QUANTUM LONGITUDINAL GALVANOMAGNETIC PHENOMENA IN THIN SEMICONDUCTOR AND SEMIMETAL FILMS

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Galvanomagnetic phenomena in thin semimetallic and semiconducting films are considered for the case when the common direction of the electric and magnetic fields is parallel to the surface of the sample. It is shown that a change of the film thickness or magnetic field strength should lead to oscillations of the conductivity at the experimentally attainable parameters. The period of the oscillations for various cases is determined. When the thickness is varied the oscillations vanish in thick films ($L \gtrsim 5000$ Å) and the same relations as those in the Shubnikov–de Haas effect are found to hold. In contrast, no oscillations of the magnetic resistance occur in very thin films. In films of intermediate thickness the oscillations can be observed in relatively weak magnetic fields. In this case the oscillation period $\Delta(1/H)$ decreases with increasing field strength and approaches the value of the period in a bulky sample.

1. INTRODUCTION. FORMULATION OF PROBLEM

THE question of the resistance of thin films in a longitudinal magnetic field investigated in a number of papers [1], where the quantization of the electron motion, connected with the Landau effect and the limited dimensions of the film, was neglected. Consequently, the obtained dependences of the resistance of the magnetic field are naturally monotonic and do not contain oscillations when the magnetic field is varied. Yet if the distance between the discrete levels is large compared with $\hbar/\tau$ (τ—electron relaxation time in the film) and k_BT, then the quantization must be taken into account when considering the kinetic phenomena. Just as in thermodynamic phenomena [2], the quasidiscrete character of the spectrum can lead under certain conditions to oscillations of the kinetic coefficients.

The quantization due to the magnetic field was confirmed experimentally. As is well known, it leads, in observations of the diamagnetic susceptibility, to de Haas-van Alphen oscillations (see, for example, [3]), and in the case of magnetoresistance it leads to the Shubnikov–de Haas effect [4].

The limited nature of the electron motion transverse to the film should also lead to the appearance of a quasidiscrete spectrum $\epsilon = \epsilon(k_x, k_y, s)$, where k_x and k_y are the longitudinal components of the quasimomentum, and the discrete quantum number s replaces the projection k_z of the quasimomentum. At fixed s, the energy runs through a discrete set of values forming the subband. In semiconductors and in semimetals, up to room temperatures, the low carrier density causes a small number of subbands to become populated [5]. For example, in the case of Bi films ($m_\perp \sim 0.01\, m_0$) at a density $\sim 10^{17}$ cm^{-3}, only subband is populated at a thickness $L \sim 5 \times 10^{-6}$; at a larger thickness, subbands with large values of s become populated. On the other hand, in the case of a metallic film, a large number of subbands become populated, which, naturally, increases the effect of the film quantization. Therefore neglect of the quantization of the electron motion in metallic films in [1] is justified.

Until recently, the question of the existence of a quasidiscrete spectrum in a film was debatable, since the relatively small value of the relaxation time of the electrons in a thin film made it difficult, as a rule, to observe such a spectrum. The quantization effect could appear only in sufficiently perfect films with small effective carrier mass. Experimental papers appeared only recently. Orgin, Lutskiĭ, and Elinson [6] observed oscillations due to film quantization, of the resistance, of the Hall constant, and of the magnetoresistance in a non-quantizing magnetic field ($\omega_c\tau < 1$) in Bi films when the thickness was varied. Subsequent investigations [7] confirmed the existence of these oscillations. Independent experiments [8] on the tunneling of electrons between Bi films through a vacuum gap also revealed a quasidiscrete character of the energy spectrum.

The quasidiscrete nature of the spectrum, due both to the limited dimensions of the film and to the magnetic field, was first taken into account by Lifshitz and Kosevich [9,10] in an analysis of the de Haas-van Alphen effect in thin films. Allowance for this quantization in galvanomagnetic phenomena was made only in the case of a transverse magnetic field [11]. It turned out then that the current depends nonlinearly on the magnitude of the electric field. On the other hand, in the case where the magnetic field does not lead to quantization, Ohm's law is satisfied [12, 13].

In this paper we consider longitudinal galvanomagnetic phenomena in semiconducting and semimetallic films (parallel electric and magnetic fields lie in the plane of the film), with allowance for the magnetic and film quantization. This quantization, of course, becomes manifest if the scattering is not very large (see the criterion (6) below). The analysis in the article is based on the kinetic equation, the applicability of which in our case is proved in the Appendix.

