RELAXATION ABSORPTION OF ELECTROMAGNETIC WAVES BY A SUPERCONDUCTOR

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Submitted April 13, 1967

Zh. Eksp. Teor. Fiz. 53, 1362–1370 (October, 1967)

The high frequency surface impedance of a pure superconductor is calculated for a stationary magnetic field near the transition point. It is shown that for low frequencies, $\omega_0 \ll \Delta^2/T_c$, an additional absorption $\sim \omega_0 \ln^2 \omega_0$ exists which is related to relaxation of the gap Δ.

LANDAU and Khalatnikov[1] have shown that, near the point of a second order phase transition, the relaxation of the ordering parameter characterizing the transition is inhibited. The relaxation time of this parameter increases according to the law

$$\tau \sim 1/(T_c - T),$$

where T_c is the transition temperature. In particular, this leads to an anomalous absorption of the sound near the λ point in liquid helium. It is of interest to determine whether or not there are similar phenomena in superconductors, where the role of the ordering parameter is played by the gap Δ.

The purpose of this research is the investigation of dissipative processes in superconductors upon relaxation of the gap Δ, for example, by the absorption of electromagnetic waves near the critical temperature. We shall see that the relaxation processes in superconductors possess a number of distinctive features and can be considered exactly on the basis of the microscopic theory. In other well known cases, the theory has made it possible only to estimate the relaxation time in order of magnitude.

1. DISSIPATIVE PROCESSES NEAR THE TRANSITION POINT

A linear equation was obtained by the author and Pitaevskiĭ[2] for the gap Δ which has the form (Δ_1 is a small deviation of the gap from the equilibrium value)

$$\begin{aligned}\Delta_1^*(\omega_0, \mathbf{k}) &= i\frac{g}{(2\pi)^4}\Delta_1^*(\omega_0, \mathbf{k})\Big\{\int d\omega d^3p \\ &\times \frac{(-\omega_+ + \xi_+)(\omega_- + \xi_-)}{(\omega_+^2 - \xi_+^2 - \Delta^2)(\omega_-^2 - \xi_-^2 - \Delta^2)} + 2\pi i \int d^3p \\ &\times \Big[\frac{2n_- u_-^2 u_+^2}{-\omega_0 - \varepsilon_+ - \varepsilon_- - i\delta} - \frac{2n_+ v_+^2 v_-^2}{-\omega_0 + \varepsilon_+ + \varepsilon_- - i\delta} \\ &+ \frac{2(n_- - n_+) u_-^2 v_+^2}{-\omega_0 + \varepsilon_+ - \varepsilon_- - i\delta}\Big]\Big\} + i\frac{g}{(2\pi)^4}\Delta_1(\omega_0, \mathbf{k})\Big\{\int d\omega d^3p \\ &\times \frac{-\Delta_0^{*2}}{(\omega_+^2 - \xi_+^2 - \Delta^2)(\omega_-^2 - \xi_-^2 - \Delta^2)} - i\pi\Delta^{*2}\int d^3p \frac{1}{\varepsilon_+\varepsilon_-} \\ &\times \Big[-\frac{n_-}{\omega_0 + \varepsilon_+ + \varepsilon_- + i\delta} - \frac{n_+}{-\omega_0 + \varepsilon_+ + \varepsilon_- - i\delta} \\ &+ \frac{n_+ - n_-}{-\omega_0 + \varepsilon_+ - \varepsilon_- - i\delta}\Big]\Big\}. \end{aligned} \tag{1}$$

Here

$$\varepsilon_\pm = \sqrt{\xi_\pm^2 + \Delta^2}, \quad \xi_\pm = \frac{(\mathbf{p} \pm \mathbf{k}/2)^2}{2m} - \mu, \quad \omega_\pm = \omega \pm \frac{\omega_0}{2}.$$

This equation contains terms with singular denominators $(-\omega_0 + \varepsilon_+ - \varepsilon_- - i\delta)$ which can vanish for all values of ω_0, including small ω_0. In particular, for $\omega_0 < 2\Delta$, the energy is absorbed by means of damping similar to Landau damping. The imaginary terms which arise from these denominators describe the relaxation of Δ that is of interest to us.

