MULTIPLASMON DECAY INSTABILITIES OF A TURBULENT PLASMA

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It is shown that the interrelationship of various three-plasmon interactions (one or two waves simultaneously participate in at least two three-plasmon interactions) may require an appreciable reevaluation of the efficiency of some nonlinear processes previously obtained and leads to qualitatively new effects. Thus, owing to the interrelationship between three-plasmon interactions, multiplasmon decay instabilities may arise already in the first order of perturbation theory with respect to the number of quasiparticles of the turbulent state. A characteristic feature of multiplasmon instabilities of the indicated type is the excitation of frequencies which may be smaller or larger than the initial ones. The influence of dissipative effects on multiplasmon decay processes is investigated. It is shown, in particular, that even in weak coupling all the coupled waves may grow at the linear instability increment of one of them. The smallness of the coupling constants determines only the relation between the intensities of the bound waves.

As is well known, nonlinear processes play an important role in different physical phenomena occurring in a plasma (dynamics of instability development, radiation, propagation of intense electromagnetic waves), in solids (nonlinear optical phenomena), etc. Most papers devoted to nonlinear wave interactions (see, for example, the reviews[1–5]) contain in the main only qualitative estimates of the efficiency of the individual ternary wave interactions, and usually pay no attention to the mutual interaction of certain processes.

The purpose of the present paper is to show that the interrelationship between different nonlinear processes in a turbulent plasma can lead to qualitatively new effects and can significantly alter the earlier estimates.[1)] We investigate here a frequently encountered situation, when one or two waves participate simultaneously in two three-plasmon interactions. Let us consider a simple example.

It is known[7, 8] that the confluence of two Langmuir waves leads to radiation from a turbulent plasma at frequencies on the order of $2\omega_{0e}$ (ω_{0e}—electron Langmuir frequency); this phenomenon has been distinctly recorded both under astrophysical conditions[9, 10] and in the laboratory.[11] The intensity of such a process, according to theoretical estimates[7, 8] obtained under the assumption that there are no other nonlinear processes to influence the given process, is relatively small.[2)] However, if there are other nonlinear processes, such as decays, which proceed very rapidly, and if in addition some of the wave indicated above participate in them, the situation may change. Simultaneously with the waves excited by the decay, there can be excited also a transverse wave with an increment determined by the fast decay process. In other words, the confluence intensity is greatly increased. We indicate for concreteness that in a nonisothermal plasma ($T_e \gg T_i$) it is possible to have a rapid decay of a Langmuir wave into a Langmuir and ion-acoustic wave, and this decay initiates intensive confluence of the Langmuir waves into a transverse one.

We emphasize that the excited waves (Langmuir, ion-acoustic, and transverse) increase with a common increment, and furthermore one of the excited waves, the transverse one, has a frequency larger than the frequency of the initial Langmuir wave.

The foregoing example illustrates the general situation which arises not only in a plasma but also in nonlinear optical media.

1. DECAY INSTABILITIES WITH EXCITATION OF THREE PLASMONS

A. The decay of one wave into three, or of two into two (called four-plasmon interaction) has been considered in a number of papers.[12–14] The probabilities of such transitions are usually smaller than the probabilities of three-plasmon processes.

We shall consider cases when simultaneous excitation of three waves by one initial wave is due not to four-plasmon interactions, but to the mutual coupling of two three-plasmon processes. Without specifying concretely the types of the waves, we investigate in general form the case when four waves participate in the following three-plasmon processes:

$$k_0 \rightleftarrows k_1 + k_2, \quad k_3 \rightleftarrows k_0 + k_1, \tag{1.1}$$

where $k_i = \{\mathbf{k}_i, \omega\}$—four-vector of the i-th wave.

Let the intensity of the wave k_0 be much larger than the intensities of the other waves. We shall show that all three waves (k_1, k_2, k_3) can be simultaneously excited upon decay of the wave k_0. (We emphasize, that we are considering here only three-plasmon processes.) It is seen from (1.1) that the frequency ω_3 is larger than the frequency of the initial wave ω_0. For the sake of simplicity we shall consider the case when the waves k_0 constitute a one-dimensional packet of waves,[3)] that

[1)]Concerning the interaction of waves with fixed phases see also [6].

[2)]At the same time, at a sufficiently high level of turbulent pulsations, the radiation due to wave confluence only can be appreciable.

