THEORY OF THE INTERACTION OF GROWING SOUND FLUCTUATIONS IN SEMICONDUCTORS

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A theory of interaction between growing sound noise under conditions of sound instability in piezoelectric semiconductors is developed. It is shown that the strongest interaction is that due to the background of strongly damped oscillations of the electron density. This interaction determines the principal nonlinear term in the equation derived for the kinetics of fluctuations. This term does not have the form of the Peierls phonon-phonon collision operator and cannot be derived from the Hamiltonian formalism. A number of problems can be studied by means of the equation of fluctuation kinetics. In the present paper, it is applied to the study of the propagation of an almost monochromatic sound beam. When the beam intensity due to nonlinear interaction tends to a fixed limit, the frequency range of the beam narrows down upon propagation of the sound.

1. INTRODUCTION

In a series of experiments on the study of sound instability in piezoelectric semiconductors in a constant electric field, an amplification of sound noise is observed, i.e., of sound fluctuations which are always present in the crystal.[1-3] The sound fluctuations were recorded either directly, by a sound receiver,[1,3] or by a secondary effect produced by it (for example, by the bend in the volt-ampere characteristic[2,3]).

The linear theory of growing fluctuations was constructed in the works of one of the authors[4,5] (see also[6]). An equation was obtained, by means of which it is possible to determine the intensity of the growing fluctuations, when it is not too high, so that it is possible to neglect their nonlinear interaction. The aim of the present work was to construct a theory which takes into account the interaction of the growing fluctuations, and to make clear the effect to which this interaction leads.

We shall consider fluctuations in a system whose macrostate generally depends on the time t. We consider the fluctuating quantity $\xi_q(t)$, which is proportional to the displacement in a traveling wave with wave vector q.

As is usually done in the derivation of the kinetic equation for phonons, and also in the theory of gases,[7,8] we shall assume that weak interaction between the large number of waves leads to a rapid randomization of the phases of quantities and a slow change in their amplitude. Therefore, we select as our characteristic of the intensity of the fluctuations the mean value $I_q = \langle \xi_q^*(t)\, \xi_q(t) \rangle$. Here the angle brackets denote averaging over the random phases. In the stationary state, I_q does not depend on t, and averaging over the phases is equivalent to taking the time average.

The phonon-phonon collision operator was first obtained by Peierls,[7] starting out from the Hamiltonian formalism. The classical derivation of this operator for weakly interacting waves was given by Galeev and Karpman, who started out from the equations of motion.[9] In the present research, we shall follow the method developed in[9].

According to the estimates given in[10], the fundamental role in piezoelectric semiconductors is played by a nonlinearity not of lattice origin (connected with the anharmonicity of the elastic forces), but of electronic origin. At first glance, it would appear simple to consider the corresponding nonlinear interaction of sound waves. Thanks to the piezoelectric coupling, each sound wave in the piezoelectric is accompanied by a variable electric field, and hence by a "stimulated" wave of electron concentration, which possesses the spatial and temporal periodicity of the sound wave passing through it. Two such distributions of the electron concentration, accompanying two sound waves, interact with one another, owing to the existence of the electron nonlinearity. This interaction leads to scattering of the sound oscillations, while in each scattering act the laws of conservation of energy (frequency) and quasimomentum (wave vector) are satisfied.[1)] Thus the situation reduces to one in which matrix elements computed with account of the electron nonlinearity must be substituted in the Peierls collision operator. This mechanism was suggested in the interesting work of Hutson,[11] and calculation of the matrix element for the process of coalescence of two phonons with the same frequency and wave vectors was carried out by Mikoshiba.[12]

In reality, however, the situation is more complicated, for the following reason. It is evident that in a piezoelectric semiconductor, in addition to the sound waves, there should exist a "free" wave of electron density, which is rapidly damped because of the processes of electrical conductivity and diffusion of the current carriers. In addition to the direct interaction

[1)] In piezoelectric semiconductors, an amplification takes place of long-wave phonons, in the collisions of which the Umklapp processes play no role.

of sound waves, their interaction through an intermediate wave—the electron density wave—is possible.[13] The corresponding collision term is proportional to a lower power of the constant of piezoelectric coupling, and therefore it is almost always larger than the Peierls term. In processes of interaction described by such a non-Peierls collision operator, only the law of conservation of the wave vector is satisfied, and not the law of conservation of energy, because of the strong damping of the intermediate wave.

The non-Peierls collision operator has the following structure:

$$\left(\frac{\partial I_{\mathbf{q}}}{\partial t}\right)_{\text{coll}} = I_{\mathbf{q}} \sum_{\mathbf{q}'} W(\mathbf{q}, \mathbf{q}') I_{\mathbf{q}'}, \tag{1.1}$$

where the expression for the kernel $W(q, q')$ is determined by the specific form of the nonlinear electron interaction. In the present work, we compute it for the so-called concentration nonlinearity, which was first proposed by Hutson,[11] although the method used by us is applicable to any other electron nonlinearity. The operator (1.1) possesses a number of unusual properties. In particular, it is capable, under definite conditions, of determining the interval of the amplified waves, producing an almost monochromatic filtering of the beam of phonons with a very small scatter in direction; the integral intensity of the fluctuations in this case remains unchanged. The Peierls operator cannot have such a property, since it specifically describes processes of the coalescence of two phonons (arrival), which broaden the range of amplified frequencies. On the other hand, if the non-Peierls collision operator predominates, the ingenious idea of Hutson relative to the fact that the limitation of amplification can be connected with the "pumping" of the acoustic energy in the processes of phonon-phonon collisions in the region of high frequency, is generally not realized.

2. FUNDAMENTAL EQUATIONS OF THE PROBLEM

The aim of the present section is the transformation of the basic equations of the problem to a form suitable for the study of the nonlinear interaction of the fluctuations.

The system of equations describing the propagation of sound in a piezoelectric semiconductor with account of the concentration linearity has been described, for example, in[10] [(Eqs. (1.21)—(1.24)]. The limits of applicability of this system were also discussed. We write down this system for the Fourier components in the coordinates:

$$u_l = \sum_{\mathbf{q}} \sum_j e_{\mathbf{q}l}^{(j)} u_{j\mathbf{q}} e^{i\mathbf{q}\mathbf{r}}, \tag{2.1}$$

$$\varphi = \sum_{\mathbf{q}} \varphi_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}}, \qquad n' = \sum_{\mathbf{q}} n_{\mathbf{q}}' e^{i\mathbf{q}\mathbf{r}}, \tag{2.2}$$

$$j_l = \langle j_l \rangle + \sum_{\mathbf{q}} j_{l\mathbf{q}} e^{i\mathbf{q}\mathbf{r}}, \tag{2.3}$$

$$\langle j_i \rangle = e n_0 V_i - e \mu_{ik} \langle n' \partial\varphi / \partial x_k \rangle. \tag{2.4}$$

Here u_l is the elastic displacement vector, φ is the variable electrostatic potential which arises under the action of the acoustic fluctuations, j_l is the current density of the conductivity, $n = n_0 + n'$ is the concentration of the conduction electrons, n_0 is its stationary value, e the electronic charge, μ_{ik} the mobility tensor,

$$V_i = \mu_{ik} E_k, \tag{2.5}$$

E_k is the constant electric field, $e_{\mathbf{q}l}^{(j)}$ are three mutually perpendicular unit vectors, which we choose to be the solutions of the system

$$(\rho \Omega_{\mathbf{q}j}^2 \delta_{il} - c_{iklm} q_k q_m) e_{\mathbf{q}l}^{(j)} = 0, \tag{2.6}$$

where $\Omega_{\mathbf{q}j}$ are the three solutions of the equation

$$|\rho \Omega_{\mathbf{q}}^2 \delta_{il} - c_{iklm} q_k q_m| = 0. \tag{2.7}$$

Such a choice is due to the fact that the corrections to the frequencies and the polarization vectors are small perturbations; first, we shall use below the smallness of the piezoelectric coupling

$$\chi = 4\pi\beta^2 / \varepsilon c \ll 1, \tag{2.8}$$

where β, ϵ, and c are respectively the components of the tensors of piezoelectric moduli, the dielectric susceptibility, and the elastic moduli. Second, we shall also assume that

$$\eta\omega / c \ll 1, \tag{2.9}$$

where η are the corresponding components of the tensor of viscosity coefficients.

