ADIABATIC TRANSFORMATION OF THE ELECTROMAGNETIC WAVE SPECTRUM IN A PLASMA

N. S. STEPANOV

Gor'kiy State University

Submitted July 7, 1967

Zh. Eksp. Teor. Fiz. 53, 2186–2193 (December, 1967)

Propagation of plane quasi-monochromatic electromagnetic waves in a nonstationary magnetoactive plasma is considered. It is assumed that the parameters of the plasma, viz., electron concentration and drift velocity of the electrons, and also the external magnetic field strength, are slow functions of the time and, generally speaking, of a single space coordinate. Variation of the frequency and wave number, amplitude and energy of the waves is investigated in the geometrical optics approximation. It is shown that these variations may be quite significant. Moreover, the ratio of the wave packet energy to the carrier frequency is an adiabatic invariant. Some concrete examples are considered and a comparison with an isotropic plasma is made.

INTRODUCTION

In the analysis of the propagation of electromagnetic waves in a plasma, the medium is usually assumed to be stationary.[1] However, both under laboratory and under natural conditions (e.g., in the sun's atmosphere) cases are possible when the parameters of the medium vary sufficiently rapidly in time. In particular, the electron density can change as a result of the motion of an inhomogeneous plasma or under the influence of powerful plasma waves. A similar nonstationary behavior of the medium can greatly influence the behavior of the waves propagating in it, leading to a change of their form and of their spectrum, to a change of the energy, and even to the transformation of certain waves into others (see, e.g., [2-7]).

Since in the general case the problem of the propagation of waves in a dispersive medium with variable parameters is quite complicated, it is of interest to consider the "adiabatic" case, when the variation of the plasma parameters is slow compared with the quasimonochromatic signal propagating in it, but is at the same time sufficiently rapid to be able to neglect dissipative processes. It is assumed, in particular, that the electron density $N(\mathbf{r}, t)$ changes little over a time of the order of ω^{-1} and over a distance k^{-1}:

$$|\partial N / \partial t| \ll \omega N, \qquad |\operatorname{grad} N| \ll kN, \tag{1}$$

where $\omega(\mathbf{r}, t)$ and $k(\mathbf{r}, t)$ are the frequency and the wave number of the electromagnetic waves; according to the continuity equation $\partial N/\partial t + \operatorname{div} N\mathbf{V} = 0$, and then the electron velocity $\mathbf{V}(\mathbf{r}, t)$ in the concentration wave satisfies the same inequalities. Under these conditions, sufficiently general results can be obtained in the geometrical-optics approximation. For nondispersive media, the laws governing the adiabatic transformation of electromagnetic waves were investigated in detail in [2]; the singularities of media with temporal dispersion were discussed in [4,7], with an isotropic plasma with variable electron density serving as a particular example.

In the present paper we consider similar problems for a magnetoactive plasma. In this case both the temporal and the spatial dispersion and the gyrotropy of the medium are important.

1. FORMULATION OF THE PROBLEM AND DERIVATION OF THE SIMPLIFIED EQUATION

For brevity we confine ourselves here to the one-dimensional problem, when both the concentration wave $N(\mathbf{r}, t)$ and the sought signal constitute plane waves propagating along the external magnetic field $\mathbf{H}_0$ (along the z axis). Assume that plasma motion (drift) also takes place in the same direction and that the electron velocity $V(z, t)$ is not small.

We introduce, as in [5], the complex variables

$$A_\pm = A_x \pm iA_y, \qquad v_\pm = v_x \pm iv_y, \tag{2}$$

where $A_{x,y}$ and $v_{x,y}$ are respectively the Cartesian projections of the vector potential $\mathbf{A}_\perp$ and the velocities of the induced oscillations of the electrons $\mathbf{v}_\perp$ under the influence of the electromagnetic wave ($\mathbf{E} = -c^{-1}\partial\mathbf{A}/\partial t$, $\mathbf{H} = \operatorname{curl}\mathbf{A}$). If we neglect the thermal motion of the electrons and the oscillations of the ions, then we can easily obtain from Maxwell's equation and from the equation of motion of the electrons in the approximation linear in the signal components

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A_\pm = -\frac{4\pi eN}{c} v_\pm,$$
$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial z} \mp i\omega_H\right)(mv_\pm) = -\frac{e}{c}\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial z}\right) A_\pm, \tag{3}$$

where c—velocity of light, e and m—charge and mass of the electron ($m = m_0(1 - V^2/c^2)^{-1/2}$, and $\omega_H = |eH_0/mc|$—gyrofrequency of the electrons.