[3)]The smearing of this packet over the directions, as a result of the reaction of the excited waves on the initial ones, is a slow process. It occurs within times much longer than the times of interest to us.

is, $N(\mathbf{k}_0) = N_0(k_{0\parallel})\,\delta(\mathbf{k}_{0\perp})$ (where $N(\mathbf{k}_i)$ is the number of waves whose wave vectors lie in the interval between $\mathbf{k}_i$ and $\mathbf{k}_i + d\mathbf{k}_i$, and $\mathbf{k}_{0\perp}$ is a vector perpendicular to the direction of propagation of the wave packet). Then the wave kinetic equations describing the processes (1.1), take the form[5]

$$\left(\frac{\partial}{\partial t}+\Gamma_1\right)N_1(\mathbf{k}_1)=\frac{w_0^{12}(\mathbf{k}_1;-\mathbf{k}_{1\perp},k_{0\parallel}-k_{1\parallel};k_{0\parallel},0)}{|\partial\omega_0/\partial k_{0\parallel}-\partial\omega_2(-\mathbf{k}_{1\perp},k_{0\parallel}-k_{1\parallel})/\partial k_{0\parallel}|}\times N_0(k_{0\parallel})[N_1(\mathbf{k}_1)+N_2(-\mathbf{k}_{1\perp},k_{0\parallel}-k_{1\parallel})]$$
$$+\frac{w_3^{10}(\mathbf{k}_1;k_{0\parallel},0;\mathbf{k}_{1\perp},k_{0\parallel}+k_{1\parallel})}{|\partial\omega_0/\partial k_{0\parallel}-\partial\omega_3(\mathbf{k}_{1\perp},k_{0\parallel}+k_{1\parallel})/\partial k_{0\parallel}|}\times N_0(k_{0\parallel})[N_3(\mathbf{k}_{1\perp},k_{0\parallel}+k_{1\parallel})-N_1(\mathbf{k}_1)], \tag{1.2}$$

$$\left(\frac{\partial}{\partial t}+\Gamma_2\right)N_2(-\mathbf{k}_{1\perp},k_{0\parallel}-k_{1\parallel})=\frac{w_0^{12}(\mathbf{k}_1;-\mathbf{k}_{1\perp},k_{0\parallel}-k_{1\parallel};k_{0\parallel},0)}{|\partial\omega_0/\partial k_{0\parallel}-\partial\omega_1(k_{0\parallel}-k_{2\parallel},-\mathbf{k}_{2\perp})/\partial k_{0\parallel}|_{\mathbf{k}_{2\perp}=-\mathbf{k}_{1\perp},\,k_{2\parallel}=k_{0\parallel}-k_{1\parallel}}}\times N_0(k_{0\parallel})[N_1(\mathbf{k}_1)+N_2(-\mathbf{k}_{1\perp},k_{0\parallel}-k_{1\parallel})], \tag{1.3}$$

$$\left(\frac{\partial}{\partial t}+\Gamma_3\right)N_3(\mathbf{k}_{1\perp},k_{0\parallel}+k_{1\parallel})=\frac{w_3^{10}(\mathbf{k}_1;k_{0\parallel},0;\mathbf{k}_{1\perp},k_{0\parallel}+k_{1\parallel})}{|\partial\omega_0/\partial k_{0\parallel}+\partial\omega_1(k_{0\parallel}+k_{3\parallel},\mathbf{k}_{3\perp})/\partial k_{0\parallel}|_{\mathbf{k}_{3\perp}=\mathbf{k}_{1\perp},\,k_{3\parallel}=k_{0\parallel}+k_{1\parallel}}}\times N_0(k_{0\parallel})[N_1(\mathbf{k}_1)-N_3(\mathbf{k}_{1\perp},k_{0\parallel}+k_{1\parallel})]. \tag{1.4}$$

In (1.2)–(1.4) we have introduced the following notation: Γ_i—decrements (increments) of the corresponding waves in the linear approximation; $w_i^{js}(\mathbf{k}_j;\ \mathbf{k}_s;\ \mathbf{k}_i)$—probability of decay of the wave $\mathbf{k}_i$ into the waves $\mathbf{k}_j$ and $\mathbf{k}_s$; N_i—number of waves of type i; $k_{0\parallel}$—solution of the equations

$$\omega_3(k_{1\parallel}+k_{0\parallel};\mathbf{k}_{1\perp})=\omega_0(k_{0\parallel},0)+\omega_1(\mathbf{k}_1),$$
$$\omega_0(k_{0\parallel},0)=\omega_1(\mathbf{k}_1)+\omega_2(-\mathbf{k}_{1\perp},k_{0\parallel}-k_{1\parallel}). \tag{1.5}$$

