ON THE PHYSICAL EQUIVALENCE OF REPRESENTATIONS RELATED BY AN AUTOMORPHISM

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The relation between field theories for which the operators can be expressed in terms of each other in a one-to-one manner is discussed. In the framework of the Wightman axiomatic approach such theories are described by Wightman functionals related by an automorphism: $W \rightleftarrows W_\tau$, where $W_\tau(a) = W(\tau^{-1}a)$ (a is an arbitrary element of the field algebra A, τ is a fixed automorphism of A). It is shown that in spite of their outward differences such theories can always be interpreted as different versions of the description of the same physical system. The symmetry groups of W_τ and W are outwardly different, but realize unitarily equivalent representations of the same abstract symmetry group. Arguments are presented in favor of the assertion that in scattering theory the definition of the Hamiltonian, as a function of the creation and annihilation operators, determines uniquely the representation of these latter operators.

1. INTRODUCTION

In the usual (Lagrangian) formulation, a quantum field theory is defined in terms of a family of equations for the field operators. The concept of symmetry group of a given family of dynamical equations is of utmost importance for physics.

In spite of the obvious importance of this concept, one cannot assert that there is complete clarity in the definition of a symmetry group, as well as in the prescription for finding such groups. As a rule, considerations are restricted to symmetries for which the presence is obvious, and the problem of finding the whole symmetry group of a given system is almost never posed.

Consider, for instance, the usual equation describing a free field

$$(\square + m^2)A_1(x) = 0. \tag{1}$$

This equation is explicitly invariant with respect to the Poincaré group: $A_1(x) \to A_1(\Lambda^{-1}(x-\alpha))$, in the sense that the operator $A_1(\Lambda^{-1}(x-\alpha))$ is again a solution of the same equation.

Let us now consider another field theory, defined by the equation

$$(\square + m^2)A_2(x) = (\square + m^2)C(x), \tag{2}$$

where C(x) is some ordinary function (not an operator). This equation will not be invariant with respect to the translation $A_2(x) \to A_2(x-\alpha)$. The question naturally arises: does Eq. (2) indeed exhibit a lower symmetry, and should one interpret the theory defined by (2) as not being invariant with respect to the Poincaré group?

The purpose of the present paper is to prove the equivalence of the theories defined by Eqs. (1) and (2). Moreover: the symmetry group of (2) is isomorphic to the symmetry group of (1), i.e., in going over from (1) to (2) no reduction of the symmetry occurs. The theories corresponding to Eqs. (1) and (2) can always be interpreted as different versions of the description of the same physical system, although such an interpretation is not unique.

The domain of applicability of this result is not restricted to the given example. A similar assertion holds for two arbitrary theories for which the field operators can be expressed in terms of each other. In the example under consideration this relation is of the form $A_2(x) = A_1(x) + C(x)$. Such a relation allows for a clear mathematical characterization within the framework of representation theory of the field algebra. It turns out that such theories correspond to representations which are related by an automorphism. We shall use this term in the sequel.

First of all it is necessary to define precisely the concept of symmetry group.

2. THE SYMMETRY GROUP OF A PHYSICAL SYSTEM

We call symmetry group of a given system of dynamical equations the group of transformations of the field operators which transforms a solution into a solution. Thus, the transformation $A_1(x) \to A_1(x-\alpha)$ is an element of the symmetry group of Eq. (1). The vagueness of this definition consists only in the absence of a complete characterization of the class of transformations of the field operators within which one looks for transformations which map solutions into solutions.

In what follows we shall make use of the formalism of axiomatic field theory. In this case considerations of mathematical rigor do not clash with the requirement of simplicity: the Wightman axiomatic approach is not only rigorous, but also the simplest and most intuitive formalism for the description of symmetry in quantum field theory.

Before formulating symmetry problems within the axiomatic framework we make three general remarks.

Firstly, the physical meaning of the results that will be obtained is completely transparent, and the results can be easily translated into any other form of description of quantum field theory (in particular, into the Lagrangian formalism).

Secondly, these results extend automatically to any scheme in which the physical theory is defined by means of a representation of a given abstract *-algebra (e.g., the Haag approach[1]).

Thirdly, we shall discuss our scheme on the example of a single scalar field, but all results can be directly extended to the case of an arbitrary number of fields with arbitrary spins.

