EXCITATION OF SURFACE WAVES BY A MOVING CHARGE

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The field of surface waves excited by an arbitrarily moving charged particle on the interface between vacuum and an absorbing nonmagnetic medium is calculated. Excitation of surface waves by a particle moving uniformly and rectilinearly at an angle with respect to the interface is considered. The particular cases of normal incidence and of motion of the charge parallel to the interface are also considered. For glancing incidence the spectral-energy density of the surface-wave excitation exceeds the transition-radiation intensity by several orders of magnitude. The estimates presented in the paper show that the experimentally-observed intense radiation produced by electrons impinging on a silver target at grazing angles can be attributed to scattering of the surface wave by irregularities of the surface.

As is well known[1], a surface H-wave can propagate along the interface of two nonmagnetic media with positive and negative dielectric constants respectively; this wave attenuates exponentially on penetrating deeper in either medium. Such a wave can be excited quite effectively by a moving charged particle, as is clearly manifest in experiments on the study of discrete energy losses[2].

The question of excitation of surface waves by a moving charge was considered by many authors[4–9], starting with Ritchie[3]. In all these investigations, dissipation was not taken into account, and the particle motion was assumed linear and uniform. In the present article we find the field of surface waves excited by an arbitrarily moving charged particle on the interface between vacuum and a nonmagnetic absorbing medium. We analyze in detail the case of uniform and linear motion at an arbitrary angle to the boundary.

1. INITIAL FORMULAS

We use as our starting point the results of[10], where the field of a arbitrarily moving charged particle was determined in the presence of a boundary between media. To find the field of the surface wave in the case of nonmagnetic media, which we shall consider here, it is necessary to use formulas (2.13) and (2.8) of[10], which describe the reflected and refracted field polarized in the plane of propagation. The electric vector of the corresponding electromagnetic waves lies in the plane passing through the wave vector and the normal to the interface, and the magnetic-field vector is parallel to the interface.

Integrating (2.13) and (2.8) of[10] along the trajectory and putting in these formulas $\epsilon_j = 1$, $\epsilon_s = \epsilon$, and $\mu_j = \mu_s = 1$, we write the Hertz vectors of the field of an arbitrarily moving charge particle in the form

$$\Pi_{\omega j}^{j} = -\frac{e\mathbf{k}}{4\pi^2\omega}\int\int\int \frac{e^{i\omega t}(\varepsilon k_{jz} - k_{sz})}{\beta\varkappa^2 k_{jz}(\varepsilon k_{jz} + k_{sz})}[\beta_z\varkappa^2 + (\beta_x k_x + \beta_y k_y)k_{jz}]$$
$$\times \exp[ik_x(x - x_\zeta) + ik_y(y - y_\zeta) + ik_{jz}(z + z_\zeta)]\,dk_x\,dk_y\,d\zeta,\quad z > 0,\ z_\zeta > 0, \tag{1.1}$$
$$\Pi_{\omega s}^{j} = -\frac{e\mathbf{k}}{2\pi^2\omega}\int\int\int \frac{e^{i\omega t}}{\beta\varkappa^2(\varepsilon k_{jz} + k_{sz})}[\beta_z\varkappa^2 - (\beta_x k_x + \beta_y k_y)k_{sz}]$$
$$\times \exp[ik_x(x - x_\zeta) + ik_y(y - y_\zeta) + ik_{jz}z - ik_{sz}z_\zeta]\,dk_x\,dk_y\,d\zeta,\quad z > 0,\ z_\zeta < 0,$$

where

$$\varkappa = \sqrt{k_x^2 + k_y^2},\quad k_{jz} = \sqrt{\omega^2/c^2 - \varkappa^2},\quad k_{sz} = \sqrt{\varepsilon\omega^2/c^2 - \varkappa^2}.$$

From the requirement that the field be finite at infinity, it follows that the imaginary parts of κ, k_{jz}, and k_{sz} are either positive or zero.

Just as in[10], we use here a rectangular system of coordinates with z axis directed into the medium j, which in our case is vacuum, and an xy plane coinciding with the interface. We omit the symbol $\parallel$, which indicates that the corresponding waves are polarized in the plane of propagation, and designate the Hertz vectors with superior indices j, which denote that the given quantities pertain to vacuum ($z > 0$). Inferior indices j or s denote that the field is determined by the particle moving respectively in vacuum ($z_\zeta > 0$) or in the medium ($z_\zeta > 0$).

Reversing the signs of the vectors (2.13) and (2.8) of[10], interchanging the indices j and s, reversing the signs in front of the components k_{jz} and k_{sz}, putting then $\epsilon_j = 1$, $\epsilon_s = \epsilon$, $\mu_j = \mu_s = 1$, and integrating along the trajectory, we obtain the reflected and refracted field in the medium ($s < 0$);

$$\Pi_{\omega s}^{s} = -\frac{e\mathbf{k}}{4\pi^2\omega}\int\int\int \frac{e^{i\omega t}(k_{sz} - \varepsilon k_{jz})}{\beta\varkappa^2 k_{sz}(\varepsilon k_{jz} + k_{sz})}[\beta_z\varkappa^2 - (\beta_x k_x + \beta_y k_y)k_{sz}]$$
$$\times \exp[ik_x(x - x_\zeta) + ik_y(y - y_\zeta) - ik_{sz}(z + z_\zeta)]\,dk_x\,dk_y\,d\zeta,\ z < 0,\ z_\zeta < 0, \tag{1.2}$$
$$\Pi_{\omega j}^{s} = -\frac{e\mathbf{k}\varepsilon}{2\pi^2\omega}\int\int\int \frac{e^{i\omega t}}{\beta\varkappa^2(\varepsilon k_{jz} + k_{sz})}[\beta_z\varkappa^2 + (\beta_x k_x + \beta_y k_y)k_{jz}]$$
$$\times \exp[ik_x(x - x_\zeta) + ik_y(y - y_\zeta) - ik_{sz}z + ik_{jz}z_\zeta]\,dk_x\,dk_y\,d\zeta,\ z < 0,\ z_\zeta > 0.$$

In formulas (1.1) and (1.2) x, y, and z are the coordinates of the point of observation of the field, and x_ζ, y_ζ, and z_ζ are the coordinate of the particle at the instant of time t.