As was already noted, the experimental investigations [6-8] prove the quantization of transverse motion of the electron in thin Bi films in the absence of a magnetic field. This means that random scattering by

lattice defects, including diffuse scattering from the boundary, is weak. Therefore we can introduce a stationary potential to take into account the limited dimensions of the film. In terms of quasiclassical approximation, the introduction of a stationary potential is equivalent to the assumption of specular reflection from the boundary. The specular character of the electron reflection from the boundaries in the Bi films was also observed earlier (see, for example,[14]), prior to the observation of the quantum size effects.

To take into account the influence of the limited nature of the film we choose, just as in[9], a model of a film potential in the form of parabolic potential well, making it possible to obtain an exact solution of Schrödinger's equation. However, the choice of the model does not affect the results substantially.

Notice should be taken, however, of the fact that when considering quantum size effects one takes into account lower levels of the energy, for which quasiclassical concepts are not applicable. Therefore the introduction of the concepts of specular and diffuse reflections from the film boundary is not valid, strictly speaking, in this case.

2. EXISTENCE OF QUASIDISCRETE SPECTRUM IN A LONGITUDINAL MAGNETIC FIELD

The complete single-particle Hamiltonian will be written in the form

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V} - eEx. \tag{1}$$

here

$$\mathcal{H}_0 = \frac{p_x^2}{2m_\parallel} + \frac{(p_y - eHz/c)^2}{2m_\parallel} + \frac{p_z^2}{2m_\perp} + \frac{m_\perp \omega_0^2 z^2}{2}, \tag{2}$$

V is the scattering potential, H and E are directed along the x axis and the normal to the film is parallel to the z axis; $m_\perp$ and $m_\parallel$ are the transverse and longitudinal masses.

The value of ω_0 does not depend on the magnetic field and is determined by the thickness of the film. In order that the chosen model describe better the lower quantum levels, ω_0 must be determined from the uncertainty relation

$$\omega_0 \sim (\hbar / m_\perp)(\pi / L)^2, \tag{3}$$

and for levels with larger quantum numbers it is better to use the relation $m_\perp \omega_0^2 L^2/2 \sim \eta$ (η—Fermi energy).

The normalized solution and the spectrum of the Schrödinger equation with Hamiltonian (2) take the form

$$\psi_{M\mathbf{k}} = \psi_\alpha = \psi_{Mk_xk_y} = \frac{1}{\sqrt{L_xL_y}} \exp\{i(k_x x + k_y y)\} \frac{1}{\sqrt{l}} \varphi_M\left(\frac{z - z^0_{k_y}}{l}\right), \tag{4}$$

$$\varepsilon_{M\mathbf{k}} = \varepsilon_\alpha = \varepsilon_{Mk_xk_y} = \left(M + \frac{1}{2}\right)\hbar\tilde{\omega} + \frac{\hbar^2 k_x^2}{2m_\parallel} + \frac{\omega_0^2}{\tilde{\omega}^2}\frac{\hbar^2 k_y^2}{2m_\parallel}. \tag{5}$$

here M is the magneto-film quantum number, k_x and k_y are the longitudinal components of the two-dimensional wave vector $\mathbf{k}$, L_x, L_y are the linear dimensions of the film, the cyclotron frequency is $\omega_c = eH/c\sqrt{m_\parallel m_\perp}$, and the magneto-film frequency is $\tilde{\omega} = \sqrt{\omega_0^2 + \omega_c^2}$; the center of the oscillations is $a k_y = (c\hbar/eH)(\omega_c/\tilde{\omega})^2 k_y$; φ_M is a Hermite function of order M; the "magneto-film" length is $l = (\hbar/m_\perp\tilde{\omega})^{1/2}$.

The singularities of the spectrum (5) consist in the manifestation of a system of equidistant magneto-film discrete levels with interval $\hbar\tilde{\omega}$, which are due to simultaneous influence on the transverse motion by the magnetic field and by the limited dimensions of the film, and also in the appearance of the dependence of the energy k_y, which is due to the lifting of the degeneracy by the film potential. In the plane of the film, the motion turns out to be quasiclassical, and the effective mass is anisotropic, its component normal to the field depends on the intensity H and on the thickness L ($m_2^* = \tilde{\omega}^2 m_\parallel/\omega_0^2$).

