POLARIZATION OF DIELECTRICS UNDER SHOCK LOAD

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The problem of the current in a short-circuited capacitor containing a dielectric is solved by assuming isotropy of the dielectric and of its polarization on the shock wave front. It is also assumed that the matter is nonconducting ahead of the shock wave front and becomes conducting behind the front. The density and dielectric constant of the matter experiences an abrupt change. The solutions presented[1-4] can be derived from the equations as particular cases.

It is shown in a number of papers[1,5-11] that shock compression of dielectrics is accompanied by their polarization. This effect is revealed experimentally by the appearance of current in an electric circuit having as one of the elements the dielectric under consideration. The question of the polarization mechanism is not fully clear, but it is obvious that the main effect is not connected with electronic or ionic displacements whose characteristic relaxation times are 10^{-15} – 10^{-13} sec. The relaxation times in the processes under consideration amount to 10^{-9} – 10^{-6} sec and more[6,10]. The polarization of the matter occurs when the shock-wave front (SWF) passes through it.

An investigation of the shock polarization processes yields information on the macroscopic characteristics of matter that is strongly compressed by a SWF - the dielectric constant, conductivity, and time of establishment of thermodynamic equilibrium. In turn, these properties of matter, when obtained simultaneously with the magnitude and the sign of the polarization on the SWF, can yield interesting information on the microstructure of the matter.

For a phenomenological description of the shock polarization process we shall assume that the dielectric is isotropic and consists of polar molecules which are oriented along the flow of the material on going through the SWF. Actually the nature of the polarization can be due also to other mechanisms, such as vacancy diffusion, charged impurities, diffusion, etc.

Behind the SWF the matter is in a non-equilibrium state and relaxes to equilibrium but with different thermodynamic parameters. A distinction must be made between two relaxation mechanisms:

1. The mechanism of thermal disorientation of the dipoles. This is the mechanical relaxation considered by Allison[2]. The time of this relaxation will be denoted by τ.

2. The mechanism of conductivity relaxation, having a time θ. This mechanism was considered by Zel'dovich[4]. It results from the fact that free carriers in the material can neutralize the oriented dipoles (surface or volume bound charges).

The existing theories[1-4] describe the experimental time dependences of the polarization current only in certain particular cases (for more details see[10]). We present below a solution for a current in a short-circuited polarization-pickup circuit with allowance for both relaxation mechanisms.

FORMULATION OF PROBLEM

Figure 1a illustrates a dielectric in which a SWF propagates with a velocity D. The SWF divides the dielectric into two regions - uncompressed and compressed matter (1 and 2 respectively). The matter behind the SWF is characterized by a constant dielectric constant ϵ_2, a volume resistivity ρ, a mass velocity u, and a compression δ. The corresponding quantities ahead of the SWF are ϵ_1, $\rho_1 = \infty$, and $u = 0$. The matter ahead and behind the SWF is isotropic. Since the circuit is closed, its time constant is zero. In this sense the problem is quasistationary.

The dielectric on the SWF is polarized to a value p_0 in the direction of motion of the matter or in the opposite direction. Since the problem is one-dimensional, the SWF, as any other surface parallel to it, is equipotential. Therefore the compressed matter bounded by the planes $x = a$ (a - initial thickness of the dielectric) and $x = dt$ can be regarded as a capacitor with capacitance

$$C_1 = \varepsilon_1[4\pi(a - Dt)]^{-1}. \tag{1}$$

(We shall solve the problem for a unit SWF area.)

We shall show that the matter behind the SWF bounded by the planes $x = ut$ and $x = Dt$ can also be represented in the form of a capacitor. To this end we consider some layer of matter of thickness x_0 with $\epsilon = \text{const}$ and arbitrary specified polarization distribution $P(x)$. The potential difference between the boundaries of such a layer is

$$V = \frac{4\pi}{\varepsilon}\int_0^{x_0} P(x)\,dx = \sigma C^{-1},$$

$$\sigma = \frac{1}{x_0}\int_0^{x_0} P(x)\,dx, \tag{2}$$

where σ is equal to the average value of the polarization, and the expression for $C = \epsilon(4\pi x_0)^{-1}$ coincides with the formula for a parallel-plate capacitor.

