FLUCTUATIONS AND SCATTERING OF ELECTROMAGNETIC WAVES IN A PLASMA LOCATED IN AN EXTERNAL ELECTRIC FIELD

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Fluctuations and scattering of electromagnetic waves in a partially ionized plasma located in an external electric field are studied. It is shown that if the frequency of the collisions between charged and neutral particles is small, $\nu \ll \omega$ (ω is the fluctuation frequency), then the correlation functions possess sharp maxima at the ion-sound oscillation frequencies. As a result, ion-sound satellites appear in the scattered radiation spectrum when an electromagnetic wave passes through the plasma. At not very large values of the external electric field strength, the intensity of these satellites increases linearly with the field strength. As the electric field approaches the critical value as defined by formula (23), the intensity of the satellites increases sharply, and tends to infinity (within the framework of the linear theory). Fluctuations and combination (Raman) scattering of electromagnetic waves in a weakly ionized plasma are also investigated and the problem of applicability of the results to other plasma media (semiconductors, electrolyte solutions) is discussed.

1. INTRODUCTION

As is well known, if weakly damped oscillations can propagate in some system, then the passage of an electromagnetic wave through such a system gives rise to short maxima—called satellites—in the spectrum of the scattering radiation. The intensity of such Raman scattering is determined by the level of the fluctuations in the system; therefore in those cases when the level of the fluctuations greatly exceeds the thermal level, the satellites are characterized usually by high intensity.

We investigate here the fluctuations and the scattering of electromagnetic waves in a partially ionized plasma situated in an external electric field. In such a plasma, as is well known[1,2], there can propagate several modes of collective oscillations. We shall show that in the presence of an external electric field, the fluctuation level in a partially ionized plasma can greatly exceed the thermal level, and therefore the intensity of the Raman scattering of the electromagnetic waves by the plasma fluctuations may also turn out to be very high.

A particularly high intensity is characteristic of the scattering of electromagnetic waves by ion-sound oscillations. At not too large values of the external electric field E_0, the cross section of this process increases linearly with increasing E_0. When the electric field approaches a certain critical value E_c, the differential scattering cross section increases rapidly, becoming infinite (within the framework of the linear theory) as $E_0 \to E_c$.

The abrupt increase in the scattering cross section when the external field approaches its critical value is connected with the fact that instability of ion-sound oscillations sets in when $E_0 = E_c$. This phenomenon is analogous in its nature to critical opalescence in a fully ionized plasma, which was considered in[3,4]. To investigate the fluctuations and the scattering of electromagnetic waves at $E_0 \geq E_c$, it is necessary to take into account the nonlinear effects that limit the growth of the amplitude of the growing ion-sound waves. In this paper we confine ourselves to the linear theory and do not deal with this question.

Besides ion-sound fluctuations and scattering of electromagnetic waves by ion sound, we investigate also low-frequency fluctuations and scattering of electromagnetic waves from them. The results obtained in this case are applicable both to a weakly ionized plasma and to other plasma media (semiconductors, solutions of electrolites). We shall show that passage of electromagnetic waves through such media give rise, in the spectrum of the scattered radiation, to satellites that are located a distance $\Delta\omega \ll \nu$ away from the main line (ν is the collision frequency), and determine the intensities of these satellites.

The main difficulty that arises in the study of fluctuations in a plasma situated in an external electric field is connected with the fact that such a system is not closed or in complete equilibrium; therefore the general methods of fluctuation theory cannot be applied to it directly. Nonetheless, we shall show that an investigation of fluctuations in the plasma in the presence of an external electric field can be carried out by generalizing somewhat the fluctuation-theory method based on the fluctuation-dissipation theorem.

2. DETERMINATION OF THE CORRELATION FUNCTIONS

Let us determine first the correlators of the fluctuations of the quantities characterizing the plasma in the presence of an external electric field. We introduce for this purpose into the equation describing the system under consideration additional extraneous quantities—the so called "random forces".[5,6] In the case of a partially ionized plasma consisting of electrons, ions, and the neutral particles, we thus start from the equations (see[7])

