MAGNETIC STRUCTURE OF THIN MAGNETIC FILMS AND FERROMAGNETIC RESONANCE

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A static and dynamic theory of fine magnetic structure ("ripples" of magnetization) in ferromagnetic films is developed by a method of canonical expansions of random functions. The probability characteristics of such a structure are determined, viz., the dispersion and spectral density of the correlation function (distribution function of magnetization wave lengths). The effect of the fine magnetic structure on FMR is considered. It is shown that the magnetic structure results in a shift, broadening, and asymmetry of the FMR line. The effective relaxation parameter contains in this case a part that depends explicitly on the frequency like $\sim\omega^{-3/4}$.

INTRODUCTION

THE main feature of thin magnetic polycrystalline films (in addition to their "two-dimensionality") is the small dimension of the crystallites, $2b \sim 10^{-6}$ cm. The effective radius of the manifestation of the volume interaction is $\delta \sim (\alpha/\beta_c)^{1/2}$, where $\alpha \sim 10^{-12}$ cm^2 is the exchange-interaction constant and β_c is the dimensionless constant of the crystallographic anisotropy. If $b \gg \delta$ (which is valid for most of bulky polycrystals), then in the absence of an external magnetic field, or if the external magnetic field is sufficiently small, the magnetization $\mathbf{M}$ in each crystallite is established along the easy axis of the anisotropy of the corresponding crystallite. Thus, the function $\mathbf{M}(\mathbf{r})$ duplicates the distribution function of the easy axes $\rho(\mathbf{r})$, with accuracy up to the degeneracy of the latter). Of course, on the boundaries between the crystallites, the exchange (and magnetostatic) interaction will lead to the formation of structures of the interdomain boundary type; however, owing to the smallness of the volume in which the deviation of the orientation of $\mathbf{M}(\mathbf{r})$ from $\rho(\mathbf{r})$ is observed, this effect has no strong influence on the physical properties of the polycrystal.

With decreasing dimensions of the crystallite, the exchange interaction between the crystallites increases, the magnetizations of the neighboring crystallites tend to become established parallel to each other, in spite of the opposite action of the crystallographic anisotropy. This gives rise to a unique magnetic structure with spatial oscillations of the magnetization $\mathbf{M}$ about a certain mean direction $\mathbf{M}_0$; with further decrease of b, the amplitude of these oscillations will decrease and the polycrystal will become ever closer in its magnetic properties to a uniaxial single crystal. A thin polycrystalline film is precisely such a unique magnetic material.

The spatial oscillations ("ripple") of the magnetization in ferromagnetic films were observed experimentally by many authors. The most rigorous and consistant theory of such a fine magnetic structure was developed by Hoffman [1].

It is of interest to see the effects that allowance for the thin magnetic structure produces in ferromagnetic resonance (FMR). However, the method used by Hoffman to calculate the static magnetic structure is not convenient for this purpose. In this connection, in Sec. 1 of this paper we develop a static theory of a fine magnetic structure on the basis of the method of canonical expansions of random functions. In addition to producing the necessary basis for going over to the theory of dynamic effects, this makes it possible to obtain certain new results. In Sec. 2 we investigate by the same method the influence of a fine magnetic structure on FMR.

The usual microscopic domain structure is disregarded in this paper. All the results are valid either for a single-domain film or for sections inside a microscopic domain.

1. STATIC THEORY OF THE FINE MAGNETIC STRUCTURE

The equation of state of the system is the Landau-Lifshitz equation in the static case

$$[\mathbf{M}\mathbf{H}^{(e)}] = 0, \tag{1.1*}$$

where

$$H_i^{(e)} = -\frac{\partial \mathcal{H}}{\partial M_i} + \frac{\partial}{\partial x_j}\left[\partial\mathcal{H}/\partial\left(\frac{\partial M_i}{\partial x_j}\right)\right].$$

The Hamiltonian of the system is chosen in the form

$$\mathcal{H} = \frac{1}{2}\alpha\frac{\partial \mathbf{M}}{\partial x_i}\frac{\partial \mathbf{M}}{\partial x_i} - \frac{1}{2}\beta(\mathbf{M}\mathbf{n})^2 + \frac{\mathbf{H}^2}{8\pi} - \frac{1}{2}\beta_c(\mathbf{M}\mathbf{l})^2 \tag{1.2}$$

and accordingly, the effective magnetic field is

$$\mathbf{H}^{(e)} = \alpha\nabla^2\mathbf{M} + \beta\mathbf{n}(\mathbf{n}\mathbf{M}) + \mathbf{H} + \beta_c\mathbf{l}(\mathbf{l}\mathbf{M}). \tag{1.3}$$

