# HEAT EXCHANGE BETWEEN A SOLID BODY AND HELIUM FILLING A NARROW GAP

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The mechanism of heat exchange between a solid body and He II filling a narrow gap is considered. It is shown that the heat exchange significantly depends on the relation between the phonon mean free path $l$ and the gap width d. The heat flux between the solid body and liquid helium below 1°K is calculated for $d \ll l$.

IT is known that heat transfer between a solid and liquid helium below the λ point has a number of unique features due to the large thermal resistance of the interface between these two media.

The main mechanisms of heat transfer between helium and a solid are elastic and inelastic scattering of rotons from the interface. The probability of the roton becoming transformed into a photon of the solid is practically nil, and that of the phonons differs from zero in a relatively narrow cone close to the normal to the interface. The transformation of a helium phonon into a phonon of the solid is in fact elastic scattering of the phonon from the helium-solid interface. The probability of elastic scattering vanishes when $\sin \chi_b = c/c_s$, where c and $c_s$ are the sound velocities in the helium and the solid, respectively. As a rule $c/c_s \ll 1$ and therefore $\chi_b \approx c/c_s \ll 1$. At the maximum (when $\chi = 0$) we have $W_{el} = 4\rho c/\rho_s c_s$[1] ($\rho$—density of the helium, $\rho_s$—density of the solid).

Although $W_{el} \ll 1$, it is nevertheless larger than the inelastic-scattering probability $W_{inel}$, which tends to zero rapidly with decreasing temperature. According to [2], the mean value of $W_{inel}$ is $(T/Mc^2)(T/\Theta)^3$, where $M = \rho_s a^3$ (a—distance between atoms of the solid, $\Theta$—Debye temperature). However, the inelastic scattering has no restrictions with respect to the angles. Since the difference between $W_{el}$ and $W_{inel}$ is quite appreciable, the following situation can arise relatively easily: The path covered by the phonons that move almost perpendicularly to the boundary,1) prior to their "escape" to the wall, turns out to be much shorter than their mean free path $l$; the analogous path of the remaining phonons is much larger than $l$, i.e.,

$$dW_{el}^{-1} \ll l \ll dW_{inel}^{-1}, \tag{1}$$

where d is half the width of the gap filled with the He II.

The phonons in He II interact with one another and with the rotons. Accordingly, there are two mean free paths, $l_{pp}$ and $l_{pr}$. According to [3], the mean free paths are (the temperature is in degrees)

$$l_{pp} \approx 10^{-3}T^{-9}\ \mathrm{cm}, \quad l_{pp} \approx 10^{-8}T^{-9/2}e^{\Delta/T}\ \mathrm{cm}. \tag{2}$$

In (1), $l$ must be taken to mean the smaller of the two paths. The inequality (1) is realized only in the case of sufficiently narrow gaps and at low temperatures (for $d \sim 10^{-5}$ cm and $0.1°K \le T < 1°K$). We note that the mean free path of the relative small-angle scattering can be of the order of the dimension of the gap or even smaller.

Let us consider a plane-parallel capillary of width 2d, filled with He II. Assume that at the initial instant of time the temperature of the He II is T and that of the capillary walls $T'$ ($T \ne T'$ and $|T - T'| \ll T$). The heat exchange between the solid and the He II cause the He II temperature to become equal to the wall temperature after a certain time $t_{eq}$.

We assume that inequality (1) is satisfied. The first to become equalized is the temperature between the "normal" phonons and the wall. In other words, the phonon temperature becomes dependent on the direction of the momentum. The situation described here is similar to that considered by Khalatnikov and Chernikova[4] and has the same nature—a sharp difference between the phonon-phonon scattering cross section at large and small angles.