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right)f^{(e)}+\frac{e}{m}\mathbf{E}\frac{\partial}{\partial \mathbf{v}}F^{(e)}+J\{f^{(e)}-y^{(e)}\}=0,$$
$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right)f^{(i)}-\frac{ze}{M}\mathbf{E}\frac{\partial}{\partial \mathbf{v}}F^{(i)}+J\{f^{(i)}-y^{(i)}\}=0, \tag{1}$$

where $F^{(\alpha)}$—equilibrium distribution function of the particles of sort α (α = e, i) in an external electric field $\mathbf{E}_0$; $f^{(\alpha)}$ and $\mathbf{E}$—deviations of the distribution function of the particles and of the electric field from the equilibrium values; $J\{f^{(\alpha)}\}$—linearized integrals of the collisions of the electrons and ions with the neutral particles, and $y^{(\alpha)}$—random forces (m, e, and M, −ze—masses and charges of the electrons and ions). The terms containing the random forces are chosen in the form J{y} in order that the introduction of the random forces not violate the particle-number conservation laws.

To find the correlation functions it is necessary to determine first the rate of change of the entropy of the system under the influence of the random forces. In the case of fluctuations with $q \neq 0$ (q—wave vector of the fluctuation), the time derivative of the entropy can be represented in the form

$$\dot{S}(t)=\sum_{\alpha}\int d\mathbf{r}\,d\mathbf{v}\,(F^{(\alpha)})^{-1}f^{(\alpha)}\frac{\partial f^{(\alpha)}}{\partial t}.$$

Using (1) and introducing the notation

$$\dot{x}^{(\alpha)}=-J\{f^{(\alpha)}-y^{(\alpha)}\},\qquad X^{(\alpha)}=(F^{(\alpha)})^{-1}f^{(\alpha)}, \tag{2}$$

we get hence

$$\dot{S}(t)=\sum_{\alpha}\int d\mathbf{r}\,d\mathbf{v}\,\dot{x}^{(\alpha)}X^{(\alpha)}. \tag{3}$$

We see that (if we confine ourselves to fluctuations with $q \neq 0$) the quantity S turns out to be (in the absence of random forces) quadratic in the deviation of the distribution functions from their equilibrium values. This makes it possible to construct the correlation function by the general method of fluctuation theory, based on the fluctuation-dissipation theorem. According to this method, we should represent the quantities $\dot{x}$ in the form

$$\dot{x}^{(\alpha)}(\mathbf{v})=-\int\gamma^{(\alpha)}(\mathbf{v},\mathbf{v}')X^{(\alpha)}(\mathbf{v}')d\mathbf{v}'+y^{(\alpha)}(\mathbf{v}). \tag{4}$$

The "kinematic coefficients" γ which enter into this relation determine directly the normalization of the random forces

$$\langle y^{(\alpha)}(\mathbf{v},\mathbf{r},t)\,y^{(\alpha')}(\mathbf{v}',\mathbf{r}',t')\rangle$$
$$=\delta_{\alpha\alpha'}\{\gamma^{(\alpha)}(\mathbf{v},\mathbf{v}')+\gamma^{(\alpha')}(\mathbf{v}',\mathbf{v})\}\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t'). \tag{5}$$

The "kinetic coefficients" γ have the simplest form if one can disregard the influence of the collisions on the fluctuations ($\omega \gg \nu$, where ω is the frequency of the fluctuations and ν is the effective collision frequency). In this case the explicit form of the collision integrals is insignificant, since it is necessary to take in the final formulas the limit as $\nu/\omega \to 0$. Making therefore in (2) the substitution $J\{f^{(\alpha)}\} \to \nu_\alpha f^{(\alpha)}$, we get

$$\gamma^{(\alpha)}(\mathbf{v},\mathbf{v}')=\delta(\mathbf{v}-\mathbf{v}')\,\nu_\alpha^{-1}F^{(\alpha)}(\mathbf{v}),$$
$$\langle y^{(\alpha)}(\mathbf{v},\mathbf{r},t)\,y^{(\alpha')}(\mathbf{v}',\mathbf{r}',t')\rangle=2\nu_\alpha^{-1}\delta_{\alpha\alpha'}F^{(\alpha)}(\mathbf{v})\,\delta(\mathbf{v}-\mathbf{v}')\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t'). \tag{6}$$

We note that in this case the averaged distribution functions F can depend on the form of the collision integrals, and therefore the correlation functions can also depend on the form of the collision integrals.