Thus, the capacitance of the layer of matter behind the SWF can be written in the form

$$C_2 = \varepsilon_2\delta(4\pi Dt)^{-1}. \tag{3}$$

Starting from this, we can reduce the problem represented by Fig. 1a at the instant of time t to the equivalent circuit of Fig. 1b where $R = \delta^{-1}\rho\,Dt$ is the resistance of the compressed matter, and C_1 and C_2 are given by (1) and (3).

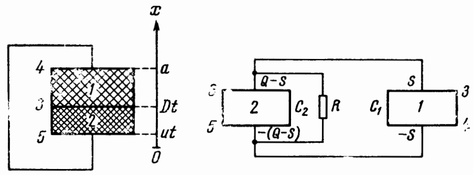

FIG. 1. a – formulation of problems, b – equivalent circuit. 1 and 2 – regions of uncompressed and compressed dielectric; 3 – SWF; 4 and 5 – stationary and moving boundaries of the dielectric. The direction of the x axis coincides with the direction of motion of the SWF and of the substance.

The condition for the equality of the voltages across C_1 and C_2 causes a charge $-S$ to flow from C_1 by the instant of time t, such as to satisfy the relation

$$(Q-S)C_1 = SC_2, \tag{4}$$

where Q is the total charge on the upper electrodes of C_1 and C_2 in the circuit of Fig. 1b. The current j in the external circuit (polarization current) is defined as the time derivative of the charge S, namely $j = dS/dt$.

SOLUTION OF PROBLEM

The quantity Q in (4) is determined by the following processes: 1) shock ionization; 2) mechanical relaxation (τ); 3) conductivity relaxation (θ). Whereas the first process increases the charges in the system under consideration, the second and third decrease them.

We calculate the change of the charge dQ during the time dt.

1) The increase of the charge (dQ_1). Since process occurs only in C_2 as a result of polarization of the additional layers of the dielectric, it follows that dQ_1 can be easily obtained by opening the circuit at the instant of time t (we note that this procedure does not change the final result). During the time dt the SWF will cover a path Ddt, and the lower electrode of the capacitor will move by an amount udt. The charge on C_2 is

$$Q-S = C_2V. \tag{5}$$

Hence,

$$dQ_1 = C_2dV + VdC_2. \tag{6}$$

Substituting here the value of dC_2 obtained from (3), we get

$$dQ_1 = (Q-S)(V^{-1}dV - t^{-1}dt). \tag{7}$$

The quantity dV is the increment of the potential difference due to the polarization of the layer of matter in the time dt:

$$dV = 4\pi D(P_0-S)(\varepsilon_2\delta)^{-1}dt. \tag{8}$$

Using (5) and (3), we get

$$V^{-1}dV = \frac{P_0-S}{Q-S}\frac{dt}{t}. \tag{9}$$

Substituting (9) in (7) we get finally

$$dQ_1 = (P_0-Q)t^{-1}dt. \tag{10}$$

2) The decrease of the charges in the system of Fig. 1b due to mechanical relaxation will be denoted dQ_2, with $dQ_2 = -p\tau^{-1}\,dt$, where P is the average value of the shock polarization. According to[2], $P = P_0\tau t^{-1}[1 - e^{-t/\tau}]$. Therefore

$$dQ_2 = -P_0[1-e^{-t/\tau}]t^{-1}dt. \tag{11}$$

3) Let us calculate the loss of charges due to conductivity (dQ_3) behind the SWF. The quantity