Assuming that N_0 is constant, the solution of the system (1.2)–(1.4) is

$$N_j(t)=\sum_{\alpha=1}^{3}N_j^{(\alpha)}e^{\nu_\alpha t},\quad j=1,2,3, \tag{1.6}$$

where ν_α are the roots of the dispersion equation

$$(\nu+\Gamma_1)(\nu+\Gamma_2)(\nu+\Gamma_3)+(\nu+\Gamma_1)(\nu+\Gamma_2)w_{31}-(\nu+\Gamma_1)(\nu+\Gamma_3)w_{21}+(\nu+\Gamma_2)(\nu+\Gamma_3)(w_{13}-w_{12})$$
$$-(\nu+\Gamma_1)w_{21}w_{31}-(\nu+\Gamma_2)w_{31}w_{12}-(\nu+\Gamma_3)w_{13}w_{21}=0. \tag{1.7}$$

Here

$$w_{21}\left|\frac{\partial\omega_0}{\partial k_{0\parallel}}-\frac{\partial\omega_1}{\partial k_{0\parallel}}\right|=w_{12}\left|\frac{\partial\omega_0}{\partial k_{0\parallel}}-\frac{\partial\omega_2(-\mathbf{k}_{1\perp};k_{0\parallel}-k_{1\parallel})}{\partial k_{0\parallel}}\right|=w_0^{12}(\mathbf{k}_1;-\mathbf{k}_{11},\ k_{0\parallel}-k_{1\parallel};\ k_{0\parallel},0)N_0(k_{0\parallel}),$$
$$w_{13}\left|\frac{\partial\omega_3}{\partial k_{0\parallel}}-\frac{\partial\omega_0}{\partial k_{0\parallel}}\right|=w_{31}\left|\frac{\partial\omega_0}{\partial k_{0\parallel}}+\frac{\partial\omega_1}{\partial k_{0\parallel}}\right|=w_3^{01}(\mathbf{k}_1;k_{0\parallel}^{(1)},\ 0;\ k_{1\parallel}+k_{0\parallel},\ \mathbf{k}_{1\perp})N_0(k_{0\parallel}).$$

B. Let us consider a case when the nonlinear build-up or attenuation of the wave are small. Then the solution of (1.7) is

$$\nu_1=0, \tag{1.8}$$
$$\nu_2={}^1\!/_2(w_{21}+w_{12}-w_{13}-w_{31})+{}^1\!/_2[(w_{21}+w_{12}-w_{13}-w_{31})^2+4(w_{21}w_{13}+w_{12}w_{31}+w_{21}w_{31})]^{1/2}, \tag{1.9}$$
$$\nu_3={}^1\!/_2[w_{21}+w_{12}-w_{13}-w_{31}]-{}^1\!/_2[(w_{21}+w_{12}-w_{13}-w_{31})^2+4(w_{21}w_{13}+w_{12}w_{31}+w_{21}w_{31})]^{1/2}. \tag{1.10}$$

As seen from (1.9), ν_2 always gives an unstable solution. However, the feasibility of such an instability calls for a special analysis. It will be shown below that such a possibility is always realized in the absence of linear damping (growth). We shall now consider different limiting cases of (1.9).

If the process $k_0 \to k_1 + k_2$ is fast, that is, if w_{21} and w_{12} are much larger than w_{31} and w_{13}, then

$$\nu_2\approx w_{12}+w_{21}. \tag{1.11}$$

All three waves build up with this maximum increment. The following relations, which can be derived from (1.2)–(1.4), are established in this case between the spectral energy densities of the different waves $I_j \equiv (2\pi)^{-3}N_j\,\omega_j$ for the time $t > 1/\nu_2$

$$\frac{I_2}{I_1}\approx\frac{w_{21}\omega_2}{w_{12}\omega_1},\qquad\frac{I_3}{I_1}\approx\frac{w_{31}\omega_3}{(w_{12}+w_{21})\omega_1}. \tag{1.12}$$