Establishment of the connection (4) between the "generalized thermodynamic velocities and the forces" x and X requires in the general case the use of the explicit form of the collision integrals, and entails in general considerable difficulties. The problem simplifies somewhat if both the equilibrium distributions of the particle velocities and the nonequilibrium increments to these distributions are close to spherically-symmetrical. Expanding in this case F and f in spherical harmonics and retaining the first two terms, we have

$$F(\mathbf{v})=F_0(v)+v^{-1}\mathbf{v}\mathbf{F}_1(v),\qquad f(\mathbf{v})=f_0(v)+v^{-1}\mathbf{v}\mathbf{f}_1(v), \tag{7}$$

with $\mathbf{F}_1 \ll F_0$ and $\mathbf{f}_1 \ll f_0$. Introducing further the zeroth and first moments of the collision integral J_0 and $\mathbf{J}_1$ and of the random force y_0 and $\mathbf{y}_1$, we rewrite (3) in the form

$$\dot{S}(t)={}^1/_3\int d\mathbf{r}\,d\mathbf{v}\,F_0^{-1}(3f_0J_0\{y_0-f_0\}-F_0^{-1}J_0\{y_0-f_0\}\,\mathbf{F}_1\mathbf{f}_1$$
$$-F_0^{-1}\mathbf{J}_1\{\mathbf{y}_1-\mathbf{f}_1\}\,\mathbf{F}_1f_0+\mathbf{f}_1\mathbf{J}_1\{\mathbf{y}_1-\mathbf{f}_1\}) \tag{8}$$

(we confine ourselves for simplicity to the contribution made to the entropy by only one sort of particles).

In the case of a sufficiently strong external field ($eE_0lM^{1/2}m^{-1/2} \gg T_0$, where T_0 is the temperature of the neutral particles and l is the electron mean free path) we can use for the electronic component of the plasma the Davydov collision integral[8]; with this

$$J_0\{f_0\}=-\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{2\frac{m}{M}v^2\nu_e\left(\frac{T_0}{m}\frac{\partial f_0}{\partial v}+vf_0\right)\right\},$$
$$\mathbf{J}_1\{\mathbf{f}_1\}=\nu_e\mathbf{f}_1. \tag{9}$$

Substituting (9) in (8) and using (5), we get

$$\langle y_{1i}(v,\mathbf{r},t)\,y_{1j}(v',\mathbf{r}',t')\rangle$$
$$=\frac{3}{2\pi}\nu_e^{-1}\delta_{ij}v^{-2}F_0(v)\,\delta(v-v')\,\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t'), \tag{10}$$

where F_0 is the zeroth harmonic of the equilibrium distribution function of the electrons in an external electric field[8],

$$F_0(v)=C\exp\left\{\frac{-3m^3v^4}{4Me^2E_0^2l^2}\right\},\qquad C=\frac{n_0}{\pi\Gamma(^3/_4)}\left(\frac{3m^3}{4Me^2E_0^2l^2}\right)^{3/4} \tag{11}$$

and n_0 is the equilibrium density of the electrons (we do not present the much more complicated expressions for the quantities $\langle y_0y_0\rangle$ and $\langle y_0\mathbf{y}_1\rangle$, since it will be shown below that to determine the correlators of the macroscopic quantities it is sufficient to know the quantity $\langle \mathbf{y}_1\mathbf{y}_1\rangle$).

We note that according to[8] the equilibrium electron velocity distribution in a strong electric field is always close to spherically-symmetrical, $\mathbf{F}_1 \ll F_0$. However, the inequality $\mathbf{f}_1 \ll f_0$, together with (10), holds true here only in the low-frequency region ($\omega \ll \nu_e$). In the high-frequency region ($\omega \gg \nu_e$) all the spherical harmonics of the function f have generally speaking the same order of magnitude; in this region, to determine the correlation functions it is necessary to use relation (6).

It is well known[2] that in the low frequency region ($\omega \ll \nu_e$) the kinematic equation for the electronic component of the plasma leads to hydrodynamic equations with a self-consistent field. Starting from the kinetic equation with random force (1) we get

$$(\mathbf{v}_0\nabla)\mathbf{v}^{(e)}+\nu_e\mathbf{v}^{(e)}=\frac{e}{m}\mathbf{E}+\mathbf{Y}^{(e)},\qquad\left(\frac{\partial}{\partial t}+\mathbf{v}_0\nabla\right)n_e+n_0\operatorname{div}\mathbf{v}^{(e)}=0,$$
$$\mathbf{Y}^{(e)}=\nu_e(3n_0)^{-1}\int v^2\mathbf{y}_1\,d\mathbf{v}, \tag{12}$$

where $v_0 = eE_0(m\nu_e)^{-1}$ is the average electron velocity and n_e and $v^{(e)}$ are the density and velocity fluctuations. The correlator of the random forces $Y^{(e)}$ has in this case, according to (10), the form

$$\langle Y_i^{(e)}(\mathbf{r},t)\,Y_j^{(e)}(\mathbf{r}',t')\rangle = 2\nu_e\delta_{ij}\frac{T^*}{m}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \tag{13}$$

where T^* is the average electron energy

$$T^* = \frac{\Gamma(1/4)}{6\Gamma(3/4)}\frac{M^{1/2}}{(3m)^{1/2}}eE_0 l. \tag{14}$$