For another choice of the film potential, the foregoing singularities will also take place, but the concrete form of the dependence of $\tilde{\omega}$ on L and on the magnetic field will change, the energy spectrum will become nonequidistant, that is, a dependence of $\tilde{\omega}$ on the quantum number M will appear. The anisotropy in the longitudinal motion will remain in this case.

The scattering of the electrons leads to a smearing of the levels, therefore, the quasidiscrete character of the spectrum must be taken into account only under the condition

$$\tilde{\omega}\tau \gg 1. \tag{6}$$

For any form of the film potential, the distance between the magnetic-film levels is larger than the distance between the discrete levels in the film in the absence of a magnetic field. Therefore the criterion (6) will certainly be satisfied for films with parameters such that quantization is observed. As will be shown below, this will bring about a situation wherein oscillations of magnetoresistance in the film can appear in relatively weak magnetic fields.

3. GENERAL RELATIONS FOR THE CURRENT: LIMITING QUANTUM CASE

The current density, as is well known, can be represented in the form $\mathbf{j} = en\mathbf{v}$, with the average electron (hole) velocity $\mathbf{v} = \mathrm{Tr}(\hat{v}\hat{\rho})$ with normalization of the density matrix $\mathrm{Tr}\,\rho = 1$, where n is the number of carrier per unit volume.

It follows from symmetry considerations that the average velocity along the y axis is equal to zero. Since $v_x = i\hbar^{-1}[\mathcal{H}x] = \hat{p}_x/m_\parallel$, the matrix element in the $\mathcal{H}_0$ representation are equal to

$$(v_x)_{M\mathbf{k},M'\mathbf{k}'} = \frac{\hbar k_x}{m_\parallel}\delta_{MM'}\delta_{\mathbf{k}\mathbf{k}'}.$$

from this follows an expression for the current

$$j = \frac{en\hbar}{m_\parallel}\sum_{M\mathbf{k}} k_x f_{M\mathbf{k}}, \tag{7}$$

where the distribution function $f_{M\mathbf{k}}$ is a diagonal element of the density matrix. Thus, the current in the film is determined only by the diagonal matrix elements of the operator $\hat{\rho}$. It is shown in the Appendix that for weak scattering, when the criterion (6) is satisfied, the diagonal elements of the density matrix satisfy the kinetic equation. In the approximation which a relaxation time exists it takes the form

$$\frac{eE}{\hbar}\frac{\partial f^{(0)}_{M\mathbf{k}}}{\partial k_x} + \frac{f^{(1)}_{M\mathbf{k}}}{\tau(\varepsilon_{M\mathbf{k}})} = 0, \tag{8}$$

where $f^{(0)}_{M\mathbf{k}}$ is the distribution function in the zeroth

approximation in the electric field, $f^{(1)}_{Mk}$ is the first-order correction in E, and the relaxation time is determined by the expression

$$\frac{1}{\tau(\varepsilon_{Mk})}=\frac{2\pi}{\hbar}\sum_{M'k'}\frac{k_x-k_x'}{k_x}|V_{Mk,M'k'}|^2\delta(\varepsilon_{Mk}-\varepsilon_{M'k'}). \qquad (9)$$

We shall first stop to discuss the case when only one magneto-film level is filled (M = 0). The problem then becomes two-dimensional in the quantum-number space, and only transitions with change of k_x and k_y are possible.

In the kinetic equation (8), the relaxation time depends in this case only on the longitudinal-motion energy ϵ_k, equal to $\hbar^2[k_x^2+(\omega_0/\tilde{\omega})^2k_y^2]/2m_{\parallel}$. Substituting into the expression for the current (7) the magnitude of the correction to the distribution function $f^{(1)}_{0,k}$ and confining ourselves to the degenerate case, we obtain the Drude formula $j = Ee^2\tau n/m_{\parallel}$.