For the Fourier components we get the following set of equations:

$$\frac{d^2 u_{j\mathbf{q}}}{dt^2} = -\Omega_{\mathbf{q}j}^2 u_{j\mathbf{q}} + \frac{1}{\rho} \beta_{j\mathbf{q}} q^2 \varphi_{\mathbf{q}} + \frac{q^2}{\rho} \sum_{j'} \eta_{\mathbf{q}}^{(jj')} \frac{du_{j'\mathbf{q}}}{dt}, \tag{2.10}$$

$$\varphi_{\mathbf{q}} = \frac{\varkappa_{\mathbf{q}}^2}{q^2} \frac{T}{en_0} n_{\mathbf{q}}' - \frac{\varkappa_{\mathbf{q}}^2}{e^2 n_0} T \sum_j \beta_{j\mathbf{q}} u_{j\mathbf{q}}, \tag{2.11}$$

$$\frac{dn_{\mathbf{q}}'}{dt} + \frac{n_0}{\tau_{\mathbf{q}}} \frac{q^2}{\varkappa_{\mathbf{q}}^2} \frac{e}{T} \varphi_{\mathbf{q}} + i\mathbf{q}\mathbf{V} n_{\mathbf{q}}' + \frac{q^2}{\varkappa_{\mathbf{q}}^2} \frac{1}{\tau_{\mathbf{q}}} n_{\mathbf{q}}' + \sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} \mu_{ik} q_i q_k' \varphi_{\mathbf{q}'} n_{\mathbf{q}''} = 0. \tag{2.12}$$

Here[2)] ρ is the density of the crystal, T the temperature in energy units,

$$\varkappa_{\mathbf{q}}^2 = \frac{4\pi e^2 n_0 q^2}{T \varepsilon_{ik} q_i q_k}, \qquad \tau_{\mathbf{q}} = \frac{\varepsilon_{ik} q_i q_k}{4\pi e n_0 \mu_{lm} q_l q_m},$$

$$\beta_{j\mathbf{q}} = \frac{1}{q^2} \beta_{i,kl} q_i q_k e_{\mathbf{q}l}^{(j)}, \qquad \eta_{\mathbf{q}}^{(jj')} = \frac{1}{q^2} \eta_{iklm} q_k q_m e_{\mathbf{q}i}^{(j)} e_{\mathbf{q}l}^{(j')}.$$

If, neglecting the nonlinear interaction, we seek a solution of the set of Eqs. (2.10)—(2.12) proportional to $\exp(-i\omega_q t)$, then we obtain an algebraic equation of seventh degree for ω_q. For $\beta_{i,kl} = 0$, this equation determines the dispersion law of six sound waves (three "direct" and three "inverse") and one wave of electron density:

$$\Omega_0 = \mathbf{q}\mathbf{V} - \frac{i}{\tau}\left(1 + \frac{q^2}{\varkappa^2}\right). \tag{2.13}$$

For $\eta_{iklm} = 0$, the frequencies of the sound waves are found from Eq. (2.7).

By virtue of the inequality (2.8), the values of the frequencies can be sought in the form of an expansion in powers of χ. The limits of applicability of this expansion are studied in detail in[5,10,14]. In first approximation, the following expressions are obtained for the

2)In order not to make the notation more difficult, we shall frequently drop the index q in symbols of this type.

frequency of vibration of the electron density, which is changed as a consequence of the interaction with the sound:

$$\omega_0 = -\frac{i}{\tau}\left[1+\frac{q^2}{\varkappa^2}+i\mathbf{qV}\tau-\sum_{j=1}^{3}\chi_j\frac{\Omega_j^2\tau^2}{(1+q^2/\varkappa^2+i\Omega_j\tau)^2+\Omega_j^2\tau^2}\right] \quad (2.14)$$

and for the frequencies of the acoustic oscillations that interact with the conduction electrons:

$$\omega_r = \Omega_r\left(1+\frac{\chi_r}{2}\frac{a_r}{1+a_r}\right)-\frac{i}{2}\frac{\eta^{(rr)}q^2}{\rho}. \quad (2.15)$$

Here

$$\chi_r = \frac{4\pi\beta_{|r|}^2 q^4}{\rho\Omega_r^2\varepsilon_{ik}q_iq_k}, \qquad a_r = \frac{q^2}{\varkappa^2}+i(\mathbf{qV}-\Omega_r)\tau. \quad (2.16)$$

The index r here runs over the values ±1, ±2, ±3, while, by definition,

$$\Omega_{\mathbf{q}-r} = -\Omega_{\mathbf{q}r}, \qquad \Omega_{-\mathbf{q}r} = \Omega_{\mathbf{q}r} \quad (2.17)$$

and, consequently,

$$\omega_{-\mathbf{q}-\lambda} = -\omega_{\mathbf{q}\lambda}^* \quad (2.18)$$

(where the index λ takes on seven values from −3 to +3). Sometimes it is more convenient to use the transformed expression (2.15), in which the real and imaginary parts are separated:

$$\omega_r = \omega_r' - i\gamma_r/2, \quad (2.15a)$$

$$\omega_r' = \Omega_r\left[1+\frac{\chi_r}{2}\frac{(q^2/\varkappa^2)(1+q^2/\varkappa^2)+(\Omega_r-\mathbf{qV})^2\tau^2}{(1+q^2/\varkappa^2)^2+(\Omega_r-\mathbf{qV})^2\tau^2}\right], \quad (2.15b)$$

$$\gamma_r = \chi_r\Omega_r\frac{(\Omega_r-\mathbf{qV})\tau}{(1+q^2/\varkappa^2)^2+(\Omega_r-\mathbf{qV})^2\tau^2}+\frac{\eta^{(rr)}q^2}{\rho} \quad (2.15c)$$