We see that in the case of low-frequency fluctuations the averaging of the random forces is in analogy with the case of ordinary hydrodynamics (see[5]), and the role of the temperature in the correlators of the quantities characterizing electrons is assumed by the average energy T^*.

It is easy to show that if $\omega \ll \nu_i$, where ν_i is the effective frequency for the collisions of ions with neutral particles, then we can start in the study of fluctuations of the quantities pertaining to the ions from hydrodynamic equations with random force $Y^{(i)}$:

$$\nu_i\mathbf{v}^{(i)} = -\frac{ze}{M}\mathbf{E} + \mathbf{Y}^{(i)}, \quad \frac{\partial n_i}{\partial t} + z^{-1}n_0\operatorname{div}\mathbf{v}^{(i)} = 0, \tag{15}$$

where $v^{(i)}$ is the velocity and n_i is the fluctuation of the ion density. The averaged product of the random forces is proportional here to the ion temperature T_i:

$$\langle Y_i^{(i)}(\mathbf{r},t)\,Y_j^{(i)}(\mathbf{r}',t')\rangle = 2\nu_i\delta_{ij}\frac{T_i}{M}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \tag{16}$$

on the other hand, the quantities $\langle Y^{(i)}Y^{(e)}\rangle = \langle Y^{(e)}Y^{(i)}\rangle$ vanish.

To determine the correlation functions, we should now, using (1) (or else (12) and (15)) and the equations of electrostatics, express the charge density, the electron density, the electric field, and other quantities characterizing the plasma in terms of the random forces, and then average over the random forces with the aid of relations (5) and (6) (or (13) and (16)).

3. HIGH-FREQUENCY FLUCTUATIONS AND CRITICAL OPALESCENCE IN A PLASMA SITUATED IN A STRONG ELECTRIC FIELD

Let us consider fluctuations in the high frequency region $\omega \gg \nu_e, \nu_i$. Using (6), we get for the charge-density correlator

$$\langle\rho^2\rangle_{\mathbf{q},\omega} = 2\pi|\varepsilon(\mathbf{q},\omega)|^{-2}\sum_\alpha (z_\alpha e)^2\int F^{(\alpha)}\delta(\omega-\mathbf{q}\mathbf{v})\,d\mathbf{v}. \tag{17}$$

where $\epsilon = 1 + 4\pi(\kappa^e + \kappa^i)$ is the longitudinal dielectric constant of the plasma and κ^e and κ^i are the electric susceptibilities of the electrons and ions,

$$\kappa^\alpha(\mathbf{q},\omega) = (z_\alpha e)^2 m_\alpha^{-1}q^{-2}\int(\omega-\mathbf{q}\mathbf{v}+i o)^{-1}\mathbf{q}\frac{\partial F^{(\alpha)}}{\partial\mathbf{v}}d\mathbf{v}. \tag{18}$$

By way of $F^{(e)}$, it is necessary to substitute into these equations the distribution function of the electrons in a strong electric field[8] $F^{(e)} = F_0^{(e)} + v^{-1}\mathbf{v}\cdot\mathbf{F}_1^{(e)}$, where $F_0^{(e)}$ is determined by formula (11) and

$$\mathbf{F}_1^{(e)}(v) = -\frac{e\mathbf{E}_0 l}{mv}\frac{dF_0^{(e)}}{dv}. \tag{19}$$

