AN ANALOG OF THE JOSEPHSON EFFECT IN NUCLEAR TRANSFORMATIONS

V. I. GOL'DANSKIĬ and A. I. LARKIN

Institute of Chemical Physics, Academy of Science, U.S.S.R.

Submitted March 30, 1967

Zh. Eksp. Teor. Fiz. 53, 1032–1037 (September, 1967)

When nuclei are bombarded by heavy ions, various processes of nucleon tunneling through the potential barrier that separates the interacting nuclei at the smallest possible classical distance are observed. It is shown that nucleon pairing may give rise to a significant increase of the cross section for the transition of neutron or proton pairs, a phenomenon which in some respects is analogous to the Josephson effect in superconductors. Pairing is taken into account in the calculation of the probability for the excitation of various levels by one-nucleon exchange, which has been calculated earlier by Breit and Ebel[1] without such corrections. The probability for two-nucleon exchange is determined. An expression is obtained for the two-proton radioactivity with account of any number of arbitrary levels, which goes over into the Galitskii-Chel'tsov formula[2] in the limiting case of a single S level.

1. INTRODUCTION

In nuclear reactions with heavy ions the energy can be chosen in such a way that the classical trajectory of the colliding nuclei corresponds to a close approach without touching. In this case one- or two-nucleon tunneling reactions are possible, where the nucleons move from one nucleus to another below the barrier. It will be shown below that the pairing of the nucleons may give rise to a significant increase of the probability for such a two-nucleon tunneling transfer, which is to some extent analogous to what is observed in the transfer of a pair of electrons from one superconductor to another through a thin barrier—a dielectric (Josephson effect[3]). This analogy was noted in[4]. An essential difference between the Josephson effect and the nucleon transfer in nuclear reactions arises from the fact that the superconductors are at rest, i.e., there is no "reserve" kinetic energy of the interacting systems which, in principle, can be used for breaking up the pair. Therefore, one-electron exchange connected with the break-up of a pair cannot occur at zero temperature. In nuclear reactions the tunneling nucleon may change its energy by an amount proportional to the inverse of the time between two collisions, ω_0, where $\hbar\omega_0 \approx 2$ to 3 MeV.[1)] This quantity is of the order of the pairing energy, so that one-nucleon transfer in nuclear reactions has a significant probability. The probability for two-nucleon transfer is of the order of the square of the probability for one-nucleon transfer. However, in some cases the relative probability for two-nucleon transfer can be strongly enhanced. This happens when there are no levels with small angular momentum within the "distance" ω_0 from the Fermi surface, since the probability for a transition from a state with large angular momentum is small owing to the centrifugal barrier.

Taking account of the pairing, we have found the probability for the excitation of various levels by one-nucleon exchange, calculated earlier by Breit and Ebel[1] without account of pairing. The probability for two-nucleon transfer is determined. An expression is obtained for the two-proton radioactivity with account of any number of arbitrary levels. This expression leads to the Galitskiĭ-Chel'tsov formula[2] in the limit of a single S level.

2. THE HAMILTONIAN OF THE NUCLEONS

Let us consider the collision of two complex nuclei ($A \gg 1$) with energies somewhat smaller than the height of the potential barrier. In this case the transfer of one or two nucleons has a small effect on the motion of the nuclei, and one can assume that the nuclei move along their classical trajectories (hyperbolas). The Hamiltonian of the nucleons at each moment depends on the distance between the nuclei as on a classical parameter, and can be written in the form

$$H = H_1 + H_2 + V. \tag{1}$$

Here H_1 and H_2 are the Hamiltonians of the isolated nuclei, and V is the tunneling Hamiltonian.

The Hamiltonians $H_{1,2}$ describe the motion of the nucleons in the self-consistent field of the core and the residual interaction, of which we shall keep that part leading to pairing. In a single-particle level representation the Hamiltonians $H_{1,2}$ have the form

$$H_{1,2} = \sum_\lambda \varepsilon_\lambda a_\lambda^+ a_\lambda + g\sum_{\lambda\lambda'} a_\lambda^+ a_{-\lambda}^+ a_{\lambda'} a_{-\lambda'}, \tag{2}$$

where a_λ^+ and a_λ are the creation and annihilation operators for a nucleon in the state λ.