We shall consider the nonlinear effects as a small perturbation. It is therefore convenient first to transform to such variables in which the types of waves of the linear theory are separated. We use the notation

$$u_{0\mathbf{q}} \equiv n_{\mathbf{q}}', \qquad u_{-j\mathbf{q}} = -\frac{1}{i\Omega_j}\frac{du_{j\mathbf{q}}}{dt} \quad (j=1,2,3) \quad (2.19)$$

and introduce the new independent variables $\xi_{\lambda\mathbf{q}}$ (λ runs through seven values) by means of the transformation

$$\xi_{\lambda\mathbf{q}} = \sum_{\lambda'} S_{\lambda\lambda'}u_{\lambda'\mathbf{q}}, \qquad u_{\lambda\mathbf{q}} = \sum_{\lambda'} S_{\lambda\lambda'}^{-1}\xi_{\lambda'\mathbf{q}}, \quad (2.20)$$

where S is a matrix diagonalizing the initial linear system. The transformed set of equations takes the form

$$\frac{d\xi_{\lambda\mathbf{q}}}{dt} = -i\omega_{\lambda\mathbf{q}}\xi_{\lambda\mathbf{q}} + \sum_{\lambda'\lambda''}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ \lambda\,\lambda'\,\lambda''\end{pmatrix}\xi_{\lambda'\mathbf{q}'}\xi_{\lambda''\mathbf{q}''}. \quad (2.21)$$

In zeroth approximation in the parameter χ, the nonzero elements of the S matrix are equal to (the corresponding contributions are similar to those obtained in[5])

$$S_{jj} = S_{-jj} = S_{j-j} = \frac{\beta_j q^2}{\sqrt{2}en_0}, \qquad S_{-j-j} = -\frac{\beta_j q^2}{\sqrt{2}en_0},$$
$$S_{00} = \frac{1}{n_0}, \qquad S_{0j} = -\frac{1+q^2/\varkappa^2+i\mathbf{qV}\tau}{(1+q^2/\varkappa^2+i\mathbf{qV}\tau)^2+\Omega_j^2\tau^2}\frac{\beta_j q^2}{en_0},$$
$$S_{0-j} = \frac{i\Omega_j\tau}{1+q^2/\varkappa^2+i\mathbf{qV}\tau}S_{0j} \quad (j=1,2,3). \quad (2.22)$$

Inasmuch as only nonlinear effects of electronic origin are considered, the sound subsystem is linear when χ = 0 and the matrix elements $B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ r\,\lambda'\,\lambda''\end{pmatrix}$ vanish. For χ ≠ θ, these elements are proportional to χ while the elements $B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ 0\,\lambda'\,\lambda''\end{pmatrix}$ do not contain this small parameter. In particular, for concentration nonlinearity,

$$B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ 0\,0\,0\end{pmatrix} = -2\pi en_0\left(\frac{\mu_{ik}q_iq_k'}{\varepsilon_{lm}q_l'q_m'}+\frac{\mu_{ik}q_iq_k''}{\varepsilon_{lm}q_l''q_m''}\right),$$

$$B\begin{pmatrix}\mathbf{q}'\mathbf{q}\mathbf{q}''\\ 0\,r\,0\end{pmatrix} = -\frac{\sqrt{2}\pi en_0}{1+a_{r\mathbf{q}'}}\left(\frac{\mu_{ik}q_iq_k''}{\varepsilon_{lm}q_l''q_m''}-\frac{\mu_{ik}q_iq_k'}{\varepsilon_{lm}q_l''q_m'}a_{r\mathbf{q}'}\right) \quad (2.23)$$

$$B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ 0\,r\,s\end{pmatrix} = \frac{\pi en_0}{(1+a_{r\mathbf{q}'})(1+a_{s\mathbf{q}''})}\left(\frac{\mu_{ik}q_iq_k'}{\varepsilon_{lm}q_l'q_m'}a_{r\mathbf{q}'}+\frac{\mu_{ik}q_iq_k''}{\varepsilon_{lm}q_l''q_m''}a_{s\mathbf{q}''}\right).$$

To calculate the elements $B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ r\,\lambda'\,\lambda''\end{pmatrix}$ we need the element

$$S_{r0} = \frac{\chi_r}{\sqrt{2}n_0}\frac{i\Omega_r\tau}{1+a_r}. \quad (2.24)$$

Then

$$B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ r\,\lambda'\,\lambda''\end{pmatrix} = \frac{\chi_r}{\sqrt{2}}\frac{i\Omega_r\tau}{1+a_r}B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ 0\,\lambda'\,\lambda''\end{pmatrix}. \quad (2.25)$$

The elements of the matrix B satisfy the relation

$$B\begin{pmatrix}-\mathbf{q}\,-\mathbf{q}'\,-\mathbf{q}''\\ -\lambda\,-\lambda'\,-\lambda''\end{pmatrix} = B^*\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ \lambda\,\lambda'\,\lambda''\end{pmatrix}, \quad (2.26)$$

which, as can be shown, holds in any approximation in the parameter χ. We then get $\xi_{-\lambda-\mathbf{q}} = \xi_{\lambda\mathbf{q}}^*$.

In concluding this section, we note that in the case of concentration nonlinearity, Eqs. (2.21) are valid for any (and not necessarily small) value of the amplitudes of the interacting waves.

3. DERIVATION OF THE EQUATIONS OF FLUCTUATION KINETICS

We shall characterize the fluctuations in the acoustic branch j by the intensity

$$I_{j\mathbf{q}} = \langle\xi_{j\mathbf{q}}^*(t)\xi_{j\mathbf{q}}(t)\rangle. \quad (3.1)$$

Our goal is to obtain a nonlinear equation describing the interaction of the fluctuations which the quantity $I_{j\mathbf{q}}$ must satisfy. In the derivation of the equation, we shall use a method proposed by Galeev and Karpman.[9] The essence of the method is that the change in the function $I_{j\mathbf{q}}$ in a time Δt brought about by the nonlinear interaction is considered. The time interval is, on the one hand, much greater than the sound period:

$$\Delta t \gg 1/\Omega_j, \quad (3.2)$$

and, on the other, much less than the damping time (or rise time) of the sound

$$\Delta t \ll 1/|\gamma_j|, \quad (3.3)$$

as well as the characteristic time in which the macrostate of the system is changed. Additional limitations imposed on the time interval Δt will be considered below.

The set of differential equations (2.21) with initial conditions

$$\xi_{\lambda\mathbf{q}}|_{t=0} = c_{\lambda\mathbf{q}} \quad (3.4)$$

is equivalent to the following set of integral equations:

$$\xi_{\lambda\mathbf{q}}(t) = c_{\lambda\mathbf{q}}\exp(-i\omega_{\lambda\mathbf{q}}t)$$
$$+\sum_{\lambda'\lambda''}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ \lambda\lambda'\lambda''\end{pmatrix}\exp(-i\omega_{\lambda\mathbf{q}}t)\int_0^t dt'\xi_{\lambda'\mathbf{q}'}(t')\xi_{\lambda''\mathbf{q}''}(t')\exp(i\omega_{\lambda\mathbf{q}}t'). \quad (3.5)$$

As in the theory of kinetics of fluctuations (see, for example,[9]), we shall assume that the phases of the complex quantities are distributely randomly, i.e., all the values of phase are equally probable. Then the calculation of the mean values reduces to averaging over these phases.