In the Wightman formalism a theory is defined not in terms of its Lagrangian, or in terms of the equations derived from it, but in terms of the vacuum expectation values of the Heisenberg field operators (Wightman functions)

$$W_n(x_1\ldots x_n) = \langle 0|A(x_1)\ldots A(x_n)|0\rangle.$$

The concept of field algebra is of utmost importance (the algebra will be denoted by A, its elements by a, b, ...). An element of A is a terminating chain of functions:

$$a \equiv (a_0,\ a_1(x_1), \ldots,\ a_k(x_1\ldots x_k), \ldots).$$

If the field theory is fixed, then the element $a \in A$ can be put in correspondence with the operator:

$$r(a) = \sum_k \int \ldots \int dx_1 \ldots dx_k a_k(x_1\ldots x_k) A(x_1)\ldots A(x_k),$$

operating on the Hilbert space of the given theory (here A(x) is the field operator). The multiplication and involution in A can be selected in such a manner that the totality R(A) of operators r(a) forms a symmetric representation of the algebra A. The rules for multiplication and involution are given by the equations:

$$(ab)_n(x_1\ldots x_n) = \sum_{k=0}^{n} a_k(x_1\ldots x_k) b_{n-k}(x_{k+1}\ldots x_n),$$

$$(a^+)_n(x_1\ldots x_n) = \bar{a}_n(x_n\ldots x_1).$$

Under these conditions any field theory is automatically a representation of the algebra A by means of operators in a Hilbert space: $r(ab) = r(a)\cdot r(b)$, $r(a)^+ = r(a^+)$.

The chain of Wightman functions can be considered as a single functional on A:

$$W(a) = \sum_k \int \ldots \int dx_1 \ldots dx_k W_k(x_1\ldots x_k) a_k(x_1\ldots x_k).$$

Wightman[2] has shown that a given representation (i.e., a field A(x)) can be completely reconstructed from the functional W (in mathematics this is the well-known Gel'fand–Naĭmark–Segal construction).

The operators of the representation R(A) are defined in H on a dense subspace L, consisting of the vectors $\psi(a) = r(a)|0\rangle$, where $|0\rangle$ denotes the vacuum vector of the theory. The functional W defines the inner product in H:

$$\langle \psi(a),\ \psi(b)\rangle = W(a^+b).$$

The consideration of symmetry starts with a discussion of the automorphisms of A. An automorphism is an arbitrary bijective mapping of A onto itself, preserving the algebraic relations (including involution, i.e., only *-automorphisms are considered):

$$\tau(\lambda_1 a + \lambda_2 b) = \lambda_1\tau a + \lambda_2\tau b, \quad \tau(ab) = \tau a\cdot\tau b, \quad \tau a^+ = (\tau a)^+.$$

The relation between automorphisms and symmetries will become evident if one notes that automorphisms of A generate the various transformations of the field operators.

Consider, for example, the translation group. It is easy to construct a group of automorphisms which implements a representation of the translation group:

$$(\tau_\alpha a)_k(x_1\ldots x_k) = a_k(x_1-\alpha, \ldots, x_k-\alpha),$$

where α denotes the translation vector. The corresponding transformation of the field operator can be found from the relation

$$r(\tau^{-1}a) = \sum_k \int \ldots \int dx_1\ldots dx_k (\tau^{-1}a)_k(x_1\ldots x_k) A(x_1)\ldots A(x_k)$$
$$= \sum_k \int \ldots \int dx_1\ldots dx_k a_k(x_1\ldots x_k) A_\tau(x_1)\ldots A_\tau(x_k). \quad (3)$$

For the translations we obtain

$$A_{\tau_\alpha}(x) = A(x-\alpha).$$

In this manner it is easy to construct the automorphisms which correspond to the classical symmetries: the Poincaré group, gauge groups, internal symmetries like the isospin (of course, the majority of these symmetries cannot be discussed in terms of a single scalar field, but the generalization is trivial: it suffices to equip the field operator in the function $a_k(x_1 \ldots x_k)$ with the appropriate indices).

However, the group of automorphisms of A is considerably richer than the group formed by these "classical" automorphisms. For example, one could construct an automorphism corresponding to the addition of a function C(x) to the field operator: $A_\tau(x) = A(x) + C(x)$. Such an automorphism looks fairly complicated, but can be constructed on the basis of (3). There also exists a multitude of other automorphisms, the relation of which to physical symmetries is not obvious. Practically always when two field theories are described by field operators $A_1(x)$ and $A_2(x)$ acting on the same Hilbert space and admitting a unique expression in terms of each other, one can construct an automorphism τ such that $A_2(x) = (A_1)_\tau(x)$. In this language one can reformulate, in particular, the linear canonical transformations to which creation and annihilation operators are subjected (as well as polynomial relations, under the condition that the equations are uniquely solvable each way).