If the process $k_3 \to k_0 + k_1$ is the faster one (w_{21} and w_{12}, w_{31} and w_{13}), then

$$\nu_2\approx\frac{w_{21}w_{13}+w_{12}w_{31}+w_{21}w_{31}}{w_{13}+w_{31}}$$

or the growth increments of all three waves is of the order of ν_2, and the intensity ratio is of the form

$$\frac{I_2}{I_1}\approx\frac{(w_{13}+w_{31})\omega_2}{w_{31}\omega_1},\qquad\frac{I_3}{I_1}\approx\frac{\omega_3}{\omega_1}. \tag{1.13}$$

In this case (at frequencies that do not differ greatly) the intensities of all waves are of the same order of magnitude, and the growth time of the waves is characterized by the time of the slow process.

Let us stop to prove that the instability defined by (1.9) always takes place. To this end it is necessary to show that if at a certain initial instant of time $t = 0$ all three N_j are positive, then in all the succeeding instants of time any N_j is positive.

From (1.2)–(1.4) we have

$$N_1^{(1)}=-N_2^{(1)}=N_3^{(1)}, \tag{1.14}$$

$$N_2^{(2)}=\frac{w_{21}}{\nu_2-w_{21}}N_1^{(2)},\qquad N_3^{(2)}=\frac{w_{31}}{\nu_2+w_{31}}N_1^{(2)}, \tag{1.15}$$

$$N_2^{(3)}=\frac{w_{21}}{\nu_3-w_{21}}N_1^{(3)},\qquad N_3^{(2)}=\frac{w_{31}}{\nu_3+w_{31}}N_1^{(3)}. \tag{1.16}$$

From (1.15) we see that if $N_1^{(2)} > 0$, then $N_2^{(2)}$ and $N_3^{(2)}$ are also positive, since $\nu_2 > w_{21}$. It follows from (1.14) and (1.16) that certain of the $N_j^{(1)}$ and $N_j^{(2)}$ have different signs. Further, we can express $N_1^{(\alpha)}$ in terms of the values of N_j at the initial instant of time ($N_j(0)$):

$$N_1^{(2)}=\frac{N_1(0)(\nu_2+w_{31})(\nu_2-w_{21})}{\nu_2(\nu_2-\nu_3)}+\frac{N_2(0)(\nu_2+w_{31})(\nu_2-w_{21})(w_{21}-\nu_3)}{\nu_2(\nu_2-\nu_3)(w_{21}+w_{31})}-\frac{N_3(0)(\nu_2+w_{31})(\nu_2-w_{21})(\nu_3+w_{31})}{\nu_2(\nu_2-\nu_3)(w_{21}+w_{31})}, \tag{1.17}$$

$$N_1^{(3)}=-\frac{N_1(0)(\nu_3-w_{21})(\nu_3+w_{31})}{\nu_3(\nu_2-\nu_3)}-\frac{N_2(0)(w_{21}-\nu_3)(\nu_2-w_{21})(\nu_3+w_{31})}{\nu_3(\nu_2-\nu_3)(w_{21}+w_{31})}-\frac{N_3(0)(w_{21}-\nu_3)(\nu_3+w_{31})(\nu_2+w_{31})}{\nu_3(\nu_2-\nu_3)(w_{21}+w_{31})} \tag{1.18}$$

$$N_1^{(1)}=N_1(0)-N_1^{(3)}-N_1^{(2)}. \tag{1.19}$$

Recognizing that $|\nu_3| > w_{31}$ and $\nu_3 < 0$, we find that for any initial distribution of the wave numbers we have $N_1^{(2)} > 0$, and consequently the remaining two solutions can develop against the "background" of the unstable solution.

It is necessary that the intensity decrease due to these solutions cannot lead to negative values of N_j. To this end it is sufficient to prove that the $N_j(t)$ curves have no points of tangency with the t axis. The point t_0 at which a tangency might take place is determined by the vanishing of one of $N_j(t)$ and its derivative $\partial N_j(t)/\partial t$. This gives two relations. For example, for $N_3(t)$ these relations are of the form

$$\nu_3 t_0 = \ln\left\{-\frac{N_1^{(1)}\nu_2}{N_1^{(3)}(\nu_2-\nu_3)}\right\},$$

$$\nu_2 t_0 = \ln\left\{\frac{N_1^{(1)}\nu_3}{N_1^{(2)}(\nu_2-\nu_3)}\right\}. \tag{1.20}$$

From (1.20) it follows that the following inequality should be satisfied

$$\nu_2 N_1^{(2)} + \nu_3 N_1^{(3)} < \nu_3 N_1(0). \tag{1.21}$$

Substituting (1.17) and (1.18) in (1.21) we verify that the inequality (1.21) is not satisfied, that is, there is no point of tangency. A similar analysis leads to the conclusion that there are no tangency points for $N_1(t)$ and $N_2(t)$.