The Fourier components of the electric and magnetic fields are determined by the vectors (1.1) and (1.2) by means of the formulas

$$\mathbf{E}_\omega^{j} = \operatorname{grad}\operatorname{div}\Pi_\omega^{j} + \frac{\omega^2}{c^2}\Pi_\omega^{j},$$
$$\mathbf{E}_\omega^{s} = \frac{1}{\varepsilon}\operatorname{grad}\operatorname{div}\Pi_\omega^{s} + \frac{\omega^2}{c^2}\Pi_\omega^{s} \tag{1.3}$$
$$\mathbf{H}_\omega^{j,s} = -i\frac{\omega}{c}\operatorname{rot}\Pi_\omega^{j,s}.$$

These are the initial results necessary to consider surface waves excited by charge particle on the interface between vacuum and the medium.

2. FIELD OF SURFACE WAVES IN THE CASE OF ARBITRARY MOTION

To find the field of the surface waves it is necessary to integrate with respect to k_x and k_y in formulas (1.1) and (1.2). A preliminary changeover to new variables φ and κ, which are connected with k_x and k_y by means of the formulas $k_x = \kappa \cos\varphi$ and $k_y = \kappa \sin\varphi$, and integration with respect to φ lead to the appearance of Bessel functions. Replacing the latter by the half-sum of Hankel functions, we can extend the integration with respect to κ from $-\infty$ to $+\infty$ and close the contour of integration in the complex plane by a semicircle of infinite radius. The integrands have a pole at $\kappa = (\omega/c)\sqrt{\epsilon/(1+\epsilon)}$ and branch points at $\kappa = \omega/c$ and $\kappa = (\omega/c)\sqrt{\epsilon}$. The pole gives a surface wave, whereas the result of integration along the edges of the cuts has a more complicated character, namely, it contains besides the surface field also part of the transition and Cerenkov radiations. In the present paper we confine ourselves to consideration of the pole contribution, and indicate the region of applicability of the corresponding result.

The values of k_{jz} and k_{sz} at the pole are

$$k_{jz} = -\frac{\omega}{c}\frac{1}{\sqrt{1+\varepsilon}}, \quad k_{sz} = \frac{\omega}{c}\frac{\varepsilon}{\sqrt{1+\varepsilon}}.$$

In the presence of absorption, the imaginary parts of κ, k_{jz}, and k_{sz} should be positive. When $\omega > 0$ this requirement is satisfied by the following choice of roots:

$$\sqrt{1+\varepsilon} = \frac{1}{\sqrt{2}}\sqrt{|1+\varepsilon|+1+\varepsilon'} + \frac{i}{\sqrt{2}}\sqrt{|1+\varepsilon|-1-\varepsilon'},$$
$$\sqrt{\varepsilon} = \frac{1}{\sqrt{2}}\sqrt{|\varepsilon|+\varepsilon'} + \frac{i}{\sqrt{2}}\sqrt{|\varepsilon|-\varepsilon'}, \tag{2.1}$$

where ϵ' is the real part of the dielectric constant: $\epsilon = \epsilon' + i\epsilon''$ ($\epsilon'' > 0$ when $\omega > 0$). When $\omega < 0$, we choose complex conjugate values of the roots.

As a result of taking the residues in (1.1) and (1.2), we obtain the field of the surface wave excited by an arbitrarily moving charged particle, in the form

$$\Pi_{\omega j}{}^{j} = -\frac{ek\varepsilon\sqrt{\varepsilon}}{c(\varepsilon^2-1)\sqrt{1+\varepsilon}}\int\frac{1}{\beta}\Big[(\beta_x\cos\varphi_\zeta+\beta_y\sin\varphi_\zeta)H_1^{(1)}\Big(\frac{\omega}{c}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\rho_\zeta\Big)$$
$$+ i\beta_z\sqrt{\varepsilon}H_0^{(1)}\Big(\frac{\omega}{c}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\rho_\zeta\Big)\Big]\exp\Big(i\omega t - i\frac{\omega}{c}\frac{z+z_\zeta}{\sqrt{1+\varepsilon}}\Big)\,d\zeta,$$
$$\Pi_{\omega s}{}^{j} = \Pi_{\omega j}{}^{j}(\beta_z\to\beta_z/\varepsilon,\ z_\zeta\to\varepsilon z_\zeta), \quad \Pi_{\omega j}{}^{s} = \Pi_{\omega j}{}^{j}(z\to\varepsilon z),$$
$$\Pi_{\omega s}{}^{s} = \Pi_{\omega j}{}^{j}(\beta_z\to\beta_z/\varepsilon,\ z\to\varepsilon z,\ z_\zeta\to\varepsilon z_\zeta). \tag{2.2}$$

Here and below the arrows in the parenthesis indicate the substitutions which must be made in $\Pi^{j}_{\omega j}$ in order to obtain the corresponding Hertz vector, and we have introduced the following notation:

$$\rho_\zeta = [(x-x_\zeta)^2+(y-y_\zeta)^2]^{1/2}, \quad \varphi_\zeta = \operatorname{arctg}\frac{y-y_\zeta}{x-x_\zeta}. \tag{2.3}$$

In the wave zone

$$\rho_\zeta\frac{\omega}{c}\left|\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right| \gg 1, \tag{2.4}$$

The results simplify and the first formula of (2.2) takes the following form:

$$\Pi_{\omega j}{}^{j} = \frac{iek\varepsilon\sqrt{-2i\omega}}{\sqrt{\pi c}\,\omega(\varepsilon^2-1)}\Big(\frac{\varepsilon}{1+\varepsilon}\Big)^{1/4}\int\frac{1}{\beta\sqrt{\rho_\zeta}}(\beta_x\cos\varphi_\zeta+\beta_y\sin\varphi_\zeta-\beta_z\sqrt{\varepsilon})$$
$$\times\exp\Big(i\omega t + i\frac{\omega}{c}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\rho_\zeta - i\frac{\omega}{c}\frac{z+z_\zeta}{\sqrt{1+\varepsilon}}\Big)d\zeta. \tag{2.5}$$

To obtain the remaining Hertz vectors it is sufficient the make the same substitutions as were indicated in (2.2).