We shall investigate in detail Eq. (1) near the transition point, i.e., when $\Delta(T) \ll \Delta_0$ (Δ_0 is the value of the gap for $T = 0$). Here we shall assume that $\omega_0 \ll \Delta(T)$. No assumptions will be made on the relation of $\mathbf{k}\cdot\mathbf{v}$ and $\Delta(T)$. We consider (1) for small ω_0. For $\omega_0 = 0$, the equation

$$\begin{aligned}&-\frac{k^2}{4m}\Delta_1^* + \frac{1}{\eta}\Big[\frac{T_c - T}{T_c} - \frac{7\zeta(3)}{4\pi^2 T_c^2}|\Delta|^2\Big]\Delta_1^* \\ &-\frac{1}{\eta}\frac{7\zeta(3)}{8\pi^2 T_c^2}\Delta_0^{*2}\Delta_1 = 0, \quad \eta = \frac{7\zeta(3)}{6\pi^2 T_c^2}\varepsilon_F\end{aligned} \tag{2}$$

follows from Eq. (1). Equation (2) coincides with the linearized Ginzburg–Landau equation, as it should.

We shall now be interested only in the first nonvanishing correction in ω_0. We see that this correction will be of the order of $\omega_0 \ln(\omega_0/\Delta)$. Such corrections arise also from residues in the zeroes of the difference denominators in Eq. (1). We rewrite Eq. (1) in the form

$$L\Delta_1^* + i\omega_0(M\Delta_1^* + N\Delta_1) = 0. \tag{3}$$

Here $L\Delta_1^*$ is the left-hand side of Eq. (2) and M and N are determined in the following way:

$$\begin{aligned}\omega_0 M &= \frac{2\pi^2}{gmp_0}\frac{\pi}{2\eta}\frac{g}{(2\pi)^3}\int d^3p(n_+ - n_-)\Big(1 - \frac{\xi_+\xi_-}{\varepsilon_+\varepsilon_-}\Big) \\ &\times \delta(-\omega_0 + \varepsilon_+ - \varepsilon_-), \\ \omega_0 N &= \frac{2\pi^2}{gmp_0}\frac{\pi}{2\eta}\frac{g}{(2\pi)^3}\int d^3p(n_+ - n_-)\frac{\Delta_0^{*2}}{\varepsilon_+\varepsilon_-}\delta(-\omega_0 + \varepsilon_+ - \varepsilon_-).\end{aligned}$$

In connection with what was said above, we have left in the equation only terms stemming from the residues. We now transform from integration over $\mathbf{p}$ to integration over ξ, making here the change of variables $x = (\mathbf{k}\cdot\mathbf{p})/p$. For M and N we obtain

$$\omega_0 M = \frac{\pi}{4}\frac{1}{\eta k}\int_{-k}^{k} dx \int d\xi(n_+ - n_-)\Big(1 - \frac{\xi_+\xi_-}{\varepsilon_+\varepsilon_-}\Big)\delta(-\omega_0 + \varepsilon_+ - \varepsilon_-),$$

$$\omega_0 N = \frac{\pi}{4}\frac{1}{k\eta}\int_{-k}^{k} dx \int d\xi(n_+ - n_-)\frac{\Delta_0^{*2}}{\varepsilon_+\varepsilon_-}\delta(-\omega_0 + \varepsilon_+ - \varepsilon_-).$$

We make the following change of variable in the integration over ξ:

$$\xi' = \xi - \frac{kv_F}{2} + \frac{k^2}{8m},$$

We then have $\xi_+ = \xi' + v_F x$. Integration over x imposes a limitation on ξ' because of the presence of the δ function: ξ' can only be larger than ξ_1, where

$$\xi_1 = -\frac{1}{2}\left\{kv_F - \omega_0\left[1 + \frac{4\Delta^2}{k^2v_F^2 - \omega_0^2}\right]^{1/2}\right\}. \quad (4)$$

In the following we shall omit the prime on ξ.