We have thus shown that in the case under consideration, for arbitrary initial conditions and relations between the probabilities w_{ij} of the different processes, an instability takes place with an increment (1.9). We emphasize that the relations between the intensities of the oscillations (1.12) and (1.13) are amenable to experimental verification.

2. MULTIPLASMON DECAYS AND RADIATION FROM A TURBULENT PLASMA

Almost-one-dimensional broad packets of Langmuir waves frequently arise in the development of two-stream instabilities. We are interested in the process whereby the Langmuir waves are transformed into transverse ones, leading in final analysis to radiation from the plasma. Using the results of the preceding section, let us consider the following processes

$$l \rightleftharpoons l' + s, \quad l(0) \rightleftharpoons l'(1) + s(2), \tag{2.1}$$

$$t \rightleftharpoons l + l', \quad t(3) \rightleftharpoons l'(1) + l(0). \tag{2.2}$$

t, l, and s are respectively the transverse, Langmuir, and ion-acoustic waves ($T_e \gg T_i$).

We note that in the absence of the process (2.1) the process (2.2) is equivalent to transformation of Langmuir waves into transverse ones on thermal Langmuir fluctuations (see, for example, [15]). This is connected with the fact that the process of coalescence of Langmuir waves (2.2), can occur only for waves propagating in almost mutually opposite directions.[4)] Therefore one of the confluent waves should be a fluctuation-thermal wave. On the other hand, if process (2.1) is "turned on," then (2.2) proceeds in a different manner. First, in the process (2.1) the excited l' wave propagates in a direction opposite to the direction of the propagation of the initial l wave, that is, the radiation power increases, since (2.2) represents coalescence of two superthermal waves. Second, owing to the mutual coupling between (2.1) and (2.2), the fast rate of the process (2.1) also leads to an increase in the plasma radiation power.

To describe the three-plasmon decay (2.1) and (2.2) we shall use the expressions for the probabilities of the processes (2.1) and (2.2) (see [5, 7, 8, 16, 17]):

$$w_0^{12}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_0) = \frac{e^2}{16\pi m_e^2 v_{Te}}\sqrt{\frac{m_e}{m_i}}|\mathbf{k}_2|\frac{(\mathbf{k}_0\mathbf{k}_1)^2}{k_0^2 k_1^2}, \tag{2.3}$$

$$w_3^{10}(\mathbf{k}_1, \mathbf{k}_0, \mathbf{k}_3) = \frac{e^2(k_0^2 - k_1^2)[\mathbf{k}_0\mathbf{k}_1]^2}{16\pi\omega_{0e}k_3^2 k_0^2 k_1^2}. \tag{2.4}$$

From the laws of conservation during the decay it follows that the decay (2.1) is possible if

$$|k_0| > \frac{1}{3}\frac{\omega_{0e}}{v_{Te}}\sqrt{\frac{m_e}{m_i}} \equiv k_{0\,min}.$$

For simplicity let us consider the limiting case $|k_0| \gg k_{0\,min}$, when all the results can be written in simple form. With the aid of (2.3) and (2.4) we get

$$w_{12} = \frac{e^2\omega_{0e}}{24\pi m_e^2 v_{Te}^3}\sqrt{\frac{m_e}{m_i}}N_0(|k_{1\parallel}|), \tag{2.5}$$

$$w_{13} = \frac{e^2\sqrt{3\omega_{0e}^2 - k_{1\perp}^2}\,k_{1\perp}^2}{6\pi\omega_{0e}^2 m_e^2}N_0(|k_{1\parallel}|);$$

$$w_{21} = \frac{1}{2}w_{12}, \qquad w_{31} = \frac{\sqrt{3\omega_{0e}^2 - k_\perp^2}}{12|k_{1\parallel}|v_{Te}^2}w_{13}. \tag{2.6}$$