We can now give an exact definition of a symmetry group of a physical theory.

We shall call symmetry group of the theory defined by the functional W the group of all automorphisms which leave the functional W invariant: $W(\tau a) = W(a)$.

In terms of the dynamical equations this will be a collection of transformations of field operators, corresponding to the automorphisms which map solutions of the equations into other solutions. In this manner we fix the class of those transformations of field operators, in which the symmetry is sought.

This definition is based upon the following reasoning: to each automorphism from the symmetry group of W, no matter how strange it looks, there corresponds a unitary operator $U_\tau \psi(a) = \psi(\tau a)$ in the Hilbert space of the representation, a fact which is easily verified. The collection of these unitary operators form a group, which leaves all inner products invariant, and the part of the group which commutes with the Hamiltonian represents the symmetries which really manifest themselves in experiments.

We make more precise the type of relation among the theories which will be considered. Let W denote the functional which defines the first theory. From this theory one can derive a whole series of other series, by replacing the field $A(x)$ with the field $A_\tau(x)$, where τ is a fixed automorphism of A. In terms of the Wightman functional this means going over from the representation defined by W to the representation corresponding to the functional W_τ:

$$W_\tau(a) = W(\tau^{-1}a).$$

3. THE RELATION BETWEEN THE REPRESENTATIONS W AND W_τ

Thus, let there be given two functionals W and W_τ related by a fixed automorphism τ of A: $W_\tau(a) = W(\tau^{-1}a)$. These functionals define representations $R(A)$ and $R_\tau(A)$ of the original *-algebra A in the Hilbert spaces H and H_τ. The operators of $R(A)$ are defined in H on a common dense domain L, formed by the vectors $\psi(a)$, and the operators of $R_\tau(A)$ are defined in H_τ on the elements $\psi_\tau(a)$ of the dense domain L_τ. We define a mapping V_τ of L into L_τ in the following manner: $V_\tau\psi(a) = \psi_\tau(\tau a)$. It is easy to see that V_τ is an isometry. Indeed

$$\begin{aligned}\langle V_\tau\psi(a),\ V_\tau\psi(b)\rangle &= \langle \psi_\tau(\tau a),\ \psi_\tau(\tau b)\rangle \\ &= W_\tau((\tau a)^+\cdot\tau b) = W_\tau(\tau(a^+b)) = W(a^+b) \\ &= \langle\psi(a),\ \psi(b)\rangle\end{aligned}$$

(here we have made use of the definitions of the inner products in H and H_τ, of the relation between W and W_τ, and of the fact that τ is an automorphism). The operator V_τ which initially maps L into L_τ be extended by continuity to an isometric operator which maps H into H_τ.

We now find the relation between the algebras $R(A)$ and $R_\tau(A)$:

$$\begin{aligned}r_\tau(\tau a)\psi_\tau(b) &= \psi_\tau((\tau a)\cdot b) = V_\tau\psi(a(\tau^{-1}b)) \\ &= V_\tau r(a)\psi(\tau^{-1}b) = V_\tau r(a)V_\tau^{-1}\psi_\tau(b).\end{aligned}$$

Since b is arbitrary, it follows that

$$r_\tau(\tau a) = V_\tau r(a)V_\tau^{-1}.$$

Without loss of generality one can inject the space H_τ into H by means of the isometry V_τ; we then obtain the relation between $R(A)$ and $R_\tau(A)$ described by Eq. ((3) (the injection identifies the vacuum vectors of the two spaces, since $|0\rangle = \psi(1)$ and $V_\tau\psi(1) = \psi_\tau(1)$). If the algebra $R(A)$ contains the mathematical representatives $r(a_i)$ of some observables, then in $R(A)$ the mathematical representatives of the same observables should be identified as the operators $r_\tau(\tau a_i)$.

It remains to discuss the connections between the symmetries of the representations $R(A)$ and $R_\tau(A)$.