After integration over x (see the similar calculations in [3]) and expansion in ω_0 for M and N, we get

$$M = -\frac{\pi}{2\eta}\frac{1}{kv_F}\int_{\xi_1}^{\infty} d\xi\frac{\partial n}{\partial \varepsilon}\left(\frac{\xi}{\varepsilon} - \frac{\varepsilon}{\sqrt{(\varepsilon+\omega_0)^2 - \Delta^2}}\right),$$

$$N = \frac{\pi}{2\eta}\frac{1}{kv_F}\int_{\xi_1}^{\infty} d\xi\frac{\partial n}{\partial \varepsilon}\frac{\Delta^{*2}_0}{\varepsilon\sqrt{(\varepsilon+\omega_0)^2 - \Delta^2}}.$$

Proceeding now to integration over ξ, we finally obtain for M and N, for the case $\omega_0 \ll \Delta$, $\Delta \ll T_c$,

$$M = \frac{\pi}{2\eta}\frac{1}{kv_F}\left\{\frac{|\Delta|}{4T}\left[\ln\frac{\varepsilon_1+|\Delta|}{\xi_1} - \ln\frac{8|\Delta|}{\omega_0}\right] - \frac{\varepsilon_1 - |\Delta|}{2T}\right\},$$
$$N = \frac{\pi}{2\eta}\frac{1}{kv_F}\frac{|\Delta|}{4T}\frac{\Delta_0^{*2}}{|\Delta|^2}\left[\ln\frac{\varepsilon_1+|\Delta|}{\xi_1} - \ln\frac{8|\Delta|}{\omega_0}\right]. \quad (5)$$

Here $\varepsilon_1 = \sqrt{|\xi_1|^2 + |\Delta|^2}$.

The region of applicability of Eqs. (5) is determined by the inequalities

$$|\xi_1| \gg \sqrt{\omega_0\Delta}, \quad \xi_1 < 0.$$

In accord with (4), this corresponds to the condition $kv_F \gg \sqrt{\omega_0\Delta}$. Here $\xi_1 \approx -\frac{1}{2}kv_F$.

For the condition

$$kv_F \gg \Delta \quad (6)$$

only the term

$$M = -\frac{\varepsilon_1 - |\Delta|}{2T}\frac{\pi}{2\eta}\frac{1}{kv_F} \approx -\frac{\pi}{8\eta T}.$$

remains in the second term of Eq. (3). It is easy to understand that this corresponds to an unlinearized equation of the form

$$-\frac{\pi}{8T\eta}\frac{\partial\Delta^*}{\partial t} = \left[\frac{\nabla^2}{4m} + \frac{1}{\eta}\left(\frac{T_c - T}{T_c} - \frac{7\zeta(3)}{8\pi^2T_c^2}|\Delta|^2\right)\right]\Delta^*. \quad (7)$$

This is the temporal Ginzburg–Landau equation. It was obtained earlier by Abrahams and Tsuneto.[3] In its properties, it is similar to a relaxation equation.[1] In a superconductor, however, it is suitable only if Δ changes sufficiently rapidly in space. Equation (6), however, is quite rigorous.

In the opposite limiting case studied in [2], when $kv_F \ll \Delta(T)$, the problem reduces to a system of kinetic equations for the excitation distribution functions and the equation for Δ. In the case in which $kv_F \sim \Delta$, the equation is seen to be nonlocal and it is impossible to represent it in any simple form. We shall see later that in our problem, just such values of k are significant. There is a significant difference here from the static case. For the application of the static Ginzburg–Landau equations, only the condition $kv_F \ll \Delta_0$ is necessary.

2. HIGH FREQUENCY IMPEDANCE IN AN EXTERNAL MAGNETIC FIELD

The dissipative terms in Eq. (3) describe the relaxation of Δ, i.e., the approach of Δ to its equilibrium value. This should lead to an anomalous absorption of acoustic and electromagnetic waves in superconductors near the transition point.

We shall consider the case of electromagnetic waves. It is already clear that it is impossible to observe the relaxation absorption of interest to us by studying the superconductor in a single (weak high-frequency) field. Actually, the change of Δ is proportional to the square of the magnetic field, so that in the linear approximation Δ does not change and the relaxation phenomena are not generally observed. It is possible to observe them if one studies the high frequency properties in an external magnetic field H_0. In this case, Δ will contain a term proportional to $H_0\tilde{H}_0$, where $\tilde{H}_0$ is the alternating magnetic field. This term leads to the anomalous absorption.