From the conservation laws in process (2.2) it follows that $k_{1\perp} < \omega_{0e}/\sqrt{3}$, which is much lower than $k_{1\parallel}$. Therefore in (2.5) and (2.6) $k_{1\perp}$ is negligibly small compared with $k_{1\parallel}$. If $v_{ph} \ll v_{Te}(9m_e/m_i)^{1/2}$ ($v_{ph} \equiv \omega_{0e}/|k_{1\parallel}|$), then the wave growth increment is determined by (1.11):

$$\nu_2 \approx \frac{e^2\omega_{0e}}{16\pi m_e^2 v_{Te}^3}\sqrt{\frac{m_e}{m_i}}N_0(|k_{1\parallel}|). \tag{2.7}$$

By way of an example let us consider the spectrum of the Langmuir oscillations excited by a beam expanding in velocity space as a result of quasilinear relaxation by an amount Δv_0, that is,

$$N_0(k_{1\parallel}) \approx \begin{cases} \dfrac{n_1 m v_0^3}{\omega_{0e}^2}8\pi^3, & |\Delta k| < \dfrac{\omega_{0e}}{v_0^2}\Delta v \\ 0, & |\Delta k| > \dfrac{\omega_{0e}}{v_0^2}\Delta v \end{cases}, \tag{2.8}$$

where Δk is $k - \omega_{0e}/v_0 > 0$; n_1 is the beam density and v_0 is the beam velocity. Substituting (2.8) in (2.7) we get

$$\nu \approx \omega_{0e}\frac{n_1}{n_0}\frac{\pi}{8}\sqrt{\frac{m_e}{m_i}}\frac{v_\Phi^3}{v_{Te}^3}. \tag{2.9}$$

The ratio of the intensities of the transverse and longitudinal waves has in accord with (1.12) the form

$$\frac{I_t}{I_{l'}} = \frac{16\sqrt{m_i/m_e}\,v_{Te}^3 k_{1\perp}^2\sqrt{3\omega_{0e}^2 - k_{1\perp}^2}}{3\omega_{0e}^3}, \qquad \frac{I_s}{I_{l'}} = \sqrt{\frac{m_e}{m_i}}\frac{v_{Te}}{v_0}. \tag{2.10}$$

With the aid of (2.10), let us estimate the integral intensity of the transverse waves:

$$\int I_t d\mathbf{k}_3 = 16\pi\int d(k_{1\perp}^2)\,dk_{1\parallel}v_{Te}^5\sqrt{\frac{m_i}{m_e}}\frac{k_{1\perp}^2\sqrt{3\omega_{0e}^2 - k_{1\perp}^2}}{\omega_{0e}^3}I_l(k_1)$$

$$\approx 16\pi n_1 m v_0^2\frac{\Delta v}{v_0}v_{Te}^5\sqrt{\frac{m_i}{m_e}}.$$

The ratio α of this intensity to the intensity of the radiation resulting from the transformation of the Langmuir waves on the thermal fluctuations is

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4) This takes place when $\omega_{0e}/kc \ll 1$ where ω_{0e} is the electronic Langmuir frequency. We henceforth put $c = 1$ for the speed of light.

$$\alpha \approx 16\pi \frac{n_1}{n_0} N_D \frac{v_0^2}{v_{Te}^2}\sqrt{\frac{m_i}{m_e}},$$

where n_0 is the plasma density and N_D is the number of particles in a sphere having the Debye radius.

3. DISSIPATIVE NONLINEAR INSTABILITIES

With the aid of (1.7) we can investigate the instability of a system of coupled plasmons for intensive dissipative processes. Let Γ_1 greatly exceed all the w_{ij}, and let Γ_2 and Γ_3 be close to zero. Then the approximation solution of (1.7) is

$$\nu_1 = -\Gamma_1 + 2(w_{21} - w_{31}) + (w_{12} - w_{13}), \tag{3.1}$$

$$\nu_2 = w_{21}(1 + w_{12}/\Gamma_1), \tag{3.2}$$

$$\nu_3 = -w_{31}(1 - w_{13}/\Gamma_1). \tag{3.3}$$

We emphasize that the waves which would be strongly damped in the absence of nonlinear effects become unstable in the presence of even weak nonlinearities (solution (3.2).[5]) The wave intensity ratios assume the following asymptotic form (for $t \gg 1/w_{21}$)

$$\frac{I_2}{I_1} = \frac{\omega_2\Gamma_1}{\omega_1 w_{12}}, \quad \frac{I_3}{I_1} = \frac{\omega_3 w_{31}}{\omega_1(w_{21} + w_{31})}. \tag{3.4}$$

