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COUPLED CYCLOTRON AND SPIN WAVES IN SEMICONDUCTORS AND METALS

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The spectra of electromagnetic waves propagating perpendicular to a static magnetic field in the electron-hole plasma of a magnetic semiconductor or metal are investigated. It is assumed that the electrons (or holes) obey a quadratic isotropic dispersion law and have an arbitrary isotropic velocity distribution. The frequency behavior of the interacting cyclotron and spin waves is determined as a function of the wave vector for arbitrary ratios between the wavelength and the electron (or hole) Larmor radius. Simple analytic expressions for the frequencies are derived for all values of the wave vector in a plasma with low gas-kinetic pressure when the mean electron (or hole) velocity is considerably slower than the Alfvén velocity.

SEVERAL branches of electromagnetic waves propagating perpendicular to a static external magnetic field can exist in the electron-hole plasma of a semiconductor or metal, just as in the case of an ordinary electron-ion plasma.[1] The frequencies of these waves are close to the cyclotron frequencies of electrons or holes (ions) and to their respective subharmonics; we shall therefore call them cyclotron waves. The existence of cyclotron waves follows from the finite Larmor radius of charge carriers.

Long-wave cyclotron frequencies (corresponding to wavelengths considerably exceeding the electron Larmor radius) have recently been studied theoretically and have been observed experimentally in the electron plasma of alkali metals by Walsh and Platzman.[2,3] Kaner and Skobov have determined frequencies of short-wave cyclotron waves (having wavelengths considerably shorter than the electron Larmor radius).[4]

We know that, in addition to the cyclotron waves in high-density plasmas, the propagation of an extraordinary electromagnetic wave is possible; at low frequencies, when there are equal numbers of electrons and holes, this wave becomes a magnetosonic wave. The cyclotron and extraordinary waves are not isolated branches; when their frequencies overlap they interact and the extraordinary wave becomes a cyclotron wave (compare with the discussion in [1]).

In the cases of magnetic semiconductors and metals, where spin waves can propagate, an investigation of electromagnetic wave propagation must take into account both the dielectric constant and magnetic permeability. The cyclotron and spin wave spectra are therefore considerably modified. The interaction between ordinary cyclotron waves and spin waves in ferromagnetic semiconductors and metals has been investigated in [5].

We note that weakly damped cyclotron waves can exist only when the effective frequency ν of collisions between carriers is considerably below the wave frequency ω; when $\omega \approx n\omega_3$ (n = 1, 2, ...) we have the condition that ν must be much smaller than the difference $|\omega - n\omega_B|$. The existence of cyclotron waves is also subject to the requirement that the angle θ between the wave vector **k** and the magnetic field **B** shall be close to $\pi/2$, so that

$$\cos\theta \approx {}^1/_2|\pi/2-\theta|^2 \ll |\omega - n\omega_B|/k\langle v\rangle.$$

In the present work we investigate the spectra of electromagnetic waves propagating perpendicular to a static magnetic field in a magnetic semiconductor or metal. We determine the frequency behavior of the interacting cyclotron and spin waves as a function of the wave vector for all ratios between the wavelength and the Larmor radius of carriers obeying a quadratic dispersion law and an arbitrary isotropic velocity distribution law. We note that a discrepancy between the actual electron (or hole) dispersion and an isotropic quadratic law leads to momentum-dependence of the cyclotron frequency and to strong damping of the cyclotron waves.

1. THE DISPERSION EQUATION

Let us consider the propagation of electromagnetic waves in an electron-hole plasma perpendicular to a magnetic field **B**. The dispersion equation relating the frequency and wave vector of these waves is separated into the two equations

$$(kc/\omega)^2 = \varepsilon_3(\omega, k)\mu_\perp(\omega), \qquad (1.1)$$

$$(kc/\omega)^2 = \mu_3(\omega, k)\varepsilon_\perp(\omega, k), \qquad (1.2)$$

where

$$\mu_\perp(\omega) = (\mu_1\mu_1' - \mu_2^2)/\mu_l, \quad \mu_l = \mu_1\cos^2\varphi + \mu_1'\sin^2\varphi,$$

$$\varepsilon_\perp(\omega, k) = [\varepsilon_1(\omega, k)\varepsilon_1'(\omega, k) - \varepsilon_2^2(\omega, k)]/\varepsilon_1(\omega, k).$$

Here ε_i and μ_i are components of the dielectric constant and magnetic permeability tensors, respectively:

$$(\varepsilon_{ij}) = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1' & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \quad (\mu_{ij}) = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1' & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \qquad (1.3)$$

The z axis is parallel to **B**, and φ is the azimuthal angle in the space of the wave-vector **k**.

In a nonmagnetic medium ($\mu_{ij} = \delta_{ij}$) where there is no spatial dispersion of the dielectric constant tensor ε_{ij}, Eq. (1.1) determines the frequency of the ordinary wave, while the frequency of the extraordinary wave is determined from Eq. (1.2). The quantities ε_i are represented by [6]

$$\varepsilon_1 = 1 - \sum_\alpha \sum_{n=-\infty}^{\infty} \frac{\omega_{p\alpha}^2 a_1}{\omega(\omega - n\omega_{B\alpha})}, \quad \varepsilon_1' = 1 - \sum_\alpha \sum_{n=-\infty}^{\infty} \frac{\omega_{p\alpha}^2 a_1'}{\omega(\omega - n\omega_{B\alpha})},$$
$$\varepsilon_2 = -\sum_\alpha \sum_{n=-\infty}^{\infty} \frac{\eta_\alpha \omega_{p\alpha}^2 a_2}{\omega(\omega - n\omega_{B\alpha})}, \quad \varepsilon_3 = 1 - \sum_\alpha \sum_{n=-\infty}^{\infty} \frac{\omega_{p\alpha}^2 a_3}{\omega(\omega - n\omega_{B\alpha})}, \tag{1.4}$$

where

$$a_1(k,n) = -\int dv \frac{v_\perp^2}{v} \frac{n^2 J_n^2(\lambda)}{\lambda^2} \frac{\partial f_0}{\partial v}$$
$$a_1'(k,n) = -\int dv \frac{v_\perp^2}{v} J_n'^2(\lambda) \frac{\partial f_0}{\partial v},$$
$$a_2(k,n) = -\int dv \frac{v_\perp^2}{v} \frac{n J_n(\lambda) J_n'(\lambda)}{\lambda} \frac{\partial f_0}{\partial v}, \tag{1.5}$$
$$a_3(k,n) = -\int dv \frac{v_\parallel^2}{v} J_n^2(\lambda) \frac{\partial f_0}{\partial v};$$

$\omega_{p\alpha} = 4\pi n_\alpha e_\alpha^2/m_\alpha^*$ and $\omega_{B\alpha} = |e_\alpha|B/m_\alpha^* c$ are the plasma and cyclotron frequencies of carriers having an effective mass m_α^*, charge e_α and equilibrium density n_α; $n_\alpha = e_\alpha/|e_\alpha|$; $f_0(v)$ is the velocity distribution of particles of kind α, normalized to unity; $J_n(\lambda)$ and $J_n'(\lambda)$ are Bessel functions and their derivatives; $\lambda = kv \sin\vartheta/\omega_{B\alpha}$; $v_\perp$ and $v_\parallel$ are the particle velocity components perpendicular and parallel to **B**.