In these expressions, the first term describes the exchange interaction; the second describes the uniaxial anisotropy β, which is common to the entire film, and $\mathbf{n}$ is the unit vector of the easy axis; the third term describes the external magnetic field and the demagnetizing field, which should be determined from the solution of the boundary-value problem of magnetostatics; the last term describes the uniaxial crystallographic anisotropy β_c, $\mathbf{l} = \mathbf{l}(\mathbf{r})$ is the unit vector of the easy axis, which has different directions in different crystallites.

The following remarks should be made with respect

*$[\mathbf{MH}] \equiv \mathbf{M}\times\mathbf{H}$.

to the last term of (1.2) and (1.3), which is the main term in our analysis. Ferromagnetic metals in thin-film states have as a rule triaxial and not uniaxial crystallographic symmetry. It would therefore be necessary to write in place of this term a term describing triaxial anisotropy. The calculation for the triaxial anisotropy involves no difficulty in principle, but leads to cumbersome expressions. On the other hand, the results of interest to us will differ only by a numerical coefficient ~ 1, which can be readily taken into account.

Let the external constant magnetic field $\mathbf{H}_0$ and the vector $\mathbf{M}_0$ be directed along the easy axis $\mathbf{n}$, which is parallel to the x axis; the z axis is normal to the plane of the film. Then the linearized equations for the vector $\mathbf{m} = \mathbf{M}/M_0$ together with the equations for the magnetostatic potential φ form a complete system

$$\alpha\nabla^2 m_y - (h + \beta_c\rho_{yy}) m_y - \frac{1}{M_0}\frac{\partial\varphi}{\partial y} + \beta_c\rho_{yz} m_z = -\beta_c\rho_{xy},$$

$$\alpha\nabla^2 m_z - (h + \beta_c\rho_{zz}) m_z - \frac{1}{M_0}\frac{\partial\varphi}{\partial z} + \beta_c\rho_{yz} m_y = -\beta_c\rho_{xz},$$

$$\nabla^2\varphi = 4\pi M_0\left(\frac{\partial m_y}{\partial y} + \frac{\partial m_z}{\partial z}\right), \qquad (1.4)$$

where

$$h = \beta + H_0/M_0, \quad \rho_{ik} = l_i l_k \ (i \neq k), \quad \rho_{ii} = l_x^2 - l_i^2,$$

and all the $\rho_{ik} = \rho_{ik}(x, y, z)$ are known stationary random functions of coordinates. Their probability characteristics can be easily determined from the distribution function of the vector $\mathbf{l}$ in a spherical coordinate system

$$f(\mathbf{l}) = f(\theta, \varphi) = \frac{1}{4\pi}\sin\theta. \qquad (1.5)$$

Consequently the mathematical expectation values, the dispersions, and the correlation moments for ρ_{ik} are

$$\mathscr{M}[\rho_{ik}] = 0, \quad \mathscr{D}[\rho_{ik}]_{i\neq k} = \mathscr{D}_0 = {}^1/_{15}, \quad \mathscr{D}[\rho_{ii}] = 4\mathscr{D}_0,$$

$$\mathscr{K}[\rho_{ii}\rho_{kk}] = 2\mathscr{D}_0, \quad \mathscr{K}[\rho_{ii}\rho_{jk}]_{j\neq k} = 0, \qquad (1.6)$$

and the autocorrelation functions and the mutual-correlation functions are equal to the corresponding $\mathscr{D}$ and $\mathscr{K}$ multiplied by a certain function of $\xi = \mathbf{r}' - \mathbf{r}$, approximately equal to unity in the interval $-b < \xi_i < b$ and equal to zero outside this interval; more detailed information concerning this function are not needed.