It follows from (1.3) that in the wave zone the electromagnetic field is determined by the obtained Hertz vectors by means of the formulas

$$\mathbf{E}_\omega{}^{j} = \frac{\omega^2}{c^2}\frac{1}{1+\varepsilon}\Pi_\omega{}^{j}[\varepsilon\mathbf{k}+\sqrt{\varepsilon}(\mathbf{i}\cos\varphi_\zeta+\mathbf{j}\sin\varphi_\zeta)],$$
$$\mathbf{E}_\omega{}^{s} = \frac{\omega^2}{c^2}\frac{1}{1+\varepsilon}\Pi_\omega{}^{s}[\mathbf{k}+\sqrt{\varepsilon}(\mathbf{i}\cos\varphi_\zeta+\mathbf{j}\sin\varphi_\zeta)], \tag{2.6}$$
$$\mathbf{H}_\omega{}^{j,s} = \frac{\omega^2}{c^2}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\Pi_\omega^{j,s}(\mathbf{i}\sin\varphi_\zeta-\mathbf{j}\cos\varphi_\zeta).$$

In the present paper we dispense with comparison of the absolute contribution from the cuts and the pole term. We note, however, that the branch points and the pole lead under certain conditions to the same field configuration, and the pole term may be completely or partly canceled. We confine ourselves to an assessment of the conditions under which such a cancellation does not take place, and consequently, a separate consideration of the pole contribution is justified.

We call attention to the fact that on the edges of the cuts the imaginary part of κ can assume arbitrary positive values, whereas the real part of κ lies in the intervals

$$0 < \operatorname{Re}\varkappa < \frac{\omega}{c}, \quad 0 < \operatorname{Re}\varkappa < \frac{\omega}{c}\operatorname{Re}\sqrt{\varepsilon},$$

which determine the regions of integration along the edges of the cuts. We can state that in the wave zone the results do not cancel each other at least under the condition when the pole lies beyond the limits of integration along the edges of the cuts, that is, when the following inequalities are satisfied:

$$\operatorname{Re}\sqrt{\frac{\varepsilon}{1+\varepsilon}} > 1, \quad \operatorname{Re}\sqrt{\frac{\varepsilon}{1+\varepsilon}} > \operatorname{Re}\sqrt{\varepsilon}. \tag{2.7}$$

In this case the pole contribution corresponds to small wavelengths which are not contained in the integrals along the edges of the cuts. A detailed examination of the region of applicability of the pole contribution entails calculation of integrals along the edges of the cuts. This problem calls for additional research.

3. EXCITATION OF SURFACE WAVES IN OBLIQUE INCIDENCE

Let us consider the field of a surface wave excited when a charged particle is incident on the interface at a certain acute angle. Assuming the motion of the particle to be uniform and linear, and choosing the plane passing through the normal to the interface and the direction of motion as the xz plane, we put $y_\zeta = 0$, $x_\zeta = v_x t$, $z_\zeta = v_z t$, $\beta_y = 0$, and $\zeta = vt$. The integration with respect to ζ in the formulas for $\Pi^{j}_{\omega j}$ and $\Pi^{s}_{\omega j}$ should be carried out for $-\infty$ to 0, and in $\Pi^{j}_{\omega s}$ and $\Pi^{s}_{\omega s}$ from zero to infinity.

In the wave zone, at sufficiently large distances from the point of intersection of the interface, where the quantities ρ_ζ in the pre-exponential factors and φ_ζ can be regarded as constant and we can confine ourselves to the linear terms of the expansion of the functions in the argument of the exponentials, we get

$$\Pi_{\omega j}{}^{j} = \frac{ek\varepsilon\,[2c\sqrt{\varepsilon(1+\varepsilon)}]^{1/2}}{\sqrt{i\pi\rho\omega^3}(\varepsilon^2-1)} \frac{(\beta_x\cos\varphi - \beta_z\sqrt{\varepsilon})\exp\{i\omega(\sqrt{\varepsilon}\rho - z)/c\sqrt{1+\varepsilon}\}}{\sqrt{1+\varepsilon} - \beta_x\sqrt{\varepsilon}\cos\varphi - \beta_z}$$

$$\Pi_{\omega s}{}^{j} = -\frac{ek\varepsilon\,[2c\sqrt{\varepsilon(1+\varepsilon)}]^{1/2}}{\sqrt{i\pi\rho\omega^3}(\varepsilon^2-1)} \frac{(\beta_x\cos - \beta_z/\sqrt{\varepsilon})\exp\{i\omega(\sqrt{\varepsilon}\rho - z)/c\sqrt{1+\varepsilon}\}}{\sqrt{1+\varepsilon} - \beta_x\sqrt{\varepsilon}\cos\varphi - \beta_z\varepsilon}$$

$$\Pi_{\omega j}{}^{s} = \Pi_{\omega j}{}^{j}(z\to\varepsilon z), \quad \Pi_{\omega s}{}^{s} = \Pi_{\omega s}{}^{j}(z\to\varepsilon z). \qquad (3.1)$$

The vectors $\Pi^{j}_{\omega j}$ and $\Pi^{s}_{\omega j}$ describe a surface wave produced when a particle moving in vacuum is instantaneously stopped on the interface, and $\Pi^{j}_{\omega s}$ and $\Pi^{s}_{\omega s}$ are the same when the particle is ejected into the medium (in the same direction and at the same velocity). If the particle moves uniformly and in a straight line through the interface, the field of the surface wave in the vacuum $(z > 0)$ is determined by the sum of the vectors $\Pi^{j}_{\omega j}$ and $\Pi^{j}_{\omega s}$. The sum of the vectors $\Pi^{s}_{\omega s}$ determines in this case the surface wave and the medium $(s < 0)$.

The spectral density of the energy flux per unit area in vacuum and in the medium is expressed in terms of the obtained Hertz vectors of the surface waves by means of the formulas

$$S_\omega{}^{j} = \frac{\omega^4}{c^3}|\Pi_\omega{}^{j}|^2\left|\frac{\varepsilon}{1+\varepsilon}\right|\mathrm{Re}\left[\sqrt{\frac{\varepsilon}{1+\varepsilon}}(\mathbf{i}\cos\varphi + \mathbf{j}\sin\varphi) - \frac{\mathbf{k}}{\sqrt{1+\varepsilon}}\right],$$

$$S_\omega{}^{s} = \frac{\omega^4}{c^3}|\Pi_\omega{}^{s}|^2\left|\frac{\varepsilon}{1+\varepsilon}\right|\mathrm{Re}\left[\frac{1}{\sqrt{\varepsilon(1-\varepsilon)}}(\mathbf{i}\cos\varphi + \mathbf{j}\sin\varphi) - \frac{\mathbf{k}}{\sqrt{1+\varepsilon}}\right]. \qquad (3.2)$$