The problem thus reduces to an investigation of the system of equations (3) with coefficients $N(z, t)$ and $V(z, t)$ which vary slowly (in accordance with a prescribed law). We shall assume that H_0 is also a slow function of z and t, although in the latter case Eqs. (3) are not satisfied exactly (since the inhomogeneous magnetic field cannot be strictly one-dimensional, and an induction electric field, which is not taken into account in (3), is produced when H_0 varies in time). However, if the function $H_0(z, t)$ satisfies conditions of the type (1), then Eqs. (3) can be used at least in the investigation of the spectrum of the waves.

We note further that for the pairs of variables A_+, v_+ and A_-, v_- (corresponding, according to (2), to right- and left-polarized waves), we obtain independent systems of equations which differ only in the sign of ω_H. We seek the solution in the form

$$A_\pm = f(z, t)e^{i\psi}, \qquad mv_\pm = g(z, t)e^{i\psi}, \tag{4}$$

where f and g are factors that vary slowly compared with $e^{i\psi}$. Introducing the small parameter $\chi \sim (\omega N)^{-1} \times \partial N/\partial t \sim (kN)^{-1}\partial N/\partial z$ (as well as for the quantities V and H_0), we can expand these factors in powers of χ:

$$f = f_0 + f_1 + f_2 + \ldots, \qquad g = g_0 + g_1 + g_2 + \ldots, \tag{5}$$

where $f_1 \sim \chi f_0$, $f_2 \sim \chi^2 f_0$, etc. (and analogously for g). Here and throughout $\omega(z, t)$ and $k(z, t)$ are defined in terms of the eikonal $\psi(z, t)$, namely $\omega = \partial\psi/\partial t$ and $k = -\partial\psi/\partial z$. Substituting (4) and (5) in the system (3) we can easily obtain the equations for the successive approximations. In particular, in the zeroth approximation we have

$$(\omega^2 - c^2k^2)f_0 - \frac{c\omega_L^2}{e}g_0 = 0, \quad (\omega - kV \mp \omega_H)g_0 - \frac{e}{c}(\omega - kV)f_0 = 0, \tag{6}$$

where $\omega_L(z, t) = (4\pi e^2 N/m)^{1/2}$ is the plasma (Langmuir) frequency, which is a variable quantity here, just as ω_H, and for the corrections of the succeeding approximation we already obtain the inhomogeneous equations

$$(\omega^2 - c^2k^2)f_1 - \frac{c\omega_L^2}{e}g_1 = i\left[2c^2k\frac{\partial f_0}{\partial z} + 2\omega\frac{\partial f_0}{\partial t} + \left(c^2\frac{\partial k}{\partial z} + \frac{\partial\omega}{\partial t}\right)f_0\right],$$
$$(\omega - kV)\frac{e}{c}f_1 - (\omega - kV \mp \omega_H)g_1 = i\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial z}\right)\left(g_0 + \frac{e}{c}f_0\right), \tag{7}$$

etc.

From the condition for the compatibility of the equations in (6) we get an equation for the eikonal $\psi(z, t)$:

$$(\omega^2 - c^2k^2)(\omega - kV \mp \omega_H) = \omega_L^2(\omega - kV), \tag{8}$$

which in the stationary case (for constant ω_L, ω_H, V) reduces to the usual dispersion equation for the high-frequency waves in a drifting magnetoactive plasma.[8] We note that, in accordance with (8), the plasma drift at $\omega_H \neq 0$ leads to spatial dispersion—the refractive index $n = ck/\omega$ depends in this case explicitly not only on the frequency ω, but also on the wave number k. This circumstance complicates the dispersion characteristics of the waves; as a result, unlike a stationary plasma, there can exist two waves of identical polarization (e.g., A_+) having the same frequency ω and propagating in the same direction with different phase and group velocities (from the point of view of the comoving coordinate system, in which the drift velocity is equal to zero, of course, these waves propagate in opposite directions, but when $V(z, t)$ is variable the transformation to the other system does not simplify the problem). On the other hand, if $H_0 = 0$ the waves "do not feel" the drift, and we obtain for the refractive index the usual expression for an isotropic plasma, $n^2 = 1 - \omega_L^2/\omega^2$, which does not contain $V(z, t)$.[1)]