As is well known, the poles of the correlation functions determine the complex frequencies of the natural oscillations of the system. Equating to zero the denominator of (17) and putting

$$T_i \ll Mq^{-2}\omega^2 \ll eF_0 lM^{1/2}m^{-1/2},$$

we obtain the frequency and the damping decrement of the ion-sound oscillations

$$\omega = V_s q(1+a^2q^2)^{-1/2},$$
$$\gamma = {}^1/_2\,\omega(\pi aq)^2(1+a^2q^2)^{-1}\Gamma^{-2}(1/4)\,(\cos\chi_c - \cos\chi), \tag{20}$$

where V_s is the speed of sound, a is the Debye electronic radius

$$V_s^2 = z\frac{2T_e\Gamma(3/4)}{M\Gamma(1/4)}, \quad a^2 = \frac{T_e\Gamma(3/4)}{2\pi e^2 n_0\Gamma(1/4)}, \tag{21}$$

χ is the angle between the vectors $\mathbf{q}$ and $\mathbf{E}_0$, $T_e = M^{1/2}(3m)^{-1/2}eE_0 l$ is the effective electron temperature, and

$$\cos\chi_c = \frac{2^{3/4}z^{1/2}}{\sqrt{3\pi}(1+a^2q^2)^{1/2}}$$
$$\times\left\{1+\frac{\sqrt{2\pi}}{\Gamma(1/4)}z\left(\frac{M}{m}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(\frac{-\Gamma(3/4)T_e z}{\Gamma(1/4)T_i(1+a^2q^2)}\right)\right\} \tag{22}$$

(the ion velocity distribution is assumed Maxwellian with a temperature T_i). It is easy to see that if $\cos\chi_c > 1$, then $\gamma > 0$ and the oscillations of the plasma are damped. If $\cos\chi_c < 1$, then oscillations propagating at an angle χ to the direction of the field larger than χ_c are attenuated. When $\chi \to \chi_c$, the damping decrement of the oscillations vanishes: in the region $\chi < \chi_c$, the plasma oscillations start to grow.

Let us trace the dependence of the character of the ion-sound oscillations on the external electric field E_0. At not too large values of the field, $\cos\chi_c > 1$, and therefore all the oscillations are damped. With increasing E_0, the first to begin to grow are the long-wave oscillations ($aq \ll 1$), propagating along the field; for the corresponding critical value of the field, using (22), we get

$$E_c = \left(\frac{3}{4}\right)^{1/2}\frac{\Gamma(1/4)}{\Gamma(3/4)}\frac{T_i}{zel}\left(\frac{m}{M}\right)^{1/2}\left\{\ln\frac{M}{m}+3\ln\left(\frac{z^{-1}(\pi z)^{2/3}}{2^{3/2}\Gamma^{4/3}(3/4)}\ln\frac{M}{m}\right)\right\}. \tag{23}$$

with further increase of the field, the oscillations with large q and large values of the angle χ between the vectors $\mathbf{q}$ and $\mathbf{E}_0$ start to increase.

Returning to relation (17), we emphasize that this relation (as well as all further formulas for the correlation functions), is valid in the case of damped oscillations ($\cos\chi < \cos\chi_c$). If the plasma oscillations increase ($\cos\chi > \cos\chi_c$), then the nonlinear effects must be taken into account in the determination of the correlation functions.

Noting that the damping decrement of the ion-sound oscillations is small, $\gamma \ll \omega$, we represent (17) in the form

$$\langle\rho^2\rangle_{\mathbf{q},\omega} = \frac{a^2q^5V_sT_ez^{1/2}}{2^{5/4}\sqrt{3}(1+a^2q^2)^2(\cos\chi_c-\cos\chi)}\delta\left(\omega^2-\frac{V_s^2q^2}{1+a^2q^2}\right). \tag{24}$$

We present one more expression for the density fluctuation correlators of the electrons and ions, separately:

$$e^2\langle n_e^2\rangle_{\mathbf{q},\omega} = (1+a^2q^2)^{-2}(ze)^2\langle n_i^2\rangle_{\mathbf{q},\omega} = -(1+a^2q^2)^{-1}ze^2\langle n_en_i\rangle_{\mathbf{q},\omega}$$
$$= \frac{qV_sT_ea^{-2}z^{1/2}}{2^{5/4}\sqrt{3}(1+a^2q^2)^2(\cos\chi_c-\cos\chi)}\delta\left(\omega^2-\frac{V_s^2q^2}{1+a^2q^2}\right). \tag{25}$$

The correlator of the electric field is connected with the correlator of the charge density by the obvious relation

$$\langle E_i E_j\rangle_{\mathbf{q},\,\omega} = (4\pi)^2 q_i q_j q^{-4} \langle \rho^2\rangle_{\mathbf{q},\,\omega}. \tag{26}$$