For a Maxwellian particle velocity distribution we have

$$a_1 = (n^2 e^{-\xi}/\xi) I_n(\xi), \quad a_1' = [(n^2/\xi + 2\xi) I_n(\xi) - 2\xi I_n'(\xi)] e^{-\xi},$$
$$a_2 = e^{-\xi} n [-I_n(\xi) + I_n'(\xi)], \quad a_3 = e^{-\xi} I_n(\xi) \tag{1.6}$$

Here $\xi = (kv_\alpha/\omega_{B\alpha})^2$, $v_\alpha = \sqrt{T_\alpha/m_\alpha^*}$ is the thermal velocity of the particles, and $I_n(\xi)$ and $I_n'(\xi)$ are a modified Bessel function and its derivative.

For a degenerate Fermi distribution we would have

$$a_1 = 3\int_0^{\pi/2} (n/\beta)^2 J_n^2(\beta \sin\vartheta) \sin\vartheta \, d\vartheta,$$
$$a_1' = 3\int_0^{\pi/2} J_n'^2(\beta \sin\vartheta) \sin^3\vartheta \, d\vartheta,$$
$$a_2 = 3\int_0^{\pi/2} (n/\beta) J_n(\beta\sin\vartheta) J_n'(\beta\sin\vartheta) \sin^2\vartheta \, d\vartheta, \tag{1.7}$$
$$a_3 = 3\int_0^{\pi/2} J_n^2(\beta\sin\vartheta)\cos^2\vartheta \sin\vartheta \, d\vartheta,$$

where $\beta = kv_F/\omega_{B\alpha}$, v_F is the limiting Fermi velocity, and ϑ is the polar angle in velocity space.

We shall now present the values of the coefficients a_i in the long-wave ($k\rho_{L\alpha} \ll 1$) and short-wave ($k\rho_{L\alpha} \gg 1$) cases; $\rho_{L\alpha} = \langle v\rangle/\omega_{B\alpha}$ is the Larmor radius and $\langle v\rangle$ is the mean particle velocity. When $k\rho_{L\alpha} \ll 1$ we have

$$a_1 = \frac{n^2}{(2^n \cdot n!)^2} \frac{2n!!}{(2n+1)!!} \left(\frac{k}{\omega_{B\alpha}}\right)^{2n-2} \Big\{ (2n+1)\langle v^{2n-2}\rangle - \frac{k^2}{\omega_{B\alpha}^2}\langle v^{2n}\rangle + \frac{1}{4(n+1)} \frac{k^4}{\omega_{B\alpha}^4} \langle v^{2n+2}\rangle \Big\},$$
$$a_1' = \frac{n^2}{(2^n \cdot n!)^2} \frac{2n!!}{(2n+1)!!} \left(\frac{k}{\omega_{B\alpha}}\right)^{2n-2} \Big\{ (2n+1)\langle v^{2n-2}\rangle - \frac{n+2}{n}\frac{k^2}{\omega_{B\alpha}^2}\langle v^{2n}\rangle + \frac{(2n^3+11n^2+16n+8)}{4n^2(n+1)(2n+3)} \frac{k^4}{\omega_{B\alpha}^4}\langle v^{2n+2}\rangle \Big\}$$
$$a_2 = \frac{n^2}{(2^n \cdot n!)^2} \frac{2n!!}{(2n+1)!!} \left(\frac{k}{\omega_{B\alpha}}\right)^{2n-2} \Big\{ (2n+1)\langle v^{2n-2}\rangle - \frac{n+1}{n}\frac{k^2}{\omega_{B\alpha}^2}\langle v^{2n}\rangle + \frac{n+2}{4n(n+1)} \frac{k^4}{\omega_{B\alpha}^4}\langle v^{2n+2}\rangle \Big\} \tag{1.8}$$
$$a_3 = \frac{(2n+3)}{(2^n \cdot n!)^2} \left[\frac{2n!!}{(2n+1)!!} - \frac{(2n+2)!!}{(2n+3)!!}\right] \left(\frac{k}{\omega_{B\alpha}}\right)^{2n} \langle v^{2n}\rangle.$$

For a Maxwellian distribution we obtain

$$a_1 = a_1' = a_2 = n^2 \xi^{n-1}/(2^n \cdot n!), \quad a_3 = \xi^n/(2^n \cdot n!).$$

For a degenerate Fermi distribution a_i is obtained from (1.8) with $\langle v^{2m}\rangle = 3v_F^{2m}/(2m+3)$.

In the short-wave case we have

$$a_1 = -2na_2 = 2\pi n^2 f_0(0)(\omega_{B\alpha}/k)^3, \; a_1' = a_3 = (\omega_{B\alpha}/2k)\langle 1/v\rangle. \tag{1.9}$$

For a Maxwellian distribution this becomes

$$a_1 = -2na_2 = n^2/(\sqrt{2\pi}\xi^{3/2}), \quad a_1' = a_3 = 1/\sqrt{2\pi\xi}. \tag{1.10}$$

For a degenerate Fermi distribution we obtain

$$a_1 = -2na_2 = 3n^2/2\beta^3, \quad a_1' = a_3 = 3/4\beta. \tag{1.11}$$

We shall consider ferromagnets and ferrites in which only one spin-wave branch exists, so that

$$\mu_\perp(\omega) = (\omega - \omega_0)(\omega + \tilde{\omega})/(\omega^2 - \omega_m^2), \tag{1.12}$$

where $\omega_0 > \omega_m$, and ω_m is the frequency of longitudinal magnetostatic oscillations. The frequencies ω_0 and ω_m are of the same order of magnitude as the cyclotron frequency of a free electron ($\omega_B = eB_0/mc$).

In ferromagnets and ferrites $\mu_3 = 1$ and the extraordinary waves do not interact with the spin waves. In antiferromagnets the magnetic susceptibility tensor $\chi_{ij} = (\mu_{ij} - \delta_{ij})/4\pi$ is proportional to $\chi_0/(\omega^2 - \omega_{res}^2)$, where $\chi_0 \sim 10^{-3}$ is the static magnetic susceptibility, and significant interactions of the ordinary and extraordinary waves with spin waves occur only when $\omega \approx \omega_{res}$. Near the resonance frequency ω_{res}, $\mu_\perp$ and $\mu_3(\omega)$ are, as previously, derived from (1.12), where ω_0, ω_m, and ω differ from ω_{res} by an amount χ_0.