When the glancing angle decreases to the limit $\beta_z \to 0$, the quantities $\Pi^{j}_{\omega j} + \Pi^{j}_{\omega s}$ and $\Pi^{s}_{\omega j} + \Pi^{s}_{\omega s}$ vanish. This means that the excited surface waves are not formed in this limiting case in the corresponding fields. In the absence of absorption in the vicinity of the angle $\varphi = \arccos(\sqrt{1+\epsilon}/\beta_x\sqrt{\epsilon})$, proportional to β_z, of coherent generation of the surface wave these quantities are inversely proportional to β_z for any arbitrarily small but finite $|\beta_z|$. This region makes the main contribution which is inversely proportional to $|\beta_z|$, to the radiation intensity. We note that in the case of glancing incidence it may be necessary to take into account the scattering of the particle in the medium. In large-angle scattering over small distances, the influence of the scattering on the generation of the surface wave can be estimated by retaining only that part of the surface-wave field which is generated along the path in the vacuum.

Let us consider now the field of the surface wave at smaller distances from the particle trajectory, when the functions ρ_ζ and φ_ζ cannot be regarded as constant. We first carry the integration in formula (2.5), rewriting it here under the assumption of uniform and linear motion of the particle in the form

$$\Pi_{\omega j}{}^{j} = \frac{iek\sqrt{-2i\omega\varepsilon}}{\sqrt{\pi c}\,\omega(\varepsilon^2-1)}\left(\frac{\varepsilon}{1+\varepsilon}\right)^{1/4}\int_{-\infty}^{0}\frac{\beta_x\cos\varphi_\zeta - \beta_z\sqrt{\varepsilon}}{\beta\sqrt{\rho_\zeta}}\exp\left(i\frac{\omega}{c}\frac{z}{\sqrt{1+\varepsilon}} + if_\zeta\right)d\zeta, \qquad (3.3)$$

where

$$f_\zeta = \frac{\omega}{v}\left(\zeta + \beta\sqrt{\frac{\varepsilon}{1+\varepsilon}}\rho_\zeta - \beta\frac{z_\zeta}{\sqrt{1+\varepsilon}}\right). \qquad (3.4)$$

In the case of glancing incidence of the particle on the interface and weak absorption of the surface wave $(\mathrm{Re}\sqrt{\epsilon/(1+\epsilon)} \gg \mathrm{Im}\sqrt{\epsilon/(1+\epsilon)})$ the exponential factor in the integrand can oscillate strongly. The integration is carried out in this case by the stationary-phase method. The stationary-phase point exists under the condition

$$|1+\varepsilon| - \mathrm{Re}(\beta_z\sqrt{1+\varepsilon^*}) < \beta_x\mathrm{Re}\sqrt{\varepsilon(1+\varepsilon^*)}. \qquad (3.5)$$

(Here and henceforth we put for concreteness $\beta_x > 0$.) The value of $\cos\varphi_\zeta$ at the extremum of the function f_ζ is equal to

$$\cos\varphi_r = \frac{|1+\varepsilon| - \mathrm{Re}(\beta_z\sqrt{1+\varepsilon^*})}{\beta_x\mathrm{Re}\sqrt{\varepsilon(1+\varepsilon^*)}}. \qquad (3.6)$$

The most essential region of integration has an order of magnitude

$$\Delta\zeta_r \sim \frac{\beta}{\beta_x}\sqrt{\left|\frac{c\rho_r}{\omega\sin^2\varphi_r}\right|}\left|\frac{1+\varepsilon}{\varepsilon}\right|^{1/4}, \qquad (3.7)$$

and the angular dimensions of the projection of this region on the interface are of the order of

$$\Delta\varphi_r \sim \sqrt{\left|\frac{c}{\omega\rho_r}\right|}\left|\frac{1+\varepsilon}{\varepsilon}\right|^{1/4}. \qquad (3.8)$$

If in the region (3.7) the imaginary part of the linear term of the expansion of f_ζ in a Taylor series in the vicinity of the stationary-phase point is much smaller in absolute magnitude than unity, that is,

$$\left|\mathrm{Im}\frac{\beta_z + \beta_x\sqrt{\varepsilon}\cos\varphi_r}{\beta_x\sqrt{1+\varepsilon}}\right|\sqrt{\left|\frac{\omega\rho_r}{c\sin^2\varphi_r}\right|}\left|\frac{1+\varepsilon}{\varepsilon}\right|^{1/4} \ll 1, \qquad (3.9)$$

and the observation point is such that distance from the corresponding ζ_r to the origin is larger than $\Delta\zeta_r$, then the integration in (3.3) leads to the following result

$$\Pi^{\ j} = \frac{2iek\varepsilon}{\omega(\varepsilon^2-1)}\frac{\beta_x\cos\varphi_r - \beta_z\sqrt{\varepsilon}}{\beta_x|\sin\varphi_r|}\exp\left[i\frac{\omega}{v}\left(\zeta_r + \beta\sqrt{\frac{\varepsilon}{1+\varepsilon}}\rho_r - \beta\frac{z+z_r}{\sqrt{1+\varepsilon}}\right)\right] \qquad (3.10)$$

where z_r is the coordinate of the point of the stationary phase, and ρ_r is the projection of the distance from the point of observation to the corresponding point of stationary phase on the interface.

The inequality (3.9) determines the spatial region of existence of the wave (3.10), due to absorption and to the inclination of the trajectory of the particle to the interface. The condition that the point of the stationary phase be far from the origin defines that part of the surface where the diffraction wave can be neglected. From the existence of a stationary-phase point it follows that the region of existence of the result (3.10) lies beyond the limits of the angle between the rays drawn from the origin at angles $|\varphi_r|$ and $-|\varphi_r|$ to the projection of the particle velocity on the surface.