To calculate the relaxation time it is necessary to propose a definite scattering mechanism. We shall assume first that the scattering of the electrons is due to randomly distributed impurities. In the case when the scattering potential of the impurity is δ-like ($V_i = V_0\delta(r - r_i)$), the matrix element can be easily calculated[15]:

$$|V_{0k,0k'}|^2=\frac{n_0V_0^2}{L_xL_yl}\left|\varphi_0\left(\frac{z^0_{k_y}}{l}\right)\varphi_0\left(\frac{z^0_{k_y'}}{l}\right)\right|^2 \qquad (10)$$

(n_0—number of impurities per unit volume). Substituting the expression (10) in (9) and going over from summation to integration over the energy of the two-dimensional motion ϵ_k and over the angle between the directions k and k′, we obtain after simple calculations

$$\frac{1}{\tau}=\frac{2\pi}{\hbar}n_0V_0^2\frac{L}{l}N_MI_0(t)e^{-t}, \qquad (11)$$

where the density of the states per unit volume at a fixed magneto-film number M is equal to

$$N_M=\frac{m_{\parallel}}{\pi\hbar^2}\frac{\tilde{\omega}}{\omega_0}\frac{1}{L}, \qquad (12)$$

$I_0(t)$ is a Bessel function of imaginary argument of zero order, $t = (\omega_c/\omega_0)^2\epsilon_k/\hbar\tilde{\omega}$. The state density (12) is a direct consequence of the dispersion law (5). It does not depend on the quantum number M or on the energy ϵ_k.

In the limiting case $H \to 0$, expression (11) goes over into $\tau^{-1} = 2\hbar^{-3}V_0^2m_{\parallel}n_0/L$, which coincides, accurate to a numerical factor, with the results of Sandomirskiĭ[16] (the difference in the coefficient is due to a different choice of the film potential). On going over to a bulky sample ($\omega_0 \to 0$), using the asymptotic expression $I_0(t)$, we obtain the well known dependence

$$\tau^{-1}\sim\omega_c\hbar^{-4}\sqrt{m_{\parallel}^3m_{\perp}}n_0V_0^2/k_x.$$

Formula (11) has a specially simple form in the region $\omega \gg \omega_c$, when the quantization is due primarily to the film potential. Since $I_0(t)e^{-t} \to 1$ as $t \to 0$, we have

$$\tau^{-1}\sim\omega_0^{1/2}[1+{}^3/_4(\omega_c/\omega_0)^2].$$

We see that the relaxation time will decrease both when the magnetic field increases and when the thickness of the films decreases.

In the case of scattering of electrons by acoustic lattice vibrations at high temperatures (a temperature above 1°C in a semiconductor or a semimetal can be regarded as high[17]), we obtain for the relaxation time a formula similar to (11):

$$\frac{1}{\tau}=\frac{(2\pi)^{3/2}}{\hbar}\frac{C^2k_BT}{\rho c_l^2}\frac{L}{l}N_MI_0\left(\frac{t}{2}\right)e^{-t/2}, \qquad (13)$$

where C is the deformation-potential constant, c_l the longitudinal speed of sound, and ρ the density of the film material. When the field H tends to zero formula (13) goes over into the expression obtained in[18] for τ^{-1}.

The carrier density in the degenerate case is equal to $n = N_M(\eta - \hbar\tilde{\omega}/2)$. Substituting in the Drude formula, we find that the conductivity does not depend on the state density and is determined by the expression

$$\sigma\sim\frac{l}{L}\left(\eta-\frac{\hbar\tilde{\omega}}{2}\right)\frac{1}{I_0(t)e^{-t}}.$$

In the region $\omega_0 \gg \omega_c$, the $\sigma(L, H)$ dependence becomes simpler, $\sigma \sim lL^{-1}(\eta - \hbar\tilde{\omega}/2)$. We see therefore that the conductivity decreases with increasing magnetic field and with decreasing L.

4. CONDUCTIVITY OSCILLATIONS UPON VARIATION OF THE MAGNETIC FIELD OR THE FILM THICKNESS

When the magnetic field or the film thickness changes, the number of populated magneto-film levels changes. This causes the quantities n and τ in the conductivity $\sigma = e^2\tau n/m_{\parallel}$ to vary nonmonotonically in the degenerate case. The carrier density will in this case be a continuous function and its oscillations will be insignificant. In this case the carrier density will be a continuous function and its oscillations will be insignificant. Indeed

$$n=\int_0^{\eta}N_{H,L}\,d\varepsilon,$$

where the total density of states is proportional to N_M and to the number of populated levels. In the chosen model it is equal to