$$dQ_3 = -R^{-1}Vdt \tag{12}$$

is determined by the conduction current, which depends linearly on the voltage. For the system 1b we have $V = C_2^{-1}(Q-S)$ and $R = \delta^{-1}\rho Dt$. Hence,

$$dQ_3 = -(Q-S)\theta^{-1}dt \quad (\theta = \varepsilon_2\rho/4\pi). \tag{13}$$

Summing (10), (11), and (13) we obtain the change in the charge in the system of two capacitors within a time dt:

$$dQ/dt = t^{-1}[P_0e^{-t/\tau} - Q] - \theta^{-1}(Q-S). \tag{14}$$

Differentiating (4) and comparing the result with (14), we obtain the initial differential equation for the charge S:

$$\frac{dS}{dt} + S\left[\frac{1-\varkappa}{\varkappa T+(1-\varkappa)t} + \frac{1}{\theta}\frac{\varkappa(T-t)}{\varkappa T+(1-\varkappa)t}\right] = \frac{P_0e^{-t/\tau}}{\varkappa T+(1-\varkappa)t}, \tag{15}$$

where $\kappa = \epsilon_1^{-1}\epsilon_2\delta$ and $T = aD^{-1}$. For $\kappa = 1$, this equation takes the simpler form:

$$\frac{dS}{dt} + S\frac{T-t}{\theta T} = P_0T^{-1}e^{-t/\tau}. \tag{16}$$

The solution of (16) with initial conditions $S = 0$ at $t = 0$ is

$$j = P_0T^{-1}e^{-t/\tau}\left[1 - \frac{T-t}{\theta T}\exp\left(-\frac{t}{\lambda}+\frac{t^2}{2\theta T}\right) \times \int_0^t \exp\left(\frac{t}{\lambda}-\frac{t^2}{2\theta T}\right)dt\right], \tag{17}$$

where $\lambda^{-1} = \theta^{-1} - \tau^{-1}$.

The solution of (15) is of the form[1)]:

$$j = \frac{P_0e^{-t/\tau}}{\varkappa T+(1-\varkappa)t}\left\{1 - \frac{\theta^{-1}\varkappa(T-t)+(1-\varkappa)}{[\varkappa T+(1-\varkappa)t]^{\varphi}} \times e^{t/\mu}\int_0^t [\varkappa T+(1-\varkappa)t]^{\varphi-1}e^{-t/\mu}dt\right\}, \tag{18}$$

where

$$\varphi = 1 + \frac{\varkappa T}{\theta(1-\varkappa)^2}, \quad \mu = \frac{\tau\theta(1-\varkappa)}{\theta+\varkappa(\tau-\theta)}.$$

Thus, Eqs. (17) and (18) solve our problem. When $t = 0$ we always have $j_0 = P_0(\kappa T)^{-1}$, that is, the initial current does not depend on the relaxation process and is inversely proportional to the initial thickness of the dielectric.

SOME CONCLUSIONS

A. Let us consider some particular cases. Putting $\tau = \infty$ in (17), we get

1)The particular solution (17) can be obtained from the general solution (18) by a rather cumbersome evaluation of the indeterminate quantities that arise in (18).

$$j = P_0 T^{-1}\left[1 - \frac{T-t}{\theta T}\exp\left(-\frac{t}{\theta} + \frac{t^2}{2\theta T}\right)\int_0 \exp\left(\frac{t}{\theta} - \frac{t^2}{2\theta T}\right) dt\right], \quad (19)$$

which agrees with the solution obtained by Zel'dovich [4].

Putting $\theta = \infty$ in (18) we get

$$j = P_0 \frac{[\varkappa T + (1-\varkappa)t]\, e^{-t/\tau} - \tau(1-\varkappa)[1 - e^{-t/\tau}]}{[\varkappa T + (1-\varkappa)t]^2}. \quad (20)$$

Expression (20) coincides with the Allison's solution. If we now put in (20) $\tau = \infty$ and $\delta = 1$, then we arrive at the equation described in [1]. Finally, the obtained solutions go over into the solutions obtained in [3] if we impose definite limitations on τ and θ in the present paper and on the impedance of the circuit in [3].