The change in the value of I_q in time Δt is equal to

$$\Delta I_{j\mathbf{q}} = \langle \xi_{j\mathbf{q}}{}^*(\Delta t)\xi_{j\mathbf{q}}(\Delta t)\rangle - \langle c_{j\mathbf{q}}{}^* c_{j\mathbf{q}}\rangle. \tag{3.6}$$

In order to separate the change of I_{jq} connected with the nonlinear interaction of the fluctuations, it is necessary to choose in (3.6) only those terms which contain the amplitude in third and higher powers. In the calculation of these terms, in accord with the inequality (3.3), we replace the factor $\exp(-\gamma_{jq}\Delta t)$ by unity. Then

$$(\Delta I_{j\mathbf{q}})_{\text{coll}} = \sum_{\lambda'\lambda''}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ \lambda\lambda'\lambda''\end{pmatrix}\int_0^{\Delta t} dt' \langle c_{j\mathbf{q}}{}^*\xi_{\lambda'\mathbf{q}'}(t')\xi_{\lambda''\mathbf{q}''}(t')\rangle$$
$$\times \exp(i\omega_{j\mathbf{q}}t') + \text{к.с.} + \sum_{\lambda'\lambda''}\sum_{\mu'\mu''}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ \lambda\lambda'\lambda''\end{pmatrix}$$
$$\times B^*\begin{pmatrix}\mathbf{q}\mathbf{k}'\mathbf{k}''\\ \lambda\mu'\mu''\end{pmatrix}\int_0^{\Delta t} dt'\int_0^{\Delta t} dt'' \langle \xi_{\lambda'\mathbf{q}'}(t')\xi_{\lambda''\mathbf{q}''}(t')\xi^*_{\mu'\mathbf{k}'}(t'')\xi^*_{\mu''\mathbf{k}''}(t'')\rangle$$
$$\cdot \exp[i\omega_{\lambda\mathbf{q}}(t'-t'')]. \tag{3.7}$$

We shall compute the right side of (3.7) in the lowest approximation in the small parameters χ and $\eta\omega/c$. For this it is sufficient to substitute for $\xi_{\lambda'q'}$ and $\xi_{\lambda''q''}$ the solutions of the set (3.5) with $\chi = \eta\omega/c = 0$. Then all $\xi_{rq}(r \neq 0)$ will satisfy linear equations whose solutions have the form

$$\xi_{r\mathbf{q}} = c_{r\mathbf{q}} \exp(-i\Omega_{r\mathbf{q}}t). \tag{3.8}$$

So far as the quantity ξ_{0q} is concerned, in a time $\lesssim \tau_q/(1 + q^2/\kappa^2)$ it ceases to depend on the initial conditions, as is seen from Eq. (2.19) for the frequency Ω_0. Therefore, if we set

$$\Delta t \gg \frac{\tau_{\mathbf{q}}}{1+q^2/\varkappa^2} \tag{3.9}$$

(as will be done in what follows), then we can assume ξ_{0q} to be independent of the initial conditions. Then the expression for this function is determined by the quantities ξ_{rq} only. For simultaneous satisfaction of the inequalities (3.3) and (3.9), satisfaction of the inequality $|\gamma_r\tau|/(1 + q^2|\kappa^2) \ll 1$ is necessary. This is a consequence of (2.8). Then

$$\xi_{0\mathbf{q}}(t) = \sum_{r'r''}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ 0\,r'\,r''\end{pmatrix} c_{r'\mathbf{q}'}c_{r''\mathbf{q}''}\frac{\exp[-i(\Omega_{r'\mathbf{q}'}+\Omega_{r''\mathbf{q}''})t]}{i(\Omega_{0\mathbf{q}}-\Omega_{r'\mathbf{q}'}-\Omega_{r''\mathbf{q}''})}$$
$$+ 2\sum_{r'}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ 0\,r'\,0\end{pmatrix} c_{r'\mathbf{q}'}\exp(-i\Omega_{0\mathbf{q}}t)\int_0^t \exp[i(\Omega_{0\mathbf{q}} -$$
$$- \Omega_{r'\mathbf{q}'})t']\xi_{0\mathbf{q}''}(t')dt' + \sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ 0\,0\,0\end{pmatrix}\exp(-i\Omega_{0\mathbf{q}}t)$$
$$\times \int_0^t \exp(i\omega_{0\mathbf{q}}t')\xi_{0\mathbf{q}'}(t')\xi_{0\mathbf{q}''}(t')dt', \tag{3.10}$$

$$(\Delta I_{j\mathbf{q}})_{\text{ст}} = 2\sum_{r'}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\,\mathbf{q}'\,\mathbf{q}''\\ j\,r'\,0\end{pmatrix}\int_0^{\Delta t}\langle c^*_{j\mathbf{q}}c_{r'\mathbf{q}'}\xi_{0\mathbf{q}''}(t')\rangle$$
$$\times \exp[i(\Omega_{j\mathbf{q}}-\Omega_{r'\mathbf{q}'})t']dt' + \sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ j\,0\,0\end{pmatrix}$$
$$\times \int_0^{\Delta t}\langle c_{j\mathbf{q}}{}^*\xi_{0\mathbf{q}'}(t')\xi_{0\mathbf{q}''}(t')\rangle \exp(i\Omega_{j\mathbf{q}}t')dt' + \text{c. c.} \tag{3.11}$$

We compute the expression (3.11) by the iteration method in lowest order in I_{rq}. From Eq. (3.10), we find, with accuracy to second order in c_{rq},

$$\xi^{(2)}_{0\mathbf{q}}(t) = \sum_{r'r''}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ 0\,r'\,r''\end{pmatrix}$$
$$\times c_{r'\mathbf{q}'}c_{r''\mathbf{q}''}\frac{\exp[-i(\Omega_{r'\mathbf{q}'}+\Omega_{r''\mathbf{q}''})t]}{i(\Omega_{0\mathbf{q}}-\Omega_{r'\mathbf{q}'}-\Omega_{r''\mathbf{q}''})}.$$

Substituting this approximation in (3.11), carrying out averaging, and dividing by Δt, we obtain the following expression for the change in the quantity I_{jq} per unit time as the result of the nonlinear interaction:

$$\left(\frac{\partial I_{j\mathbf{q}}}{\partial t}\right)_{\text{coll}} = 4I_{j\mathbf{q}}$$
$$\sum_{r'}\sum_{\mathbf{q}'+\mathbf{q}''=\mathbf{q}} \cdot \left\{\frac{B\begin{pmatrix}\mathbf{q}\;\mathbf{q}'\;\mathbf{q}''\\ j\;r'\;0\end{pmatrix}B\begin{pmatrix}\mathbf{q}''\;\mathbf{q}\;-\mathbf{q}'\\ 0\;j\;-r'\end{pmatrix}}{i(\Omega_{0\mathbf{q}''}+\Omega_{r'\mathbf{q}'}-\Omega_{j\mathbf{q}})} + \text{c.c.}\right\} I_{r'\mathbf{q}'}. \tag{3.12}$$

This expression is unusual in that the frequency $\Omega_{0q''}$ has a large imaginary part, thanks to which the frequency denominator cannot be replaced by a δ function. The appearance of such a non-Peierls collision operator, which can have any sign, is brought about by the existence of a strongly damped zero branch. This operator describes the processes of interaction of phonons in terms of the zero branch, in which the energy is not conserved. The terms of the detailed structure are also encountered in the theory of weakly turbulent plasma (see [15,16]).