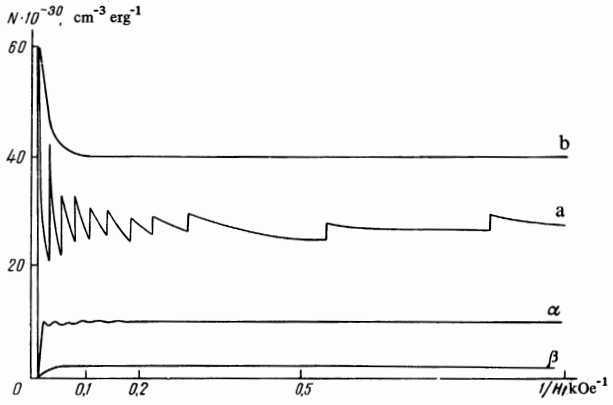
$$N_{H,L}=\frac{m_{\parallel}}{\pi\hbar^2}\frac{\tilde{\omega}}{\omega_0}\frac{1}{L}\left[\frac{\varepsilon}{\hbar\tilde{\omega}}+\frac{1}{2}\right] \qquad (14)$$

(The number of populated levels $[\epsilon/\hbar\tilde{\omega} + {}^1/_2]$ is the integer part of the argument). Substituting (14) in the expression for the concentration, we obtain

$$n=\frac{m_{\parallel}}{\pi\hbar^2}\frac{\tilde{\omega}}{\omega_0}\frac{1}{L}\left[\frac{\eta}{\hbar\tilde{\omega}}+\frac{1}{2}\right]\left\{\eta-\frac{1}{2}\left[\frac{\eta}{\hbar\tilde{\omega}}+\frac{1}{2}\right]\hbar\tilde{\omega}\right\} \qquad (15)$$

As follows from (15), the dependence of the concentration on the magnetic field is a continuous function. A plot of it is shown in the figure, under the assumption that the Fermi level is constant (see below). The dependence of n on L is also continuous.

The relaxation time (9) depends on L and H via the density of states and the matrix element of the scattering potential. The matrix elements are monotonic functions, and therefore the oscillatory character of the $\tau(L, H)$ dependence will be determined for all the scattering mechanisms by the jumps of the state density (14), which arise when the number of populated magneto-film levels changes. With increasing film thickness or decreasing magnetic field, the state



Dependence of the electron density of states $N_{H,L}$ in Bi films on the magnetic field H: a – thickness of film L = 2000 Å. For the same thicknesses we show, in an arbitrary scale, the dependence of the electron concentration on H (curves α and β).

density will decrease until a new level begins to be populated. This is accompanied by jumplike increase in the density of states by an amount N_M, and then $N_{H,L}$ will again start to decrease. Thus, the function of the density of states, and with it the conductivity, will oscillate.

The period of the oscillations will obviously depend on how the Fermi level position changes with changing L and H. For a quantitative analysis of the indicated oscillatory dependence we shall assume that the Fermi energy is constant. This situation takes place, for example, for an isotropic dispersion law in a semimetal [12], in which the concentrations of the electrons and holes are equal. This will also occur in the presence of anisotropy, if the mass of one of the carriers is much larger than the mass of the other (for example, $m_h \gg m_e$). In this case the hole Fermi energy is $\eta_h \ll \eta_e$, since $\eta \sim (m_1 m_2 m_3)^{1/3}$, and can be neglected. Since the overlap of the electron and hole bands is constant (this takes place when the film potential and the magnetic field do not change appreciably the effective masses), then the Fermi energy of the electrons will not depend on L and H.

Calculations of the density of the electronic state by formulas (12) and (14) for two values of Bi film thickness lead to the curves shown in the figure. For thick films ($L \gtrsim 5000$ Å), since $\omega_c \gg \omega_0$, the period of the oscillations $\Delta(1/H)$ does not depend on the magnetic field and is equal to

$$\Delta(1/H) = e\hbar/\eta c\sqrt{m_{\parallel} m_{\perp}}.$$

It turns out to be the same as in the case of a bulky sample. The difference would occur only in weak fields, but in this region the criterion (8) of the quantizing field is violated. For very thin films ($L \approx 500$ Å), the change of the magnetic field affects the energy spectrum little and no oscillations arise at attainable fields.