B. As follows from (17) and (18), in the general case the density of the polarization current is a function of a number of parameters—ρ, ϵ_2, τ, and P_0. Therefore the theory can be verified if the values of the indicated parameters are known beforehand. Unfortunately, the present available literature contains individual information only on the values of ρ for certain substances [9,12,13]. The comparison made in [6] of the results of an experiment on organic glass and polystyrene with Allison's theory is apparently not quite correct, particularly at high pressures, where no account was taken of the possible influence of the conductivity.

C. Figure 2 shows a number of curves calculated by means of formulas (17) and (18) at certain combinations of the parameters. A comparison of these curves with the experimental time dependences of the polarization current, obtained with quartz [1], organic glass [5,6], epoxy resin and water [5], ionic crystals [7-11], and also polybutylmethacrylate and the explosive TNT (Fig. 3), shows a qualitative agreement of the curves. We took into account here the fact that the value of the polarization current at t = 0 cannot be determined experimentally [10].

D. The solution for the polarization-pickup circuit was obtained under the assumption that there is only one mechanism of polarization of the substance by the shock wave front. At the same time, the fact of anomalous polarization in ionic crystals and, in some cases, the reversal of the sign of the polarization current during the passage of SWF through the sample 2) [10,11] can be attributed to the existence of at least two independent processes of polarization with opposite signs. Each of these processes should be characterized by its own values of τ and P_0 [11].

In conclusion the authors consider it their present duty to thank Ya. B. Zel'dovich for critical remarks and interest in their work.

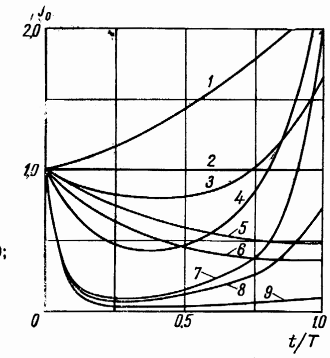

FIG. 2. Plots of $j/j_0 = f(t/T)$. The values of the parameters t/T and θ/T are respectively: 1 — 1.5; ∞; ∞; 2 — 1.0; ∞; ∞; 3 — 2.0; 1.0; 1.0; 4 — 2.0; 5.0; 0.2: 5 — 0.9; 2.0; 2.0; 6 — 1.0; 1.0; 1.0; 7 — 2.0; 5.0; 0.05; 8 — 1.0; 5.0; 0.05; 9 — 2.0; 0.05; 5.0

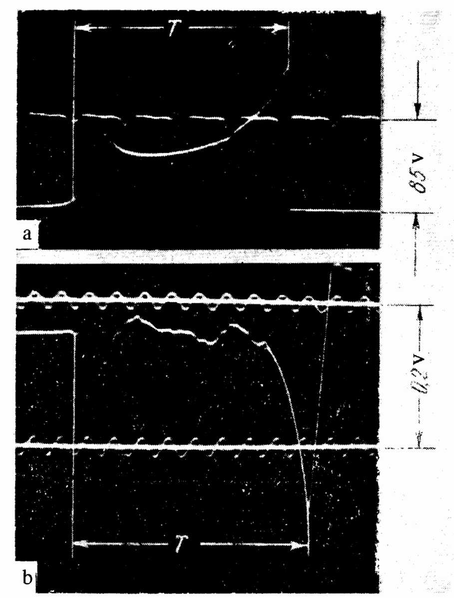

FIG. 3. Oscillograms of the polarization current from the samples: a – polybutylmethacrylate (pressure p ≈ 180 kbar); b – TNT (p = 190 kbar) time scale – 0.1 μsec.

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2) The corresponding oscillograms in [11] are not described by the given solution.

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