2. THE ORDINARY WAVE

We shall first study the dispersion equation (1.1), from which the frequencies of ordinary cyclotron waves and spin waves are derived. It is easy to determine the frequency behavior of these waves as a function of the wave vector, by solving (1.1) graphically. We represent (1.1) in the form

$$f_1(\omega, k) = f_2(\omega, k),$$

where

$$f_1 = \varepsilon_3, \quad f_2 = \frac{k^2 c^2 (\omega^2 - \omega_m^2)}{\omega^2(\omega - \omega_0)(\omega + \tilde{\omega})}.$$

With $f_1(\omega)$ increasing monotonically with the frequency and vanishing at $\pm\infty$ for $\omega \to n\omega_{B\alpha} \mp 0$, and with $f_2(\omega)$ decreasing monotonically as ω increases, while $f_2 \to +\infty$ for $\omega \to 0$ and $f_2 \to \pm\infty$ for $\omega \to \omega_0 \pm 0$, it is easily shown that each interval $\omega_j < \omega < \omega_{j+1}$ contains one intersection point of the $f_1(\omega)$ and $f_2(\omega)$ curves that corresponds to the solution $\omega = \omega^{(j+1)}(k)$ ($j = 0, 1, \ldots$). Here

$$\omega_1 < \omega_2 < \omega_3 < \ldots \tag{2.1}$$

comprise an increasing sequence of frequencies from the set $\{n\omega_{B\alpha}, \omega_0\}$. In the interval $\omega_s < \omega_m < \omega_{s+1}$

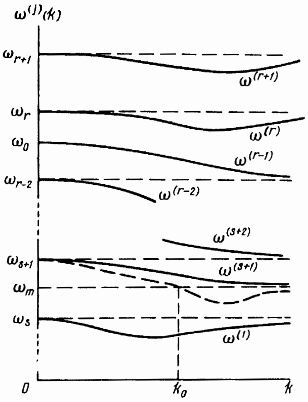

FIG. 1

the abscissa of the intersection point $\omega = \omega^{(s+1)}(k)$ of the f_1 and f_2 curves can lie in either one of the two intervals (ω_s, ω_m) and (ω_m, ω_{s+1}).

The behavior of the eigenfrequencies $\omega^j(k)$ as a function of the wave vector is shown schematically in Fig. 1. Let $\omega_r < \omega_{r+1} < \ldots$ be frequencies of the sequence (2.1) that exceed $\omega_0 = \omega_{r-1}$, and let $\omega_s > \omega_{s-1} > \ldots$ be frequencies of (2.1) that are below ω_m. The eigenfrequencies $\omega^{(j)}(k)$ ($j \le s$ or $j \ge r$) behave as functions of the wave vector in qualitatively the same manner as for nonmagnetic media: As k increases these frequencies at first decrease from the value $\omega^{(j)} = \omega_j$ for $k = 0$; after reaching a minimum point they again approach ω_j as $k \to \infty$ (see Fig. 1 in [1]).

The eigenfrequencies $\omega^{(j)}(k)$ ($s + 2 \le j \le r - 1$) decrease monotonically, as k increases, from the value ω_j for $k = 0$; ω_{j+1} is approached as $k \to \infty$. The frequency $\omega^{(s+1)}(k)$ also decreases monotonically as k increases if $\varepsilon_3(\omega_m, k) > 0$ for all k; in this case the abscissa of the intersection point of $f_1(\omega)$ and $f_2(\omega)$ always lies in the interval (ω_m, ω_{s+1}). If for some value $k = k_0$ we have $\varepsilon_3(\omega_m, k_0) = 0$, then $\omega^{(s+1)}(k)$ decreases, as k increases, from $\omega = \omega_{s+1}$ for $k = 0$; a minimum is reached at $k = k_m < k_0$, followed by a monotonic approach to $\omega = \omega_m$ as $k \to \infty$; in this case $\omega^{(s+1)}(k)$ is represented by the dashed line in Fig. 1.

We shall now derive analytic expressions for the eigenfrequencies. In a low-pressure plasma when $\langle v \rangle \ll v_{A\alpha}$, where $v_{A\alpha} = B_0/\sqrt{4\pi m_\alpha n_\alpha}$ is the Alfvén velocity, for not very small values of $k\rho_{L\alpha}$ we find that $\omega^{(j)}(k)$ is close to $n\omega_{B\alpha}$ or to ω_m:

$$\omega = n\omega_{B\alpha}[1 - \omega_{B\alpha}^2 a_3(n)\mu_\perp(n\omega_{B\alpha}) / (kv_{A\alpha})^2] \quad (n = 1, 2, \ldots), \tag{2.2}$$

$$\omega = \omega_m[1 + \varepsilon_3(\omega_m)(\omega_m - \omega_0)(\omega_m + \tilde{\omega}) / 2(kc)^2]. \tag{2.3}$$

The correction terms in (2.2) and (2.3) are of the order $\langle v^2 \rangle / v_{A\alpha}^2$. When $k\rho_{L\alpha} \ll 1$, Eq. (1.1) easily yields

$$\omega = n\omega_{B\alpha}[1 - \omega_{p\alpha}^2 a_3(n)\mu_\perp(n\omega_{B\alpha}) / (k^2c^2 + \omega_p^2\mu_\perp(n\omega_{B\alpha}))], \tag{2.4}$$

or $\omega = \omega_+$, or $\omega = \Omega_{1,2}$, where $\omega_p^2 = \sum_\alpha \omega_{p\alpha}^2$ and

$$\omega_\pm = \frac{1}{2(k^2c^2 + \omega_p^2)}\{\omega_p^2(\omega_0 - \tilde{\omega}) \pm [\omega_p^4(\omega^0 - \tilde{\omega})^2 + 4(k^2c^2 + \omega_p^2)(k^2c^2\omega_m^2 + \omega_p^2\omega_0\tilde{\omega})]^{1/2}\}, \tag{2.5}$$

$$\Omega_{1,2} = \frac{\omega_+ + n\omega_{B\alpha}}{2} \pm \frac{1}{2}\left[(\omega_+ - n\omega_{B\alpha})^2 + \frac{4\omega_{p\alpha}^2 a_3(n)\omega_+(\omega_0 - \omega_+)(\omega_+ + \tilde{\omega})}{(k^2c^2 + \omega_p^2)(\omega_+ - \omega_-)}\right]^{1/2}. \tag{2.6}$$

The frequencies $\omega^{(j)}(k)$ ($j \ge r$ or $j \le s$) are determined from (2.2) for $k^2c^2 \ll \omega_p^2$ and from (2.4) from $k\rho_{L\alpha} \ll 1$. These two expressions become converted into each other in the region of $k^2c^2 \ll \omega_p^2$ and $k\rho_{L\alpha} \ll 1$, where both (2.2) and (2.4) are applicable.

When $k\rho_{L\alpha}$ is not very small, $\omega^{(s+1)}(k)$ is determined from (2.3). If none of the frequencies $n\omega_{B\alpha}$ is contained within the interval (ω_m, ω_0), then $\omega^{(s+1)}(k)$ ($s + 1 = r - 1$) for small $k\rho_{L\alpha}$ is determined from Eq. (2.5) for ω_+. Equations (2.5) and (2.3) coincide for $k^2c^2 \gg \omega_p^2$ and $k\rho_{L\alpha} \ll 1$.

If the interval (ω_m, ω_0) contains one or more of the frequencies $n\omega_{B\alpha}$, then for very small $k\rho_{L\alpha}$ we find that $\omega^{(j)}(k)$ ($s + 1 \le j \le r - 1$) is determined from (2.2), for $k\rho_{L\alpha} \to 0$ the frequencies $\omega^{(j)}(k)$ ($s + 1 \le j \le r - 2$) approach $n\omega_{B\alpha}$ and are determined from (2.4) and $\omega^{(r-1)}(k)$ is determined from (2.5) for ω_+. For $k\rho_{L\alpha} \ll 1$ the frequencies $\omega^{(j)}(k)$ ($s + 1 \le j \le r - 1$) are determined from (2.5) for ω_+ if ω_+ is not too close to $n\omega_{B\alpha}$, and from (2.6) if $\omega_+ \approx n\omega_{B\alpha}$. With increasing distance from the point where $\omega_+ = n\omega_{B\alpha}$ the expressions (2.6) for $\Omega_{1,2}$ "are matched" to (2.4) and (2.2).