Further equalization of the temperature (energy exchange between the solid and the helium) is effected either by collisions between the "normal" phonons and the remaining quasiparticles, or by inelastic collisions with the walls. The latter mechanism was considered in detail by Khalatnikov,[2] who showed that the energy flux between the wall and helium due to the inelastic collisions is equal to $q_{inel} = \alpha_{inel}(T - T')$, where $\alpha_{inel}$ is the sum of the phonon ($\alpha_{inel}^p$) and roton ($\alpha_{inel}^r$) terms (in cgs units):

$$\alpha_{inel}^p \approx 10^4 T^7, \tag{3}$$

$$\alpha_{inel}^r \approx 10^7 T^3 e^{-\Delta/T} \tag{4}$$

(T is in degrees).

Let us calculate the energy flux q connected with the scattering of the "normal" phonons. We assume that q, like $q_{inel}$, is calculated per unit area and per unit time; then it is obvious that

$$q = dQ, \tag{5}$$

where Q is the energy transferred from one subsystem

1)We shall call them "normal."

to the other in a unit volume per unit time. The value of Q is governed here by three scattering mechanisms: spin-roton collisions, large angle phonon-phonon scattering, and collisions between phonons with close momentum directions. The latter, in accordance with the energy and momentum conservation laws, are not accompanied by noticeable changes in the directions of the momenta of the colliding phonons, but occur very frequently ($t_{pp} \ll \tau_{pp}$, where $t_{pp}$ and $\tau_{pp}$ are, in accordance with the notation of [3], the times characterizing the phonon scattering at small and large angles).

The collisions between phonons with close momentum directions leads to unique diffusion of the phonons in the space of the angles $\chi$. Knowing $t_{pp}$[3] it is easy to obtain the diffusion coefficient D characterizing this process. Indeed, from the energy and momentum conservations laws it follows that in collisions at zero angle the phonons are deflected from their initial direction by an average $\Delta\chi = kTc^{-1}\sqrt{\gamma}$ (where $\gamma$ is a parameter describing the deviation of the phonon dispersion from linear). Then the order of magnitude of the diffusion coefficient is

$$D \approx (\Delta\chi)^2/t_{pp}. \tag{6}$$

From this we get for the energy flux $q_d = dQ_d$ connected with the diffusion mechanism[2)]

$$q_d \approx \frac{(\Delta\chi)^2}{t_{pp}} C_p d(T-T'). \tag{7}$$

Substituting in (7) the numerical values of all the parameters (in cgs units) we have

$$\alpha_d \approx d\cdot 10^{10} T^{12}. \tag{8}$$

Let us calculate now the energy flux q connected with the large-angle spin-phonon scattering and with the phonon-roton collisions. If we denote by I(n) the phonon collision integral, then

$$Q = \int\limits_{X} I(n)\,\varepsilon\, d\tau_r, \tag{9}$$

where $\varepsilon = cp$, $d\tau_r = p^2 dp dO/(2\pi\hbar)^3$, the symbol X denotes that the integration is carried out in the narrow cone $\chi < \chi_b$, and $I(n) = I_{pr}(n) + I_{pp}(n)$.

Assuming that the phonons have a Planck distribution (with temperature T′) and the rotons a near-Maxwellian distribution (with temperature T), and recognizing that when a phonon is scattered by a roton the energy of the latter remains practically unchanged, we get

$$I_{pr} = (T-T')\frac{\partial n}{\partial T} c\sigma_{pr} N_r, \tag{10}$$

where $N_r$ is the number of rotons per unit volume and $\sigma_{pr}$ is the phonon-roton scattering cross section.[3] Substituting the expression for $I_{pr}$ into (9) we get the part of the energy flux due to scattering by the rotons:

$$q_{pr} = \frac{\chi_b{}^2}{\tau_{pr}} C_p d(T-T'), \tag{11}$$

where $C_p$ is the specific heat of the phonon gas[3] and $\tau_{pr}$ is the phonon-roton relaxation time:

