SUPERFLUIDITY OF EXCITONS IN SEMICONDUCTORS

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The problem of distribution of exciton concentration in a pure semiconductor sample is considered in the case of local exciton injection. It is shown that at temperatures below the Bose condensation temperature the exciton distribution differs considerably from the diffusion distribution. In some conditions imposed by the geometry of the sample the depth of penetration of excitons into the semiconductor considerably exceeds the diffusion length. It is suggested that this phenomenon can be used for an experimental confirmation of Bose condensation of excitons.

1. It was shown in [1,2] that large radius excitons in semiconductors, at concentrations n such that $nr_e^3 \ll 1$ (r_e—exciton radius), can form a Bose condensate. In this connection, interest attaches to a clarification of the experimentally observed effects to which Bose condensation of excitons can lead. In semiconductors in which the extrema of the valence band and of the conduction band correspond to the same value of quasimomentum (for example in GaAs), Bose condensation of the excitons causes the appearance of negative light absorption at a frequency close to the exciton-absorption line [1]. We can point to another phenomenon due to the existence of Bose condensate of excitons. Since the spectrum of the elementary excitations becomes acoustic in the presence of the low-density condensate, such a system has the property of superfluidity, which can become manifest for example in the following manner. If excitons are created at some place of the semiconductor, then in the absence of a condensate they will penetrate inside the crystal to a distance on the order of the diffusion length $L_d = \sqrt{D\tau}$, where D is the diffusion coefficient and τ is the lifetime of the exciton. Since in Ge at $T < 10^\circ K$ we have $D \sim 100$ cm^2/sec and $\tau \sim 10^{-4}$ sec [1], we get $L_d \sim 10^{-1}$ cm.

It can be expected that after the transition into the superfluid state the excitons can propagate over a distance on the order of $L = v_{cr}\tau$, where v_{cr} is the critical velocity at which superfluidity becomes destroyed. The length L can turn out to be much larger than the diffusion length. Then the registration of excitons at such large distances from the point of their injection will serve as experimental proof of the existence of the exciton condensate. The calculation presented below confirms the correctness of these qualitative considerations. The indicated effect, unlike the negative absorption of light predicted in [1], arises in superconductors with indirect transitions, such as Ge or Si.

2. A superfluid gas is characterized by a macroscopic wave function $\Psi = \sqrt{n_s}\, e^{i\varphi}$ where n_s is the concentration and $\hbar m^{-1} \nabla\varphi = v_s$ is the velocity of the superfluid component. When T = 0 [1)] we have $n_s = n$, where n is the total concentration of the excitons. The function Ψ satisfies the differential equation [3,4,5]

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + \alpha\Psi + \beta|\Psi|^2\Psi - \frac{i\hbar}{2\tau}\Psi, \qquad (1)$$

Here m - exciton mass, $\alpha + \beta|\Psi|^2 = \alpha + \beta n$ - chemical potential of the excitons, with $\beta = 4\pi\hbar^2 f/m$, where f is the amplitude of exciton scattering by one another. The term $i\hbar\Psi/2\tau$, where τ is the exciton lifetime, takes into account the exciton annihilation. Substituting Ψ in the form $\Psi = \sqrt{n}\, e^{i\varphi}$ in (1), we get equations for the quantities n and $v = \hbar m^{-1}\nabla\varphi$:

$$\frac{\partial n}{\partial t} = -\operatorname{div}(nv) - \frac{n}{\tau},$$

$$m\frac{\partial v}{\partial t} = -\nabla\left\{-\frac{\hbar^2}{2m}\frac{\Delta\sqrt{n}}{\sqrt{n}} + \frac{mv^2}{2} + \beta n\right\}. \qquad (2)$$

The first equation in (2) represents the continuity equation with allowance for exciton annihilation, and the second determines the change of the macroscopic velocity with time.