We have thus found that Eqs. (2.2)–(2.6) determine the frequencies $\omega^{(j)}(k)$ of ordinary waves in a low-pressure plasma for the entire range of the wave vector k.

We shall now determine the frequencies of ordinary cyclotron waves in a high-pressure plasma when $\langle v \rangle \gg v_{A\alpha}$. Since $\langle v \rangle \gg v_{A\alpha}$ we can neglect the left-hand side of (1.1) in zeroth approximation. We then find that the spin-wave frequency equals ω_0, while the frequency of the ordinary cyclotron waves is determined from the equation $\varepsilon_3 = 0$, i.e.,

$$\sum_\alpha \sum_{n=-\infty}^{\infty} \frac{\omega_{p\alpha}^2 a_3(n)}{\omega - n\omega_{B\alpha}} = 0. \tag{2.7}$$

For small values ($k\rho_{L\alpha} \ll 1$) of the wave vector Eq. (2.7) yields

$$\omega = n\omega_{B\alpha}[1 - \omega_{p\alpha}^2 a_3(n) / \omega_p^2] \quad (n = 1, 2, \ldots), \tag{2.8}$$

where a_3 is given by (1.8). Equation (2.8) is a special case of (2.4), which with $k \to 0$ can be applied to a plasma where $\langle v \rangle / v_{A\alpha}$ can assume any arbitrary value. Equation (2.8) was obtained in [2] for $n = 1$ in the case of a degenerate Fermi distribution.

When the coupling of cyclotron and spin waves is taken into account we find that the frequencies of these coupled spin and ordinary cyclotron waves in the case $\langle v \rangle \gg v_{A\alpha}$ are determined from

$$\omega = \omega_0[1 + k^2c^2(\omega_0^2 - \omega_m^2) / \omega_0^3\varepsilon_3(\omega_0)(\omega_0 + \tilde{\omega})], \tag{2.9}$$

$$\omega = \omega^{(0)}(k)\left[1 + k^2c^2/\omega^{(0)3}\frac{\partial\varepsilon}{\partial\omega^{(0)}}\mu_\perp(\omega^{(0)})\right], \tag{2.10}$$

where $\omega^{(0)}$ is the solution of (2.7).

In the region of very short waves subject to the condition $(k\rho_{L\alpha})^3 \gg \langle v \rangle^2 / v_{A\alpha}^2 \gg 1$ we easily obtain from (1.1) the result

$$\omega = n\omega_{B\alpha}[1 - \omega_{p\alpha}^2\omega_{B\alpha}\mu_\perp(n\omega_{B\alpha})\langle 1/v\rangle_\alpha / 2k^3c^2] \qquad (n = 1, 2, \ldots), \tag{2.11}$$

or

$$\omega = \omega_m\left[1 + \frac{(\omega_0 - \omega_m)(\omega_m + \tilde{\omega})}{4\omega_m k^3c^2}\sum_\alpha \sum_{n=-\infty}^{\infty}\left(\frac{\omega_{p\alpha}^2\omega_{B\alpha}}{\omega_m - n\omega_\alpha}\right)\left\langle\frac{1}{v}\right\rangle_\alpha\right]. \tag{2.12}$$

Equation (2.11) was obtained in [4] for a degenerate Fermi distribution with $\mu_\perp = 1$.

To conclude this section we shall consider the interaction of spin and ordinary cyclotron waves in antiferromagnets, where $\mu_\perp(\omega)$ differs from unity by the small quantity $\sim\chi_0$ in the entire frequency range except within narrow regions near the poles $\omega = \omega_{res}$ of the magnetic susceptibility tensor components μ_1, μ_1', and μ_2. By taking the magnetic susceptibility into account we obtain the correction $\sim\chi_0$ to the cyclotron-wave frequencies $\omega^{(j)}(k)$ in a nonmagnetic medium:

$$\omega = \omega^{(j)}(k) + [\mu_\perp(\omega) - 1] / \eta'(\omega)\,|_{\omega=\omega^{(j)}}, \tag{2.13}$$

where $\eta(\omega) = k^2c^2/\omega^2\varepsilon_3(\omega, k)$, $\eta' = d\eta(\omega)/d\omega$, $\omega^{(j)}(k)$ is the solution of (1.1) for $\mu_\perp = 1$, and $\eta(\omega^{(j)}) = 1$.

The spin-wave frequency is easily obtained from (1.1) when we consider that for $\omega \approx \omega_0 \approx \omega_m$ we have $\mu_\perp(\omega) \approx (\omega - \omega_0)/(\omega - \omega_m)$ to terms of the order $\sim\chi_0$:

$$\omega = \omega_0\{1 + \eta(\omega_0)(\omega_0 - \omega_m) / [1 - \eta(\omega_0)]\omega_0\}. \tag{2.14}$$

The correction to ω_0 in (2.14) is of the order $(\omega_0 - \omega_m)/\omega_0 \sim \chi_0$ if ω_0 is not near $\omega^{(j)}(k)$.

The interaction of cyclotron and spin waves becomes significant when $\omega_0 \approx \omega^{(j)}(k)$. Equations (2.13) and (2.14) may then be inapplicable, and the frequencies of spin and cyclotron waves are obtained using

$$\omega = {}^1\!/_2(-\omega_0 + \omega^{(j)}) \pm {}^1\!/_2[(\omega_0 - \omega^{(j)})^2 - 4(\omega_0 - \omega_m) / \eta'(\omega_0)]^{1/2}. \tag{2.15}$$

At the intersection point $\omega_0 = \omega^{(j)}$ of noninteracting waves the frequency correction is of the order $\sqrt{(\omega_0 - \omega_m)\omega_0} \sim \omega_0\chi_0^{1/2}$ As the difference $\omega_0 - \omega^{(j)}$ increases (2.15) is transformed into (2.13) and (2.14) when in the latter equations we assume $\omega_0 \approx \omega^{(j)}$ and neglect terms of the order $(\omega_0 - \omega^{(j)})/\omega_0$ compared with unity.

3. EXTRAORDINARY WAVES IN A ONE-COMPONENT PLASMA

We shall now investigate the dispersion equation (1.2), and shall consider first a nonmagnetic medium ($\mu_3 = 1$) and a plasma consisting of electrons as the only kind of charge carrier. To determine how the solutions of (1.2) behave qualitatively we investigate this equation graphically in the case when the plasma frequency ω_p considerably exceeds the cyclotron frequency ω_B of the electrons. We set $f_1(\omega) = k^2c^2/\omega^2$; and $f_2 = \varepsilon_\perp(\omega, k)$, and becomes infinite for $\omega = n\omega_B$ and ω_n', where ω_n' represents the zeros of $\varepsilon_1(\omega)$, i.e., the solutions of

$$f(\omega) = \sum_{n=1}^{\infty}\omega_p^2 a_1(n)/(\omega^2 - n^2\omega_B^2) = 0. \tag{3.1}$$

Considering that $f(0) < 0$ and $f(\omega) \to \mp\infty$ for $\omega \to n\omega_B \mp 0$ and $\partial f(\omega)/\partial\omega < 0$, i.e., $f(\omega)$ is a decreasing function, it is easily proved that (3.1) has the solutions $\omega = \omega_1', \omega_2', \ldots$, where $n\omega_B < \omega_n' < (n+1)\omega_B$.