Let us consider a superconductor occupying the half space $z > 0$ and located in a constant magnetic field H_0 (H_0 is the value of the field on the surface), directed along the y axis. The problem consists of calculating the surface impedance ξ relative to a high-frequency field $\tilde{H}$ of frequency ω_0. We shall assume that it is also directed along the y axis. For another polarization, the effect in which we are interested is absent.

Solution of the problem is complicated by the nonlinearity of the effect and the non-local character of the equations. However, it is essential that in most pure type I superconductors the penetration depth of the field, δ, is much less than the correlation length $\hbar v_F/\Delta$:

$$\delta \sim \varkappa\hbar v_F/\Delta, \quad \varkappa \ll 1.$$

There are two terms in the variable portion of Δ. The first of these changes at distances $\sim\delta$. The dependence of this term on the coordinates is determined in the fundamental magnetic field. The temporal effects are not generally significant for low frequencies and one can use the ordinary Ginzburg–Landau equation for the determination of this term. The other term changes at distances $\sim\hbar v_F/\Delta$. At these distances, the field is lacking, the nonlinear effects are unimportant, and one can use the general linear equation (3) directly. We note beforehand that the dissipation of energy in the main takes place at these large distances.

One can show that the first nonlinear region gives us boundary conditions for Δ in the second, nonlocal dissipative region. In the first region, the superconductor is described by a set of Ginzburg–Landau equations which it is convenient to represent in the form

$$d^2\psi/dz'^2 = \varkappa^2[-(1 - A'^2)\psi + \psi^3],$$
$$d^2A'/dz'^2 = \psi^2A'. \quad (9)$$

Here

$$\psi = \Delta/\Delta_\infty, \quad A' = A/\sqrt{2}H_{cm}\delta,$$
$$z' = z/\delta, \quad \delta = \varkappa v_F/\sqrt{6}\Delta,$$

where δ is the penetration depth of the magnetic field, Δ_∞ is the equilibrium value of the gap in the absence of a magnetic field, A is the vector potential, directed along the x axis. The magnetic field H is expressed in terms of A by the formula $H = dA/dz$. H_{cm} is the critical field in the bulk of the superconductor

$$H_{cm} = \frac{\hbar c}{\sqrt{2}\, e} \frac{\varkappa}{\delta^2}.$$

In correspondence with the setup of the problem, we write

$$A' = A_0'(z') + \tilde{A}'(z') e^{-i\omega_0 t}, \quad \tilde{A}' \ll 1,$$
$$\psi = \psi_0(z') + \tilde{\psi}(z') e^{-i\omega_0 t}, \quad \tilde{\psi} \ll 1.$$

As has already been mentioned, we consider here the case of a type I superconductor with small $\kappa \ll 1$. In this case, one can neglect the dependence of ψ_0 on the coordinates and set $\psi_0 \equiv 1$. Solving Eq. (9) with $\psi_0 = 1$, we obtain the expression

$$A_0' = -H_0' e^{-z'}, \quad H_0' = H_0 / \sqrt{2} H_{cm} \quad (\psi_0 = 1), \tag{10}$$

for A_0', where H_0 is the constant magnetic field on the surface. By linearizing the set (8), (9) in $\tilde{A}'$ and $\tilde{\psi}$, and taking (10) into account, we get the equations

$$d^2\tilde{\psi} / dz'^2 - \varkappa^2[2\tilde{\psi} + 2A_0'\tilde{A}' + A_0'^2\tilde{\psi}] = 0, \tag{11}$$
$$d^2\tilde{A}' / dz'^2 = \tilde{A}' + 2A_0'\tilde{\psi}' \tag{12}$$

for $\tilde{\psi}'$ and $\tilde{A}'$.