We now consider combination scattering of electromagnetic waves in a plasma situated in an external electric field. We use for this purpose the well known expression for the differential cross section for scattering of electromagnetic waves (see [6,7])

$$d\Sigma = \left(\frac{e^2}{mc^2}\right)^2 (1+\cos^2\vartheta)\langle n_e^2\rangle_{\mathbf{q},\,\Delta\omega}\frac{d\omega' do'}{4\pi}, \tag{27}$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$; $\Delta\omega = \omega - \omega'$; ω, ω' and $\mathbf{k}$, $\mathbf{k}'$ are the frequencies and wave vectors of the incident and scattered waves; ϑ is the scattering angle (the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$), and do' is the solid-angle element of the vector $\mathbf{k}'$. Substituting here (25) and confining ourselves to the case of long-wave incident radiation ($ak \ll 1$), we get

$$d\Sigma = \frac{\Gamma(^1/_4)z^{1/2}n_0}{2^{13/4}\sqrt{3}\,\Gamma(^3/_4)}\left(\frac{e^2}{mc^2}\right)^2(1+\cos^2\vartheta)(\cos\chi_c - \cos\chi)^{-1}$$
$$\times\left\{\delta\left(\Delta\omega - 2V_s k\sin\frac{\vartheta}{2}\right) + \delta\left(\Delta\omega + 2V_s k\sin\frac{\vartheta}{2}\right)\right\} d\omega' do', \tag{28}$$
$$\cos\chi = \left(2\sin\frac{\vartheta}{2}\right)^{-1}(\cos\theta - \cos\theta'),$$

where θ (θ') is the angle between $\mathbf{k}(\mathbf{k}')$ and $\mathbf{E}_0$. We see that two satellites of equal intensity, with frequencies $\omega' = \omega \pm 2V_S k \sin(\vartheta/2)$, appear in the spectrum of the scattered radiation.

Integrating in (28) with respect to the frequency ω', we determine the intensity of scattering per unit solid-angle interval

$$\frac{d\Sigma}{do'} = \frac{\Gamma(^1/_4)z^{1/2}n_0}{2^{9/4}\sqrt{3}\,\Gamma(^3/_4)}\left(\frac{e^2}{mc^2}\right)^2(1+\cos^2\vartheta)(\cos\chi_c - \cos\chi)^{-1}. \tag{29}$$

We see that the angular distribution of the scattered radiation depends in essential fashion on the magnitude of the external electric field. When $E_0 \ll E_c$, this distribution is almost isotropic (if we disregard the factor $1 + \cos^2\vartheta$). With increasing E_0, the intensity of the waves scattered at angles close to θ_0' increases sharply; here

$$\theta_0' = \arccos\{\cos\theta - 2\sin(\vartheta/2)\}. \tag{30}$$

As $E_0 \to E_c$, the intensity of scattering at angle θ_0' becomes infinite (within the framework of the linear theory). The total cross section for the scattering of the electromagnetic waves by ion sound also increases with the increasing E_0 and tends to infinity (logarithmically) as $E_0 \to E_c$. The dependence of the scattering intensity on the angle χ and on the external field E_0 is shown in Figs. 1 and 2.

The sharp increase of the electromagnetic wave scattering intensity at $E_0 \to E_c$ is connected with the ion-sound instability which sets in when $E_0 \geq E_c$. This phenomenon is similar in its nature to the well known phenomenon of critical opalescence.[1)]

[1)]In the absence of an external electric field, the critical opalescence and the plasma was investigated in [3,4].

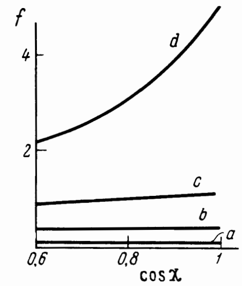

FIG. 1. Angular distribution of scattered radiation. The vertical axis represents the function

$$f\left(\cos\chi, \frac{E_0}{E_c}\right) = \frac{d\Sigma}{do'}\left\{\frac{\Gamma(^1/_4)n_0}{2^{9/4}\sqrt{3}\,\Gamma(^3/_4)}\left(\frac{e^2}{mc^2}\right)^2(1+\cos^2\vartheta)\right\}^{-1},$$

and the horizontal axis the function

$$\cos\chi = (2\sin(\vartheta/2))^{-1}(\cos\theta - \cos\theta').$$

Curves a, b, c, and d pertain to the cases E_0/E_c = 0.5, 0.7, 0.8, and 0.9. In this case $M/m = 10^4$ and $z = 1$.