Let us consider a plate of sufficiently pure semiconductor of length $l = l_1 + l_2$, in the left side of which ($-l_2 < x < 0$) there are produced in some manner g excitons per unit volume per second. We assume the surface of the plate to be so treated that it is possible to neglect the annihilation of the excitons on the surface compared with the volume annihilation. Therefore the exciton flux to the surface is equal to zero and the problem becomes one-dimensional. In addition, we neglect in (2) the quantum correction term, which is proportional to $\hbar^2$, since the concentration of the excitons turns out to be a sufficiently slowly varying function of the coordinates. Then, under stationary conditions, the equations (2) take the form

$$\frac{d}{dx}(nv) = -\frac{n}{\tau} + g\,\theta(-x), \qquad \frac{mv^2}{2} + \beta n = \alpha';$$

$$\theta(-x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x < 0 \end{cases}; \qquad (3)$$

α' is the integration constant. The boundary conditions are

$$nv|_{x=l_1} = nv|_{x=-l_2} = 0. \qquad (4)$$

Expressing v in terms of n, we get

$$\frac{d}{dx}\left(n\sqrt{1 - \frac{\beta}{\alpha'}n}\right) = -\frac{n - n_0\theta(-x)}{L}, \qquad (5)$$

where the characteristic length is $L = \tau\sqrt{2\alpha'/m}$, and $n_0 = g\tau$.

[1)] For simplicity we consider the system only at T = 0. The obtained results are qualitatively valid at finite temperatures that are not too close to the temperature of the Bose condensation T_C.

Equation (5) is valid only at velocities

$$v = \sqrt{\frac{2\alpha'}{m}\left(1-\frac{\beta}{\alpha'}n\right)},$$

that are lower than critical. The critical velocity connected with the creation of ring vortices is

$$v_{cr} = \frac{\hbar}{Rm}\ln\frac{R}{a},$$

where R is the radius of the vortex and a is the radius of its core. Assuming that a is of the same order as the exciton radius (10^{-6} cm) and R is equal to the thickness of the film, which can be of the order of 10^{-4} cm, we get $v_{cr} \approx 6 \times 10^4$ cm/sec. The velocity of sound in the condenstate, on the other hand, $v_s = \sqrt{\beta n/m}$, amounts to 10^6 cm/sec at $n \sim 10^{16}$ cm^{-3} [1]. Since the root in (5) is simply $v/\sqrt{2}v_s$, and $v < v_{cr} < v_s$, Eq. (5) can be solved approximately. As a result we get

$$n = \frac{\alpha'}{\beta}\left\{1-\left(\frac{x-l_1}{L}\right)^2\right\} \quad \text{if } x>0,$$

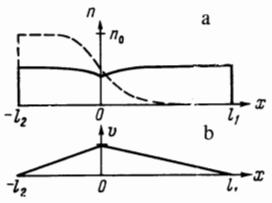
$$n = \frac{\alpha'}{\beta}\left\{1-\left(\frac{x+l_2}{L}\right)^2\right\} \quad \text{if } x<0, \qquad (6)$$

where $\widetilde{L} = L/(\beta n_0/\alpha' - 1)$. Making these solutions continuous at x = 0, we determine the constant α', and the exciton concentration turns out to be (see the figure)

$$n = n_0\frac{l_2}{l_1+l_2}\left\{1-\left(\frac{x-l_1}{L}\right)^2\right\}. \qquad (7)$$

The maximum value of the velocity, which is equal to l_1/τ, is reached at the section x = 0. It is clear that with increasing length l_1 an increase takes place in the number of excitons annihilating in the region $0 < x < l$. Therefore the flux increases and with it the velocity of the excitons in the cross section x = 0. Finally, when $l_1 > v_{cr}\tau$, the velocity necessary to maintain the exciton concentration stationary at the origin should be larger than critical. At such velocities the condition of superfluidity is violated and the solution obtained by us is no longer valid. Thus, if the sample length l_1 is smaller than $v_{cr}\tau$ and larger than the diffusion length, then a transition of the excitons to the superfluid state corresponds to a sharp increase of their concentration on the edge of the sample opposite to the injector.