The most interesting region is that of intermediate thicknesses, 1000–3000 Å. For these thicknesses, the quantization criterion (see Sec. 2) is satisfied for fields exceeding several kG. In such magnetic fields, as seen from curve b, oscillations arise in the state-density function, and this should lead even in the case of weak fields to the Shubnikov–de Haas effect. It must be noted, however, that the period of the oscillations $\Delta(1/H)$ will be larger than in a bulky sample. With increasing field H, the period of the oscillations will decrease and in the limit of strong magnetic fields it will tend to the value of the period for a bulk sample.

The oscillatory dependence of the relaxation time and of the conductivity upon change in film thickness will also be determined by the state density function. The period of the oscillations will depend, naturally, on the value of the field H, and, as seen from (14), will increase with increasing magnetic field. This is connected with the fact that the influence of the limited size of the film will be weaker in the strong field.

The relations obtained for the oscillations of the conductivity with variation of L and H do not pretend, of course, to be in exact quantitative agreement with experiment. This is connected primarily with the fact that the chosen parabolic potential in our paper does not coincide with the true potential in the film. However, even for another choice of the film potential, the dependence $\sigma(L, H)$ retains its oscillatory character. All the quantitative relations obtained above also remain in force, since the form of the potential was not used at all in their derivation.

The quantization criterion (6) can be satisfied not by turning on a strong magnetic field, but by preparing a sufficiently thin film. In this case $\omega_0 \gg \omega_c$, and the magnetic field leads only to a perturbation of the film levels of the energy. The change of the period of the oscillations with increasing H can be readily determined by recognizing that the number of the populated magneto-film levels is determined by the condition $M = [\eta/\hbar\tilde{\omega} + 1/2]$ (the square brackets denote the integer part of the argument). For an arbitrary $\omega_0(L)$ dependence we have

$$\frac{\Delta L}{L} = \left(\frac{\Delta L}{L}\right)_{H=0}\left[1 + \beta^2\left(M + \frac{1}{2}\right)^2 H^2\right], \qquad (16)$$

where the constant $\beta = l\hbar/\eta c\sqrt{m_{\parallel} m_{\perp}}$. Thus, the period of the conductivity oscillations with change in the film thickness will increase with increasing H.

The conductivity oscillations can arise, of course, only in the presence of strong degeneracy, when the condition

$$\hbar\tilde{\omega} \gg k_B T \qquad (17)$$

is satisfied.

The criterion (17) is not a strong limitation on the temperature. In weak magnetic fields, for thicknesses smaller than $L < 5 \times 10^{-6}$ cm, at $m_{\perp} \sim 0.1\, m_0$ (for example, bismuth), the criterion is satisfied already at nitrogen temperatures. An increase in the magnetic field leads to satisfaction of (17) at higher temperatures or else for thicker films.

Thus, in thin semimetals and degenerate-semiconductor films, one can expect the appearance of conductivity oscillations both when the film thickness is varied and when the magnetic field is varied. Although the condition for the existence of Landau levels in the not too strong magnetic fields is, as a rule, violated in a film as a result of the small carrier mobility, the aforementioned oscillations will take place, since the criteria (6) and (17) are not stringent in a thin film.

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APPENDIX

The quantum equation for the density matrix is

$$i\hbar\partial\rho / \partial t = [\mathcal{H}\rho], \tag{A.1}$$

where $\mathcal{H}$ is the complete Hamiltonian (1). We confine ourselves to the calculation of the density matrix in the linear approximation in the electric field, that is, to a calculation of the magnetoresistance in the ohmic region. This is always possible, at least for weak electric fields. Then the release of Joule heat can be neglected, and the electron-gas temperature can be regarded as constant.

Following the method of Kohn and Luttinger [19], we represent the density matrix in the form

$$\rho = \rho_0 + \rho_\varepsilon, \tag{A.2}$$

where ρ_0 is the density-matrix operator in the absence of an electric field and ρ_c is matrix correction term which is linear in the field.

