CONCERNING THE THERMAL CONDUCTIVITY OF SUPERCONDUCTORS IN THE INTERMEDIATE STATE

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It is shown that if $T \ll T_c$ and the superconducting and normal layers have thicknesses $a_s \ll a_n$ the thermal resistance of the intermediate state of superconductors is determined mainly by tunneling of the thermal excitations through the s-layers, and not by jumps over the barrier.

As shown by Andreev[1], thermal electronic excitations in normal layers of the intermediate state, with energy $\omega < \Delta$ (Δ is the size of the gap, $\hbar = 1$) are reflected from the n-s interface between the normal and the superconducting half-spaces. This result was used in[1] to calculate the thermal conductivity of the intermediate state in a direction perpendicular to the stratification lines. The mechanism of the thermal conductivity consists of successive over-the-barrier jumps of the thermal electronic excitations from one normal layer to another. However, taking into account the finite thickness of the layers forming the intermediate state, one can expect in principle that, besides the over-the-barrier transitions, some contribution to the thermal conductivity will be made by excitations that tunnel through the superconducting layers. Although the thickness of the superconducting layers is quite large (compared with ξ, the radius of the Cooper pair), calculation shows that this question is not trivial, and the tunnel effects cannot be neglected in the general case.

Let us estimate the probability of tunneling for a barrier of rectangular form of height Δ and width a_s (a_s—width of superconducting layer, a_n—width of normal layer). Let

$$\Delta = \begin{cases} 0 & -\infty < x \leqslant 0, \\ \Delta & 0 \leqslant x \leqslant a_s, \\ 0 & a_s \leqslant x < +\infty. \end{cases} \tag{1}$$

The assumed model is valid if the inequalities $\xi \ll a_s \ll a_n$ are satisfied, that is, $10^{-3} \lesssim a_s < 10^{-2}$ cm. The corresponding smoothly-varying parts of the wave functions from[1] are

$$\begin{pmatrix}\eta\\ \chi\end{pmatrix} = A\begin{pmatrix}1\\0\end{pmatrix}e^{i\mathbf{kr}} + B\begin{pmatrix}0\\1\end{pmatrix}e^{-i\mathbf{kr}}, \qquad -\infty < x \leqslant 0;$$

$$\begin{pmatrix}\eta\\ \chi\end{pmatrix} = \frac{C}{\sqrt{2}}\begin{pmatrix}\sqrt{1+u\mathbf{n}\mathbf{k}_1/\omega}\\ -i\sqrt{1-u\mathbf{n}\mathbf{k}_1/\omega}\end{pmatrix}e^{i\mathbf{k}_1\mathbf{r}} + \frac{D}{\sqrt{2}}\begin{pmatrix}\sqrt{1-u\mathbf{n}\mathbf{k}_1/\omega}\\ +i\sqrt{1+u\mathbf{n}\mathbf{k}_1/\omega}\end{pmatrix}e^{-i\mathbf{k}_1\mathbf{r}},$$
$$0 \leqslant x \leqslant a_s; \tag{2}$$
$$\begin{pmatrix}\eta\\ \chi\end{pmatrix} = E\begin{pmatrix}1\\0\end{pmatrix}e^{i\mathbf{kr}} + F\begin{pmatrix}0\\1\end{pmatrix}e^{-i\mathbf{kr}}, \qquad a_s \leqslant x < +\infty.$$

Here $\begin{pmatrix}\eta\\ \chi\end{pmatrix}$ is a two-component wave function describing the excitation in the intermediate state, u is the Fermi velocity, and

$$\mathbf{k} = \mathbf{n}\frac{\omega}{u}; \quad \mathbf{n} = \frac{\mathbf{r}}{|r|}, \quad \mathbf{k}_1\mathbf{n} = \begin{cases} u^{-1}\sqrt{\omega^2-\Delta^2}, & n_x > 0 \\ -u^{-1}\sqrt{\omega^2-\Delta^2} & n_x < 0 \end{cases}$$

The case $n_x < 0$ corresponds to a "hole" (in the terminology of[1]) incident on the barrier from the side of negative x; the case $n_x > 0$ corresponds to incidence of a "particle"; A, B, C, D, E, and F are arbitrary constants.

