sections. It is equal to

$$q \approx \theta(\tau'/\theta)^{9(1-\alpha)/4(3-\alpha)}, \quad \tau' < \theta. \tag{28}$$

At the instant $t = q$ the ratio of the densities and the ratio of the average energies e (for small values of α, $\alpha > (\tau/q)^{(1-\alpha)/3}$):

$$\frac{n_\nu}{n_{\gamma,e^\pm}} \approx \frac{\bar{E}_{\gamma,e^\pm}}{\bar{E}_\nu} \approx \left(\frac{\tau'}{\theta}\right)^{3(3+5\alpha)/8(3-\alpha)}. \tag{29}$$

After $t = q$, the expansion proceeds in all directions, and the neutrinos cease to interact with the antineutrinos. For $t > q$ we have

$$S(t) \approx S(q), \quad \frac{n_\nu}{n_{\gamma,e^\pm}} \approx \frac{n_\nu}{n_{\gamma,e^\pm}}\Big|_{t=q}, \quad \frac{E_\nu}{E_{\gamma,e^\pm}} \approx \frac{E_\nu}{E_{\gamma,e^\pm}}\Big|_{t=q}.$$

For values of α that are not small, it follows from (26) that the energy densities of the neutrinos, the quanta, and of the pairs are of the same order at the

[5] More accurately, at the instant of the "separation" of the neutrinos, the density of their energy is much smaller than ϵ_1.

[6] The separation is considered in [17], and for a correct conclusion concerning the contemporary density of ν and ϵ_ν in the Friedmann model see [18].

[7] The interaction of ν_μ and ν_μ leads to the production of muon pairs, which become thermalized and subsequently decay into e^+, e^-, $\bar{\nu}_\mu$, ν_e, and $\bar{\nu}_e$. The energy of such secondary neutrinos is ~ 30 MeV and is much smaller than the ever increasing energy of the primary neutrinos moving along the x_1 axis.

instant t = q. The large anisotropy soon disappears, and the isotropization time is on the order of q. However, since $n_\nu \ll n_\gamma, e^\pm$ (see (29)), we get $\bar{E}_\nu \gg \bar{E}_{\gamma, e^\pm}$, and the distribution of the neutrino momenta is sharply anisotropic.

We have likewise not considered here the possible isotropization of a directed flux of neutrinos. The relations written out do not take into account the factor β, which in the case of neutrinos can range from $1/4$ to 10^{-2} (depending on τ).

It was assumed earlier in this section that the neutrino annihilation cross section (see (24)) satisfies the condition $\sigma_\nu \sim E_\nu^2$ for all energies. However, this condition is apparently satisfied only for $E_\nu \leq 300$ GeV. At higher energies, the neutrino annihilation cross section either remains constant or even decreases with increasing energy. Therefore the results obtained above are fully applicable only when $q \leq 10^4$ sec ($\theta \leq 3 \times 10^{10}$ sec, $\alpha = 1/3$). For $q = 10^4$ sec we get $\theta \approx 3 \times 10^{10}$ sec, $\tau \approx 10^{-5}$ sec, $E_\nu|_{t=\tau} \approx 3$ GeV, $E_\nu|_{t=q} = 300$ GeV, and $S(q) \approx 10^3 S(\tau)$. With this, the present-day neutrino energy is $E_\nu \approx 10^4$ eV.

For $\theta > 3 \times 10^{10}$ sec ($E_\nu|_{t=\tau} > 3$ GeV), the opposing neutrino and antineutrino annihilate effectively into e^+, e^- pairs only so long as $E \leq 300$ GeV. The larger θ, the sooner the neutrino energy reaches 300 GeV (we denote this instant by t_3), the sooner the annihilation of the neutrino and the growth of the entropy cease. For $\theta > 3 \times 10^{10}$ sec, the formulas of this section are applicable only for a time

$$t \leqslant t_3 \approx 10^{15/\alpha} \tau' \left(\frac{\tau'}{\theta} \right)^{(3+5\alpha)/4\alpha} \qquad (\alpha \sim 1/3).$$

When $t_3 \leq t$ it is necessary to use again the formulas of Sec. 4. With increasing θ ($\theta > 3 \times 10^{10}$ sec) the isotropization time increases, and the growth of the entropy during the course of deformation and the final energy of the present-day neutrinos decrease. The parameter θ cannot exceed $\approx 10^{16}$ sec, for otherwise the expansion would not have time to become isotropic at the present time (10^{14} with allowance for [19, 20]).