In turn, we then get from the vanishing of the determinant of the system (7) the following equation for the amplitude factor f_0:

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right)f_0 = \delta(z, t)f_0, \tag{9}$$

where $u(z, t) = d\omega/dk$ is the group velocity of the waves:

$$u = q[2c^2k \pm V\omega_H\omega_L^2(\omega - kV \mp \omega_H)^{-2}],$$
$$q = [2\omega \pm \omega_H\omega_L^2(\omega - kV \mp \omega_H)^{-2}]^{-1},$$
$$\delta = q\left\{-\left(c^2\frac{\partial k}{\partial z} + \frac{\partial\omega}{\partial t}\right)\right.$$
$$\left.\mp \omega_L^2(\omega - kV \mp \omega_H)\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial z}\right)[\omega_H(\omega - kV \mp \omega_H)^{-1}]\right\}. \tag{10}$$

Neglecting in the expansions (5) all the terms with the exception of f_0 and g_0, we obtain the solution in the geometrical-optics approximation. In this approximation the different group fronts of the quasimonochromatic signal move independently of each other with velocity $u(z, t)$ and the spreading of the wave packets due to the dispersion is not taken into account. The nonstationary nature of the medium, nonetheless, leads to a parametric variation of both the form of the envelope and of the duration of the wave packet, as well as of the carrier (the values of $\omega(z, t)$ and $k(z, t)$.[2)]

2. TRANSFORMATION OF THE WAVE SPECTRUM

We consider first the variation of the carrier frequency ω and of the wave number k. Although, generally speaking, it is impossible to obtain the eikonal $\psi(z, t)$ in explicit form, owing to the complexity of the nonlinear equation (8), the main laws can be clarified with sufficient detail. Thus, differentiating (8) with respect to t, we get

$$\frac{d\omega}{dt} = \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right)\omega = \alpha\frac{\partial\omega_L}{\partial t} + \beta k\frac{\partial V}{\partial t} + \gamma\frac{\partial\omega_H}{\partial t}, \tag{11}$$

where

$$\alpha = 2q\omega_L(\omega - kV)(\omega - kV \mp \omega_H)^{-1}, \quad \beta = \pm q\omega_H\omega_L^2(\omega - kV \mp \omega_H)^{-2},$$
$$\gamma = \pm q\omega_L^2(\omega - kV)(\omega - kV \mp \omega_H)^{-2}.$$

Similarly, differentiating (8) with respect to z, we get an equation for $k(z, t)$:

$$-\frac{dk}{dt} = \alpha\frac{\partial\omega_L}{\partial z} + \beta k\frac{\partial V}{\partial z} + \gamma\frac{\partial\omega_H}{\partial z}, \tag{12}$$

with the same dimensionless coefficients α, β, and γ as in (11).

Thus, the changes of the parameters ω_L, V, and ω_H (which enter into the expression for the refractive index) in time and in space lead respectively to perturbation of the time and space structure of the carrier of the quasimonochromatic signal, as is natural. On the other hand, in a stationary and homogeneous medium, for a fixed group of front, the quantities ω and k remain unchanged in this approximation, although in a modulated wave $\partial\omega/\partial t$ and $\partial\omega/\partial z$ can separately be different from zero.

Let us dwell in greater detail on the effect connected with the change of the electron density $N(z, t)$, corre-

1) In the relativistic case $V(z, t)$ enters implicitly into the expression for n via the mass $m = m_0(1 - V^2/c^2)^{-1/2}$.