4. LOW FREQUENCY FLUCTUATIONS AND THE SCATTERING OF ELECTROMAGNETIC WAVES IN A WEAKLY IONIZED PLASMA

Let us consider in our fluctuations in the high-frequency region, $\omega \ll \nu_e, \nu_i$. Expressing with the aid of the hydrodynamic equations (12) and (15) and with the aid of the electrostatic equations the charge density in terms of the random forces, and averaging over the random forces with the aid of (13) and (16), we get

$$\langle\rho^2\rangle_{\mathbf{q},\,\omega} = \frac{q^2}{2\pi}\varepsilon_0{}^3\frac{T^*\omega^2\omega_e + T_i(\omega - \mathbf{q}\mathbf{v}_0)^2\omega_i}{|\varepsilon(\mathbf{q},\omega)|^2\omega^2(\omega - \mathbf{q}\mathbf{v}_0)^2}, \tag{31}$$

where T^* is the average electron energy, determined by formula (14), ϵ is the longitudinal dielectric constant of the system,

$$\varepsilon(\mathbf{q},\omega) = \varepsilon_0\{1 + i\omega_e(\omega - \mathbf{q}\mathbf{v}_0)^{-1} + i\omega_i\,\omega^{-1}\};$$
$$\omega_\alpha = 4\pi e^2 z_\alpha n_0(\varepsilon_0 m_\alpha \nu_\alpha)^{-1} \tag{32}$$

and ϵ_0 is the dielectric constant of the neutral component of the plasma.

In order to determine the complex frequencies of the natural oscillations of the plasma, let us determine the poles of the correlator (31). Using (32), we obtain $\omega = \omega_\pm$, where

$$\omega_\pm = {}^1/_2(\mathbf{q}\mathbf{v}_0 - i\omega_e - i\omega_i) \pm$$
$$\pm\, {}^1/_2\{(\mathbf{q}\mathbf{v}_0 - i\omega_e - i\omega_i)^2 + 4i\mathbf{q}\mathbf{v}_0\omega_i\}^{1/2}. \tag{33}$$

We note that relations (31) and (33) are not applicable if the wave propagates almost perpendicular to the constant electric field

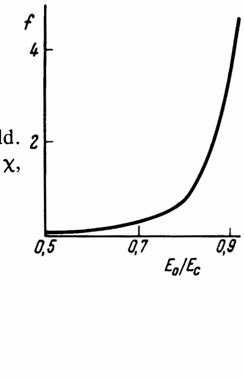

FIG. 2. Scattering intensity vs. electric field. The vertical axis represents the function $f(\cos\chi, E_0/E_c)$ at $\cos\chi = 1$ (for $M/m = 10^4$ $z = 1$); the horizontal axis represents E_0/E_c.

$$\left|\frac{\pi}{2}-\chi\right| \ll \frac{v_0}{c^2 q} \min\{\omega_e, \omega_i\}.$$

In this range of angles, as shown in[2], it is necessary to use in lieu of the electrostatic equations the complete system of Maxwell's equations and the oscillations with $\mathbf{q} \cdot \mathbf{v}_0 \ll \omega_\alpha (v_0/c)^2$ turn out to grow. It is essential, however, that both the growth increment and the volume in wave-vector space corresponding to the growing oscillations are very small ($\sim c^{-2}$), so that the influence of the growing oscillations on the fluctuations in the stability region can be disregarded.

Using (33), we can represent (31) in the form

$$\langle\rho^2\rangle_{\mathbf{q},\omega} = (2\pi)^{-1} q^2 \varepsilon_0 |\omega-\omega_+|^{-2} |\omega-\omega_-|^{-2} \times\{T^* \omega^2 \omega_e + T_i (\omega - \mathbf{q}\mathbf{v}_0)^2 \omega_i\}. \quad (34)$$

A similar formula is used to determine the electron density correlator:

$$e^2 \langle n_e^2\rangle_{\mathbf{q},\omega} = (2\pi)^{-1} q^2 \varepsilon_0 |\omega-\omega_+|^{-2} |\omega-\omega_-|^{-2} \times \omega_e \{T^* (\omega^2+\omega_i^2) + T_i \omega_e \omega_i\}. \quad (35)$$

We see that if the imaginary part of any of the complex frequencies $\omega_\pm$ is small, then the correlation functions have sharp maxima at $\omega = \mathrm{Re}\,\omega_\pm$, corresponding to the possible propagation of weakly damped oscillations in the plasma.