Assuming that $\mathbf{m}$ does not depend on z (this is valid if the characteristic wavelength of the magnetization is much larger than the thickness of the film), averaging the first two equations of the system (1.4) with respect to z, and taking Fourier transforms with respect to x and y, we obtain for the Fourier transforms the system

$$(\alpha\varkappa^2 + h)\hat{m}_y + \frac{ik_2}{M_0}\langle\hat{\varphi}\rangle + \beta_c[g(\rho_{yy}m_y) - g(\rho_{yz}m_z)] = \beta_c\hat{\rho}_{xy},$$

$$(\alpha\varkappa^2 + h)\hat{m}_z + \frac{1}{M_0}\left\langle\frac{\partial\hat{\varphi}}{\partial z}\right\rangle + \beta_c[g(\rho_{zz}m_z) - g(\rho_{yz}m_y)] = \beta_c\hat{\rho}_{xz},$$

$$\frac{\partial^2\hat{\varphi}}{\partial z^2} - \varkappa^2\hat{\varphi} = 4\pi M_0 ik_2\hat{m}_y, \qquad (1.7)$$

where

$$\varkappa^2 = k_1^2 + k_2^2, \quad g(\rho_{ik}m_j) = \frac{1}{(2\pi)^2}\int\int\rho_{ik}m_j e^{-i(k_1x+k_2y)}dx\,dy, \qquad (1.8)$$

and all the ρ_{ik} and $\hat{\rho}_{ik}$ are averaged over z, which can be taken into account by multiplying their $\mathscr{D}$ and $\mathscr{K}$ by the factor b/d.

Solving the third equation of the system (1.7) with the usual conditions for the continuity of the potential on the surface of the film, and substituting the solution in the first two equations, we get

$$\left[\alpha\varkappa^2 + h + 4\pi\frac{k_2^2}{\varkappa^2}(1 - V)\right]\hat{m}_y + \beta_c[g(\rho_{yy}m_y) - g(\rho_{yz}m_z)] = \beta_c\hat{\rho}_{xy},$$

$$[\alpha\varkappa^2 + h + 4\pi V]\hat{m}_z + \beta_c[g(\rho_{zz}m_z) - g(\rho_{yz}m_y)] = \beta_c\hat{\rho}_{xz}, \qquad (1.9)$$

where $V = (1 - e^{-2\kappa d})/2\kappa d$, and 2d is the thickness of the film.

We neglect all the $g(\rho_{ik}m_j)$; the limits of applicability of such an approximation will be considered later. Then the system (1.9) breaks up into two independent equations, from which we can determine $\hat{m}_y$ and $\hat{m}_z$:

$$\hat{m}_y = \beta_c\hat{\rho}_{xy} / [\alpha\varkappa^2 + h + 4\pi k_2^2\varkappa^{-2}(1 - V)], \qquad (1.10)$$

$$\hat{m}_z = \beta_c\hat{\rho}_{xz} / [\alpha\varkappa^2 + h + 4\pi V]. \qquad (1.10a)$$

As expected, owing to the large demagnetizing field perpendicular to the plane of the film, $\hat{m}_z \ll \hat{m}_y$ and is of no further interest to us in this section.

Let us determine the probability characteristics of the solution for m_y. The mathematical expectations $\mathscr{M}[\hat{m}_y] \sim \mathscr{M}[\rho_{xy}] = 0$. The spectral density of the correlation function for the function m_y is

$$S_m(k_1, k_2) = \beta_c^2 S_\rho(k_1, k_2) / [\alpha\varkappa^2 + h + 4\pi k_2^2\varkappa^{-2}(1 - V)]^2. \qquad (1.11)$$

The spectral density of the correlation function for the function ρ_{xy}, which enters here, is given by the expression

$$S_\rho(k_1, k_2) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_\rho(\xi, \eta)e^{-i(k_1\xi + k_2\eta)}d\xi d\eta, \qquad (1.12)$$

where the correlation function K_ρ, in accordance with the remark made above, can be approximated, for example, by the expression

$$K_\rho \approx \mathscr{D}_0 \exp\left(-\frac{|\xi|}{b} - \frac{|\eta|}{b}\right). \qquad (1.13)$$

For long waves ($k_1 b \ll 1$, $k_2 b \ll 1$), regardless of the concrete form of K_ρ, we have

$$S_\rho(k_1, k_2) \approx (b/\pi)^2\mathscr{D}_0, \qquad (1.14)$$

i.e., it represents the spectral density of white noise.