The function $f_1 = k^2c^2/\omega^2$ decreases monotonically as ω increases; $f_1(\omega) \to \infty$ for $\omega \to 0$ and $f_2(\omega) \to \pm 0$ for $\omega \to n\omega_B \mp 0$ and $\omega \to \omega_n' \mp 0$.

The interval $\{m\omega_B, (m+1)\omega_B\}$ (m = 1, 2, ...) contains two intersection points of the $f_1(\omega)$ and $f_2(\omega)$ curves, which correspond to the solutions $\omega_{m+1}^{(1)}(k)$ and $\omega_{m+1}^{(2)}(k)$, m = 1, 2, ...; $\omega_{m+1}^{(1)} > \omega_{m+1}^{(2)}$; the interval $(0, \omega_B)$ contains only the single solution $\omega_1^{(1)}(k)$.

As k increases in the small-k region the frequencies $\omega_n^{(1)}(k)$ (n = 1, 2, ...) decrease monotonically from $n\omega_B$ at k = 0; after a minimum is reached $n\omega_B$ is again approached. The frequencies $\omega_n^{(2)}(k)$ (n = 2, 3, ...) decrease monotonically as k increases, from $\omega = n\omega_B$ at k = 0 to $(n-1)\omega_B$ as $k \to \infty$. The behavior of the frequencies $\omega_m^{(1,2)}(k)$ as a function of the wave vector in the present case of arbitrary isotropic electron velocity distributions is qualitatively the same as in the case of a Maxwellian velocity distribution (see Fig. 5 in [1]). We derive explicit expressions for the frequencies $\omega_n^{(1)}(k)$ in a low-pressure plasma. The expressions for $\omega_n^{(1)}(k)$ (n = 1, 2, ...) are derived from (1.2), retaining only resonance terms $\sim 1/(\omega - n\omega_B)$ in ε_1, ε_1', and ε_2 for $k\rho_L \gg \kappa \equiv \langle v^2\rangle/v_{A\alpha}^2$:

$$\frac{\omega_n^{(1)} - n\omega_B}{n\omega_B} = -\frac{\omega_p^2}{k^2c^2}\frac{a_1a_1' - a_2^2}{a_1}. \tag{3.2}$$

In the short-wave region ($k\rho_L \gg 1$) we shall have $a_1, a_2 \ll a_1' \approx a_3$, and (3.2) coincides with (2.2) and (2.13) for the frequency of the ordinary cyclotron wave.

Equation (3.2) cannot be used for $\omega_n^{(1)}$ (n = 2, 3, ...) when we have $k\rho_L \ll 1$ in the region $k\rho_L \lesssim \kappa$. In this case we must retain terms $\sim 1/(\omega + \omega_B)$ in ε_1, ε_1', and ε_2 in addition to the resonance terms. We thus obtain

$$\frac{\omega_n^{(1)} - n\omega_B}{n\omega_B} = -\frac{\omega_p^2(a_1a_1' - a_2^2)(n+1)}{[k^2c^2(n+1) + 2n\omega_p^2]a_1}$$

$(n = 2, 3, \ldots)$, where

$$\frac{a_1a_1' - a_2^2}{a_1} \approx \left(\frac{k}{\omega_B}\right)^{2n+2}\frac{2n!!}{(2^n n!)^2(2n+1)!!}\left[\frac{n+2}{(n+1)(2n+3)}\langle v^{2n+2}\rangle - \frac{1}{2n+1}\frac{\langle v^{2n}\rangle^2}{\langle v^{2n-2}\rangle}\right] \tag{3.3}$$

Equations (3.2) and (3.3) determine the ordinary cyclotron-wave frequencies $\omega_n^{(1)}(k)$ (n = 1, 2, ...) in a low-pressure plasma.

We obtain the frequencies $\omega_n^{(2)}(k)$ (n = 2, 3, ...) for $k\rho_L \ll 1$ when $a_1 \approx a_1' \approx a_2$ and we have retained in ε_1, ε_1', and ε_2 the resonance terms and terms $\sim 1/(\omega + \omega_B)$:

$$\frac{\omega_n^{(2)} - n\omega_B}{n\omega_B} = -\frac{a_1(n-1)[(n+1)k^2c^2 + 2n\omega_p^2]}{n^2(k^2c^2 + \omega_p^2)} \quad (n = 2, 3, \ldots), \tag{3.4}$$

where $a_1 \sim (k\rho_L)^{2n-2}$. It is obvious that for identical values of $k\rho_L$ the frequencies represented by (3.3) are considerably closer to $n\omega_B$ than the frequencies represented by (3.4). The difference $\omega_n^{(2)} - n\omega_B$ is enhanced as the wave vector is increased. In the region $k^2c^2 \gg \omega_p^2$ (but $k\rho_L \ll 1$), Eq. (3.4) has the simpler form

$$(\omega_n^{(2)} - n\omega_B)/n\omega_B = -a_1(n^2 - 1)/n^2. \tag{3.5}$$

This last equation represents longitudinal (electrostatic) waves with the frequency $\omega_n^{(2)}$; it is the solution of the dispersion equation $\varepsilon_1(\omega, k) = 0$ for longitudinal waves. Equation (3.5) can no longer be used when $k\rho_L$

~ 1; to determine $\omega_n^{(2)}(k) = \omega_{n-1}'$ we must then solve (3.1) numerically. For $k\rho_L \gg 1$ the equation $\varepsilon_1 = 0$ has the solution

$$\omega_n^{(2)} = (n-1)\omega_B[1 + 2\pi f_0(0)\omega_B(\omega_p^2/k^3c^3)] \quad (n = 2, 3, \ldots), \quad (3.6)$$

and approaches $(n-1)\omega_B$ for $k \to \infty$.

Equations (3.2)–(3.4) were derived in [1] for plasmas having a Maxwellian electron velocity distribution. In the cases of high-pressure plasmas ($\kappa \gtrsim 1$), Equations (3.2) and (3.4) can also be used when $k\rho_L \to 0$ and (2.13) and (3.6) can be used when $k\rho_L \to \infty$.

4. EXTRAORDINARY WAVES IN A TWO-COMPONENT PLASMA

We now proceed to investigate the dispersion equation (1.2) in the case of a plasma that consists of two kinds of carriers having charges e_1 and e_2 (with $e_1e_2 < 0$), masses m_1 and m_2 (with $m_1 > m_2$), and the equilibrium densities n_1 and n_2. Neglecting spatial dispersion of the dielectric constant, we obtain

$$\begin{aligned} \varepsilon_1 = \varepsilon_1' &= -\omega_{p1}^2/(\omega^2 - \omega_{B1}^2) - \omega_{p2}^2/(\omega^2 - \omega_{B2}^2), \\ \varepsilon_2 &= -\eta_1\omega_{p1}^2\omega_{B1}/\omega(\omega^2 - \omega_{B1}^2) - \eta_2\omega_{p2}^2\omega_{B2}/\omega(\omega^2 - \omega_{B2}^2) \end{aligned} \quad (4.1)$$