The ordinary (Peierls) collision operator appears in our theory only in the next order in the small parameter χ. It arises under consideration in (3.7) of terms containing the square of the elements of $B\begin{pmatrix}\mathbf{q}\mathbf{q}'\mathbf{q}''\\ j\,\lambda'\lambda''\end{pmatrix}$. In order to give it the usual form, we write it down in terms of the number of phonons:

$$N_{\mathbf{q}} = \frac{n_0\vartheta}{\chi}\frac{T}{\hbar\Omega_{\mathbf{q}}}\frac{\varkappa^2}{q^2}I_{\mathbf{q}}, \tag{3.13}$$

$$\left(\frac{\partial N_{\mathbf{q}}}{\partial t}\right)_{\text{coll}} = \frac{\pi}{8}\frac{\chi^3\hbar}{Tn_0}\frac{1}{\vartheta}\sum_{\mathbf{q}'}\left\{\frac{(q^2/\varkappa^2)(q'^2/\varkappa^2)[(\mathbf{q}-\mathbf{q}')^2/\varkappa^2]\Omega_{\mathbf{q}}\Omega_{\mathbf{q}'}\Omega_{\mathbf{q}-\mathbf{q}'}}{(1+q^2/\varkappa^2)^2(1+q'^2/\varkappa^2)^2[1+(\mathbf{q}-\mathbf{q}')^2/\varkappa^2]^2}\right.$$
$$\times (N_{\mathbf{q}'}N_{\mathbf{q}-\mathbf{q}'} - N_{\mathbf{q}}N_{\mathbf{q}-\mathbf{q}'} - N_{\mathbf{q}}N_{\mathbf{q}'})\delta(\omega_{\mathbf{q}}-\omega_{\mathbf{q}'}-\omega_{\mathbf{q}-\mathbf{q}'})$$
$$+ 2\frac{(q^2/\varkappa^2)(q'^2/\varkappa^2)[(\mathbf{q}+\mathbf{q}')^2/\varkappa^2]\Omega_{\mathbf{q}}\Omega_{\mathbf{q}'}\Omega_{\mathbf{q}+\mathbf{q}'}}{(1+q^2/\varkappa^2)^2(1+q'^2/\varkappa^2)^2[1+(\mathbf{q}-\mathbf{q}')^2/\varkappa^2]^2} \tag{3.14}$$
$$\left.\times (N_{\mathbf{q}'}N_{\mathbf{q}+\mathbf{q}'} + N_{\mathbf{q}}N_{\mathbf{q}+\mathbf{q}'} - N_{\mathbf{q}}N_{\mathbf{q}'})\delta(\omega_{\mathbf{q}}+\omega_{\mathbf{q}'}-\omega_{\mathbf{q}+\mathbf{q}'})\right\}.$$

Here ϑ is the volume of the crystal. For simplicity, we have considered the case here in which only one branch is excited; therefore the index j is omitted. The expression (3.14) was obtained in the work of Mikoshiba for the case $\mathbf{q}' = 2\mathbf{q}$.[12]

From what has been pointed out, it is evident that, generally speaking, the Peierls collision operator (3.14) plays no role. In comparison with the non-Peierls operator (3.12), it could be appreciable only for such values of the parameters, (if such exist) for which the latter vanish. One could verify that, in the case of a concentration nonlinearity, this takes place close to $V = w$[3]), when a narrow sound beam is excited.

[3])It should be noted that corrections to the non-Peierls term exist of the same order in χ as (3.14). They lead to a small change in the value of the parameters for which (3.12) vanishes.

In what follows, we shall not consider this special case and shall therefore discard the term (3.14).

The complete equation of fluctuation kinetics has the form

$$\frac{\partial I_{j\mathbf{q}}}{\partial t}+\frac{\partial \Omega_{j\mathbf{q}}}{\partial \mathbf{q}}\frac{\partial I_{j\mathbf{q}}}{\partial \mathbf{r}}+\gamma_{j\mathbf{q}}I_{j\mathbf{q}} = -I_{j\mathbf{q}}\frac{\mathcal{V}}{(2\pi)^3}\sum_{j'}\int W\begin{pmatrix} j & j' \\ \mathbf{q} & \mathbf{q}'\end{pmatrix} I_{j'\mathbf{q}'}d^3q', \tag{3.15}$$

where

$$W\begin{pmatrix} j & j' \\ \mathbf{q} & \mathbf{q}'\end{pmatrix} = 2\sqrt{2}\chi_{j\mathbf{q}}\left\{\frac{\Omega_{j\mathbf{q}}\tau_{\mathbf{q}}}{1+a_{j\mathbf{q}}}\left[\frac{B\begin{pmatrix}\mathbf{q},\mathbf{q}',\mathbf{q}-\mathbf{q}'\\ 0,j',\ 0\end{pmatrix}B\begin{pmatrix}\mathbf{q}-\mathbf{q}',\mathbf{q},-\mathbf{q}'\\ 0,\ j,\ -j'\end{pmatrix}}{\Omega_{0\mathbf{q}-\mathbf{q}'}+\Omega_{j'\mathbf{q}'}-\Omega_{j\mathbf{q}}} + \frac{B\begin{pmatrix}\mathbf{q},-\mathbf{q}',\mathbf{q}+\mathbf{q}'\\ 0,-j',\ 0\end{pmatrix}B\begin{pmatrix}\mathbf{q}+\mathbf{q}',\mathbf{q},\mathbf{q}'\\ 0,\ j,\ j'\end{pmatrix}}{\Omega_{0\mathbf{q}+\mathbf{q}'}-\Omega_{j'\mathbf{q}'}-\Omega_{j\mathbf{q}}}\right]+\text{c.c.}\right\} \tag{3.16}$$

We assume that the intensity of the fluctuations depends on the spatial coordinate **r**. This means that wave packets are considered rather than plane waves (cf.[7], and also [17]).

It is necessary to add boundary and initial conditions to Eq. (3.15). The general rule is that one must make the solution of this equation continuous with the solution of the linear equation of fluctuation kinetics which was obtained in [4,5] and which is valid for a low level of fluctuations. As an example, let us consider the case of stationary fluctuations in a crystal bounded by the plane x = 0. We shall assume the drift velocity to be directed from the surface into the depth of the crystal. We shall assume the fluctuations to be increasing along the x axis, and neglect the waves reflected from the opposite boundary of the crystal. The linear equation of the fluctuation kinetics isolated in [4,5], has the form

$$\frac{\partial I_{j\mathbf{q}}}{\partial t}+\frac{\partial \Omega_{j\mathbf{q}}}{\partial \mathbf{q}}\frac{\partial I_{j\mathbf{q}}}{\partial \mathbf{r}}+\gamma_{j\mathbf{q}}I_{j\mathbf{q}} = \gamma_{j\mathbf{q}}^{(0)}I_{j\mathbf{q}}^{(0)}. \tag{3.17}$$

In the right side of this equation is the power of the fluctuation source. According to [4,5]

$$\gamma_{j\mathbf{q}}^{(0)} = \frac{1}{\rho}\eta^{(jj)}q^2+\chi_{j\mathbf{q}}\Omega_{j\mathbf{q}}^2\frac{\tau_{\mathbf{q}}}{|1+a_{j\mathbf{q}}|^2}, \qquad I_{j\mathbf{q}}^{(0)} = \frac{\chi_{j\mathbf{q}}}{n_0\mathcal{V}}\frac{q^2}{\varkappa^2}. \tag{3.18}$$