Taking into account the averaging over z, we obtain finally

$$S_m(k_1, k_2) = \beta_c^2\frac{b}{d}\left(\frac{b}{\pi}\right)^2\mathscr{D}_0[\alpha\varkappa^2 + h + 4\pi k_2^2\varkappa^{-2}(1 - V)]^{-2}. \qquad (1.15)$$

The most complete characteristic of the solution, namely the correlation function for m_y, is given by the relations

$$K_m(\xi, \eta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S_m(k_1, k_2)e^{i(k_1\xi + k_2\eta)}dk_1dk_2, \qquad (1.16)$$

and the dispersion is

$$\mathscr{D}_m = K_m(0, 0) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S_m(k_1, k_2)\,dk_1dk_2. \qquad (1.17)$$

The integration in the last expression was performed approximately: V was expanded in a series in $\kappa d < 1$, and subsequently we used the fact that the major contribution to the integral is made only by $k_2 \ll k_1$. As a result, we obtain the expression

$$\mathcal{D}_m = 0.42\pi^{-3/2}\beta_c^2\mathcal{D}_0(b/d)^{3/2}(b^2/\alpha h)^{1/4}. \quad (1.18)$$

The mean square deviation $|m| = \mathcal{D}_m^{1/2}$ corresponds to the expression obtained earlier by Hoffman [1], and differs only by an inessential numerical coefficient and by the factor b/d, which was taken into account in the averaging over z.

We did not calculate $K_m(\xi, \eta)$, since the function $S_m(k_1k_2)$ itself is more convenient for comparison with experiment. Indeed, $S_m(k_1, k_2)$ is proportional to the two-dimensional distribution function of the wave numbers, determined for the positive values of k_1 and k_2:

$$f(k_1, k_2) = \frac{4}{\mathcal{D}_m} S_m(k_1, k_2), \quad \int\limits_0^\infty\int\limits_0^\infty f(k_1, k_2)\, dk_1 dk_2 = 1. \quad (1.19)$$

The distribution function of the wave number k_1

$$f(k_1) = \int\limits_0^\infty f(k_1, k_2)\, dk_2 \approx \frac{\beta_c^2 b^3 \mathcal{D}_0}{2(\pi d)^{3/2}\mathcal{D}_m} \frac{\sqrt{k_1}}{(\alpha k_1^2 + h)^{3/2}}, \quad (1.20)$$

has a maximum at a wavelength

$$\lambda_{1m} = 2\pi / k_{1m} = 2\pi(5\alpha/h)^{1/4}. \quad (1.21)$$

In magnetooptic polarization experiments at a suitable experimental geometry, the intensity of the light $I \sim m_y$. Therefore in such experiments $\mathcal{D}_m$ and $f(k_1, k_2)$ can be measured directly. In electron-optical experiments $I \sim |\partial m/\partial x|$ and the distribution function of the wave numbers $f_e(k_1, k_2) \sim k_1^2 f(k_1, k_2)$ has a maximum at a wavelength

$$\lambda_{1m}{}^e = 2\pi(\alpha/5h)^{1/4}, \quad (1.22)$$

which is $1/5$ of λ_{1m}. Thus, the pictures of the magnetization wavelengh distributions, obtained from magneto-optical and electron-optical experiments, should be significantly different.

Let us find the limits of applicability of the employed approximation. Since $m_z \ll m_y$, we write down the first equation of the system (1.9) in the form

$$B(k)\hat{m}_y + \beta_c g(\rho_{yy}m_y) = \beta_c\hat{\rho}_{xy}, \quad (1.23)$$

where

$$B(k) = \alpha\varkappa^2 + h + 4\pi k_2^2\varkappa^{-2}(1 - V).$$

Substituting here m_y in the form of a series in β_c:

$$\hat{m}_y = \beta_c\hat{m}_0 + \beta_c^2\hat{m}_1 \ldots, \quad (1.24)$$

we get

$$\hat{m}_0 = \hat{\rho}_{xy}/B(k), \quad \hat{m}_1 = g(\rho_{yy}m_0)/B(k). \quad (1.25)$$

From the convolution theorem we get

$$g(\rho_{yy}m_0) = \int dk'\hat{\rho}_{yy}(k-k')\hat{m}_0(k') = \int\frac{dk'}{B(k')}\hat{\rho}_{yy}(k-k')\hat{\rho}_{xy}(k'). \quad (1.26)$$