(where we shall assume $|\varepsilon_{1,2}| \gg 1$ and shall neglect the displacement current). The solution of the dispersion equation $(kc/\omega)^2 = \varepsilon_\perp(\omega)$ now becomes

$$\omega(k) = \left[\frac{(\omega_{p1}^2\omega_{B2} - \omega_{p2}^2\omega_{B1})^2 + k^2c^2(\omega_{p1}^2\omega_{B2}^2 + \omega_{p2}^2\omega_{B1}^2)}{(\omega_{p1}^2 + \omega_{p2}^2)(\omega_{p1}^2 + \omega_{p2}^2 + k^2c^2)}\right]^{1/2}. \quad (4.2)$$

With increase of the wave vector the extraordinary-wave frequency grows monotonically and approaches the ("hybrid") plasma resonance frequency

$$\omega(\infty) = [(\omega_{p1}^2\omega_{B2}^2 + \omega_{p2}^2\omega_{B1}^2)/(\omega_{p1}^2 + \omega_{p2}^2)]^{1/2}. \quad (4.3)$$

With decrease of the wave vector in a noncompensated plasma, $\omega(k)$ approaches $\omega(0)$, which is defined by

$$\omega(0) = |\omega_{p1}^2\omega_{B2} - \omega_{p2}^2\omega_{B1}|/(\omega_{p1}^2 + \omega_{p2}^2). \quad (4.4)$$

We note that $\omega_{B1} < \omega(\infty) < \omega_{B2}$, and that $\omega(0)$ can be either smaller or greater than the cyclotron frequency ω_{B1}.

For small k in a compensated plasma the ordinary wave becomes a magnetosonic wave:

$$\omega = kv_A, \quad (4.5)$$

where

$$v_A = c\frac{(\omega_{p1}^2\omega_{B2}^2 + \omega_{p2}^2\omega_{B1}^2)^{1/2}}{\omega_{p1}^2 + \omega_{p2}^2} = \left[\frac{|e_1|B_0^2}{4\pi n_2(|e_1|m_2 + |e_2|m_1)}\right]^{1/2}. \quad (4.6)$$

Besides the extraordinary wave with the frequency (4.2), for small k the dispersion equation (1.2) possesses two solutions that approach $n\omega_{B\alpha}$ ($n = 2, 3, \ldots$) for $k \to 0$ and one solution that approaches $\omega_{B\alpha}$ for $k \to 0$.

If $k\rho_{L\alpha}$ is not very small and only the resonance terms are retained in (1.4) we obtain

$$(\omega - n\omega_{B\alpha})/\omega = -\omega_{p\alpha}^2(a_1a_1' - a_2^2)/a_1k^2c^2 < 0 \ (n = 1, 2, \ldots). \quad (4.7)$$

With $n = 1$ this equation is also valid for $k \to 0$.

With increase of the wave vector, the frequency given by (4.7) for small $k\rho_{L\alpha}$ decreases from $n\omega_{B\alpha}$ to a minimum for $k\rho_{L\alpha} \gtrsim 1$; this represents a maximum deviation of

$$|(\omega - n\omega_{B\alpha})/\omega| \sim \varkappa_\alpha \ll 1,$$

which is followed by a reapproach to $n\omega_{B\alpha}$. When $k\rho_{L\alpha} \gg 1$ we have a simplified form of (4.7):

$$(\omega - n\omega_{B\alpha})/\omega \approx -\omega_{p\alpha}^2a_1'/(kc)^2, \quad (4.7')$$

where a_1' is given by (1.9). Since $a_1' = a_3$, for short waves the extraordinary cyclotron wave obeys the same dispersion law as the ordinary cyclotron wave with the frequency (2.2).

In addition to (4.7), Eq. (1.2) possesses solutions that correspond (for not very small $k\rho_{L\alpha}$) to longitudinal cyclotron waves. The dispersion equation for these branches is

$$\varepsilon_1(k, \omega) = -\sum_\alpha \sum_{n=-\infty}^{\infty} \omega_{p\alpha}^2a_1/\omega(\omega - n\omega_{B\alpha}) = 0. \quad (4.8)$$

Considering that

$$\varepsilon_1(0, k) = \sum_\alpha \sum_{n=-\infty}^{\infty} 2\omega_{p\alpha}^2a_1/n^2\omega_{B\alpha}^2 > 0,$$

$\partial\varepsilon_1/\partial\omega > 0$, and $\varepsilon_1(\omega, k) \to \mp\infty$ for $\omega \to n\omega_{B\alpha} \pm 0$, we find that a zero of $\varepsilon_1(\omega, k)$ corresponding to a longitudinal cyclotron wave is found between two neighboring poles. As the wave vector increases, the frequency of the longitudinal cyclotron wave approaches $n\omega_{B\alpha}$:

$$(\omega - n\omega_{B\alpha})/\omega = \omega_{p\alpha}^2a_1/n^2\omega_{B\alpha}^2 \quad (n = 1, 2, \ldots) \quad (4.9)$$

[where a_1 is given by (1.9)].

For small $k\rho_{L\alpha}$ the longitudinal-wave frequency obtained from (4.8) is

$$(\omega - n\omega_{B\alpha})/\omega = \omega_{p\alpha}^2a_1/n^2\omega_{B\alpha}^2\varepsilon_1 \quad (n = 2, 3, \ldots), \quad (4.10)$$

where ε_1 is given by (4.1).

Thus, when $k\rho_{L\alpha}$ is not very small, between two neighboring poles $\omega = n\omega_{B\alpha}$ and $l\omega_{B\beta}$ of $\varepsilon_1(\omega, k)$ we find two solutions of (1.2), corresponding to an extraordinary cyclotron wave and a longitudinal cyclotron wave, respectively.

The spectral pattern is greatly complicated for small values of $k\rho_{L\alpha}$, in which case we must take into account the interaction of the extraordinary wave having the frequency $\omega(k)$ given by (4.2), with the extraordinary and longitudinal cyclotron waves. If $\omega \approx \omega_{B\alpha}$ and ω is not near $\omega(k)$, then (4.7) can be used, as previously, for the frequency of the extraordinary cyclotron wave. If $\omega \approx n\omega_{B\alpha}$ where $n = 2, 3, \ldots$, then to determine the corrections to the frequencies $n\omega_{B\alpha}$, we must take into account a term $\sim 1/(\omega - \omega_{B\alpha})$ besides the resonance terms $1/(\omega - n\omega_{B\alpha})$ in the tensor ε_{ij}. In this last case we have

$$\begin{aligned} \varepsilon_1(\omega, k) &= \varepsilon_1 - \omega_{p\alpha}^2a_1/\omega(\omega - n\omega_{B\alpha}), \\ \varepsilon_1'(\omega, k) &= \varepsilon_1 - \omega_{p\alpha}^2a_1'/\omega(\omega - n\omega_{B\alpha}), \\ \varepsilon_2(\omega, k) &= \varepsilon_2 - \eta_\alpha\omega_{p\alpha}^2a_2/\omega(\omega - n\omega_{B\alpha}). \end{aligned} \quad (4.11)$$

Inserting (4.11) into (1.2), we obtain

$$\frac{\omega - n\omega_{B\alpha}}{\omega} = -\frac{\omega_{p\alpha}^2}{k^2c^2 - 2\omega^2(\varepsilon_1 - \eta_\alpha\varepsilon_2)}\frac{a_1a_1' - a_2^2}{a_1} \quad (n = 2, 3, \ldots), \quad (4.12)$$

or

$$\frac{\omega - n\omega_{B\alpha}}{\omega} = -\frac{\omega_{p\alpha}^2a_1(2\varepsilon_1 - 2\eta_\alpha\varepsilon_2 - k^2c^2/\omega^2)}{k^2c^2\varepsilon_1 - \omega^2(\varepsilon_1^2 - \varepsilon_2^2)}. \quad (4.13)$$

This last equation is inapplicable to the case of $k \to k_n$, where

$$k_n^2 = (\omega/c)^2(\varepsilon_1 - \varepsilon_2^2/\varepsilon_1)|_{\omega = s\omega_{B1}}.$$

When $k = k_n$, the cyclotron and extraordinary branches obviously "intersect" at $\omega(k) = s\omega_{B1}$, so that their interaction must be taken into account at $k = k_n$.