The tunneling Hamiltonian has the form

$$V = \sum T_{\lambda_1\lambda_2}(a_{\lambda_1}^+ a_{\lambda_2} + a_{\lambda_2}^+ a_{\lambda_1}). \tag{3}$$

Here $a_{\lambda_1}^+$ and a_{λ_1} are the creation and annihilation operators for particles in the state λ_1 of the first nucleus, and $a_{\lambda_2}^+$ and a_{λ_2} are the creation and annihilation operators for particles in the state λ_2 of the second nucleus. The overlap integral $T_{\lambda_1\lambda_2}$ is equal to

$$T_{\lambda_1\lambda_2} = \int \psi_{\lambda_1}^* \left(\frac{p^2}{2m} + U_1 + U_2\right)\psi_{\lambda_2} dv, \tag{4}$$

where $U_{1,2}$ are the self-consistent potentials of the first and second nuclei. For relatively sharp boundaries of the well the overlap integral is up to a factor given by

[1)]In the following we assume $\hbar = c = 1$.

$$T_{\lambda_1\lambda_2}(t) \sim \exp\left\{-\int dr\sqrt{2m|\varepsilon_i| + l_i^2/R_i^2}\right\}, \tag{5}$$

where the integral is taken between the boundaries of the nuclei, and ϵ_i and l_i are the energy and angular momentum of that of the states λ_1 and λ_2 for which the exponential (5) is larger. The factor in front of the exponential (5) depends on the detailed form of the diffuseness of the potential at the nuclear boundary and cannot be calculated theoretically. However, it can be determined from experiments on one-nucleon exchange, and the value obtained can be used for the calculation of the probability of two-nucleon exchange.

Taking the moment of closest approach of the nuclei as the zero of the time coordinate, we obtain for near head-on collisions

$$T_{\lambda_1\lambda_2}(t) = T_{\lambda_1\lambda_2}(0)\exp[-\omega_0^2t^2/2], \quad \omega_0 = [\sqrt{2m\varepsilon_i}Z_1Z_2e^2/\mu R^2]^{1/2} \approx 2\text{–}3\ \mathrm{MeV} \tag{6}$$

Here μ is the reduced mass of the colliding nuclei with the charges Z_1 and Z_2, and R is the closest classical distance (between the centers of the nuclei).

3. ONE-NUCLEON TRANSFER

In the interaction picture the S matrix for the transition is written in the form

$$S = \hat{T}\exp\left\{i\int_{-\infty}\hat{V}(t)\,dt\right\}, \tag{7}$$

where $\hat{T}$ is the time ordering operator, and the operator $V(t)$ has the form

$$V(t) = \sum_{\lambda_1\lambda_2} T_{\lambda_1\lambda_2}(t)[a_{\lambda_1}^+(t)a_{\lambda_2}(t) + a_{\lambda_1}(t)a_{\lambda_2}^+(t)], \quad a_{\lambda_1}(t) = e^{-iH_1t}a_{\lambda_1}e^{iH_1t}, \quad a_{\lambda_2}(t) = e^{-iH_2t}a_{\lambda_2}e^{iH_2t}. \tag{8}$$

The probability for one-nucleon transfer is given by the square of the modulus of the matrix element

$$W = \left|\int\langle A_1+1, A_2-1\,|V(t)|\,A_1, A_2\rangle\,dt\right|^2. \tag{9}$$

Let us substitute here $V(t)$ from (8) and use for the matrix elements of the operators a and a^+ the expressions obtained in the superconducting model of the nucleus[5] for even-even nuclei:

$$|\langle a_\lambda\rangle|^2 = 1 - |\langle a_\lambda^+\rangle|^2 = (E_\lambda - \varepsilon_\lambda)/2E_\lambda. \tag{10}$$

Here ϵ_λ is the energy of a single-particle level without account of pairing found, for example, in the Nilsson scheme; $E_\lambda = \sqrt{\epsilon_\lambda^2 + \Delta^2}$ is the single-particle energy with account of pairing.

As a result we obtain for the probability of the transition of a nucleon from the state λ_1 of one nucleus to the state λ_2 of another

$$W_1 = 2\pi\omega_0^2|T_{\lambda_1\lambda_2}(0)|^2\frac{E_{\lambda_1}+\varepsilon_{\lambda_1}}{2E_{\lambda_1}}\frac{E_{\lambda_2}-\varepsilon_{\lambda_2}}{2E_{\lambda_2}} \times\exp\left\{-\frac{(M_{A_1+1}-M_{A_1}+M_{A_2-1}-M_{A_2}+E_1+E_2)^2}{\omega_0^2}\right\}. \tag{11}$$

Here M_A is the mass of nucleus A; E_1 and E_2 are the excitation energies of the nuclei after the collision.