Substituting (A.2) in (A.1) and linearizing with respect to the intensity E, we obtain

$$i\hbar\frac{\partial\rho_0}{\partial t} = [\mathcal{H}_0 + V, \rho_0], \tag{A.3}$$

$$i\hbar\frac{\partial\rho_\varepsilon}{\partial t} = -eE[x\rho_0] + [\mathcal{H}_0 + V, \rho_\varepsilon]. \tag{A.4}$$

The condition (6) allows us to regard the potential V as a perturbation to the Hamiltonian $\mathcal{H}_0$, we therefore get from (A.3)

$$\langle\alpha'|\rho_0|\alpha\rangle = \begin{cases} f^0(\varepsilon_\alpha) = f_\alpha{}^0 & \alpha' = \alpha \\ \langle\alpha'|V|\alpha\rangle(f_{\alpha'}{}^0 - f_\alpha{}^0)/(\varepsilon_{\alpha'} - \varepsilon_\alpha) & \alpha' \neq \alpha. \end{cases} \tag{A.5}$$

Here the equilibrium function of the electron distribution

$$f_\alpha{}^0 = [1 + \exp(\varepsilon_\alpha - \eta) / k_BT]^{-1}.$$

Substituting the matrix elements (A.5) in (A.4) we obtain the equation

$$(\varepsilon_{\alpha'} - \varepsilon_\alpha - i\hbar s)\langle\alpha'|\rho_\varepsilon|\alpha\rangle = ieE\frac{\partial\langle\alpha'|\rho_0|\alpha\rangle}{\partial k_x} + \langle\alpha'|[\rho_\varepsilon V]|\alpha\rangle, \tag{A.6}$$

The small parameter s indicates the rule for circuiting around the poles. In determining the first term of (A.6), we used the form of the operator x in the representation $\mathcal{H}_\rho(x = i\partial/\partial k_x)$. Let us write out separately the equation (A.6) for the diagonal and the nondiagonal matrix elements:

$$\begin{aligned}(\varepsilon_{\alpha'} - \varepsilon_\alpha - i\hbar s)\langle\alpha'|\rho_\varepsilon|\alpha\rangle &= ieE\frac{\partial\langle\alpha'|\rho_0|\alpha\rangle}{\partial k_x} \\ &+ \langle\alpha'|V|\alpha\rangle(\langle\alpha'|\rho_\varepsilon|\alpha'\rangle - \langle\alpha|\rho_\varepsilon|\alpha\rangle) \\ &+ \sum_{\alpha''\neq\alpha,\,\alpha'}\{\langle\alpha'|\rho_\varepsilon|\alpha''\rangle\langle\alpha''|V|\alpha\rangle - \langle\alpha'|V|\alpha''\rangle\langle\alpha''|\rho_\varepsilon|\alpha\rangle\}\end{aligned} \tag{A.7}$$

for $\alpha' \neq \alpha$ and

$$\begin{aligned}-i\hbar s\langle\alpha|\rho_\varepsilon|\alpha\rangle &= ieE\frac{\partial f_\alpha{}^0}{\partial k_x} + \sum_{\alpha'\neq\alpha}\{\langle\alpha|\rho_\varepsilon|\alpha'\rangle\langle\alpha'|V|\alpha\rangle \\ &- \langle\alpha|V|\alpha'\rangle\langle\alpha'|\rho_\varepsilon|\alpha\rangle\}.\end{aligned} \tag{A.8}$$

in formula (A.7) we left out the terms containing the diagonal matrix elements of the scattering potential V, which lead to a shift of the levels ϵ_α by an amount $\langle\alpha|V|\alpha\rangle$. This quantity does not depend on the states α if V does not contain differentiation and integration operators. By varying the reference level from which the energy is reckoned, we can exclude this shift without loss of generality.

We seek a solution of (A.7) and (A.8) in powers of the potential V. It is easy to see that the diagonal element of the matrix, as in the ordinary kinetic theory, are of the order of V^{-2}. Consequently, the principal role in the right side of (A.7) is played by the second term. Retaining only this term, we get from (A.7) an expression for the nondiagonal elements and substitute it in the (A.8). We then get

$$eE\frac{\partial f_\alpha{}^0}{\partial k_x} + 2\pi\sum_{\alpha'(\alpha'\neq\alpha)}|\langle\alpha|V|\alpha'\rangle|^2\delta(\varepsilon_\alpha - \varepsilon_{\alpha'})(f_\alpha^{(1)} - f_{\alpha'}^{(1)}), \tag{A.9}$$

where $f_\alpha^{(1)} = \langle\alpha|\rho_\epsilon|\alpha\rangle$. We have taken here the limit as $s \to 0$. Expression (A.9) is the usual kinetic equation, which includes the field and the collision terms.

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