For $k \approx k_n$, we find, in place of (4.13) and (4.2), that $\omega = \omega_\pm$, where

$$\frac{\omega_\pm - n\omega_{B\alpha}}{\omega} = \frac{1}{2}\left[\frac{\omega(k)}{n\omega_{B\alpha}} - 1\right] \pm \frac{1}{2}\left\{\left[\frac{\omega(k)}{n\omega_{B\alpha}} - 1\right]^2 + 4\sigma\right\}^{1/2}; \quad (4.14)$$

$$\sigma = \left.\frac{\omega_{p\alpha}{}^2 a_1 [k^2c^2/\omega^2 - 2(\varepsilon_1 - \eta_\alpha\varepsilon_2)]}{\omega \dfrac{\partial}{\partial\omega}[k^2c^2\varepsilon_1 - \omega^2(\varepsilon_1{}^2 - \varepsilon_2{}^2)]}\right|_{\omega = n\omega_{B\alpha}} \quad \text{for } n = 2, 3\ldots, \quad (4.15a)$$

$$\sigma = \left.\frac{2\omega_{p1}{}^2(a_1a_1' - a_2{}^2)}{\omega \dfrac{d}{d\omega}[\omega^2(\varepsilon_1{}^2 - \varepsilon_2{}^2)/\varepsilon_1]}\right|_{\omega = \omega_{B_1}} \quad \text{for } n = 1; \quad (4.15b)$$

$$2(a_1a_1' - a_2{}^2) \approx \frac{k^4}{\omega_{B1}{}^4}\left(\frac{1}{20}\langle v^4\rangle - \frac{1}{18}\langle v^2\rangle^2\right). \quad (4.16)$$

Equations (4.12) and (4.13) cannot be used for $k \to q_n$, where

$$q_n{}^2 = 2(\omega/c)^2(\varepsilon_1 - \eta_\alpha\varepsilon_2)\,|_{\omega = n\omega_{B\alpha}}.$$

In this case we find, in place of (4.12) and (4.13),

$$\omega \approx \omega_\pm,$$

where

$$\tilde{\omega}_\pm = n\omega_{B\alpha} + \frac{\omega_{p\alpha}{}^2 a_1}{2\omega}\,\frac{k^2c^2/\omega^2 - 2\varepsilon_1 + 2\eta_\alpha\varepsilon_2}{(\varepsilon_1 - \eta_\alpha\varepsilon_2)^2}$$

$$\pm\left[\frac{\omega_p{}^{\alpha 4}a_1{}^2(k^2c^2/\omega^2 - 2\varepsilon_1 + 2\eta_\alpha\varepsilon_2)^2}{4\omega^2(\varepsilon_1 - \eta_\alpha\varepsilon_2)^2} + \frac{\omega_{p\alpha}{}^2(a_1a_1' - a_2{}^2)}{\varepsilon_1 - \eta_\alpha\varepsilon_2}\right]^{1/2}. \quad (4.17)$$

Figure 2 shows the behavior scheme of the eigenfrequencies as functions of the wave vector when $\omega(0) > s\omega_{B1}$; this last case is possible only in a charged plasma where $n_1 - n_2 > s[n_2 + (m_2/m_1)n_1]$.

We shall denote the increasing sequence of the frequencies $n\omega_{B\alpha}$ by $\omega_1, \omega_2, \ldots$ and shall assume that s resonance frequencies $\omega_{B1}, 2\omega_{B1}, \ldots, s\omega_{B1}$ are located between zero and $\omega(0)$; that the frequencies $(s+1)\omega_{B1}, \ldots, l\omega_{B1}$ are located between $\omega(0)$ and $\omega(\infty)$; and that the frequencies $(l+1)\omega_{B1}, \ldots, r\omega_{B1}$ are located between $\omega(\infty)$ and $\omega_{B2} = \omega_{r+1}$. The two eigenfrequencies between ω_s and ω_{s-1} will be denoted by $\omega_s^{(1)}(k)$ and $\omega_s^{(2)}(k)$ (with $\omega_s > \omega_s^{(1)} > \omega_s^{(2)}$). The equations determining $\omega_j^{(1,2)}(k)$ in the case of $\omega(0) > \omega_{B1}$ are listed in the accompanying table.

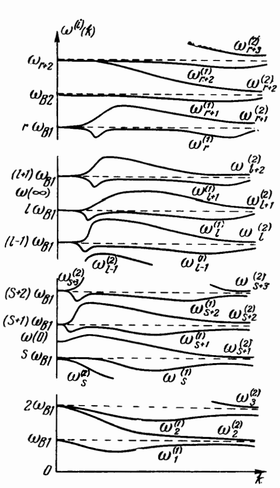

FIG. 2

Eigenfrequency	Region of wave-vector variation	Equation
$\omega_1^{(1)}(k)$, $\omega_{r+1}^{(1)}(k)$	$0 < k < \infty$	(4.7)
$\omega_m^{(1)}(k)$ $m = 2, \ldots, s$	$0 < k < \infty$	(4.12)
$\omega_m^{(2)}(k)$ $m \geqslant r + 2$	$k\rho_L \ll 1$	(4,13)
$\omega_{s+1}^{(2)}(k)$	$k < q_{s+1}$	(4.2)
	$k \approx q_{s+1}$	(4.14) for ω_-, $n = s + 1$
	$k > q_{s+1}$, $k\rho_L \ll 1$	(4.13)
$\omega_m^{(2)}(k)$, $m = s + 2, \ldots, l$	$k < q_{m-1}$	(4.12)
	$k \approx q_{m-1}$	(4.17) for $\tilde{\omega}_+$, $n = m - 1$
	$q_{m-1} < k < k_{m-1}$	(4.13), $n = m - 1$
	$k \approx k_{m-1}$	(4.14) for ω_+, $n = m - 1$
	$k_{m-1} < k < k_m$	(4.2)
	$k \approx k_m$	(4.14) for ω_-, $n = m$
	$k > k_m$, $k\rho_L \ll 1$	(4.13), $n = m$
$\omega_{l+1}^{(2)}(k)$	$k < q_l$	(4.12)
	$k \approx q_l$	(4.17) for $\tilde{\omega}_+$, $n = l$
	$q_l < k < k_l$	(4.13), $n = l$
	$k \approx k_l$	(4.14) for ω_+, $n = l$
	$k > k$	(4.2)
$\omega_m^{(1)}(k)$, $m = l + 1, \ldots, r$	$k < q_m$	(4.13), $n = m$
	$k \approx q_m$	(4.17) for $\tilde{\omega}_-$, $n = m$
	$k > q_m$	(4.12), $n = m$
$\omega_m^{(2)}(k)$, $m = l + 2, \ldots, r + 1$	$k < q_{m-1}$	(4.12), $n = m - 1$
	$k \approx q_{m-1}$	(4.17) for $\tilde{\omega}_+$, $n = m - 1$
	$k > q_{m-1}$, $k\rho_L \ll 1$	(4.13), $n = m - 1$

If $\omega(0)$ is smaller than ω_{B1} [which occurs for $n_1(1 - m_2/m_1) < 2n_2$] then $\omega_1^{(1)}(k)$ (in Fig. 3) is obtained from (4.2) for $k < k_1$, from (4.14) and (4.15b) for $k \approx k_1$, and from (4.12) for $k > k_1$.