We shall solve the set (11)–(12) by the method of successive approximations, which corresponds to an expansion in κ. In other words, we assume $\tilde{A}' = \tilde{A}'_0 + \tilde{A}'_1$ where $\tilde{A}'_1 \lesssim \kappa \tilde{A}'_0$. For $\tilde{A}'_0$, we have the equation

$$d^2\tilde{A}_0' / dz'^2 = \tilde{A}_0',$$

such that

$$\tilde{A}_0' = -\tilde{H}_0' e^{-z}, \tag{13}$$

$\tilde{H}'_0$ is the alternating magnetic field on the surface. Substituting (13) in (11), and neglecting terms $\sim \kappa^2$, we get for $\tilde{\psi}$ the equation

$$d^2\tilde{\psi} / dz'^2 = 2\varkappa^2 H_0' \tilde{H}_0' e^{-2z'}.$$

The particular solution of this equation has the form

$$\tilde{\psi} = \psi_n = {}^1\!/_2 \varkappa^2 H_0' \tilde{H}_0' e^{-2z'}.$$

However, this solution still does not satisfy the boundary condition $d\psi/dz = 0$ at $z = 0$. Therefore, one must add ψ_{hom} to it—the solution corresponding to the homogeneous problem, i.e., the problem without the field. The solution of the homogeneous problem changes, however, at large distances $\sim \hbar v_F/\Delta$, or, in dimensionless units, $\sim 1/\kappa$. Therefore, for the determination of ψ_{hom}, we should use the general equation (3). In other words, if we represent $\tilde{\psi}$ in the form

$$\tilde{\psi} = \psi_n + \psi_{hom},$$

then ψ_{hom} should satisfy the boundary condition

$$\frac{d\psi_{hom}}{dz'} = -\frac{d\psi_n}{dz'} = \varkappa^2 H_0' \tilde{H}_0' \quad \text{for } z = 0,$$

or, in dimensional variables,

$$\frac{d\psi_{hom}}{dz} = -\frac{d\psi_n}{dz} = \frac{\varkappa^2}{\delta} \frac{H_0 \tilde{H}_0}{2H_{cm}^2} \quad \text{for } z = 0. \tag{14}$$

Equation (3) was introduced for the unbounded superconductor, while here we are dealing with a half-space. This difficulty can be avoided if we assume $\psi_{hom}(z)$ to be an even function of z. On the boundary $z = 0$, the derivative of this function, in correspondence with (14), should undergo a jump, equal to

$$a = -2\frac{d\psi_n}{dz}\bigg|_{z=0}.$$

If we assume that ψ can be considered real for the selected gauge of A,[1)] then the equation for ψ_{hom} can be rewritten in the form

$$\hat{B}\psi_{hom}(z) = \frac{a}{4m}\delta(z),$$

where $\hat{B}$ is the operator corresponding to the left side of (3). The right δ-function part guarantees the jump of any necessary quantity. Here it is taken into account that the operator becomes $\nabla^2/4m$ for large **k**.

Carrying out a Fourier transformation, we get

$$[L_1 + i\omega_0(M+N)]\psi_{hom} = a/4m,$$

or

$$\psi_{k\,hom} = -\frac{\varkappa^2 H_0 \tilde{H}_0 / H_{cm}^2}{k^2 + 2\varkappa^2/\delta^2 + i\omega_0 4m(M+N)},$$

where M and N are expressed by Eqs. (5). Now it is necessary to perform the inverse transformation from ψ_{khom} to $\psi_{hom}(z)$. In what follows, however, we shall need only $\psi_{hom}(z = 0)$:

$$\psi_{hom}(0) = \frac{\varkappa^2 H_0 \tilde{H}_0}{\pi H_{cm}^2} \int_0^\infty dk \frac{1}{k^2 + 2\varkappa^2/\delta^2 + i\omega_0 4m(M+N)}.$$

Introducing the new variable $y = kv_F/2\Delta$ and making several transformations, we get for $\psi_{hom}(0)$

$$\psi_{hom}(0) = \frac{\sqrt{6}\,\varkappa}{4\pi} \frac{H_0 \tilde{H}_0}{H_{cm}^2} \int_0 dy \frac{1}{y^2 + 3 + 3i\omega_0\tau f(y)/y}. \tag{15}$$

Here τ is a parameter having the meaning of a relaxation time:[2)]

$$\tau = \frac{\pi}{14\zeta(3)} \frac{T_c}{\Delta^2} = 1{,}84 \frac{T_c}{\Delta^2},$$

while

$$f(y) = \ln\frac{1 + \sqrt{y^2 + 1}}{y} - \sqrt{y^2 + 1} + 1 - \ln\frac{8\Delta}{\omega_0}.$$

In (15), for $\omega_0\tau \sim 1$, the values of $y \sim 1$ are important, i.e., the values $kv_F \sim \Delta$, of which we have already spoken above.