According to (2.2), $\Pi^{s}_{\omega j} = \Pi^{j}_{\omega j}(z \to \epsilon z)$ and therefore calculation of the vector $\Pi^{s}_{\omega j}$ coincides fully with that given above, and the result is determined by means of formula (3.10), in which we will replace z by ϵz. Similar calculations of the field of a surface wave generated when a particle moves in a medium leads to formulas that differ from (3.5)–(3.9) in that β_z is replaced in the latter by $\epsilon\beta_z$. We ultimately get $\Pi^{j}_{\omega s} = \Pi^{j}_{\omega j}(\varphi_r \to \varphi_r', \beta_z \to \beta_z/\epsilon, z \to \epsilon z_r)$, where $\varphi_r' = \varphi_r(\beta_z \to \epsilon\beta_z)$, and according to (2.2) $\Pi^{s}_{\omega s} = \Pi^{s}_{\omega s}(z \to \epsilon z)$. Unlike $\Pi^{j}_{\omega j}$ and $\Pi^{s}_{\omega j}$, these results take place within the limits of the angle produced by rays drawn from the point of intersection of the particle and the interface at angles $|\varphi_r'|$ and $-|\varphi_r'|$.

Using these results, let us find the spectral density of the Poynting vector through the plane $y = \rho_r |\sin\varphi_r|$ and $y = -\rho_r |\sin\varphi_r|$ (and accordingly, $y = \rho_r |\sin\varphi_r'|$) of a surface wave generated when a particle moves in vacuum and in a medium, in the form

$$W_{\omega j} = \frac{2e^2}{c}\left|\frac{\varepsilon}{1-\varepsilon^2}\right|^2\left|\frac{\varepsilon}{1+\varepsilon}\right|\frac{|\beta_x\cos\varphi_r - \beta_z\sqrt{\varepsilon}|^2}{|\beta_x\beta_z\sin\varphi_r|}\left(\operatorname{Re}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\operatorname{Im}\sqrt{1+\varepsilon} + \operatorname{Re}\frac{1}{\sqrt{\varepsilon(1+\varepsilon)}}\operatorname{Im}\frac{\sqrt{1+\varepsilon}}{\varepsilon}\right)\operatorname{Im}\sqrt{1+\varepsilon}\exp\left(-2\frac{\omega}{c}\rho_r\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right), \tag{3.11}$$

$$W_{\omega s} = -\frac{2e^2}{c}\left|\frac{\varepsilon}{1-\varepsilon^2}\right|^2\frac{|\beta_x\sqrt{\varepsilon}\cos\varphi_r' - \beta_z|^2}{|(1+\varepsilon)\beta_x\beta_z\sin\varphi_r'|}\left(\operatorname{Re}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\operatorname{Im}\sqrt{1+\varepsilon} + \operatorname{Re}\frac{1}{\sqrt{\varepsilon(1+\varepsilon)}}\operatorname{Im}\frac{\sqrt{1+\varepsilon}}{\varepsilon}\right)\operatorname{Im}\frac{\sqrt{1+\varepsilon}}{\varepsilon}\exp\left(-2\frac{\omega}{c}\rho_r\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right). \tag{3.12}$$

In the absence of absorption these results greatly simplify and take on the following form:

$$W_{\omega j} = \frac{2e^2\varepsilon_1(\varepsilon_1 - 1 + \beta_z^2\varepsilon_1^2)}{c|\beta_z|\sqrt{\beta_x^2\varepsilon_1 - \varepsilon_1 + 1}\sqrt{\varepsilon_1 - 1}(\varepsilon_1^2 - 1)}, \tag{3.13}$$

$$W_{\omega s} = \frac{2e^2(\varepsilon_1 - 1 + \beta_z^2)}{c|\beta_z|\sqrt{\beta_x^2\varepsilon_1 - \varepsilon_1 + 1}\sqrt{\varepsilon_1 - 1}(\varepsilon_1^2 - 1)}, \tag{3.14}$$

where $\epsilon_1 = -\epsilon > 1$, $\beta_x^2\epsilon_1 > \epsilon_1 - 1$. The total energy flux for uniform and linear motion of the particle through the interface is determined by the sum $W_{\omega j} + W_{\omega s}$.

The spectral density of the excitation energy of the surface wave is inversely proportional to the glancing angle. In the case of a nonrelativistic particle, this quantitative dependence was indicated by Stern and Ferrell[5]. In the limit as $\beta_z \to 0$, the resultant infinite intensity is connected with the assumption of uniform and linear motion of the particle and coherent interaction of the particle with the surface wave on the entire infinite path—the Cerenkov effect on surface waves.

In concluding this section we note that if an inequality that is the inverse of the (3.9) is satisfied, and is also the inverse of (3.9) when β_z in the latter is replaced by $\epsilon\beta_z$, then the results are given by formulas (3.1).

4. SURFACE WAVES AT NORMAL INCIDENCE

In the simplest particular case of uniform and linear particle motion from vacuum into a medium perpendicular to the interface, the field of the surface wave in the wave zone is determined by formulas (3.1), in which we must put $\beta_x = 0$ and $\beta_z = -\beta$. Using these results, we obtain the spectral density of the energy flux in the vacuum and in the medium, respectively, through a cylindrical surface of radius ρ in the case of instantaneous stopping of the particle on the interface:

$$W_{\omega j}^{j} = \frac{2e^2\beta^2|\varepsilon|^2}{c|(\sqrt{1+\varepsilon}+\beta)(\varepsilon - 1)|^2}\left|\frac{\varepsilon}{1+\varepsilon}\right|^{5/2}\frac{\operatorname{Re}\sqrt{\varepsilon(1+\varepsilon^*)}}{\operatorname{Im}\sqrt{1+\varepsilon}}\times\exp\left(-2\frac{\omega}{c}\rho\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right),$$

$$W_{\omega j}^{s} = \frac{-2e^2\beta^2|\varepsilon|}{c|(\sqrt{1+\varepsilon}+\beta)(\varepsilon - 1)|^2}\left|\frac{\varepsilon}{1+\varepsilon}\right|^{5/2}\frac{\operatorname{Re}\sqrt{\varepsilon(1+\varepsilon)}}{\operatorname{Im}(\varepsilon^*\sqrt{1+\varepsilon})}\times\exp\left(-2\frac{\omega}{c}\rho\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right), \tag{4.1}$$