Let $R(A)$ possess a symmetry group G. This means that there exists a group $\tilde{G}$ of automorphisms of A, which is a representation of the abstract group G, such that the functional W is invariant with respect to $\tilde{G}$: $W(\tilde{g}a) = W(a)$, $\tilde{g} \in \tilde{G}$. In the Hilbert space H the symmetry group $\tilde{G}$ (and consequently also G) is represented by the group of unitary operators $U(g)$:

$$U(g)\psi(a) = \psi(\tilde{g}a).$$

If the automorphism τ does not commute with the automorphisms in $\tilde{G}$, the functional W_τ is not invariant with respect to $\tilde{G}$. But this does not mean that W_τ is less symmetric than W. It is easy to note that W_τ is invariant with respect to the group $\tilde{G}_\tau = \tau\tilde{G}\tau^{-1}$. Indeed,

$$W_\tau(\tau\tilde{g}\tau^{-1}a) = W(\tilde{g}\tau^{-1}a) = W_\tau(a).$$

This implies that in the space H_τ (or in H, if one uses the injection) there exists a group of unitary operators $U_\tau(g)$:

$$U_\tau(g)\psi_\tau(a) = \psi_\tau(\tau\tilde{g}\tau^{-1}a),$$

which is a representation of the automorphism group $\tilde{G}_\tau$.

The groups $\tilde{G}_\tau$ and $\tilde{G}$ are isomorphic and are equally well qualified as representations of the same symmetry group G. In the same manner the unitary groups $\{U(g)\}$ in H, and $\{U_\tau(g)\}$ in H are representations of G. It is easy to establish that these representations are unitarily equivalent:

$$U_\tau(g) = V_\tau U(g)V_\tau^{-1}$$

(and if one makes use of the injection, they can be identified).

Thus, in spite of the external distinction between the symmetries of W and W_τ, these functionals implement unitarily equivalent representations of the same group G.

As an example we consider the translation group. Usually the group of automorphisms which represents the translations is the following: $A(x) \to A(x-\alpha)$ (in terms of the corresponding transformations of the field operator). A theory is considered translation-invariant if there exists a unitary group U_α, such that

$$A(x-a) = U_a A(x) U_a^{-1}.$$

From this point of view the field $A'(x) = A(x) + C(x)$ ($C(x)$ is a nonconstant function) is not translation-invariant. Moreover, the automorphism $A'(x) \to A'(x-\alpha)$ does not reduce to a unitary similarity transformation. Instead there exists the automorphism

$$A'(x) \to A'(x-a) - C(x-a) + C(x),$$

which realizes a representation of the same translation group, and

$$A'(x-a) - C(x-a) + C(x) = U_a A'(x) U_a^{-1}.$$

(This automorphism was obtained according to the general rule $\tau g\tau^{-1}$, where g is the automorphism corresponding to the translation, and τ is the automorphism corresponding to the addition of the function $C(x)$ to the field.) The field $A'(x)$ is invariant with respect to this new automorphism, in the same manner as the field $A(x)$ is invariant with respect to the old automorphism. And in this sense both fields are equally invariant with respect to the translation group.

In conclusion it should be stressed that the physical equivalence of the theories W and W_τ should in no case be understood as the invariance of the theory with respect to the automorphism τ. The relation between these two concepts reminds one of the relation between concepts of covariance and invariance of an equation.

Note. The preceding reasoning is easily extended to the case of an arbitrary isomorphism of representations. Let R(A) and R'(A) be isomorphic representations with kernels M and M', respectively. This means that there is a mapping T of R(A) into R'(A): $Tr(a) = r'(a_T)$, which implements this isomorphism. The functional W allows us to construct the functional W_T:

$$W_T(Tr(a)) = W(r(a))$$

(this notation takes into account the fact that the functionals W and W_T are defined essentially on the quotient algebras A/M and A/M', respectively. The proof of the equivalence of W and W_T does not differ from the preceding proof. Therefore the problem of comparing arbitrary isomorphic representations of W and W' can always be reduced to the comparison of W_T and W', which have the same kernel.

4. REPRESENTATIONS OF THE CANONICAL COMMUTATION RELATIONS

The preceding discussion referred to the case when the representation R(A) completely determines the physical theory. This is true for the axioms of quantum field theory.

The representation theory of the canonical commutation relations is not of this type. In this case, in order to determine a physical theory it is necessary to specify, in addition to the representation of the creation and annihilation operators a and a^+, also the expressions of the operators describing the dynamical quantities in terms of a and a^+. The physical contents of theories corresponding to different representations of a and a^+ can be compared only if there is given a unique prescription for writing down these expressions for any representation of a, a^+. The following can be adopted as one of these prescriptions: the total Hamiltonian of the the system is the same function of a and a^+ in any of their representations.