In the stationary case, the solution of (3.18) has the form

$$I_{j\mathbf{q}} = I_{j\mathbf{q}}(0)\exp\left(-\frac{\gamma_{j\mathbf{q}}x}{\partial\Omega_{j\mathbf{q}}/\partial q_x}\right)+I_{j\mathbf{q}}^{(0)}\frac{\gamma_{j\mathbf{q}}^{(0)}}{\gamma_{j\mathbf{q}}}\left[1-\exp\left(-\frac{\gamma_{j\mathbf{q}}x}{\partial\Omega_{j\mathbf{q}}/\partial q_x}\right)\right]. \tag{3.19}$$

We are now interested in waves whose wave vectors lie in the "cone of amplification" $\gamma_{j\mathbf{q}} < 0$. For what follows, it is important that, as the intensity of these fluctuations increases with increased separation from the surface of the crystal, a region exists where, first, the processes of creation of new fluctuations no longer play a role and only the amplification of "old" fluctuations is important, and second, the nonlinear effects are still negligibly small. The first of these conditions is satisfied for such x for which[4)]

$$\exp\left(-\frac{\gamma_{j\mathbf{q}}x}{\partial\Omega_{j\mathbf{q}}/\partial q_x}\right)\gg 1. \tag{3.20}$$

Actually, in this region, the quantity

$$I_{j\mathbf{q}} = \left[I_{j\mathbf{q}}(0)-\frac{\gamma_{j\mathbf{q}}^{(0)}}{\gamma_{j\mathbf{q}}}I_{j\mathbf{q}}^{(0)}\right]\exp\left(-\frac{\gamma_{j\mathbf{q}}x}{\partial\Omega_{j\mathbf{q}}/\partial q_x}\right) \tag{3.21}$$

satisfies Eq. (3.17) with the right side equal to zero.

In order to formulate the second condition, we introduce the characteristic value of the intensity of fluctuations I_{nl}, at which the nonlinear effects are already beginning to play a significant role. The order of magnitude of I_{nl} is determined from the condition that the nonlinear term in Eq. (3.15) be of the order of $\gamma_{j\mathbf{q}}I_{j\mathbf{q}}$, i.e., that the nonlinear damping be comparable with the linear one. Then

$$I_{nl}\sim\frac{1}{\mathcal{V}\varkappa^3}\frac{\gamma}{W}. \tag{3.22}$$

The estimate of W from (3.16) gives

$$I_{nl}\sim\frac{1}{\mathcal{V}\varkappa^3}\frac{\Omega}{\tau}\frac{1}{|B|^2}. \tag{3.22a}$$

The second characteristic feature of the region of values of x that are of interest is that $I_{j\mathbf{q}} \ll I_{nl}$, in it. Consequently, the condition for the existence is the inequality $I^0 \ll I_{nl}$, which, with the help of (3.18), takes on the form

$$\frac{\Omega}{\tau}\frac{1}{|B|^2}\gg\chi\left(\frac{4\pi e^2}{\varepsilon T}n_0^{1/3}\right)^{3/2}. \tag{3.23}$$

The latter inequality is satisfied in practice for all possible mechanisms of nonlinearity under the condition that

$$\left(\frac{4\pi e^2}{\varepsilon T}n_0^{1/3}\right)\ll 1. \tag{3.24}$$

We shall assume this condition to be satisfied. It will be shown below that it is one of the criteria of applicability of linear fluctuation theory.

Thus, at a sufficient distance from the boundary, we can neglect the source of thefluctuations and in place of the boundary condition for x = 0 we have the boundary condition at some plane $x = x_0$:

$$I_{j\mathbf{q}}(x_0) = \left[I_{j\mathbf{q}}(0)-\frac{\gamma_{j\mathbf{q}}^{(0)}}{\gamma_{j\mathbf{q}}}I_{j\mathbf{q}}^{(0)}\right]\exp\left(-\frac{\gamma_{j\mathbf{q}}x_0}{\partial\Omega_{j\mathbf{q}}/\partial q_x}\right). \tag{3.25}$$

The value of x_0 should satisfy the inequality (3.20). This means that for $x > x_0$ we can use Eq. (3.15), which is equally suitable both in the region in which the nonlinear effects are important and where they can be neglected.

In conclusion, we note the connection which exists (with accuracy up to terms of higher order in χ) between the intensity of the fluctuations $I_{j\mathbf{q}}$ and the sound energy density of the wave of branch j with vector **q**, which is determined in the following way:

$$W_{j\mathbf{q}} = \rho\left\langle\left|\frac{du_{j\mathbf{q}}}{dt}\right|^2\right\rangle. \tag{3.26}$$

By means of Eq. (2.22), we easily obtain

$$I_{j\mathbf{q}} = \chi_{j\mathbf{q}}\frac{q^2}{\varkappa^2}\frac{1}{Tn_0}W_{j\mathbf{q}}. \tag{3.27}$$

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4) For simplicity, we shall consider the case in which the ratio $\gamma_{j\mathbf{q}}^{(0)}/|\gamma_{j\mathbf{q}}| \approx \Omega/|\Omega - \mathbf{qV}|$ is of the order of unity. These estimates are easily extended to the general case, but the formulas become very cumbersome.

4. SOLUTION OF THE EQUATIONS OF FLUCTUATION KINETICS

Let us consider a typical experimental situation: 1) the voltage drop on the specimen is given and does not depend on I_q; 2) I_q changes only in one direction (the x axis), which for simplicity we shall consider to be the axis of symmetry of the crystal.

Let us determine the dependence of the drift field on x (cf[4-6]). The mean density of the total current $\langle j \rangle$ can be represented in the form of a sum of density of the ohmic current en_0V and the density of the sound-electric current j^{ac}. We shall define the latter as the difference between the values of the current in the presence and in the absence of sound for a given value of the electric field. From the condition

$$\frac{\partial}{\partial x}\langle j\rangle = 0 \tag{4.1}$$

we have

$$V = \overline{V} + \frac{\overline{j}^{ac} - j^{ac}}{en_0}. \tag{4.2}$$

Here

$$\overline{j}^{ac} = \frac{1}{L}\int_0^L j^{ac}dx, \qquad \overline{V} = \frac{\mu_{xx}}{L}\int_0^L E\,dx, \tag{4.3}$$

L is the length of the specimen. The mean value of the drift velocity depends only on the potential difference between the ends of the specimen and is thus determined by the experimental conditions.

In the lowest order in the intensity, we have

$$j^{ac} = \sum_j \frac{\mathcal{V}}{(2\pi)^3}\int F_j(\mathbf{q}) I_{j\mathbf{q}} d^3q. \tag{4.4}$$

For concentration nonlinearity, as follows from (2.4),

$$j^{ac} = -e\mu_{xx}\langle n'\partial\varphi / \partial x\rangle, \tag{4.5}$$

$$F_j(\mathbf{q}) = Tn_0\mu_{xx}q_x \frac{(\Omega_j - \overline{V}_l q_l)\tau}{(1+q^2/\varkappa^2)^2 + (\overline{V}_l q_l - \Omega_j)^2\tau^2}\frac{\varkappa^2}{q^2}. \tag{4.6}$$

Thus the problem of finding the distribution of the intensity of fluctuations in the specimen (in the case in which this intensity is not too large) reduces to the simultaneous solution of the set of equations (3.15), (4.2), and (4.4). To this set must be added the boundary condition (3.25).