3. Let us clarify now several problems connected with the realization of such an experiment. The effective source of excitons may be a semiconducting sandwich structure consisting of injecting contacts of type p and n and a layer of pure semiconductor between them. When current is made to flow in the forward direction, the electrons and holes are injected into the layer of pure material and recombine with one another.[2] The time of binding of the electron and hole into an exciton and the time of relaxation of the exciton energy are much shorter than the exciton lifetime [1]. Therefore the distribution of the excitons with respect to



Distribution of exciton concentration along the length of the sample (a): solid curve–$T \ll T_C$, dashed–$T > T_C$. Dependence of exciton flux velocity (b) on the coordinate at $T \ll T_C$.

energy barely differs from equilibrium. Simple estimates show that when a pure Ge or Si semiconductor layer has a thickness on the order 10^{-4} cm and the current density in it is 10^{-2} A/cm^2, a stationary exciton concentration on the order of 10^{17} cm^{-3} is obtained. At such concentrations, the temperature of the Bose condensate turns out to be of the order of 1°K.

Let us see whether the heat released in the nonradiative annihilation of the exciton leads to heating of the sample above the critical temperature. In sufficiently pure Ge or Si, at helium temperatures, the phonon mean-free path is 10^{-1} cm. If the thickness of the sample is much smaller than this quantity, the phonons produced upon annihilation of the excitons readily reach the boundary of the sample, where they give up their energy. Equating the power released in the volume by exciton annihilation, which is equal to $E_g n/\tau$ (E_g–width of the forbidden band of the semiconductor), to the energy flux carried away by the phonons to the walls, we get

$$T^4 = \frac{30\hbar^3}{\pi^2 k^4}\frac{E_g n}{\tau}\frac{d}{(2/u_t^2 + 1/u_l^2)}, \qquad (8)$$

where d is the thickness of the semiconductor and u_l and u_t are the longitudinal and transverse sound velocities. For example, for Ge we have $E_g = 0.7$ eV, $u_l = 6 \times 10^5$ cm/sec, $u_t = 3 \times 10^5$ cm/sec, $n = 10^{17}$ cm^{-3}, and $\tau = 10^{-4}$ sec. Therefore, even in the continuous mode we get T = 1°K. The heating can be greatly reduced in the pulsed mode.

In obtaining the foregoing results we neglected crystal defects. However, all the conclusions concerning the superfluid behavior of the excitons remain valid also at sufficiently small impurity concentrations N, such that the uncertainty in the exciton momentum due to the scattering by impurities, $\hbar N\sigma$ (σ-cross section for the scattering by the impurity) is much smaller than the characteristic momentum mv_s which is connected with the interaction of the excitons with one another. This requirement leads to the condition

$$Nf^3 \ll \frac{4\pi f^2}{\sigma}\sqrt{nf^3}.$$

At realistically attainable impurity concentrations in Ge, namely, $N \sim 10^{12}$–10^{13} cm^{-3}, this condition is satisfied with the a large margin, even if the excitons are scattered by ionized impurities and the cross section σ exceeds $4\pi f^2$.

Let us now discuss the influence of the impurities on the exciton lifetime. As is well known [6], impurities with shallow levels do not make a contribution to the recombination of the electrons and holes, and consequently also excitons. As to their recombination via impurities with deep levels, it must be borne in mind here that the electron and hole connected with the large-radius exciton interact with these impurities

[2] The exciton detector on the opposite end of the sample may be a structure of the p-i-n type, biased in the inserse direction. The exitons entering the structure under the influence of the field decay into electrons and holes and this increases the inverse current through the structure.

practically as if they were free. The cross section for the capture of the electron and of the hole by the impurity are usually quite different. Therefore, the impurity will capture primarily the exciton-forming particle having the larger cross section. The other particle can either be captured on the Bohr orbit of large radius next to this impurity, or else go over into the band. In the former case, the particle captured on the Bohr orbit recombines quite rapidly, returning the impurity to the initial state. The lifetime of the exciton is then determined by the larger of the captured cross section. Recombination in the second case does not differ in any way from the recombination of free electrons and holes with concentrations equal to concentration of the excitons, which is much larger in our case than the impurity concentration N. Then, according to [6], the lifetime will be determined by the smaller capture cross section. When $N \sim 10^{12}$ cm^{13}, the longest and the shortest lifetimes will be respectively [6] 10^{-3} and 10^{-4} sec. Thus, the foregoing estimates remain valid also when recombination of the excitons on the impurities is taken into account.

In conclusion we note that the results of this paper pertain to an equal degree also to the case of Bose condensation of exciton molecules, with obvious replacement of the corresponding parameters.

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