The mathematical expectation is $\mathcal{M}[m_1] \sim \mathcal{K}[\rho_{yy}\rho_{xy}] = 0$; calculating S_{m1} and then $\mathcal{D}[m_1]$ we get

$$\mathcal{D}[m_1] \lesssim \frac{8}{\beta_c^4}\mathcal{D}_m^2, \quad (1.27)$$

where $\mathcal{D}_m$ is determined by (1.18). From the requirement $\beta_c^4\mathcal{D}[m_1] \ll \beta_c^2\mathcal{D}[m_0]$ we get the condition under which the approximation connected with neglecting the functions $g(\rho_{ik}m_j)$ in the system (1.9) is valid:

$$8\mathcal{D}_m \ll 1. \quad (1.28)$$

2. DYNAMIC THEORY OF THE FINE MAGNETIC STRUCTURE. FERROMAGNETIC RESONANCE

The equation of motion of the system is the Landau-Lifshitz equation

$$\dot{M} = g[MH^{(e)}] - \frac{\xi}{M_0}[M\dot{M}], \quad (2.1)$$

where g is the gyromagnetic ratio and ξ is a dimensionless relaxation parameter. The effective magnetic field $H^{(e)}$ is defined as before by (1.3), but H now includes also high-frequency magnetic fields.

Let us consider the same geometry as in the first section. After linearization, going over to Fourier components, and calculating the demagnetizing fields, and averaging over the thickness of the film, the system of equations for $m = M/M_0$ takes the form

$$\frac{1}{gM_0}(\dot{\hat{m}}_y - \xi\dot{\hat{m}}_z) = A(k)\hat{m}_z + \beta_c[g(\rho_{zz}m_z) - g(\rho_{yz}m_y)] - \beta_c\hat{\rho}_{xz},$$
$$\frac{1}{gM_0}(\dot{\hat{m}}_z + \xi\dot{\hat{m}}_y) = -B(k)\hat{m}_y - \beta_c[g(\rho_{yy}m_y) - g(\rho_{yz}m_z)] + \frac{\hat{H}_y}{M_0} + \beta_c\hat{\rho}_{xy}, \quad (2.2)$$

where

$$A(k) = \alpha\varkappa^2 + h + 4\pi V,$$
$$B(k) = \alpha\varkappa^2 + h + 4\pi k_2^2\varkappa^{-2}(1 - V),$$

$\hat{H}_y$ is the Fourier component of the external high frequency magnetic field.

Representing $m(r, t)$ in the form

$$m(r, t) = m(r) + \mu(r, t), \quad (2.3)$$

we obtain for $m(r)$ the system (1.9), which was investigated in the first section, and for $\mu(r, t)$ we obtain the system

$$\frac{1}{gM_0}(\dot{\hat{\mu}}_y - \xi\dot{\hat{\mu}}_z) - A(k)\hat{\mu}_z - \beta_c[g(\rho_{zz}\mu_z) - g(\rho_{yz}\mu_y)] = 0,$$
$$\frac{1}{gM_0}(\dot{\hat{\mu}}_z + \xi\dot{\hat{\mu}}_y) + B(k)\hat{\mu}_y + \beta_c[g(\rho_{yy}\mu_y) - g(\rho_{yz}\mu_z)] = \frac{\hat{H}_y}{M_0}, \quad (2.4)$$

which describes (in the linear approximation) all the dynamic properties of the fine magnetic structure.

Let the system be acted upon by a high frequency field

$$H_y = a\cos\omega t, \quad \hat{H}_y = a\cos\omega t\cdot\delta(k), \quad \delta(k) = \delta(k_1, k_2). \quad (2.5)$$

replacing $\cos\omega t$ by $\exp(i\omega t)$, we get for $\mu \sim \exp(i\omega t)$

$$i\sigma\hat{\mu}_y - [A(k) + i\xi\sigma]\hat{\mu}_z = \beta_c g(\rho_{zz}\mu_z) - \beta_c g(\rho_{yz}\mu_y),$$
$$[B(k) + i\xi\sigma]\hat{\mu}_y + i\sigma\hat{\mu}_z = -\beta_c g(\rho_{yy}\mu_y) + \beta_c g(\rho_{yz}\mu_z) + r\delta(k), \quad (2.6)$$

where $\sigma = \omega/gM_0$ $r = a/M_0$.