On the other hand, if wave 1 is unstable in the linear approximation (that is, $\Gamma_1 < 0$), then all three waves build up with a linear increment $-\Gamma_1$ as a result of the nonlinear coupling between the waves. Thus, the processes of nonlinear spectral energy redistribution occur within times that are determined by the linear increment. The nonlinear coupling coefficients determine only the wave intensity ratios, which take the form

$$\frac{I_2}{I_1} = \frac{\omega_2 w_{21}}{-\Gamma_1\omega_1} \quad \frac{I_3}{I_1} = \frac{w_{31}\omega_3}{-\Gamma_1\omega_1}. \tag{3.5}$$

If intense absorption or buildup take place for wave 2 ($\Gamma_1 \approx \Gamma_3 \approx 0$), then the roots of (1.7) are

$$\begin{aligned} &\nu_1 \approx -\Gamma_2,\\ &\nu_2 = {}^1\!/_2(w_{12} - w_{31} - w_{13}) + {}^1\!/_2[(w_{12} - w_{31} - w_{13})^2 + 4w_{31}w_{12}]^{1/2},\\ &\nu_3 = {}^1\!/_2(w_{12} - w_{31} - w_{13}) - {}^1\!/_2[(w_{12} - w_{31} - w_{13})^2 + 4w_{31}w_{12}]^{1/2}. \end{aligned} \tag{3.6}$$

In the case of strong damping of wave 2, the intensity ratios of the unstable solution are

$$\frac{I_2}{I_1} = \frac{\omega_2 w_{21}}{\omega_1\Gamma_2}, \quad \frac{I_3}{I_1} = \frac{w_{31}\omega_3}{(\nu_2 + w_{31})\omega_1}. \tag{3.7}$$

In (3.7), ν_2 is determined by (3.6).

For the example considered in the preceding section, we find that even in the region of strong damping of the ion-acoustic waves, namely if

$$\frac{n_1}{n_0} \ll 48\sqrt{2\pi}\sqrt{\frac{m_e}{m_i}}\frac{v_{Te}^4}{v_0^4} \tag{3.8}$$

(see [16]), the buildup of all the waves has an increment w_{12} which amounts to $2/3$ of the increment (2.9), and the intensity ratio determined by (3.7) takes the form

$$\frac{I_s}{I_{l'}} = \frac{n_1 v_0^3\sqrt{2\pi}}{12 n_0 v_{Te}^3},$$

$$\frac{I_t}{I_{l'}} = \frac{8\sqrt{m_i/m_e}\, v_{Te}^3 k_{1\perp}^2 \sqrt{3\omega_{0e}^2 - k_{1\perp}^2}}{\omega_{0e}^3}. \tag{3.9}$$

We note that by virtue of (3.8) the intensity ratio I_2/I_1 determined by (3.9) is always smaller than this ratio in the absence of strong damping (see (2.10)).

Finally, if strong dissipation (or buildup) takes place only for the wave k_3, then the solution of (1.7) can be written in the form

$$\begin{aligned} &\nu_1 = -\Gamma_3,\\ &\nu_2 = {}^1\!/_2(w_{21} + w_{12} - w_{13}) + {}^1\!/_2[(w_{21} + w_{12} - w_{13})^2 + 4w_{13}w_{21}]^{1/2},\\ &\nu_3 = {}^1\!/_2(w_{21} + w_{12} - w_{13}) - {}^1\!/_2[(w_{21} + w_{12} - w_{13})^2 + 4w_{13}w_{21}]^{1/2}, \end{aligned} \tag{3.10}$$

and we get for the intensity ratio

$$\frac{I_2}{I_1} = \frac{\omega_2 w_{21}}{\omega_1(\nu_2 - w_{21})}, \quad \frac{I_3}{I_1} = \frac{\omega_3 w_{31}}{(-\Gamma_3)\omega_1}, \quad \Gamma_3 < 0. \tag{3.11}$$

In conclusion we note that the concrete example presented by us covers by far not all the possibilities of the manifestations of the effects considered here. These effects are particularly important for a magnetoactive plasma, since the presence of a large number of branches leads to the possible occurrence of various nonlinear couplings.

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5) A wave buildup effect of this type, in the presence of damping, was considered for a somewhat different case in [18].