DRIFT APPROXIMATION FOR A FIELD WITH A NONRESONANT HIGH-FREQUENCY COMPONENT

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The effect of a high-frequency field on the drift of a particle is considered. The field frequency is assumed not to be very close to the cyclotron frequency. It is shown that the averaged force due to the high-frequency field acting on the particle is equivalent to the gradient of a certain effective potential. Under similar assumptions, a hydrodynamic equation for smooth motion is derived, in which collisions are also taken into account.

THE drift theory of motion of a charged particle in a slowly varying electromagnetic field has been developed in the literature in considerable detail (see, for example, [1,2]). If in addition to the aforementioned field there is present also a high frequency (hf) field, then the drift motion is influenced also by a force due to the averaged action of the hf field[3]. The determination of the integrals of the drift motion is greatly simplified if this force is potential. Morozov and Solov'ev[2] cite the simplest case, when the frequency of the hf field is much larger than the cyclotron frequency. A more general analysis by Miller[4] has led to an incorrect conclusion, which will be discussed below. The main result of the proposed paper is to prove the potential character of the average hf field under rather general conditions, which are typical of the solution of certain problems in plasma physics.

Let us assume that a particle having a charge e and a mass m moves in the electromagnetic field, which can be resolved into a slowly varying or dc field $\mathbf{E}_0$, $\mathbf{B}_0$ and an hf field $\mathbf{E}_1$, $\mathbf{B}_1$ ($|\mathbf{B}_1| \ll |\mathbf{B}_0| = B_0$) with frequency ω. The amplitudes and the frequency of the hf field can in general also vary slowly. In order to formulate exactly the limiting assumptions, let us introduce the following notation:

$$\tau_\sim = \max\left\{\frac{2\pi}{\omega}, \frac{2\pi}{|\Omega|}, \left|\frac{2\pi}{\omega-|\Omega|}\right|\right\},$$

$$\delta = \max\left\{\frac{\tau_\sim}{\tau_0}, \frac{|\mathbf{v}|\tau_\sim}{L}\right\},$$

$\Omega = eB_0/mc$ is the cyclotron frequency ($|\Omega| \neq \omega$), τ_0 is shortest characteristic time of the slowly varying values of the electromagnetic field, v is the particle velocity ($|\mathbf{v}| \ll c$), and L is the shortest characteristic length of the inhomogeneity of the electromagnetic field. It is assumed that $\delta \ll 1$ and quantities on the order of δ^N, $N \geq 3$, are neglected.

If the particle in question is one of a certain ensemble (say a plasma), then the fields $\mathbf{E}_0$, $\mathbf{B}_0$, $\mathbf{E}_1$, and $\mathbf{B}_1$ can be excited both by external sources and by collective interactions of the particles. It is therefore assumed that the particle can also be located in regions where the space charge and the current density are different from zero.

We resolve the radius vector of the particle position $\mathbf{r}(t)$ into a smoothly varying part $\mathbf{R}(t)$ and a rapidly oscillating part $\rho(t)$: $\mathbf{r} = \mathbf{R} + \rho$, $\mathbf{R} = \overline{\mathbf{r}}$. The bar denotes smoothing, that is, discarding rapidly oscillating terms (with frequencies $\geq 2\pi/\tau$). The smoothing of the equation of motion yields

$$m\frac{d^2\mathbf{R}}{dt^2} = e\mathbf{E}_0(\mathbf{R}) + e\overline{\mathbf{E}_1(\mathbf{r})}$$

$$+\frac{e}{c}\left(\left[\frac{d\mathbf{R}}{dt}\mathbf{B}_0(\mathbf{R})\right] + \overline{\left[\frac{d\boldsymbol{\rho}}{dt}\mathbf{B}_0(\mathbf{r})\right]} + \overline{\left[\frac{d\boldsymbol{\rho}}{dt}\mathbf{B}_1(\mathbf{R})\right]}\right).$$

Within the framework of the approximation under consideration, it is sufficient to put $\rho = \rho_c + \rho_1$, where ρ_c is the cyclotron rotation, which conserves its magnetic moment, $\rho_1 = i\mathbf{v}_1/\omega$, and $\mathbf{v}_1$ satisfies the equation*

$$-i\omega m\mathbf{v}_1 = e\mathbf{E}_1(\mathbf{R}) + ec^{-1}[\mathbf{v}_1\mathbf{B}_0(\mathbf{R})].$$

The possible nonlinear and parametric resonances of the oscillating motion are disregarded.