In order to find the correction to the impedance, it is necessary to determine the correction to the vector potential. $\tilde{A}'_1$ satisfies the equation

$$d^2\tilde{A}_1' / dz'^2 - \tilde{A}_1' = 2A_0'\tilde{\psi} \tag{16}$$

with the boundary conditions

$$\tilde{A}_1'|_{z=\infty} = 0, \quad d\tilde{A}_1' / dz'|_{z=0} = 0, \tag{17}$$

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1) The fact that ψ is real automatically guarantees the constancy of the density of electrons for a change in Δ. Actually, according to [2], the change in the density connected with the nonstationarity of Δ

$$\delta\rho \sim i|\Delta|^2 \frac{\partial}{\partial t} \ln\frac{\Delta^*}{\Delta}$$

vanishes for real Δ. In the opposite case, a change in Δ would generate electric fields which would change the result.

2) In our calculations, we have actually assumed that $\omega_0\tau \sim 1$, or more precisely, that $\omega_0\tau \ll 1/\kappa^2$. In the opposite case, we should have used in place of Eq. (8), the time-dependent equation (7).

where $A_0' = -H_0' e^{-z'}$ is the vector potential of the constant magnetic field.

The right-hand side of Eq. (16) differs from zero only for $z \lesssim \delta$. At these distances, ψ_{hom} changes little and it can be replaced by $\psi_{hom}(z=0)$. The term ψ_n which enters into $\tilde{\psi}$, can generally be omitted from the right-hand side of (16), so that it $\sim \kappa^2$, in contrast with $\psi_{hom} \sim \kappa^2$. We thus have

$$d^2\tilde{A}_1' / dz'^2 - \tilde{A}_1' = -2H_0' e^{-z} \psi_{hom}(0).$$

Solving this equation with the boundary conditions (17), we get

$$\tilde{A}_1'(z') = H_0' \psi_{hom}(0)(1+z')e^{-z'}.$$

The impedance of the superconductor in the region of not very high frequencies is expressed in terms of the penetration depth in the following way:

$$\zeta = -i\frac{\omega_0}{c}\delta.$$

The penetration depth in which we are interested is computed from the formula

$$\tilde{\delta} = \tilde{A}_0 / \tilde{H}_0, \tag{18}$$

where $\tilde{A}_0$ and $\tilde{H}_0$ are the values of the potential and the magnetic field on the surface.

Substituting the value of $\tilde{A}$ on the surface in (18), we get for $\tilde{\delta}$

$$\tilde{\delta} = \delta_0\left[1 - \frac{H_0}{\tilde{H}_0}\psi_{hom}(0)\right].$$

We then have for the addition to the impedance

$$\zeta = \zeta_0\left(1 + \frac{\varkappa\sqrt{6}}{4\pi}\frac{H_0^2}{H_{cm}^2} I\right), \tag{19}$$

where $\zeta = -i\omega_0\delta_0/c$ is the impedance in the absence of an external magnetic field and

$$I = \int_0^\infty dy \frac{1}{y^2+3+3i\nu f(y)/y}.$$

Here the notation $\nu = \omega_0\tau$ has been introduced.

It is seen from (19) that the real part of ζ, which determines the absorption, is equal to

$$\zeta' = -\zeta_0\frac{\varkappa\sqrt{6}}{4\pi}\frac{H_0^2}{H_{cm}^2}3\nu I', \tag{20}$$

where $(-3\nu I')$ is the imaginary part of I,

$$I' = \int_0^\infty dy \frac{y f(y)}{y^2(y^2+3)^2 + 9\nu^2 f^2(y)}. \tag{21}$$

We now investigate the behavior of the integral (21) at low frequencies,[3)] i.e., when $\omega_0\tau = \nu \ll 1$. We now break up the integral (21) into a sum of 2 integrals

$$I = \int_0^{y_1} + \int_{y_1}^\infty = I_1 + I_2, \tag{22}$$

where $\nu \ll y_1 \ll 1$.