The matrix element $T_{\lambda_1\lambda_2}$ decreases with increasing angular momentum of the states λ_1 and λ_2. Although its value is known only up to the factor in front of the exponential, expression (11) allows one to connect the relative probabilities for the excitation of different levels with the admixture of a one-nucleon state at these levels. Except for the last three factors, formula (11) was obtained by Breit and Ebel.[1]

4. TWO-NUCLEON TRANSFER

The probability amplitude for two-nucleon transfer is determined by formula (7) in second order in the operator V:

$$M = \left\langle A_1+2, A_2-2\,\middle|\int_{-\infty}^{\infty}dt\int_{-\infty}^{t}dt'V(t)V(t')\,\middle|\,A_1, A_2\right\rangle = \frac{1}{2}\sum_{\lambda_1\lambda_2}\int\int dt\,dt'T_{\lambda_1\lambda_2}(t)T_{-\lambda_1-\lambda_2}(t')F_{\lambda_1}(t,t')F_{\lambda_2}(t',t). \tag{12}$$

Here F is the Green's function introduced by Gor'kov:[6] $F_\lambda(t,t') = \langle A-2|\hat{T}a_\lambda(t)a_\lambda(t')|A\rangle =$

$$= \frac{\Delta}{2E_\lambda}\exp\left\{i\left[(M_A - M_{A-2})\frac{t+t'}{2} - E_\lambda|t-t'|\right]\right\}. \tag{13}$$

Substituting (13) in (12), we obtain for the transition probability:

$$W_2 = |M|^2 = \frac{\pi}{4\omega_0^2}\left|\sum_{\lambda_1\lambda_2}\frac{\Delta_1\Delta_2T^2_{\lambda_1\lambda_2}(0)}{4E_{\lambda_1}E_{\lambda_2}(E_{\lambda_1}+E_{\lambda_2})}\int_0^\infty\exp\left\{iy - \frac{\omega_0^2y^2}{4(E_{\lambda_1}+E_{\lambda_2})}\right\}dy \times\exp\left\{-\frac{(M_{A_1}+M_{A_2}-M_{A_1+2}-M_{A_2-2})^2}{4\omega_0^2}\right\}\right|^2. \tag{14}$$

The comparison of the probabilities for two- and one-nucleon transfer must be carried out for each nucleus using the actual location of the single-particle levels. The comparison is simplified in the case when there is only one level with a small angular momentum near the Fermi surface and when the probability for a transition from a state with large angular momentum can be neglected because of the centrifugal barrier. In this case the unknown overlap integral in the ratio of the probability for two-nucleon exchange over the square of the probabilities for one-nucleon exchange cancels out. As a result we obtain

$$\frac{W_2}{W_1^2} = \frac{1}{\pi}\left|\frac{\omega_0\Delta_1\Delta_2}{4(E_{\lambda_1}+E_{\lambda_2})(E_{\lambda_1}+\varepsilon_{\lambda_1})(E_{\lambda_2}-\varepsilon_{\lambda_2})} \times\int_0^\infty\exp\{iy - \omega_0^2y^2/4(E_{\lambda_1}+E_{\lambda_2})\}dy\right|^2 \times\exp\left\{-\frac{(M_{A_1}+M_{A_2}-M_{A_2-2}-M_{A_1+2})^2}{2\omega_0^2}\right\} \times\exp\left\{+\frac{2(M_{A_1+1}-M_{A_1}+M_{A_2-1}-M_{A_2}+E_1+E_2)^2}{\omega_0^2}\right\}. \tag{15}$$

The quantities Δ, E, ω_0, $M_i - M_k$ entering in this expression have the same order of magnitude. Therefore the ratio (15) is of order unity for the majority of nuclei. The last factor, and hence the whole expression (15), become exponentially large when the excitation energy of one of the nuclei E_1 or E_2 is large compared to ω_0 in one-nucleon transfer. If ω_0 tends to zero, one-nucleon becomes impossible. This case is realized in the Josephson effect in superconductors and in the two-proton radioactivity in that variant (cf.[4]) where the binding energy of the last (even) proton is positive.

5. TWO-PROTON RADIOACTIVITY

The two-proton radioactivity was predicted by one of the authors.[4,7] A quantitative theory for a single S level has been given by Galitskiĭ and Chel'tsov.[2] The method of the tunneling Hamiltonian considered above allows one to obtain in a simple way a more general formula applicable to real nuclei.

Like the amplitude for two-nucleon exchange, the amplitude for two-proton radioactivity is determined by the matrix element of the S matrix (7) in second order in the tunneling Hamiltonian V. The vacuum plays the role of one of the colliding nuclei to which the two nucleons are transferred. Without account of the interaction of the protons in the final state the matrix element of the operator $a^{+}_{\lambda_i}(t)$ between the vacuum and a state of the proton in the Coulomb field of the nucleus with energy ϵ is equal to $e^{-i\epsilon t}$. Formula (12) takes the form

$$M = \frac{1}{2}\sum_{\lambda}\iint_{-\infty}^{\infty} dt\,dt' F_{\lambda}(t,t')\,T_{\lambda\lambda_1}T_{\lambda\lambda_2}e^{-i(\varepsilon_1 t+\varepsilon_2 t')}. \tag{16}$$