The equation of the smooth motion $\mathbf{R}(t)$ can be transformed to a simpler form, using the relations curl $\mathbf{E}_1 = i\omega\mathbf{B}_1/c$ and div $\mathbf{B}_0 = 0$:

$$m\frac{d^2\mathbf{R}}{dt^2} = e\mathbf{E}_0(\mathbf{R}) + \frac{e}{c}\left(\left[\frac{d\mathbf{R}}{dt}\mathbf{B}_0(\mathbf{R})\right] + \overline{\left[\frac{d\boldsymbol{\rho}_c}{dt}(\boldsymbol{\rho}_c\nabla)\mathbf{B}_0\right]}\right) - \nabla U, \quad (1)$$

$$U = \frac{m}{4}(\mathbf{v}_1\mathbf{v}_1^*) + \frac{im}{4\omega}(\boldsymbol{\Omega}[\mathbf{v}_1\mathbf{v}_1^*]), \quad (2)$$

where $\Omega = e\mathbf{B}_0/mc$. In explicit form,

$$U = \frac{e^2}{4m\omega^2}\left(1-\frac{\Omega^2}{\omega^2}\right)^{-1}\left\{(\mathbf{E}_1^*\mathbf{E}_1) - \frac{1}{\omega^2}(\mathbf{E}_1^*\boldsymbol{\Omega})(\boldsymbol{\Omega}\mathbf{E}_1) + \frac{i}{\omega}(\boldsymbol{\Omega}[\mathbf{E}_1^*\mathbf{E}_1])\right\}. \quad (3)$$

It can be shown that the longitudinal component (parallel to $\mathbf{B}_0$) of Eq. (1) coincides with an equation derived for longitudinal motion of a particle[5]. Comparing (1) with the relations obtained by Miller[4], we arrive at the conclusion that the nonpotential part of the averaged hf force in Eq. (22) of[4] is superfluous. In the particular case when the field $\mathbf{B}_0$ is homogeneous and $\mathbf{E}_0 = 0$, Eq. (1) goes over to the well known result of[6].

It follows from (1) that for the smooth motion $\mathbf{R}(t)$ we can simply use the results of the drift approximation, which are known for the case without the hf field. It is only necessary to replace the electric field. It is only necessary to replace the electric field $\mathbf{E}_0$ formally by the field $\mathbf{E}_0 - \text{grad}(U/e)$. In the particular

*$[\nu_1\mathbf{B}_0(\mathbf{R})] \equiv \nu_1 \times \mathbf{B}_0(\mathbf{R})$.

case when $\omega \gg |\Omega|$ the same result follows from the relations of Morozov and Solov'ev[2].

It is easy to modify the conclusions for a description of the motion of the particle in the field of a quasimonochromatic wave packet moving with velocity $v_g \approx \dot{R}$, $|v_g| \ll c$. Assuming that $|\dot{R} \times E_0| \ll cB_0$, we again obtain Eq. (1), where

$$U = \frac{e^2}{4m}[(\omega')^2 - \Omega^2]^{-1}\left((E_1'^* E_1') - \frac{1}{(\omega')^2}(E_1'^*\Omega)(\Omega E_1') + \frac{i}{\omega'}(\Omega[E_1'^* E_1'])\right). \quad (4)$$

In Eq. (4), $E_1' = E_1 + \dot{R} \times \dot{B}_1 c^{-1}$, $\omega' = \omega - k \cdot \dot{R}$, $|\omega'| \neq |\Omega|$, and k is the wave vector. In the analogous problem considered in Sec. 7 of[5], the assumption $v_g \approx \dot{R}$ should be satisfied.