"Solving" the system with respect to $\hat{\mu}_y$ and $\hat{\mu}_z$, substituting in the form of a series in powers of β_c,

$$\mu = \mu_0 + \beta_c\mu_1 + \beta_c^2\mu_2 + \ldots \quad (2.7)$$

and equating the terms of equal powers of β_c, we obtain for the zeroth approximation

$$\hat{\mu}_{y0}=\frac{\tilde{A}(\mathbf{k})}{\Delta(\mathbf{k})}r\delta(\mathbf{k}),\qquad \hat{\mu}_{z0}=\frac{i\sigma}{\Delta(\mathbf{k})}r\delta(\mathbf{k}), \tag{2.8}$$

and for all the succeeding approximations the recurrence relations

$$\begin{aligned}\hat{\mu}_{yn}&=\frac{i\sigma}{\Delta(\mathbf{k})}[g(\rho_{zz}\mu_{z\,n-1})-g(\rho_{yz}\mu_{y\,n-1})]\\&-\frac{\tilde{A}(\mathbf{k})}{\Delta(\mathbf{k})}[g(\rho_{yy}\mu_{y\,n-1})-g(\rho_{yz}\mu_{z\,n-1})],\\ \hat{\mu}_{zn}&=-\frac{i\sigma}{\Delta(\mathbf{k})}[g(\rho_{yy}\mu_{y\,n-1})-g(\rho_{yz}\mu_{z\,n-1})]\\&-\frac{\tilde{B}(\mathbf{k})}{\Delta(\mathbf{k})}[g(\rho_{zz}\mu_{z\,n-1})-g(\rho_{yz}\mu_{y\,n-1})],\end{aligned} \tag{2.9}$$

where

$\Delta(\mathbf{k})=\tilde{A}(\mathbf{k})\tilde{B}(\mathbf{k})-\sigma^2,\ \tilde{A}(\mathbf{k})=A(\mathbf{k})+i\xi\sigma,\ \tilde{B}(\mathbf{k})=B(\mathbf{k})+i\xi\sigma.$

Let us find the mathematical expectations of the terms of the series (2.7). The zeroth-order terms do not contain random functions and their mathematical expectations are equal to $\hat{\mu}_{y0}$ and $\hat{\mu}_{z0}$, respectively. The first-order terms are proportional to ρ_{ik} and their mathematical expectations are equal to zero. The mathematical expectations of the second-order terms are proportional to terms of the type $\mathscr{M}[g(\rho_{ik}m_{j1})]$; let us consider one such term:

$$\begin{aligned}\mathscr{M}[g(\rho_{yy}\mu_{y1})]&=\int d\mathbf{k}'\mathscr{M}[\hat{\rho}_{yy}(\mathbf{k}-\mathbf{k}')\hat{\mu}_{y1}(\mathbf{k}')]\\&=-\frac{r}{\Delta_0}\int\frac{d\mathbf{k}'}{\Delta(\mathbf{k}')}\{\sigma^2\mathscr{M}[\hat{\rho}_{yy}(\mathbf{k}-\mathbf{k}')\hat{\rho}_{zz}(\mathbf{k}')]\\&+\tilde{A}_0\tilde{A}(\mathbf{k}')\mathscr{M}[\hat{\rho}_{yy}(\mathbf{k}-\mathbf{k}')\hat{\rho}_{yy}(\mathbf{k}')]\\&+i\sigma[\tilde{A}_0-\tilde{A}(\mathbf{k}')]\mathscr{M}[\hat{\rho}_{yy}(\mathbf{k}-\mathbf{k}')\hat{\rho}_{yz}(\mathbf{k}')]\}\\&=-\frac{r}{\Delta_0}\int\frac{d\mathbf{k}'}{\Delta(\mathbf{k}')}\{\sigma^2S_{\rho_{yy}\rho_{zz}}(\mathbf{k}')\}+\tilde{A}_0\tilde{A}(\mathbf{k}')S_{\rho_{yy}\rho_{yy}}(\mathbf{k}')\\&+i\sigma[\tilde{A}_0-\tilde{A}(\mathbf{k}')]S_{\rho_{yy}\rho_{yz}}(\mathbf{k}')\}\delta(\mathbf{k}-2\mathbf{k}')=-\frac{r}{4\Delta_0\Delta_2}\Big\{\sigma_2S_{\rho_{yy}\rho_{zz}}\Big(\frac{\mathbf{k}}{2}\Big)\\&+\tilde{A}_0\tilde{A}_2S_{\rho_{yy}\rho_{yy}}\Big(\frac{\mathbf{k}}{2}\Big)+i\sigma(\tilde{A}_0-\tilde{A}_2)S_{\rho_{yy}\rho_{yz}}\Big(\frac{\mathbf{k}}{2}\Big)\Big\}.\end{aligned} \tag{2.10}$$