Assuming for concreteness $n_x > 0$ (tunneling of a "particle," wherein only a "particle" moving from the barrier is present in the region $a_s \le x < \infty$, that is, $E \neq 0$ and $F = 0$), joining the wave function on the interface, and using the usual definition of the transparency of the barrier $d(\omega)$, we get

$$d(\omega) = \frac{|E|^2}{|A|^2} = \frac{4e^{-2\varkappa a_s}}{1-(\Delta^2-\omega^2)/\omega^2}, \qquad \varkappa = u^{-1}\sqrt{\Delta^2-\omega^2}, \quad \omega < \Delta. \tag{3}$$

We now calculate the heat flux W_t directed from the normal to the superconducting phase and due to the tunnel excitations,

$$W_t = \int_{v_x > 0} 2\omega n_0(\omega) d(\omega) \frac{d^3\mathbf{p}}{(2\pi)^3}, \tag{4}$$

where $\mathbf{v} = d\omega/dp$, $\omega = |\zeta|$, $\zeta = u(p - p_0)$, p_0—Fermi momentum, $n_0(\omega)$—equilibrium distribution function (when $T \ll \Delta$ we have $n_0(\omega) \approx e^{-\omega/T}$). The integration of (4) is in analogy with the procedure used in[1]. Only the expressions for the transparency of the barrier $d(\omega)$ are different, and accordingly the limits of integration with respect to ω. In[1] $\Delta \le \omega < +\infty$, whereas here $0 \le \omega \le \Delta$. As a result of the integration we get

$$W_t \approx 4\left(\frac{p_0}{\pi}\right)T^2\exp\left(-\frac{2\Delta a_s}{u}\right).$$

Comparing the flux W_t with the flux W_T due to the above-the-barrier transitions, calculated in[1], we find

$$\frac{W_t}{W_T} \approx \gamma\frac{T^2\exp(-2\Delta a_s/u)}{T\sqrt{\Delta T}\exp(-\Delta/T)}$$
$$(\gamma \sim 1, \quad \hbar = 1, \quad k = 1).$$

Thus, the fluxes W_t and W_T become comparable when

$$2\Delta a_s / u = \Delta / T, \tag{5}$$

which corresponds to a temperature

$$T \approx \hbar u / k a_s, \tag{5a}$$

k is Boltzmann's constant, and u is the Fermi velocity. Putting $a_s \sim 10^{-2}$–10^{-3} cm and $u \sim 10^8$ cm/sec, we get $T \sim 0.1$–1°K. This estimate allows us to conclude that the exponential growth of the thermal resistivity with temperature in the intermediate state, predicted in[1], should be observed only up to a definite quite low but experimentally attainable temperature, $T \sim 0.1-1^{\circ}K$. Below this temperature, the principal role in the heat flux is assumed by tunneling thermal excitations, and the temperature dependence of the thermal resistance assumes a power-law form.

It should be noted that the exponential variation of the thermal resistance with decreasing temperature can be violated also as a result of phonon thermal conductivity, boundary effects, etc. However, unlike other factors leading to the power-law dependence of the thermal conductivity on the temperature, the tunneling thermal resistance, which replaces the above-the-barrier resistance at low temperatures, depends exponentially on the external magnetic field (the external field determines the thickness of the superconducting and normal layers of the intermediate state). Therefore the tunneling thermal resistance can be easily separated against the background of the remaining possible power-law dependences.

In conclusion we emphasize that the results are valid when $a_s/a_n \ll 1$. In the case when $a_s \sim a_n$ it is necessary to take into account the quantization of the excitation energy in normal layers[2], which becomes appreciable at the same temperatures as the effect considered above.

[1] A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].
[2] A. F. Andreev, ibid. **49**, 655 (1965) [**22**, 455 (1966)].

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