We obtain $\omega_2^{(2)}(k)$ from (4.12) for $\omega(\infty) > 2\omega_{B1}$ and $\omega(0) < \omega_{B1}$ and when $k < k_1$; from (4.14) and (4.15b) for ω_+ when $k \approx k_1$; from (4.2) when $k_1 < k < k_2$; from (4.14) for ω_- with $n = 2$ when $k \approx k_2$; and from (4.13) with $n = 2$ when $k > k_2$ and $k\rho_L \ll 1$. This wave becomes longitudinal when $k^2c^2 \gg \omega^2|\varepsilon_1 \pm \eta\varepsilon_2|$.

When $\omega_{B1} < \omega(0) < \omega(\infty) < 2\omega_{B1}$, we obtain $\omega_2^{(2)}(k)$ from (4.12) for $k < k_1$, from (4.14) and (4.15b) for ω_+ with $k \approx k_1$, from (4.2) for $k > k_1$ and $k\rho_L \ll 1$. This wave becomes longitudinal when $k^2c^2 \gg \omega^2|\varepsilon_1 \pm \eta\varepsilon_2|$.

The behavior of $\omega_m^{(1)}(k)$ $(m \geq 2)$ and $\omega_m^{(2)}(k)$ $(m \geq 3)$ when $\omega(0) < \omega_{B1}$ is the same as when $\omega(0) > \omega_{B1}$ and is derived from the same equations.

We shall now consider the interaction of spin waves with the extraordinary wave and cyclotron waves in an antiferromagnetic semiconductor or metal. Since the dispersion equation (1.2) becomes (1.1) following the substitutions $\varepsilon_\perp \to \varepsilon_3$ and $\mu_3 \to \mu_\perp$, and $\mu_3(\omega)$ has the same form as (1.12), the frequencies of the interacting extraordinary wave and cyclotron waves will be obtained from (2.13)–(2.17), in which we must insert $\eta(\omega) = (k^2c^2/\omega^2)\varepsilon_\perp(\omega, k)$ and replace $\mu_\perp(\omega)$ with $\mu_3(\omega)$.

The behavior exhibited by the frequencies of the interacting waves near the intersection of the noninteracting branches is shown qualitatively in Fig. 4 for a plasma consisting of two kinds of carriers, when

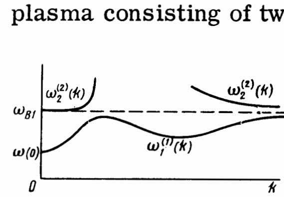

FIG. 3

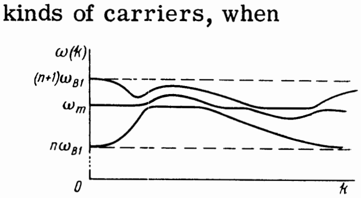

FIG. 4

$$(n-1)\omega_{B1} < \omega_m \lesssim \min \omega_n^{(1)}(k) < \max \omega_n^{(2)}(k) < n\omega_{B1} < \omega(\infty).$$

5. CONCLUSION

We shall now discuss briefly the feasibility of investigating cyclotron waves experimentally in alkali metals, where electrons obey a quadratic isotropic dispersion law. Since in these metals $m^* \sim 10^{-27}$ g, $n_0 \sim 10^{22}/cm^3$, and $v_F \sim 10^8$ cm/sec, we find that we here have a high-pressure plasma ($v_F > v_A$) in magnetic fields $H_0 < 10^6$ gauss. In this case when $k\rho_L < 1$ the ordinary cyclotron wave frequencies are given by (2.8):

$$(\omega - n\omega_B) / \omega_B = -\alpha_n (k\rho_L)^{2n} \quad (n = 1, 2, 3, \ldots), \quad (5.1)$$

but in the case of the extraordinary cyclotron wave the frequency is given by Eq. (3.2) for the fundamental resonance:

$$(\omega - \omega_B) / \omega_B = -\beta_1 v_F^2 / v_A^2 (k\rho_L)^2 \quad (n = 1) \quad (5.2)$$

and by (3.3) for integral multiple harmonics:

$$(\omega - n\omega_B) / \omega_B = -\beta_n (k\rho_L)^{2n+2} \quad (n = 2, 3, \ldots); \quad (5.3)$$

the frequencies of the plasma cyclotron wave are given by (3.4):

$$(\omega - n\omega_B) / \omega_B = -\gamma_n (k\rho_L)^{2n-2} \quad (n = 2, 3, \ldots). \quad (5.4)$$

The coefficients α_n, β_n, and γ_n here decrease very rapidly like $(n!)^{-1}$ as n increases.

The experiments reported in [2] were performed on a pure sodium slab of thickness $L = 10^{-3} - 10^{-2}$ cm, in a field $H_0 \sim 10^3 - 10^4$ gauss at low temperature $T = 1.3°K$ with $\nu/\omega \sim 0.1$ and $\omega \sim 10^{10}$/sec. The resonance at $\omega \approx \omega_B$ was observed when the electric field **E** of the wave was parallel to the magnetic field, while the resonance at $\omega \approx 2\omega_B$ was observed with **E** perpendicular to the magnetic field; in the latter case resonance at ω_B was not observed. Standing waves were excited in the sodium plate, with a wave vector $k = 2\pi(l - 1/4)/L$ ($l = 1, 2, \ldots$).

The experimental dependence of the ordinary cyclotron wave frequency $\omega \approx \omega_B$ on the wave vector is well represented by (5.1).[2] It follows from (5.2) that the resonance of the extraordinary wave at ω_B could not be observed in the experiments of Walsh and Platzman, where $(v_F/v_A)^2 \sim 10^4$ and $\rho_L \sim 10^{-3}$ cm while the minimum wave vector was $k_{min} \sim 2\pi/L \sim 600$ cm. At ω_B waves subject to these conditions could be observed only in thick slabs ($L \gtrsim 1$ cm).

The resonance at $2\omega_B$ that was observed by Walsh and Platzman must be attributed to the excitement of a plasma cyclotron wave having the frequency given by (5.4). [We note that the corrections $\sim (k\rho_L)^2$ to the frequency of this wave are of the same order as for an ordinary wave with $\omega \approx \omega_B$.]

The higher harmonics ($n \geq 2$ for the ordinary and extraordinary cyclotron waves and $n \geq 3$ for the plasma wave) have frequencies very close to $n\omega_B$. The observation of these harmonics under the conditions of [2] was evidently impeded because the condition $|\omega - n\omega_B| > \nu$ was violated.

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