where

$$\Delta_0=\tilde{A}_0\tilde{B}_0-\sigma^2,\ \tilde{A}_0=\tilde{A}(0),\ \tilde{B}_0=\tilde{B}(0),$$
$$\Delta_2=\tilde{A}_2\tilde{B}_2-\sigma^2,\ \tilde{A}_2=\tilde{A}(\mathbf{k}/2),\ \tilde{B}_2=\tilde{B}(\mathbf{k}/2).$$

We similarly calculate all the terms of the type $\mathscr{M}[g(\rho_{ik}m_{j1})]$. According to (1.6) and (1.12),

$$S_{\rho_{yy}\rho_{yy}}\Big(\frac{\mathbf{k}}{2}\Big)=S_{\rho_{zz}\rho_{zz}}\Big(\frac{\mathbf{k}}{2}\Big)=4S_2,\quad S_{\rho_{yy}\rho_{zz}}\Big(\frac{\mathbf{k}}{2}\Big)=2S_2,$$
$$S_{\rho_{yy}\rho_{yz}}\Big(\frac{\mathbf{k}}{2}\Big)=S_{\rho_{zz}\rho_{yz}}\Big(\frac{\mathbf{k}}{2}\Big)=0, \tag{2.11}$$

where S_2, with allowance for averaging over z, is determined (for $\kappa b\ll 1$) by the expression

$$S_2\approx\frac{b}{d}\Big(\frac{b}{\pi}\Big)^2\mathscr{D}_0. \tag{2.12}$$

Finally, putting for symmetry $\Delta(\mathbf{k})=\Delta_1$, $\tilde{A}(\mathbf{k})=\tilde{A}_1$, and $\tilde{B}(\mathbf{k})=\tilde{B}_1$, we get

$$\mathscr{M}[\hat{\mu}_{y2}]=\frac{rS_2}{4\Delta_0\Delta_1\Delta_2}[\sigma^2(\tilde{A}_0+\tilde{A}_1+\tilde{A}_2+4\tilde{B}_2)+\tilde{A}_0\tilde{A}_1(\tilde{B}_2+4\tilde{A}_2)]. \tag{2.13}$$

We obtain $\mathscr{M}[\mu_{z2}]$ analogously.

Thus, a homogeneous high-frequency magnetic field excites both the homogeneous magnetization precession described by the terms (2.8), and inhomogeneous modes whose mathematical expectations (accurate to second-order terms) are described by the terms $\beta_e^2\mathscr{M}[\hat{\mu}_{y2}]$ and $\beta_c^2\mathscr{M}[\mu_{z2}]$.

The effective complex susceptibility of the system, which is observed in the ordinary geometry of ferromagnetic resonance, is determined by the expression

$$\chi_{yy}=\chi'-i\chi''=\frac{M_0}{a}\int d\mathbf{k}\mathscr{M}[\hat{\mu}_y(\mathbf{k})]. \tag{2.14}$$

The imaginary part of the susceptibility χ'' in the approximation

$$A_1\approx A_2\approx A_0=4\pi+h,\ \ B_0=h\ll A_0,\ \ \sigma^2\sim A_0B_0\ll A_0^2$$

is determined by the expression

$$\chi''\approx\frac{\varepsilon}{x^2+\varepsilon^2}\Big\{1+\beta_c^2\int d\mathbf{k}S_2T(\mathbf{k})\Big\}, \tag{2.15}$$

where

$$T(\mathbf{k})=\frac{(B_1-P)(B_2-P)+x(B_1+B_2-2P)-\varepsilon^2}{[(B_1-P)^2+\varepsilon^2][(B_2-P)^2+\varepsilon^2]}$$

$\epsilon=\sigma\xi$, $x=h-P$, $P=\sigma^2/A_0$; $x=0$ corresponds to the point of ferromagnetic resonance at $\beta_c=0$.