In the case of ejection of the particle into the medium:

$$W_{\omega s}^{j} = \frac{2e^2\beta^2}{c|(\sqrt{1+\varepsilon}+\beta\varepsilon)(\varepsilon - 1)|^2}\left|\frac{\varepsilon}{1+\varepsilon}\right|^{5/2}\frac{\operatorname{Re}\sqrt{\varepsilon(1+\varepsilon^*)}}{\operatorname{Im}\sqrt{1+\varepsilon}}\times\exp\left(-2\frac{\omega}{c}\rho\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right),$$

$$W_{\omega s}^{s} = -\frac{2e^2\beta^2}{c|(\sqrt{1+\varepsilon}+\beta\varepsilon)(\varepsilon - 1)|^2}\left|\frac{\varepsilon}{1+\varepsilon}\right|^{5/2}\frac{\operatorname{Re}\sqrt{\varepsilon(1+\varepsilon)}}{|\varepsilon|\operatorname{Im}(\varepsilon^*\sqrt{1+\varepsilon})}\times\exp\left(-2\frac{\omega}{c}\rho\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right) \tag{4.2}$$

and in the case of motion of the particle through the interface

$$W_{\omega} = \frac{2e^2\beta^2|1+\beta\sqrt{1+\varepsilon}|^2|\varepsilon|}{c|(\sqrt{1+\varepsilon}+\beta)(\sqrt{1+\varepsilon}+\beta\varepsilon)|^2}\left|\frac{\varepsilon}{1+\varepsilon}\right|^{3/2}\frac{\operatorname{Re}\sqrt{\varepsilon(1+\varepsilon^*)}}{\operatorname{Im}\sqrt{1+\varepsilon}}\times\exp\left(-2\frac{\omega}{c}\rho\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right).$$

$$W_{\omega}^{s} = -\frac{2e^2\beta^2|1+\beta\sqrt{1+\varepsilon}|^2}{c|(\sqrt{1+\varepsilon}+\beta)(\sqrt{1+\varepsilon}+\beta\varepsilon)|^2}\left|\frac{\varepsilon}{1+\varepsilon}\right|^{3/2}\frac{\operatorname{Re}\sqrt{\varepsilon(1+\varepsilon)}}{\operatorname{Im}(\varepsilon^*\sqrt{1+\varepsilon})}\times\exp\left(-2\frac{\omega}{c}\rho\operatorname{Im}\sqrt{\frac{\varepsilon}{1+\varepsilon}}\right). \tag{4.3}$$

In the absence of absorption, the results greatly simplify and take the form

$$W_{\omega j}^{j} = \frac{2e^2\beta^2\varepsilon_1^5}{c(\beta^2+\varepsilon_1-1)(\varepsilon_1^2-1)^2\sqrt{\varepsilon_1-1}},$$

$$W_{\omega s}^{j} = \frac{2e^2\beta^2\varepsilon_1^3}{c(\beta^2\varepsilon_1^2+\varepsilon_1-1)(\varepsilon_1^2-1)^2\sqrt{\varepsilon_1-1}}, \tag{4.4}$$

$$W_{\omega}^{j} = \frac{2e^2\beta^2(\beta^2\varepsilon_1-\beta^2+1)}{c(\beta^2+\varepsilon_1-1)(\beta^2\varepsilon_1^2+\varepsilon_1-1)(\varepsilon_1-1)^{3/2}},$$

where $\epsilon_1 = |\epsilon| = -\epsilon > 1$, and the quantities $W_{\omega j}^{s}$, $W_{\omega s}^{s}$, and W_{ω}^{s} are negative and their absolute values are smaller by a factor ϵ_1^2 than $W_{\omega j}^{j}$, $W_{\omega s}^{j}$, and W_{ω}^{j}, respectively.

The spectral density of the excitation energy of the surface waves is determined by the sum of the fluxes in vacuum and in the medium. When the particle moves through the interface, this density is equal to

$$W_{\omega} = \frac{2e^2\beta^2(\beta^2\varepsilon_1-\beta^2+1)(\varepsilon_1+1)\varepsilon_1}{c(\beta^2+\varepsilon_1-1)(\beta^2\varepsilon_1^2+\varepsilon_1-1)\sqrt{\varepsilon_1-1}}. \tag{4.5}$$

Excitation of surface waves in the absence of absorption, in the case of uniform and linear motion of the particle perpendicular to the interface between vacuum and a medium, was also considered by Eĭdman[7]. Our results for the quantities W_{ω}^{j}, W_{ω}^{s}, and W_{ω} exceed by a factor of 2 those obtained in[7] (the corresponding rather complicated formulas (4) and (5) of[7] for the energy flux of the surface wave in the medium and for the total flux (in the vacuum and in the medium) can be simplified and one can verify that their integrands coincide with ours apart from the factor 2). The reason for the discrepancy lies in the fact that the region of variation of the frequencies was arbitrarily limited in [7] to positive values only.

5. COHERENT GENERATION OF A SURFACE WAVE

Let us consider the excitation of a surface wave by a charge particle moving uniformly and in a straight line along an interface. Let the particle move in vacuum at a distance z_0 from the boundary in the xz plane in the positive direction of the x axis. In this case the surface-wave field in vacuum, in the wave zone, is determined by the following integral:

$$\Pi^j_{\omega j} = \frac{iek\sqrt{-2i\omega\varepsilon}}{\sqrt{\pi c}\,\omega(\varepsilon^2-1)}\left(\frac{\varepsilon}{1+\varepsilon}\right)^{1/4}\int\frac{\cos\varphi_\zeta}{\sqrt{\rho_\zeta}}\exp\left[i\frac{\omega}{v}x_\zeta\left(1-\beta\sqrt{\frac{\varepsilon}{1+\varepsilon}}\cos\varphi_\zeta\right)+i\frac{\omega}{c}\sqrt{\frac{\varepsilon}{1+\varepsilon}}(x\cos\varphi_\zeta+y\cos\varphi_\zeta)-i\frac{\omega}{c}\frac{z+z_0}{\sqrt{1+\varepsilon}}\right]dx_\zeta, \tag{5.1}$$

where the quantities ρ_ζ and φ_ζ are determined from formulas (2.3), in which we must put $y_\zeta = 0$.