Consider, for example, nonrelativistic scattering theory. Then

$$H_0 = \int d\mathbf{p}\, p_0 a^+(\mathbf{p}) a(\mathbf{p}), \quad p_0 = \mathbf{p}^2/2m,$$

$$H_{in} = g \int\int\int d\mathbf{p}_1\, d\mathbf{p}_2\, d\mathbf{q}\, a^+\left(\mathbf{p}_1 + \frac{\mathbf{q}}{2}\right) a^+\left(\mathbf{p}_2 - \frac{\mathbf{q}}{2}\right) \times V(\mathbf{q}) a\left(\mathbf{p}_1 - \frac{\mathbf{q}}{2}\right) a\left(\mathbf{p}_2 + \frac{\mathbf{q}}{2}\right).$$

Usually one adopts for a and a^+ the Fock representation: $a(\mathbf{p}) = \alpha(\mathbf{p})$ and $a^+(\mathbf{p}) = \alpha^+(\mathbf{p})$ (here and in the sequel α and α^+ denote the Fock representation creation and annihilation operators). There exists a vector Φ_0 such that $\alpha(\mathbf{p})\Phi_0 = 0$, and the vectors $\alpha^+(\mathbf{p})\Phi_0$, $\alpha^+(\mathbf{p}_1) \times \alpha^+(\mathbf{p}_2)\Phi_0$, etc. are interpreted as one-particle, two-particle, etc. states. The invariance of the Hamiltonian under the transformation $\alpha \to e^{i\delta}\alpha$, $\alpha^+ \to e^{-i\delta}\alpha^+$ corresponds to particle-number conservation.

Let us assume now that a non-Fock representation has been selected for the a and a^+. For definiteness let us take the case when the a and a^+ are obtained from the Fock-representation operators α and α^+ by means of a Bogolyubov transformation:

$$a(\mathbf{p}) = \lambda_1(\mathbf{p})\alpha(\mathbf{p}) + \lambda_2(\mathbf{p})\alpha^+(-\mathbf{p}), \qquad (4)$$
$$a^+(\mathbf{p}) = \lambda_1(\mathbf{p})\alpha^+(\mathbf{p}) + \lambda_2(\mathbf{p})\alpha(-\mathbf{p});$$

$\lambda_{1,2}$ are ordinary functions, such that $\lambda_{1,2}(\mathbf{p}) = \lambda_{1,2}(-\mathbf{p})$, $|\lambda_1(\mathbf{p})|^2 - |\lambda_2(\mathbf{p})|^2 = 1$.

If one defines one-particle, two-particle, etc., states as before by $\alpha^+(\mathbf{p})\Phi_0$, $\alpha^+(\mathbf{p}_1)\alpha^+(\mathbf{p}_2)\Phi_0 \ldots$, the Hamiltonian H' expressed in terms of the α and α^+ ($H'(\alpha, \alpha^+) = H(a, a^+)$) describes a theory without particle number conservation, since this Hamiltonian is not invariant under the transformation $\alpha \to e^{i\delta}\alpha$, $\alpha^+ \to e^{-i\delta}\alpha^+$.

The preceding reasoning is incorrect for three reasons.

Firstly, it should not be forgotten that the operators a and a^+ are essentially operator-valued distributions, i.e., in any representation only the smeared-out operators $\int d\mathbf{p}\, f(\mathbf{p})\, a(\mathbf{p})$ are meaningful, for sufficiently smooth f. The correctness of expressions of the form $\int d\mathbf{p}\, p_0\, a^+(\mathbf{p})\, a(\mathbf{p})$ depends on the representation chosen for a and a^+.

It is easy to see that such an operator is well-defined only for the Fock representation. If one expresses a and a^+ in terms of α and α^+ by means of Eqs. (4), there appear additional terms of the form

$$\int d\mathbf{p}\, p_0 \lambda_1(\mathbf{p}) \bar{\lambda}_2(\mathbf{p}) \alpha^+(\mathbf{p}) \alpha^+(-\mathbf{p}),$$

$$\int d\mathbf{p}\, p_0 |\lambda_2(\mathbf{p})|^2 \alpha(\mathbf{p}) \alpha^+(\mathbf{p}),$$

which are meaningless as operators. This incorrectness cannot be removed by a simple normalization (i.e., by removing $\delta(0) \int d\mathbf{p}\, p_0 |\lambda_2(\mathbf{p})|^2$) since this leaves an incorrect expression, containing the product of two creation operators (and its adjoint). Consequently, the transition to another representation of a and a^+ necessitates additional justification, of the type of a renormalization of some incorrect (meaningless) expressions which are obtained, and any assertions about symmetry are meaningful only for the renormalized Hamiltonian (if the Hamiltonian can be at all reasonably renormalized).