As an example of effects to which the non-Peierls collision term can lead, we consider the problem of stationary fluctuations near the threshold of amplification, when only one branch is amplified. Substituting (4.2) and (4.4) in (3.15), we get the equation

$$w_x\frac{dI_\mathbf{q}}{dx} + \gamma_\mathbf{q} I_\mathbf{q} - I_\mathbf{q}\Big\{\frac{\mathcal{V}}{(2\pi)^3}\int P(\mathbf{q},\mathbf{q}')I_{\mathbf{q}'}d^3q' \tag{4.7}$$

$$-\frac{1}{L}\int_0^L dx\frac{\mathcal{V}}{(2\pi)^3}\frac{1}{en_0}\frac{\partial\gamma_\mathbf{q}}{\partial\overline{V}}\int F(\mathbf{q}')I_{\mathbf{q}'}d^3q'\Big\} = 0,$$

where

$$\mathbf{w} = \frac{\partial\Omega}{\partial\mathbf{q}}, \qquad P(\mathbf{q},\mathbf{q}') = W(\mathbf{q},\mathbf{q}') + \frac{1}{en_0}\frac{\partial\gamma_\mathbf{q}}{\partial\overline{V}}F(\mathbf{q}'). \tag{4.8}$$

γ_q in these formulas must be assumed to be a function of $\overline{V}$.

In cases of concentration nonlinearity it is easy to deduce qualitatively, from Eq. (4.7) the effect on the amplification of the sound-electric current. Inside the cone of amplification, $F(\mathbf{q}) < 0$. Let amplification take place at the beginning of the specimen, where the intensity of the fluctuations is small. Then the second component in (4.8) increases the amplification with growth of the fluctuations along the specimen, if the inequality $\partial\gamma_q/\partial\overline{V} < 0$, or $\overline{V}_q < \Omega + \tau^{-1}(1 + q^2/\kappa^2)$ is satisfied for the significant region of values of $\mathbf{q}$. In the case of the opposite inequality, this component decreases the amplification. From Eq. (4.7) it is also seen that the terms connected with the sound-electric current are by no means the only nonlinear terms affecting the amplification of the fluctuations.

The qualitative behavior of $I_q(x)$ depends on the sign of $P(\mathbf{q}, \mathbf{q}')$ in the significant range of values of $\mathbf{q}$. If $P(\mathbf{q}, \mathbf{q}') < 0$, then the nonlinear interaction plays a stabilizing role, limiting the total amplification. In the opposite case, the nonlinear effects increase the amplification to the point where terms of higher order in I_q begin to play a role. We shall not discuss the more complicated case, in which $P(q, q')$ is an alternating function.

For $P(\mathbf{q}, \mathbf{q}') < 0$, an interesting effect is possible—a contraction of the range of values of q in which amplification occurs. We shall illustrate this effect by the simplest example, in which $P(\mathbf{q}, \mathbf{q}') \equiv P = \text{const}$. Here Eq. (4.7) takes the form

$$w_x dI_\mathbf{q} / dx + (\tilde{\gamma}_\mathbf{q} - PI)I_\mathbf{q} = 0, \tag{4.9}$$

where

$$\tilde{\gamma}_\mathbf{q} = \gamma_\mathbf{q} + \frac{1}{L}\int_0^L dx\frac{\mathcal{V}}{(2\pi)^3}\frac{1}{en_0}\frac{\partial\gamma_\mathbf{q}}{\partial\overline{V}}\int F(\mathbf{q}')I_{\mathbf{q}'}d^3q', \tag{4.10}$$

$$I = \frac{\mathcal{V}}{(2\pi)^3}\int I_\mathbf{q}\, d^3q. \tag{4.11}$$

Integrating (4.9), we find

$$I_\mathbf{q} = I_\mathbf{q}(x_0)\exp\Big[-\frac{\tilde{\gamma}_\mathbf{q}}{w_x}(x-x_0) + \frac{P}{w_x}\int_{x_0}^x I(x')dx'\Big]. \tag{4.12}$$

For larger values of x, the value of I_q has a sharp maximum at the point $q = q_m$, where $\tilde{\gamma}_q$ reaches a minimum. We expand $\tilde{\gamma}_q$ in a series in powers of $\mathbf{q} - \mathbf{q}_m$ with accuracy to terms of second order

$$\tilde{\gamma}_\mathbf{q} = -a\Big[1 - \frac{(q_x - q_m)^2}{q_\parallel} - \frac{q_y^2 + q_z^2}{q_\perp^2}\Big], \qquad a > 0, \tag{4.13}$$

for $ax_0/w_x \gg 1$, we obtain

$$I(x) = C_m\frac{\mathcal{V}}{(2\pi)^3}q_\parallel q_\perp^2\Big(\frac{\pi w_x}{ax}\Big)^{3/2}\exp\Big[\frac{ax}{w_x} + \frac{P}{w_x}\int_{x_0} I(x')dx'\Big], \tag{4.14}$$

where

$$C_m = \Big[I_\mathbf{q}(0) - \frac{\gamma_\mathbf{q}^{(0)}}{\gamma_\mathbf{q}}I_\mathbf{q}^{(0)}\Big]_{\mathbf{q}=\mathbf{q}_m}. \tag{4.15}$$

The differential equation for $I(x)$ then follows:

$$\frac{dI}{dx} - \Big(\frac{a}{w_x} - \frac{3}{2x}\Big)I - \frac{P}{w_x}I^2 = 0. \tag{4.16}$$

Its solution is the following:

$$\frac{1}{I} = x^{3/2}\exp\Big(-\frac{ax}{w_x}\Big)\Big[\frac{1}{I_0}x_0^{-3/2}\exp\Big(\frac{ax_0}{w_x}\Big) - \frac{P}{w_x}\int_{x_0}^x\exp\Big(\frac{ax'}{w_x}\Big)\frac{dx'}{x'^{3/2}}\Big]. \tag{4.17}$$

Here $I_0 = I(x_0)$. The condition $I_q(x_0) \ll I_{nl}$ in the given case can be rewritten in the form

$$\frac{ax_0}{w} \ll \ln \frac{I_\infty}{I_0}, \tag{4.18}$$

where

$$I_\infty = a/|P|. \tag{4.19}$$

By virtue of (4.18), the nonlinear term in Eq. (4.16) can be neglected at $x = x_0$, and the following equation is obtained for the integration constant I_0:

$$I_0 = C_m \mathscr{V} q_\| q_\perp^2 \left(\frac{\pi w_x}{ax_0}\right)^{3/2} \exp\left(\frac{ax_0}{w_x}\right). \tag{4.20}$$

For large values of x, when

$$\frac{ax}{w_x} \gg \ln \frac{I_\infty}{I_0},$$

$I(x)$ approaches a constant limit equal to I_∞. Thus, for a sufficiently large distance from the beginning of the specimen $(x = 0)$, the total intensity of the sound beam $I(x)$ ceases to increase—the amplification is limited by nonlinear effects. For a determination of the asymptotic form of the phonon distribution function I_q one must find the asymptote of the integral appearing in (4.12). It is easiest to do this with the help of Eq. (4.14). As a result, we obtain

$$I_q = I_\infty \left(\frac{ax}{\pi w_x}\right)^{3/2} \frac{(2\pi)^3}{q_\| q_\perp^2 \mathscr{V}} \exp\left[-\frac{(q_x - q_m)^2}{q_\|^2} ax - \frac{q_y^2 + q_z^2}{q_\perp^2} ax\right]. \tag{4.21}$$

This expression shows that with increasing x, the distribution function $I_q(x)$ describes a sound beam which is continuously contracting in **q** space. The contraction continues until the beam reaches the boundary of the crystal or until the present theory ceases to be applicable (see the next section).