Let the particle be ejected from the point x = 0 and let it suddenly stop at the point x = a. Then the integration in (5.1) should be carried out from zero to a. At sufficiently large distances from the particle trajectory, when the quadratic term of the expansion of the function in the argument of the exponential in (5.1) in powers of x_ζ is much smaller than unity in the entire region of integration, the integration leads to a result which is the analog of a three dimensional spherical wave:

$$\Pi^j_{\omega j} = \frac{ekv\sqrt{-2i\omega}\,\varepsilon\cos\varphi}{\sqrt{\pi c\rho}\,\omega^2(\varepsilon^2-1)}\left(\frac{\varepsilon}{1+\varepsilon}\right)^{1/4}\exp\left(i\frac{\omega}{c}\frac{\sqrt{\varepsilon}\rho-z-z_0}{\sqrt{1+\varepsilon}}\right)\times\left[1-\beta\sqrt{\frac{\varepsilon}{1+\varepsilon}}\cos\varphi\right]^{-1}\left[1-\exp\left\{i\frac{\omega}{v}\left(1-\beta\sqrt{\frac{\varepsilon}{1+\varepsilon}}\cos\varphi\right)a\right\}\right], \tag{5.2}$$

where $\rho = \sqrt{x^2+y^2}$ and $\varphi = \tan^{-1}(y/x)$.

At smaller distances from the trajectory of the particle, the integration is by the stationary-phase method. As a result we obtain the analog of the three-dimensional cylindrical Cerenkov radiation wave:

$$\Pi^j_{\omega j} = -\frac{2iek\varepsilon|\mathrm{ctg}\,\varphi_r|}{\omega(\varepsilon^2-1)}\exp\left[i\frac{\omega}{v}\left(x_r+\beta\sqrt{\frac{\varepsilon}{1+\varepsilon}}\rho_r-\beta\frac{z+z_0}{\sqrt{1+\varepsilon}}\right)\right],$$
$$\varphi_r = \arccos\frac{|1+\varepsilon|}{\beta\,\mathrm{Re}\sqrt{\varepsilon(1+\varepsilon^*)}}. \tag{5.3}$$

This result holds also in the case when the stationary-phase point lies in the integration region, and the essential integration region does not contain the points of the particle emission and its sudden stopping. It follows hence that the wave (5.3) exists in the interval between rays drawn at identical angles $|\varphi_r|$ and $-|\varphi_r|$ respectively from the point of ejection and stopping, with the exception of the regions with angular dimensions on the order of $\Delta\varphi_r$ adjacent to these rays, where the diffraction of the surface wave is significant. At distances satisfying the following relation

$$\sqrt{c\rho_r}|1+\varepsilon|^{1/4} \sim a|\varepsilon|^{1/4}\sqrt{\omega\sin^2\varphi_r}, \tag{5.4}$$

the diffraction regions overlap and the wave is transformed into an analog of a three dimensional spherical wave (5.2).

Using (5.3), let us find the spectral energy flux density in the absence of absorption in vacuum, per unit path of the particle, in the form

$$\frac{dW^j_{\omega j}}{dx} = \frac{4\omega e^2\varepsilon_1{}^3\exp(-2\omega z_0/c\sqrt{\varepsilon_1-1})}{vc(\varepsilon_1{}^2-1)^2\sqrt{\beta^2\varepsilon_1-\varepsilon_1+1}} \tag{5.5}$$

where $\epsilon_1 = |\epsilon| = -\epsilon > 1$ and $\beta^2\epsilon_1 > \epsilon_1 - 1$. For the spectral density of the energy flux in the medium, the result is negative, and is smaller in absolute magnitude than (5.5) by a factor ϵ_1^2. Thus, the spectral density of the excitation energy of the coherent surface wave per unit path of the particle is equal to

$$\frac{dW_{\omega j}}{dx} = \frac{dW^j_{\omega j}}{dx} + \frac{dW^s_{\omega j}}{dx} = \frac{4\omega e^2\varepsilon_1\exp(-2\omega z_0/c\sqrt{\varepsilon_1-1})}{vc(\varepsilon_1{}^2-1)\sqrt{\beta^2\varepsilon_1-\varepsilon_1+1}}. \tag{5.6}$$

Sitenko and Tkalich [4] indicated the existence of a pole corresponding in the nonrelativistic approximation to the generation of a surface wave when a particle moves along the interface. The first to consider coherent generation of a surface wave on an interface between vacuum and a non-absorbing medium were Barsukov and Naryshkina[9]. It must be noted that the spectral densities of the energy flux in vacuum and in the medium and of the total flux turn out to be larger by a factor of 2 than those derived from the corresponding formulas (19) and (22) of [9] (it is assumed that the integration is carried there both over positive and negative frequencies; in the second formula of (19) of [9] there is a misprint: the term $(|\epsilon| - 1)$ in the denominator of the integrand should be replaced by the square of this term). This difference is connected with the loss of a factor 2 in [9] when taking the residue at the pole¹⁾.

6. WORK OF THE DECELERATING FORCE

The energy lost by a charged particle can be determined as the work performed by the moving charge against the reaction force exerted by the field. We are interested in that part of the work, which is connected with the excitation of surface waves. We confine ourselves to the case of uniform and linear motion. When the particle is stopped on the interface, the formula for the work over the entire path of motion in the vacuum has in this case the form

$$A_j = \frac{e^2}{4\pi^2}\int_{-\infty}^{+\infty}\frac{d\omega}{\omega}\int_{-\infty}^{0}d\zeta\int_{-\infty}^{0}d\zeta'\int_0^\infty d\varkappa\int_{-\pi}^{+\pi}d\varphi\frac{\varkappa}{k_{jz}v^2}\left(\frac{\varepsilon k_{jz}-k_{sz}}{\varepsilon k_{jz}+k_{sz}}\right)(\varkappa^2v_z{}^2-k_{jz}{}^2v_x{}^2\cos^2\varphi)\exp\{iv^{-1}[(\omega-\varkappa v_x\cos\varphi)(\zeta-\zeta')+k_{jz}v_z(\zeta+\zeta')]\}. \tag{6.1}$$

Integrating first with respect to ζ and ζ', in order to avoid the appearance of diverging expressions when changing the order of integration, we must exclude from the integration the small interval near the origin, letting it go to zero after the calculations are completed. The subsequent integration with respect to φ is elementary. The integration with respect to κ can

¹⁾Formulas (15) and (17) of [9] for the field components of the particle contain misprints (see, for example, [11,12]).