Secondly, there exists a simple consideration which strongly restricts the selection of a representation: the interaction Hamiltonian must not contain terms describing transitions of two, three, or more particles into the vacuum. For Hamiltonians which are polynomials in the creation and annihilation operators this consideration determines the representation uniquely.

The first objection could be dropped on the basis of its "unphysical" character; the second one can be removed if one refuses to interpret $\alpha^+(\mathbf{p})\Phi_0$, $\alpha^+(\mathbf{p}_1)\alpha^+(\mathbf{p}_2)\Phi_0$, etc. as one-particle, two-particle, etc., states. But then one can raise a third objection, directly related to the discussion in the preceding section.

Let H be expressed in terms of a and a^+, for which the representation is not specified. We assume that H is invariant under a group G of transformations of a and a^+. Then the generators J_i of this symmetry group can be expressed in the standard fashion in terms of the a and a^+. These expressions will be of the same kind as the expression of H and will commute with the latter. In addition they form the Lie algebra of the group G. It should be stressed that all commutation relations depend only on the form of H and J_i (i.e., essentially they depend only on H), but not on the representation chosen for a and a^+ (since the commutation relations of a and a^+ are the same in any representation).

Let us assume that all generators J_i are correct (i.e., meaningful) in any representation (their incorrectness is of the same nature as the one of H). Then one can assert that for any representation of a and a^+ the physical theory possesses a symmetry group for which the Lie algebra does not depend on the representation.

It is easy to observe that the modification obtained in going over from one representation to the other is the following. In changing representation the expressions a and a^+ in terms of the Fock operators α and α^+ are changed, and therefore the expression of the Hamiltonian in terms of the Fock operators changes its form: $H'(\alpha, \alpha^+) = H(a, a^+)$ where H' is representation-dependent. Together with the form of H' the expression of its symmetry group in terms of the Fock operators changes (i.e., of the transformations of α and α^+ which leave H' invariant). This situation is completely analogous to the one discussed in the preceding section. The symmetry groups of H' in different representations differ in their external form, but they are isomorphic and their unitary representations are equivalent, i.e., they realize the unitary representations of the same abstract group G.

We now discuss the interpretation of this symmetry.

In scattering theory, where one deals with the vacuum, one-particle, two-particle, etc., states, the only creation and annihilation operators can be the Fock operators α and α^+. The symmetries which are observed in scattering are always related to the conservation of particle quantum numbers. To symmetries of this kind there corresponds the invariance of the Hamiltonian with respect to well defined transformations of the Fock operators.

It has been shown that under a change of representation only the external form of the invariance group of the Hamiltonian changes, as expressed in terms of the Fock operators. Let us assume that there exists a representation such that all the elements of the symmetry group have the form of transformations related to the ordinary conservation laws for some quantum numbers. Such a representation is optimal. Indeed, for any other representation a part of the usual symmetries is violated, but new ones appear, having the same Lie algebra. The nature of such symmetries is completely unexplainable, to say nothing of the fact that they do not show up in experiments. Thus the requirement of the "usualness" of the symmetries also leads for a given H to a selection of the representation of the a and a^+.

One arrives at the same representation by a direct identification of the generators J_i with conserved observables (this identification is representation-independent, starting from the Lie-algebra).

5. CONCLUSION

The main conclusion of the present paper is the following: one should not identify an abstract symmetry group of a given physical system with a concrete group of automorphisms which represents it. From the practical point of view two conclusions are important.

1. If a field theory is given in such a manner that the Wightman functions are not invariant with respect to some group of automorphisms, this does not necessarily mean that the symmetry, which was traditionally associated with this automorphism, is absent. It is possible that there exists a group of automorphisms which differs from the traditional one, but represents the same symmetry and leaves the Wightman functional invariant.

2. Attempts to obtain a theory which violates a certain symmetry by means of a transition to another representation $R_\tau(A)$ by means of an automorphism which does not commute with the symmetries form the automorphism group of $R(A)$ are useless (in other words, it is of no use to replace the field by a new field $A_\tau(x)$ which is one-to-one correspondence with the original field). In reality no restriction of the symmetry group occurs, but the group of automorphisms which represents it is replaced by another. Therefore the symmetry breaking in the theory described by $R_\tau(A)$ is purely external.

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