In the case of concentration nonlinearity, $W(\mathbf{q}, \mathbf{q}')$ is an alternating function. Its form is so complicated that we shall not write it out, for lack of space and necessity. As an illustration, we write down its value for $\mathbf{q} \parallel \mathbf{q}' \parallel \mathbf{V}$, $q = q' = \kappa$:

$$W(\varkappa, \varkappa) = \chi \Omega_\varkappa \frac{(V - w)\varkappa\tau}{[4 + (V-w)^2 \varkappa^2 \tau^2]^3} \frac{36[1 - 2(V-w)^2 \varkappa^2 \tau^2]}{25 + 4(V-w)^2 \varkappa^2 \tau^2}. \tag{4.22}$$

It is then seen that for small values of the difference $V - w$, we have $P > 0$. This corresponds to a nonlinear amplification of the fluctuations.

In conclusion of this section, we enumerate several types of problems whose solution can be obtained with the help of the equation of fluctuation kinetics.

1. Finding the spectral composition of the fluctuations (i.e., function I_q) with account of their nonlinear interaction and also, if this is necessary, account of waves reflected from the boundary of the crystal.

2. Determination of the form of the volt-ampere characteristic in the initial part, after the bend, in the presence of a sound instability (far from the bend, the intensity of the fluctuations can increase, so that it is not possible for it to be limited by the lowest terms of the expansion in I_q in the expressions (3.12) and (4.4)).

3. Investigation of the stability of the stationary distribution of the fluctuations and construction of a theory of current oscillations passing through a piezoelectric.

4. Construction of a theory of interaction of two sound beams and a study of the possibilities of the experimental determination of the function $W(\mathbf{q}, \mathbf{q}')$.

There is still one more important problem: the interaction of the sound signal and sound noise. For its solution, further development of the theory is necessary.

5. LIMITS OF APPLICABILITY OF THE THEORY

We shall first point out the criterion (3.24), which is necessary for the applicability of the linear fluctuation theory, although it was not mentioned in[4,5]. Actually, from the linearized equations (4.1b) and (4.1c) of[4], for $\beta = 0$, we can determine the mean square fluctuation of the electron concentration. Comparing the results of these calculations with the results of the thermodynamic approach[18] for the computation of the same quantity, we see that the linearized equations are applicable only upon satisfaction of the inequality (3.24).

One of the criteria of applicability of the nonlinear theory has already been pointed out in the derivation of Eq. (3.15):

$$\gamma / \Omega \ll 1. \tag{5.1}$$

A similar criterion exists also for the nonlinear interaction. It should change slightly the amplitude of the wave over the time of a single period. If we mean by τ_n the characteristic time of change of the amplitude because of the nonlinear interaction, then this condition leads to the inequality

$$\Omega_q \tau_n \gg 1. \tag{5.2}$$

The time τ_n is determined by the relation

$$\frac{1}{\tau_n} = \frac{\mathscr{V}}{(2\pi)^3} \left| \int W(\mathbf{q}, \mathbf{q}') I_{q'} d^3 q' \right|. \tag{5.3}$$

In the case of concentration nonlinearity near the amplification threshold, when there is a narrow sound beam, we get, with the help of (4.26)

$$\chi(V - w)\varkappa\tau \frac{\mathscr{V}}{(2\pi)^3} \int I_q d^3 q \ll 1. \tag{5.4}$$

In the derivation of Eq. (3.15), we have carried out an expansion in the fluctuation intensity. One can show that, for example, for $\Omega\tau \ll 1$ this expansion is permissible if $|n'|/n_0 \ll 1$. By means of (3.10), one easily obtains the corresponding condition on the intensity

$$\frac{\mathscr{V}}{(2\pi)^3} \int I_q d^3 q \ll 1.$$

There exists still another group of criteria connected with the random-phase approximation that has been used. The fact that such criteria can play an important role is seen from the following consideration. Equation (3.15) allows us to study the amplification of a narrow sound beam (by the width of the beam we mean the value of the region δq occupied by the beam in **q** space), i.e., of an almost monochromatic wave. On the other hand, the problem of the amplification of a monochromatic sound wave was solved by another method, for which the phase of this wave was assumed to be a completely determined quantity.[10] These two methods give significantly different results. The fact is that these methods have different, and in a well known sense, contradictory limits of applicability, and the width of the beam enters into the corresponding criteria.

In constructing the method of iteration with respect to the amplitude for a monochromatic wave,[10] it was assumed that resonance terms are absent from the constraining force in second order in the amplitude and hence secular terms are also absent from the solution. Here the dispersion of the sound velocity, a measure of which is the difference

$$(\Delta q)_d = \frac{\omega(2\mathbf{q}) - 2\omega(\mathbf{q})}{w},$$

must exceed the beam width δq:[5)]

$$(\Delta q)_d \gg \delta q. \qquad (5.5)$$

The second criterion is connected with the fact that the nonlinear correction to the wave vector $(\Delta q)_{nl}$, found by means of the iteration method, must be greater than δq:

$$(\Delta q)_{nl} \gg \delta q. \qquad (5.6)$$

In the case considered by us of a weakly nonlinear interaction we have $(\Delta q)_{nl} \ll (\Delta q)_d$ and therefore it is actually necessary to take into account only the inequality (5.6).

We shall now make clear when the narrow beam can be described by means of the equations of fluctuation kinetics, i.e., when the random phase approximation is valid. For iterations of weakly damped (sound) waves by means of the random phase method,[9] it is necessary that even the second approximation include solution terms proportional to Δt, i.e., the criterion (5.5) is replaced by the opposite:

$$(\Delta q)_d \ll \delta q. \qquad (5.7)$$

This same criterion can be formulated differently: one can show that it is the condition of the vanishing of the argument of the δ function. In our case, when weakly damped sound waves interact with the strongly damped wave of the zero branch, such a criterion is not needed: the damping of the zero branch is so large that the terms of second approximation, which increase with time, appear even in the case of comparatively strong dispersion. The criterion (5.7) is necessary also in our case, when the Peierls collision operator plays a role.

It is evident that a second criterion exists under all circumstances, in a certain sense the inverse of (5.6):[19]

$$\delta\Omega\tau_n \gg 1, \qquad \delta\Omega = \frac{\partial\Omega}{\partial q}\delta q. \qquad (5.8)$$

The meaning of this criterion is the following. In order that the phases of the waves be regarded as uncorrelated, they must be completely randomized after a characteristic time of nonlinear interaction. The random phase results after a time of the order of $1/\delta\Omega$.

We note that the contraction of the sound beam, considered in the previous section, will take place according to Eq. (4.25) under the criterion (5.8) is violated. Then the approximation of random phase becomes inapplicable. To track the subsequent behavior of the sound beam is one of the most interesting and, as yet unresolved problems of fluctuation kinetics.

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5)The criterion (5.5) is written for the case of a weakly nonlinear interaction, a measure of which, as has been pointed out,[10] is the ratio of the "nonlinearity" to the "dispersion." In the case of a strong nonlinear interaction, the dispersion can be neglected[19] and the corresponding criterion is somewhat different (see[20]).

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