The maximum growth of entropy ($S(q)/S(\tau) \approx 10^3$) and the maximum energy of the neutrino today ($\sim 10^4$ eV) correspond to values $\theta = 3 \times 10^{10}$ sec and $q \approx 10^4$ sec. It is necessary to repeat Thorne's calculations[6] of the chemical composition of prestellar matter with allowance for a different time dependence of the temperature of the medium, owing to the neutrino effects, and to determine in this manner the interval of the parameter θ, which is certainly excluded by an unacceptable chemical composition (large amounts of He, D). On the other hand, if at the instant of neutrino separation the graviton energy density becomes much larger than ε_1, then the kinematics of the expansion will be determined by the gravitons, and the processes with the neutrinos will be determined by this kinematics.

Thermodynamic equilibrium at ultrahigh densities in anisotropic models takes place only under the condition $f = \sigma n c t > 1$, $t \to 0$ (21), where σ is the interaction cross section of arbitrary particles. It is obvious that this is possible only if 1) the cross section does not decrease with the increasing energy and 2) the initial particle density is high enough. If $\sigma(E) \to$ const as $E \to \infty$, and the initial density of all the particles is not sufficient for rapid establishment of the equilibrium ($f < 1$), or else the cross section $\sigma(E)$ decreases with increasing energy at high energies, then obviously equilibrium at ultrahigh densities will not have time to become established. In this case all particles behave like noninteracting ones, and the composition of matter at the early stages is determined by the initial conditions. In accordance with the results of Sec. 4, the expansion is determined by the particles that contain the maximum energy. Under these conditions, on the one hand, the present-day neutrino energies can be quite high (all that is satisfied is the condition $E_\nu n_\nu / c^2 \leq \rho_m$, where ρ_m is the density of matter), and on the other hand, at the earlier stage the important role can be played by different collective processes that lead to rapid isotropization.[8)]

After isotropization, the energy of all the particles decreases rapidly and the γ quanta, electron-positron pairs, and nucleons can enter in equilibrium. At the same time, the entropy of the matter increases strongly.

6. CONCLUSIONS

1. The dynamics of anisotropic cosmological models and the physics of processes in them are closely connected with the presence of weakly interacting particles and with the possible nonequilibrium nature of matter.

2. The energy of weakly interacting particles (neutrinos, gravitons) can at present greatly differ from that predicted by the isotropic model and can be quite large.

3. In anisotropic models, a strong increase of the initial entropy of matter can take place.

4. It is possible that allowance for collective interactions leads to a rapid isotropization of the solution. If the isotropization occurs at a sufficiently early stage ($t < 1$ sec), then by now such anisotropic models do not differ from the isotropic one.

5. Finally, it is not excluded that further development of the considerations advanced here will lead to the conclusion that the anisotropic (and inhomogeneous) cosmological models are incompatible with the observed properties of metagalaxies.

All the possible variants of the anisotropic models, particularly with account taken of different instabilities, the magnetic field, and the possible nonequilibrium nature of the initial stages, still call for an analysis. The main purpose of the present paper was to show that the role of weakly interacting particles must be taken into account in the theory of anisotropic cosmological models.

Note added in proof (18 July 1967). In the time interval between our preliminary note [11] and the present article, notes and preprints were published by Misner [21-23] on the same topic. Without stopping on the common features, we note that Misner considered the stage dur-

8)In the analysis of the collective interactions it is necessary to take into account the fact that near the singular point the "horizon" of the particles tends to zero.

ing which the viscosity caused by the neutrinos is significant, and, in addition, always regards the instant of separation of the neutrino as corresponding to $T = 2 \times 10^{10}$ °K. Therefore his physical conclusions pertain only to the case when the anisotropy is small at the stage $T \sim 2 \times 10^{10}$°K.

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