$$\frac{1}{\tau_{pr}} = \frac{(2\pi^3)^{1/2}\mu^{1/2}P_0{}^4}{\hbar^7\rho^2c^5}\left[\frac{2}{9}+\frac{1}{25}\left(\frac{P_0}{\mu c}\right)^2+\frac{2A}{9}\frac{P_0}{\mu c}+A^2\right](kT)^{3/2}e^{-\Delta/T},$$

where

$$A = \frac{\rho^2}{P_0 c}\left[\frac{\partial^2\Delta}{\partial\rho^2}+\frac{1}{\mu}\left(\frac{\partial P_0}{\partial\rho}\right)^2\right],$$

and $\Delta$, $P_0$, and $\mu$ are the parameters of the energy spectrum of the He II excitations. Using the numerical values of all the parameters in (11) we obtain (in the same units)

$$\alpha_{\gamma}{}^{r} \approx d\chi_b{}^2\cdot 10^{16} T^{15/2} e^{-\Delta/T}. \tag{12}$$

To calculate the phonon-phonon part of the flux, $q_{pp}$, we use the relaxation-time approximation,[4] i.e., we write

$$I_{pp}(n) = -\delta n/\tau_{pp}, \tag{13}$$

where $\delta n = -(T-T')\,\partial n/\partial T$. Hence

$$q_{pp} = \frac{1}{2}\frac{\chi_b{}^2}{\tau_{pp}} C_p d(T-T'). \tag{14}$$

Substituting in (14) the numerical value of all the parameters, we get

$$\alpha_{\gamma}{}^{p} \approx d\chi_b{}^2\cdot 10^{11} T^{12}. \tag{15}$$

Comparing (8) and (15), we see that $\alpha_d$ and $\alpha_{el}^{p}$ have the same temperature dependence and are of the same order of magnitude.

Comparison of formulas (8), (12), and (15) with formulas (3) and (4) shows that in the temperature interval $0.6°K < T < 1°K$ it is necessary to take into account the energy transfer mechanisms; at lower temperatures the principal role is assumed by inelastic scattering of the phonons from the interface.

The obtained formulas for the heat flux allow us to determine the effective temperature equalization time $t_{eq} = C_p d/\alpha_{eff}$, where $\alpha_{eff}^{-1}$ is the thermal resistance of the interface ($\alpha_{eff} = \Sigma_j \alpha_j$, where the sum is taken over all the heat-transfer mechanisms). The temperature relaxation time in the capillary, which is one of the kinetic characteristics of the helium in the capillary, can determine in a number of cases the magnitude of the damping of the fourth sound.

We have slightly "oversimplified" the entire analysis. The equalization of the helium-phonon and wall temperatures is actually described by an integro-differential equation that takes into account simultaneously two circumstances: the smearing of the step (gradual equalization of the temperatures of the "normal" and remaining phonons) and the indirect transfer of energy from the "normal" phonons to the remaining ones by scattering through large angles.

Our analysis enables us only to estimate the role of the different mechanisms and to determine the temperature dependence of the corresponding coefficients. The agreement of the temperature dependences of $\alpha_d$ and of $\alpha_{el}^{p}$ shows that a more accurate analysis can only refine the numerical factors.

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2)We note that D does not depend on the parameter $\gamma$. This is connected with the fact that both $(\Delta\chi)^2$ and $t_{pp}$ are proportional to $\gamma$.

[1] I. M. Khalatnikov, Vvedenie v teoriyu sverkhtekuchesti, Nauka, 1965 [Introduction to the Theory of Superfluidity, Benjamin, 1965].
[2] I. M. Khalatnikov, Usp. Fiz. Nauk **60**, 69 (1956).
[3] I. M. Khalatnikov, ibid. **59**, 672 (1956).
[4] I. M. Khalatnikov and D. M. Chernikova, Zh. Eksp. Teor. Fiz. **49**, 1957 (1965) [Sov. Phys.-JETP **22**, 1336 (1966)].

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