2) When frequency-modulated signals propagate, as is clear from (9), distortion of the envelope takes place also in a stationary medium.

sponding to the first term in the right sides of (11) and (12). We note that in an isotropic plasma ($H_0 = 0$) the remaining two terms in general vanish, which agrees with the results obtained in [7]. In particular, if $\partial\omega_L/\partial t$ does not depend on z, then Eq. (11) can be easily integrated:

$$\omega^2(z, t) = \omega_0^2(\xi) + [\omega_L^2(t) - \omega_L^2(0)], \qquad (13)$$

where $\omega_0(\xi)$ is the initial frequency of the signal, which is an arbitrary function of the "group" characteristic $\xi(z, t)$ (ξ—first integral of the equation $\partial\xi/\partial t + u\partial\xi/\partial z = 0$). Consequently, with increasing concentration N(t) the frequency of the fixed group front (ξ = const) also increases (and vice-versa), and the increment $\Delta\omega^2 = \omega^2(t) - \omega_0^2$ is in this case simply equal to the drop of the square of the plasma frequency $\Delta\omega_L^2$ during the time of motion of this front in the nonstationary plasma. Inasmuch as the drops of the concentration N in the plasma can be sufficiently large (e.g., in the already mentioned solar atmosphere), it follows that the indicated adiabatic shifts of the frequency of the electromagnetic waves can be appreciable (when $\omega \sim \omega_L$ it is quite possible to have a frequency increment $\Delta\omega \sim \omega_0$). In addition, as is clear from the form of Eqs. (11) and (12), the effect can accumulate if the perturbations of the plasma parameters propagate themselves in the form of waves that are synchronous with the group velocity of the signal; now we can already have $\Delta\omega \gg \Delta\omega_L$ and $\Delta\omega \gg \omega_0$. In fact, when $H_0 = 0$ and $N = N(\zeta)$, where $\zeta = z - at$, it is easy to show that the complete integral of (11) satisfies the relation $(d\omega^2/d\omega_L^2) = (1 - u/a)^{-1}$.

In a magnetoactive plasma ($\omega_H \neq 0$) the coefficient a in Eqs. (11) and (12) already depends on the ratio of ω to the gyrofrequency ω_H, the dependence being different for right- and left-polarized waves. Thus, when $\omega - kV > 0$ and $\omega - kV \mp \omega_H > 0$ the effect for a left-polarized wave (A_-) is larger than for a wave A_+ of the same frequency. We note also that at the gyroresonance frequency (corresponding in a drifting plasma to $\omega - kV \mp \omega_H = 0$) we have $\alpha = 0$. In addition, there appears here also a term $\beta k\partial V/\partial t$, connected with the spatial dispersion, i.e., the change in the drift velocity also influences the frequency of the waves (with different signs for A_+ and A_-).

In order to estimate the role of this term, let us consider an example in which the perturbations of N and of V are due to a plasma wave of frequency $\Omega \ll \omega$, propagating in a medium which on the whole is at rest. Specifying $N = N_0[1 + p\cos(\Omega t - Kz)]$, we get for V(z, t) from the continuity equation, accurate to p^2, $V = p\Omega K^{-1} \times \cos(\Omega t - Kz)$. It is clear that electromagnetic waves propagating in such medium are modulated in amplitude and phase at a frequency Ω.[3)] The ratio of the two terms in (11) will in this case be equal to (we assume that $\omega \gg kV$)

$$\frac{\beta k(\partial V/\partial t)}{\alpha(\partial\omega_L/\partial t)} = \frac{\Omega k\omega_H}{K\omega(\omega \mp \omega_H)}. \qquad (14)$$

Depending on the values of ω and ω_H and on the phase velocities of the signal (ω/k) and of the "pump" (Ω/K), this ratio can vary between wide limits. Thus, it is small for high-frequency waves ($\omega \gg \omega_H$), but can also exceed unity for the extraordinary waves in the region $\omega < \omega_H$. Consequently, allowance for the spatial dispersion in the analysis of parametric phenomena in a gyrotropic plasma can be important.

Further, the change of the intensity of the magnetic field H_0 in time and in space also influences the values of ω and k, a fact reflected in the last term of (11) or (12). The contribution made by this term to the change of the frequency can be estimated from the expression

$$\Delta\omega = \int_{t_0}^{t} \gamma(\omega)\frac{\partial\omega_H}{\partial t}\,dt, \qquad (15)$$