To study the macroscopic behavior of a "liquid" of identical particles (say the electronic or ionic component of a plasma), it is necessary to have the hydrodynamic equation of the smooth motion of such a liquid. It can be obtained from the Euler equation by a method similar to that described above. All the phases of the cyclotron rotation of the particles are assumed to be equally probable, so that the only rapidly oscillating component of the macroscopic velocity is the component with frequency ω. Collisions with the remaining sorts of particles are taken into account by the force $-\nu m V$ (ν—the corresponding effective collision frequency, V—velocity of the liquid). The result can be written in the form

$$m\frac{dV_L}{dt} = m\left(\frac{\partial V_0}{\partial t} + (V_0\nabla)V_0\right) = -\frac{\operatorname{div} P_0}{n_0} + eE_0 + \frac{e}{c}[V_L B_0] - \nu m V_L - \nabla W - \frac{\nu m}{2\omega}\operatorname{Re}\{i(\{\nabla V_1\}V_1^*)\}, \quad (5)$$

where $V_L = V_0 - \operatorname{Re}\{i(V_1^* \cdot \nabla)V_1\}(2\omega)^{-1}$ is the velocity, smoothed out and expressed in Lagrange coordinates; the zero subscript denotes a quantity smoothed at a stationary point (that is, in Euler coordinates), n is the concentration, and P is the pressure tensor; $V_1 = V - V_0$, with

$$V_1 = \frac{ie}{\omega_\nu m}\left(1 - \frac{\Omega^2}{\omega_\nu^2}\right)^{-1}\left(E_1 + \frac{i}{\omega_\nu}[E_1\Omega] - \frac{1}{\omega_\nu^2}\Omega(\Omega E_1)\right),$$
$$\omega_\nu = \omega + i\nu \quad (6)$$

and W is given by (2), with v_1 replaced by V_1. In (5) we used the notation $(\{\nabla V_1\}V_1^*) = V_1^* \times \operatorname{curl} V_1 + (V_1^*\nabla)V_1$. We see, for example, that when $\nu \ll \omega$ the averaged hf force is potential (see (1)), and consequently the equation of motion (5) has a relatively simple form.

The physical meaning of the velocity V_L introduced above can be explained with the aid of the expression for the smooth density of the electric current $j_0 = en\overline{V}$ due to the motion of the liquid under consideration. Using the continuity equation and the connection between V_L and V_0, we obtain

$$j_0 = en_0 V_L + c \operatorname{rot} M_0, \qquad M_0 = \frac{ien_0}{4c\omega}[V_1 V_1^*].$$

M_0 is the smoothed magnetic moment of the oscillating motion V_1 per unit volume.

In the derivation of (5) we assumed that the pressure tensor is a second-order quantity. Therefore, the expression (6) for V_1 does not contain the proper pressure of the liquid, and the foregoing hydrodynamic description is in general compatible with the theory of wave propagation only in the case when the liquid under consideration has a low temperature. The foregoing limitation can be estimated in simple fashion in the case of a plane wave with phase velocity v_f:

$$-i\omega V_1 = \frac{e}{m}E_1 + [V_1\Omega] - \nu V_1 - i\omega\frac{u^2}{v_f^2}\frac{k(kV_1)}{(kk)}; \quad (7)$$

$u \approx (T/m)^{1/2}$ is the speed of sound in the liquid under consideration. If the last term in (7) cannot be neglected it is necessary to turn to the more complicated theory of[7]. In such a case we should have $u \gtrsim v_f$; then, for infrequent collisions, it is necessary to take into account the Doppler effect and, of course, the hydrodynamic description is inadequate.

It is convenient sometimes to write the potential W in the form

$$W = -\frac{ie}{4\omega}(E_1^* V_1) + \frac{i\nu m}{4\omega}(V_1^* V_1) = \frac{e}{4\omega}\operatorname{Im}\{(E_1^* V_1)\}. \quad (8)$$

From the connection that W be real we get the natural relation

$$\nu m(V_1^* V_1) = e\operatorname{Re}\{(E_1^* V_1)\}.$$

The influence of the collisions on the averaged high frequency force was taken into account also by Johnston[8] (zero magnetostatic field) and by Perel' and Pinskiĭ[9] (homogeneous magnetostatic field). The expressions introduced there follow from the last two terms of (5) as particular cases. It should be noted that, unlike Johnston[8], we assumed $B_0 \neq 0$. Summing the average high frequency forces acting on the electrons and ions of the quasineutral plasma, we obtain for $\nu = 0$ agreement between (8) and the result of the thermodynamic theory of a transparent dielectric[10].

A more detailed exposition of the presented theory, together with other results, will be published later in the Czechoslovak Journal of Physics, Section B.

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