If $\mathbf{q} \cdot \mathbf{v}_0 \gg \omega_e, \omega_i$, then $\omega_+ = \mathbf{q} \cdot \mathbf{v}_0 - i\omega_e$ and $\omega_- = -i\omega_i$; relations (34) and (35) then take the form

$$\langle\rho^2\rangle_{\mathbf{q},\omega} = e^2 \langle n_e^2\rangle_{\mathbf{q},\omega} = {}^1\!/_2\, q^2 \varepsilon_0 T^* \delta(\omega - \mathbf{q}\mathbf{v}_0). \quad (36)$$

If $\omega_e \gg \max\{\mathbf{q} \cdot \mathbf{v}_0, \omega_i\}$, then the weakly damped oscillations are those with frequency ω_-, near which we have

$$\langle\rho^2\rangle_{\mathbf{q},\omega} = e^2 \langle n_e^2\rangle_{\mathbf{q},\omega} \left(\frac{\mathbf{q}\mathbf{v}_0}{\omega_e}\right)^2 = \frac{1}{2} q^2 \varepsilon_0 \delta\left(\omega - \mathbf{q}\mathbf{v}_0 \frac{\omega_i}{\omega_e}\right) \left\{T^* \frac{\omega_i}{\omega_e} + T_i\right\}. \quad (37)$$

Let us consider, finally, the scattering of electromagnetic waves by low-frequency fluctuations. Substituting (35) in (27), we get

$$d\Sigma = \frac{e^2 q^2}{8\pi^2 (mc^2)^2} \varepsilon_0 |\omega-\omega_+|^{-2} |\omega-\omega_-|^{-2} \omega_e \{T^* (\omega^2+\omega_i^2) + T_i \omega_e \omega_i\} (1+\cos^2\vartheta)\, d\omega'\, do'. \quad (38)$$

If $\mathbf{k} \cdot \mathbf{v}_0 \gg \omega_e, \omega_i$, then

$$d\Sigma = \frac{e^2 k^2}{2\pi (mc^2)^2} \varepsilon_0 T^* (1+\cos^2\vartheta) \sin^2\frac{\vartheta}{2}\, \delta(\Delta\omega - (\mathbf{k}-\mathbf{k}')\mathbf{v}_0)\, d\omega'\, do'. \quad (39)$$

We see that a single sharp maximum with frequency $\omega' = \omega + (\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}_0$ appears in the spectrum of the scattered radiation when $\mathbf{k} \cdot \mathbf{v}_0 \gg \omega_{e,i}$.

When $\omega_e \gg \max\{\mathbf{k} \cdot \mathbf{v}_0, \omega_i\}$, formula (38) takes the form

$$d\Sigma = \frac{e^2 k^2}{2\pi (mc^2)^2} \varepsilon_0 \left\{T^* \frac{\omega_i}{\omega_e} + T_i\right\} \times \left(\frac{\omega_i}{\Delta\omega}\right)^2 (1+\cos^2\vartheta) \sin^2\frac{\vartheta}{2}\, \delta\left(\Delta\omega - \frac{\omega_i}{\omega_e}(\mathbf{k}-\mathbf{k}')\mathbf{v}_0\right) d\omega'\, do'. \quad (40)$$

In this case there appears in the spectrum of the scattered waves a satellite with frequency $\omega' = \omega + \omega_i \omega_e^{-1} (\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}_0$. In both cases, both the frequency and the intensity of the scattered radiation depend strongly on the direction of propagation of the incident and scattered waves.

We note in conclusion that the results obtained in this section are applicable not only to a weakly-ionized plasma, but also to other plasma media (semiconductors, electrolyte solutions) placed in an external electric field. In such media there should also occur combination scattering of electromagnetic waves, described by formulas (38)—(40). If the external electric field is in this case sufficiently strong, and the carrier mass ratio is also large, then the effective temperature T^*, which determines the scattering intensity, is given by the same formula (14) as in the case of a plasma. Relations (38)—(40) remain in force also when $M/m \sim 1$; in this case one must take the temperatures T^* and T_i to be quantities on the order of the average carrier energies. In the case of plasma media, an important factor may also turn out to be the deviation from unity of the quantity ϵ_0, which represents in this case the dielectric constant of the medium in the absence of carriers.

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