Here the function F is determined by (13), and the overlap integrals $T_{\lambda\lambda_i}$ do not depend explicitly on the time but depend only on the angular momentum l of the initial state λ and the energy ϵ_i of the final state λ_i with exponential accuracy:

$$T_{\lambda\lambda_i} = \exp\left\{-\int_R^{r_0} dr\left[2m\left(\varepsilon_i - \frac{Ze^2}{r}\right) - \frac{l(l+1)}{r^2}\right]^{1/2}\right\}, \tag{17}$$

where R is the nuclear radius, and r_0 is the zero of the expression under the root sign. For $l = 0$ and $R \ll r_0$ we have

$$T_{\lambda\lambda_i} \sim \exp\{-\pi Ze^2/\hbar v_i\}. \tag{18}$$

As is usually done in perturbation theory, we first regard the time during which the perturbation is present as finite in the continuous spectrum, and then let it tend to infinity. As a result we obtain for the probability per unit time for a transition to a state with two protons emitted with the energies ϵ_1 and ϵ_2

$$\frac{dW}{dt} = \frac{2\pi}{\hbar}\left|\sum_{\lambda} T_{\lambda\lambda_1}T_{-\lambda\lambda_2}\frac{\Delta}{(\varepsilon_1-\varepsilon_2)^2/4 - E_\lambda^2}\right|^2 \times \delta(M_A - M_{A-2} - 2m_p - \varepsilon_1 - \varepsilon_2). \tag{19}$$

In the model of a single S level, where $E_\lambda = \Delta = g^2$, formula (19) goes over into the expression obtained by Galitskii and Chel'tsov.[2] In the general case one must take all levels into account. The main contribution to (19) comes from the state with maximal angular momentum, even if its energy E_λ is large.

6. CONCLUSION

The pairing of nucleons has a large effect on the probability for nuclear transitions. Besides the usual effect due to the difference in binding energy of the nucleons in even and odd nuclei, there exists an additional effect which enhances strongly the probability for two-proton radioactivity and for two-nucleon exchange. The point is that even if the nuclear levels are far apart, the model of a single j level may turn out to be ill-suited for a description of such transitions. Usually the upper level has a large angular momentum, and the probability for a transition from it is small because of the centrifugal barrier. The pairing has the effect that the angular momentum of an individual nucleon is no longer well defined, and there is a significant probability that the nucleon is in a level with small angular momentum. Since the overlap integral depends strongly on the angular momentum, it may happen that the total transition rate is determined by the transition from a level with small l.

It should be kept in mind that formulas (14) and (19) describe the pairing effect only qualitatively. In order to obtain quantitative results, one must determine the factors in front of the exponentials in the overlap integrals and take account of the quantum mechanics of the motion of the colliding nuclei. In addition, several physical effects must also be considered. It is possible, for example, that the excitation of collective vibrational or rotational states plays an appreciable role. In classical terms, this corresponds to the appearance of a tidal wave during the collision of the nuclei, which may influence the value of the overlap integrals. In the case of far-away collisions one must possibly take account of the interaction of the nucleons in the sub-barrier region.

a b

In order to take account of these effects and for the consideration of more complicated reactions, it is convenient to develop a diagram technique which would combine the diagram technique of Migdal for the theory of finite superfluid fermi systems[5] with the diagram technique of Shapiro[8] for direct nuclear reactions. The reaction amplitude for a two-nucleon exchange is shown in Fig. a, and that for two-proton radioactivity in Fig. b. The solid lines inside the nuclei represent the Green's functions F of a superfluid fermi liquid, and the solid lines outside the nuclei illustrate the free motion of the nucleons in the sub-barrier region.

[1] G. Breit and M. E. Ebel, Phys. Rev. **103**, 679 (1956).
[2] V. M. Galitskiĭ and V. F. Chel'tsov, Nucl. Phys. **56**, 86 (1964).
[3] B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[4] V. I. Gol'danskiĭ, Usp. Fiz. Nauk **87**, 255 (1965) [Sov. Phys.-Uspekhi **8**, 770 (1966)].
[5] A. B. Migdal, Teoriya konechnykh fermi-sistem (Theory of Finite Fermi Systems), Nauka, 1965, Engl. Transl. Interscience Publ., N. Y., 1967.
[6] L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **34**, 735 (1958) [Sov. Phys.-JETP **7**, 505 (1958).
[7] V. I. Gol'danskiĭ, Zh. Eksp. Teor. Fiz. **39**, 497 (1960) [Sov. Phys.-JETP **12**, 348 (1961)].
[8] I. S. Shapiro, Teoriya pryamykh yadernykh rektsiĭ (Theory of Direct Nuclear Reactions) Atomizdat, 1963.

Translated by R. Lipperheide
118