If the dimension of the crystallites is large compared with the effective radii of the exchange and the magnetostatic interaction (which may hold true for a bulky material), and $h\gg\beta_c$, then $T(\mathbf{k})\approx T(0)$ and

$$\chi''\approx\frac{\varepsilon}{x^2+\varepsilon^2}\Big\{1+4\beta_c^2\mathscr{D}_0\frac{3x^2-\varepsilon^2}{(x^2+\varepsilon^2)^2}\Big\}. \tag{2.16}$$

The maximum of χ'' is situated in this case, as before (accurate to terms β_c^2), at the point $x=0$, the FMR line is symmetrical, and the line width expressed in terms of the magnetic-field scale is

$$\Delta h\approx 2\sigma\xi[1+6\mathscr{D}_0(\beta_c/\sigma\xi)^2]. \tag{2.17}$$

The quantity

$$\xi^{(e)}=\xi[1+6\mathscr{D}_0(\beta_c/\sigma\xi)^2] \tag{2.18}$$

can be regarded in this case as a certain effective relaxation parameter, which determines the observed line width of the FMR in the polycrystal. In addition to the term that does not depend explicitly on the frequency, $\xi^{(e)}$ contains also a term $\sim\omega^{-2}$.

For a thin magnetic film, the dimension of the crystallites is small and S_2 can be taken outside the integral sign. The integral of $T(\mathbf{k})$, after changing over to a polar system, was transformed as follows: since only $\varphi\ll 1$ contributes to the integral, we set $\sin\varphi$ equal to φ and replaced the integration with respect to φ from 0 to $\pi/2$ by integration from 0 to ∞. Then, after suitable renormalization, the remaining integral was calculated by numerical methods. As a result we obtained

$$\chi''\approx\frac{\varepsilon}{x^2+\varepsilon^2}\{1+\beta_c^2Rf(x)\}, \tag{2.19}$$

where

$$R=2\pi^{-5/2}\mathscr{D}_0\Big(\frac{b}{d}\Big)^{3/2}\Big(\frac{b^2}{a\varepsilon}\Big)^{3/4},$$

$$f(x)\approx\begin{cases}-0.3,\ x=-\varepsilon\\-0.7,\ x=0,\\+0.8,\ x=+\varepsilon\end{cases}\qquad\Big(\frac{\partial f}{\partial x}\Big)_{x=0}\approx 2{,}2/\varepsilon$$

Denoting the point of the maximum of χ'' by x_m, and the points corresponding to half the height of the resonance line by x_- and x_+, we obtained in the linear approximation

$$\begin{aligned} x_- &\approx -\varepsilon(1 + 0.4\beta_c^2 R), \\ x_m &\approx 1.1\varepsilon\beta_c^2 R, \\ x_+ &\approx \varepsilon(1 + 1.6\beta_c^2 R), \end{aligned} \tag{2.20}$$

and the line width in the magnetic field scale is

$$\Delta h \approx 2\sigma\xi[1 + \beta_c^2 R]. \tag{2.21}$$

Thus, the presence of a fine magnetic structure deforms the FMR line in a complicated fashion: besides broadening, it is possible to observe a sharp line asymmetry and a shift of the FMR frequency. The effective relaxation parameter

$$\xi^{(e)} = \xi[1 + \beta_c^2 R], \tag{2.22}$$

contains besides the term that does not depend explicitly on the frequency also a term $\sim\omega^{-3/4}$.

CONCLUSION

In conclusion we make three remarks.

1. The static and dynamic properties of the fine magnetic structure were considered for the case when the film was magnetized along the easy anisotropy axis. The transition to the case of a film magnetized along the difficult axis is effected by reversing the sign of β (which enters only in h) in all the preceding expressions.

2. The dispersion (1.18) has a singular point at $h = 0$, i.e., $H_0 = -H_a$, corresponding to the approximation in which the magnetic structure does not influence the coercive force of the film. Since this result follows from the neglect of the terms $g(\rho_{ik}m_j)$ in the system (1.9), the solution of the problem with allowance for these terms can be reduced to an estimate of the contribution made by the magnetic structure to the coercive force.

3. The magnetic structure of the type considered here can be due not only to the polycrystalline nature of the film, but to arbitrary inhomogeneities which are randomly distributed in the film (for example, internal elastic stresses, inclusions, etc.). All the obtained results are valid also in this case, provided $\beta_c M_0$ is now taken to mean the effective magnetic field of such inhomogeneities, and 2b is taken to mean their characteristic dimension.

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