be extended to the entire real axes going over simultaneously to integration with respect to the frequencies in the interval from zero to infinity. Closing the contour of integration with respect to κ in the upper complex half-plane ($\mathrm{Im}\,\kappa > 0$), confining ourselves to calculation of the contribution from the pole, and then letting the excluded interval of integration with respect to ζ and ζ' go to zero and leaving out the integration with respect to the frequencies, we obtain the spectral density of the excitation energy of the surface waves in the form

$$W = \frac{e^2}{c}\,\mathrm{Im}\,\frac{\varepsilon}{\beta_z(\varepsilon^2-1)\sqrt{1+\varepsilon}}\left[\frac{(\sqrt{1+\varepsilon}-\beta_z)^2-\beta_z^2\varepsilon^2}{[(\sqrt{1+\varepsilon}-\beta_z)^2-\beta_x^2\varepsilon]^{1/2}} - \frac{(\sqrt{1+\varepsilon}+\beta_z)^2-\beta_z^2\varepsilon^2}{[(\sqrt{1+\varepsilon}+\beta_z)^2-\beta_x^2\varepsilon]^{1/2}} + 2\beta_z\right]. \qquad (6.2)$$

Analogous calculations for the spectral density of the excitation energy of the surface waves when a particle is ejected into the medium from the interface leads to a result that differs from (6.2) in the fact that β_Z is replaced in the latter by $\epsilon\beta_Z$ everywhere except $\beta_Z^2\epsilon^2$ in the numerators, where ϵ^2 should be omitted.

Calculation of the spectral density of the energy of excitation of the surface waves when a particle moves through the interface leads to the following result:

$$W_\omega = \frac{e^2}{c\beta_z}\,\mathrm{Im}\,\frac{1}{(1+\varepsilon)^2\sqrt{1+\varepsilon}}\left[\varepsilon\frac{(\sqrt{1+\varepsilon}-\beta_z)^2-\beta_z^2\varepsilon(2+\varepsilon)}{\sqrt{(\sqrt{1+\varepsilon}-\beta_z)^2-\beta_x^2\varepsilon}} - \varepsilon\frac{(\sqrt{1+\varepsilon}+\beta_z)^2-\beta_z^2\varepsilon(2+\varepsilon)}{\sqrt{(\sqrt{1+\varepsilon}+\beta_z)^2-\beta_x^2\varepsilon}} - \frac{(\sqrt{1+\varepsilon}-\beta_z\varepsilon)^2-\beta_z^2(1+2\varepsilon)}{\sqrt{(\sqrt{1+\varepsilon}-\beta_z\varepsilon)^2-\beta_x^2\varepsilon}} + \frac{(\sqrt{1+\varepsilon}+\beta_z\varepsilon)^2-\beta_z^2(1+2\varepsilon)}{\sqrt{(\sqrt{1+\varepsilon}+\beta_z\varepsilon)^2-\beta_x^2\varepsilon}}\right]. \qquad (6.3)$$

In (6.2) and (6.3) it is assumed that the real parts of the roots in the denominator terms in the square brackets, which remain after the introduction of the factors of the type $(\sqrt{1+\epsilon} \pm \beta_Z)$ and $(\sqrt{1+\epsilon} \pm \beta_Z\epsilon)$, are positive.

CONCLUSION

In concluding our analysis, let us discuss certain consequences of the results for the case of glancing incidence of the charge. This problem is of particular interest because recently radiation was observed [13,14] of approximately double the intensity of either transition radiation or bremsstrahlung in a number of experiments with fast electrons (with energy on the order of 30 keV) in silver and under similar conditions. This radiation has an intensity maximum whose position does not coincide with the transparency band in the silver, but is somewhat shifted towards longer wavelengths and corresponds quite accurately to the position of the maximum of the surface-wave intensity [13,14]. It is obvious, however, that the surface waves do not form a spherical wave and cannot be observed directly in the form of radiation at large distances. Nonetheless, they can make a contribution to the radiation, if there exists an effective mechanism for their transformation into three-dimensional waves—for example, scattering by irregularities of the surface, nonlinear effects, etc. The spectral density of excitation of the surface wave should then be much larger than the observed radiation intensity, since the transformation coefficient is in general small. An estimate shows that the ratio of the intensity of the surface waves to the intensity of the transition radiation, for a glancing angle of 0.5° and for $\beta = 0.33$, is $\sim 10^4$. Thus, any transformation mechanism with efficiency on the order of several per cent can yield the experimentally observed radiation intensity.

It is known [15] that the intensity of scattering by a statistically rough surface is proportional to the quantity $k^2\zeta^2$, where ζ is the roughness parameter and k the wave vector of the wave. From estimates of the latter concerning the field configuration of the surface wave in the wave zone it follows that the transformation turns out to be sufficiently effective even in the case of a quite good surface.

[1] L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred, Gostekhizdat, 1957, p. 364 [Electrodynamics of Continuous Media, Addison Wesley, 1960].

[2] C. J. Powell and J. B. Swan, Phys. Rev. **115**, 869 (1959); **118**, 640 (1960).

[3] R. H. Ritchie, ibid. **106**, 874 (1957).

[4] A. G. Sitenko and V. S. Tkalich, Zh. Tekh. Fiz. **29**, 1074 (1959) [Sov. Phys.-Tech. Phys. **4**, 981 (1959)].

[5] E. A. Stern and R. A. Ferrell, Phys. Rev. **120**, 130 (1960).

[6] Yu. A. Romanov, Izv. Vuzov, Radiofizika **7**, 242 (1964); Dissertation, Gor'kiy State University, 1966.

[7] V. Ya. Éĭdman, Izv, Vuzov Radiofizika **8**, 188 (1965).

[8] K. A. Barsukov and L. G. Naryshkina, Zh. Tekh. Fiz. **36**, 225 (1966) [Sov. Phys.-Tech. Phys. **11**, 166 (1966)].

[9] K. A. Barsukov and L. G. Naryshkina, ibid. **36**, 800 (1966) [**11**, 596 (1966)].

[10] V. E. Pafomov, Izv. Vuzov Radiofizika **10**, 240 (1967).

[11] B. M. Bolotovskiĭ, Usp. Fiz. Nauk **75**, 295 (1961) [Sov. Phys.-Usp. **4**, 781 (1962)].

[12] V. E. Pafomov, Trudy FIAN (Physics Institute, Academy of Sciences) **16**, 94 (1961).

[13] H. Boersch, P. Dobberstein, D. Fritzsche, and G. Sauerbrey, Z. Physik **187**, 97 (1965).

[14] G. E. Jones, L. S. Cram, and E. A. Arakawa, Phys. Rev. **147**, 515 (1966).

[15] S. M. Rytov, Vvedenie v statisticheskuyu radiofiziku (Introduction to Statistical Radiophysics), Nauka, 1966.

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