where the integral is taken along the characteristic $\xi(z, t)$. We note that when $\partial\omega_H/\partial t > 0$ the frequency of the right-polarized waves increases, and that of the left-polarized waves, to the contrary, decreases. For those sections where the derivative $\partial\omega_H/\partial t$ can be regarded as constant, we have in order of magnitude $\Delta\omega \sim \gamma(\partial\omega_H/\partial t)\Delta t$ (Δt—time of motion of the group front on this section). In particular, if the magnetic field can be assumed to be homogeneous in space ($\partial\omega_H/\partial z = 0$), then $\Delta\omega \sim \gamma\Delta\omega_H$, where $\Delta\omega_H$ is the change of the gyrofrequency of the electrons for a fixed group front (ξ = const), i.e., the shift of the carriers is determined here, too, by the drop of the field intensity H_0 during the time of motion of the front under consideration. In this case the coefficient $\gamma(\omega) \ll 1$ if $\omega \gg \omega_L, \omega_H$, but can be of the order of or larger than unity at lower frequencies.

In particular cases, of course, the value of $\omega(z, t)$ can be calculated here in explicit form, too. For example, when ω_L = const, V = 0, $\partial H_0/\partial z = 0$ and $\omega \gg \omega_L, \omega_H$ we get $\gamma(\omega) \pm (\omega_L^2/2\omega^2)$ and Eq. (11) can be integrated:

$$\omega^3(z, t) = \omega_0^3(\xi) \pm {}^3\!/_2\omega_L^2[\omega_H(t) - \omega_H(0)], \qquad (16)$$

where $\omega_0(\xi)$, just as in formula (13), is the initial value of the frequency of the group front under consideration, and $\omega_H(0)$ is the corresponding value of the gyrofrequency.

Thus, all three aforementioned factors in a nonstationary magnetoactive plasma can play an important role, leading to a deep frequency modulation of the electromagnetic waves, transforming and mixing their spectra.

3. CHANGE OF AMPLITUDE AND ENERGY OF THE WAVES

Once the quantities $\omega(z, t)$ and $k(z, t)$ are known, the determination of the amplitude factor f_0 from Eq. (9) no longer entails any difficulty. We shall assume further that H_0 = const, for otherwise, as shown earlier, the problem is not strictly one-dimensional and it is necessary in the calculation of the energy quantities, for example, to take into account the change in the diameter of the ray tube. In this case the function f_0 can be obtained in the form

$$f_0(z, t) = F(\xi)\Phi^{1/2}\left[k \pm \frac{V\omega_H\omega_L^2}{2c^2(\omega - kV \mp \omega_H)^2}\right]^{-1/2}, \qquad (17)$$

where $F(\xi)$ is an arbitrary (but, of course, slow) function, determined by the boundary and initial conditions; $\Phi(z, t)$ is a factor that depends on the law governing the variation of the group velocity of the waves:

3) Resonance effects for $\Omega \gtrsim \omega$ were considered in [5,6].

$$\Phi = \exp\left(\int_{t_0}^{t} \frac{\partial \ln u}{\partial t} dt \right), \quad (18)$$

where the integral is taken along the curve ξ = const. Then from relations (6) we can easily obtain, in the same approximation, the quantity g_0, and from formulas (2)—the projections of the vectors $\mathbf{A}_\perp$ and $\mathbf{v}_\perp$ and the intensities of the fields **E** and **H**. In particular, the amplitudes of the latter are equal to *

$$E_{0x} = -\frac{i\omega f_0}{2c}, \quad E_{0y} = \mp \frac{\omega f_0}{2c}, \quad \mathbf{H}_{0\perp} = \frac{c}{\omega}[\mathbf{kE}_0].$$

It is of interest to consider also the change of the energy of the electromagnetic waves in a nonstationary plasma. The energy density in a gyrotropic medium, as is well known, is[9]

$$w = \frac{1}{16\pi}\left\{ \mathbf{H}_{0\perp}^* \mathbf{H}_{0\perp} + E_{0i}^* E_{0j} \frac{\partial}{\partial \omega}[\omega \varepsilon_{ij}(\omega, \mathbf{k})] \right\}, \quad (19)$$

where ϵ_{ij} are the components of the dielectric tensor.[4)] For the drifting plasma, this tensor was calculated in [8]. Using the results obtained there, we can easily ascertain that in our case

$$w = \omega |f_0^2| / 32\pi c^2 q. \quad (20)$$

The energy flux density is then equal to wu. We note that in a drifting magnetoactive plasma this quantity is not equal to the modulus of the Poynting vector $S = (c/4\pi)\mathbf{E}_0 \times \mathbf{H}_{0\perp}$, the difference $S_k = wu - S \sim V\omega_H\omega_L^2$ obviously corresponding to the kinetic flux carried by the moving medium. On the other hand, in an isotropic plasma $S_k = 0$ and $S = wu$.[7]