We first consider the second integral. Inasmuch as $y \gg \nu$ here, we can write

$$I_2 = \int_{y_1}^\infty dy \frac{f(y)}{y(y^2+3)^2}.$$

After some calculations, we get

$$I_2 = \frac{1}{18}\ln^2 y_1 + \frac{1}{9}\ln y_1 \ln\frac{4\Delta}{\omega_0} + C_1 - C_2,$$

where

$$C_1 = \frac{1}{2}\int_0^\infty dy \ln^2 y \frac{d}{dy}\frac{1}{(y^2+3)^2},$$

$$C_2 = \int_0^\infty dy \ln y \frac{d}{dy}\left[\frac{\varphi(y)}{(y^2+3)^2}\right],$$

$$\varphi(y) = \ln(1+\sqrt{y^2+1}) - \sqrt{y^2+1} + 1 - \ln\frac{8\Delta}{\omega_0}.$$

In the first integral in (22), we make the change of variables $y^2 = \nu^2 t$. We obtain

$$I_1 = -\frac{1}{9}\int_0^{t_1} dt \frac{\ln \alpha t}{4t + \ln^2 \alpha t},$$

where

$$t_1 = y_1^2/\nu^2, \quad \alpha = \nu^2(4\Delta/\omega_0)^2.$$

Taking it into account that $t_1 \gg 1$, I_1 can reduce to

$$I_1 = -\frac{1}{9}\left[\ln y_1 \ln\frac{4\Delta}{\omega_0} + \frac{1}{2}\ln^2 y_1 - \ln\frac{\nu}{2}\ln\frac{4\Delta}{\omega_0} - \frac{1}{2}\ln^2\frac{\nu}{2}\right] - \frac{F}{36}.$$

Here,

$$F(\ln\alpha) = \int_0^1 dx \frac{\ln(\alpha x/4)}{x + \ln^2(\alpha x/4)} - \int_1^\infty dx \frac{\ln^3(\alpha x/4)}{x(x+\ln^2(\alpha x/4))}.$$

Adding I_1 and I_2, we now obtain

$$I = \frac{1}{18}\left[\ln\frac{\nu}{2}\ln\frac{\alpha}{\nu^2} + \ln^2\frac{\nu}{2}\right] - \frac{1}{36}F + C_1 - C_2.$$

Thus, for $\omega_0\tau \ll 1$, the absorption $\zeta' \sim \omega_0^2, \ln^2\omega_0$. In the general Landau–Khalatnikov scheme, it would have been simply $\zeta' \sim \omega_0^2$. The appearance of the additional factor $\ln^2 \omega_0\tau$ increases the absorption at low frequencies. This is connected with the nonlocal character of the dissipative terms or, in other words, with the dependence of the relaxation time on **k**.

The region of applicability of the resultant formulas, as has already been noted, is limited to temperatures near the critical temperature, where the superconductor is a London superconductor. For this case, it is necessary that the condition

$$\Delta(T)/\Delta(0) \ll 4\varkappa$$

be satisfied,[4] or

$$\alpha \gg 1/\varkappa^2.$$

It must be noted that the resultant formulas are connected with the very delicate mechanism of dissipation in superconductors and their experimental verification represents considerable interest. Here, however, it must be kept in mind that the addition to the impedance ζ' that has been calculated is small in comparison with the real part of the impedance, which exists even in the absence of an external magnetic field.

The author expresses his gratitude to L. P. Pitaevskiĭ for help in the research and to A. A. Abrikosov for useful discussions.

[3)]In reality, however, it is necessary, both in (19) and in (21), to go to the limit of low frequencies, since Eq. (13) for the distribution of the magnetic field is effective only in the frequency region $\omega_0 < \Delta^3/T_c^2$. For higher frequencies, the anomalous skin effect begins to be significant.

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Translated by R. T. Beyer
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