It is best to explain the energy relations by using as an example short wave packets with a uniform carrier and with a length that is small compared with the space and time scales of the inhomogeneity of the medium. To calculate the total energy it is necessary in this case to determine more precisely the change of the duration of the pulse τ. Recognizing that, in accordance with (9), these changes are determined by the behavior of the group characteristics $\xi(z, t)$, it is easy to show that

$$\tau = \tau_0 / \Phi, \quad (21)$$

where τ_0 is the initial pulse duration (at $t = t_0$).

According to formulas (17), (20), and (21), the energy per unit area of the wave front, for a rectangular energy pulse, will be equal to

$$W = wu\tau = \omega\tau_0 F^2 / 32\pi c^2, \quad (22)$$

so that the ratio of the energy of the packet to the carrier frequency is constant:

$$I = W / \omega = \text{const}, \quad (23)$$

i.e., the quantity I is an adiabatic invariant. It is easy to generalize this relation to include also an arbitrary form of the envelope of the pulse. Knowledge of the invariant (23) makes it possible, in particular, when solving the concrete problems, to confine oneself to an investigation of the eikonal equation and by the same token determine also the amplitudes of the fields, without considering the first-approximation equation for the envelope f_0. The validity of the invariant (23) was demonstrated earlier for weakly dispersive media in [2] and for an isotropic plasma in [7].

Knowing the τ (z, t) dependence, we can estimate also the width of the spectral band of the signal, which obviously is of the order of τ^{-1}; in particular, if in some region the group velocity u increases with time ($\partial u/\partial t > 0$) then, according to relations (18) and (21), the pulse duration decreases, and the corresponding frequency band broadens, and vice versa. In view of the independence of the group fronts in the approximation under consideration, relation (21) is valid also for an arbitrary separated short train in the wave, and characterizes in this case the compression or the extension of individual sections of the signal envelope. From (11) and (21) it also follows that the parametric change of the quantity τ in a dispersive medium occurs generally speaking not in inverse proportion to the frequency ω, as is the case in the absence of dispersion.[5)][2] As a result the carrier frequency and the signal energy can increase in such a medium simultaneously with the duration, and by the same token simultaneously with a narrowing of the spectral band.[7] The latter circumstance is of practical interest, and in this respect a magnetoactive plasma offers more opportunities than an isotropic medium (e.g., synchronism of the signal with the concentration wave can be ensured here by choosing the proper magnetic field intensity).

*$[\mathbf{kE}_0] \equiv \mathbf{k} \times \mathbf{E}_0$.

4)The asterisk in this formula denotes the complex conjugate.

[1] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Fizmatgiz, 1960 [Addison-Wesley, 1964].

[2] N. S. Stepanov, Izv. vuzov, Radiofizika **3**, 672 (1960).

[3] G. I. Freĭdman, Zh. Eksp. Teor. Fiz. **41**, 226 (1961) [Sov. Phys.—JETP **14**, 165 (1962)].

[4] L. A. Ostrovskiĭ, Izv. vuzov, Radiofizika **4**, 293 (1961).

[5] L. A. Ostrovskiĭ and N. S. Stepanov, Zh. Eksp. Teor. Fiz. **45**, 1473 (1963) [Sov. Phys.—JETP **18**, 1018 (1964)].

[6] J. Askne, Proc. IEEE **54**, 1292 (1966).

[7] N. S. Stepanov, Izv. vuzov, Radiofizika, in press.

[8] G. Kh. Kamenetskaya and M. S. Kovner, ibid. **7**, 596 (1964).

[9] V. M. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasmalike Media), Gosatomizdat, 1961 [Transl. in prep., Gordon and Breach].

Translated by J. G. Adashko
248

5)It was assumed in error in [3] that upon reflection from a moving mirror the duration of the wave packet changes in inverse proportion to the frequency, and therefore the expressions obtained for the energy of the packet (e.g., formula (15)) are incorrect.