EFFECTS OF PENETRATION OF AN EXTERNAL ELECTRIC FIELD INTO SEMIMETALS AND METALS

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It is shown that as a result of the well-known singularity of the dielectric constant $\epsilon(k)$ in a degenerate electron gas, slowly damped oscillations of the potential with a period $1/2p_0$ (p_0 is the Fermi momentum) arise during penetration of an external longitudinal electric field into a sample, just as around a charged impurity. This quasiperiodic potential leads to the appearance of a gap in the one-electron spectrum. Various manifestations of this singularity in the spectrum are considered.

1. The derivative of the dielectric constant $\partial\epsilon/\partial k$ in a degenerate electron gas has at $\omega = 0$ a logarithmic singularity at $k = 2p_0$, where p_0 is the Fermi momentum ($\hbar = 1$). This singularity, as is well known,[1] leads to the result that the potential of the charged center, contains, in addition to the Debye screened part, an oscillating component of the form $r^{-3}\cos(2p_0 r)$.

It is significant that the effect of the potential extends over distances that are very large in comparison with the screening radius r_D. This circumstance has great importance for the understanding of phenomena connected with the interaction between impurities and with the Knight shift in alloys.[2] As has been shown by Brovman and Kagan,[3] this long-range part of the potential plays a principal role in the formulation of the phonon spectrum of the metal. As is well known, the Kohn effect is also associated with this same singularity in $\partial\epsilon/\partial k$.[2]

It is evident that the singularity in $\partial\epsilon/\partial k$ under study should appear in various boundary problems of the penetration of an external electric field into a solid with a degenerate electron gas. Above all, it should be noted that at comparatively large distances from the boundary, where the field is usually considered to be absent, a potential will exist which falls off slowly with increasing distance from the boundary, and which oscillates with a period $1/2p_0$. We shall consider the effect of this oscillating field on the electronic properties of the solid. The fundamental effect which should evidently take place as the result of such a character of the penetrating field is the formation of a one-dimensional gap at the Fermi surface in the spectrum of one-particle electron excitations. The appearance of the gap and the change in the density of states of the electrons can lead to singularities in the superconductivity, in the static electrical conductivity, and in the absorption of light. Furthermore, the periodic potential produces periodic modulation of the ion density and, consequently, the appearance of a new period in the lattice.

2. Let us first consider the coordinate dependence of the potential. Let a semi-infinite specimen with plane surfaces $z = 0$ occupy the region $0 \leq z < \infty$. A constant external electric field with intensity $\mathbf{E} = (0, 0, E)$ is applied on the surface of the sample. (Actually, such a statement of the problem means that the sample of semimetal (metal) considered is one of the plates of a plane capacitor.) Then, as was shown by Shafranov,[4] the potential inside the sample ($z > 0$) has the form

$$V(z) = \frac{E}{\pi}\int_{-\infty}^{+\infty} dk\,\frac{e^{ikz}}{k^2\varepsilon(k)} \tag{1}$$

under the condition of specular reflection of the electrons from the surface.

In the derivation of Eq. (1), no assumptions were made regarding the properties of the electron system; therefore, it is valid both for degenerate and nondegenerate systems. We shall use as $\epsilon(k)$ the expression for the compressed electron system at a temperature of $T = 0°K$:

$$\varepsilon(k) = 1 + \varkappa_D^2\Pi(k)/k^2$$

with the polarization operator

$$\Pi(k) = \frac{1}{2} + \frac{p_0}{2k}\left(1 - \frac{k^2}{4p_0^2}\right)\ln\left|\frac{k+2p_0}{k-2p_0}\right|. \tag{2}$$

Here $\varkappa_D = r_D^{-1}$ is the reciprocal of the Debye radius.

Calculation of the integral (1) for the given form of $\Pi(k)$ leads approximately to the following expression for $V(z)$:

$$V(z) = E\exp(-\varkappa_D z)/\varkappa_D, \quad z < \bar{z}; \tag{3a}$$

$$V(z) = \frac{E}{2p_0}\left(\frac{\varkappa_D}{2p_0}\right)^2\frac{\sin 2p_0 z}{(2p_0 z)^2}, \quad z > \bar{z}. \tag{3b}$$

The value of $\bar{z}$ is determined from the condition that the amplitude of the oscillating part becomes equal to the Debye screening part of the potential

$$\exp(-\varkappa_D\bar{z}) = (\varkappa_D/2p_0)^3(2p_0\bar{z})^{-2}.$$

From the form of $V(z)$ for the region $z > \bar{z}$ it follows that, at distances from the surface greater than $1/2p_0$, the amplitude of the quasiperiodic potential $(E/2p_0)(\varkappa_D/2p_0)^2(2p_0 z)^{-2}$ changes little over the wavelength of the oscillation $1/2p_0$.

We note that the reduced form of the potential obviously holds under the condition

$$2p_0 l \gg 1, \tag{4}$$

where l is the length of the free path of the electrons. We also note that the form of the potential (3a), (3b)

differs from the form of the potential near the charged impurity by the absence of divergence as $z \to 0$ and by the slower decay of the oscillating part. The potential $V(z)$ was computed from Eq. (1) and (2) for the value $\kappa_D/2p_0 = 1$ on a computer and the result is shown in the drawing.

3. We proceed to the study of the energy spectrum. To be precise, we consider the region of the crystal $z > \bar{z}$, where the amplitude of the oscillating part changes little in the distance $1/2p_0$, and we find the spectrum of the one-particle excitations in the potential $V(z)$, given by Eq. (3b). For a strictly periodic potential, the solution is known—it is a band spectrum, while in the motion along the p_z axis in p space, the first gap automatically lies on the Fermi surface. In our case, we clearly get a similar result, but only the "gap" will be damped in motion inside the sample.

Actually, we write down the Hamiltonian in the form

$$\mathcal{H} = \sum_{\mathbf{k}} \varepsilon_0(\mathbf{k}) a_{\mathbf{k}}^+ a_{\mathbf{k}} + U(z) \sum_{\mathbf{k}} (a_{\mathbf{k}}^+ a_{\mathbf{k}-2\mathbf{p}_{0z}} - a_{\mathbf{k}}^+ a_{\mathbf{k}+2\mathbf{p}_{0z}}). \quad (5)$$

Here

$$U(z) = \frac{i}{2} V_0(z), \quad V_0(z) = \frac{eE}{2p_0} \left(\frac{\varkappa_D}{2p_0}\right)^2 \frac{1}{(2p_0 z)^2},$$

e is the charge of the electron; the vector $\mathbf{p}_{0Z}$ has the components $(0, 0, p_0)$.

Taking $V_0(z)$ out from under the summation sign in (5) means that we assume the amplitude of the quasiperiodic potential to be a slowly varying function at the distance $\sim 1/2p_0$.

The expression for the spectrum can now be obtained immediately by using the well-known results; however, for what follows, it is more convenient for us to complete this simple calculation.

The equation of motion for the Fourier forms of the Green's function $\langle a_{\mathbf{k}}(t) | a_{\mathbf{k}'}^+(t') \rangle$ have the form

$$\begin{aligned} \{\omega - \varepsilon_0(\mathbf{k})\} G(\mathbf{k}, \omega) &= (2\pi)^{-1} + U(z)\{\Phi_1(\omega) - \Phi_2(\omega)\}, \\ \{\omega - \varepsilon_0(\mathbf{k} - 2\mathbf{p}_{0z})\} \Phi_1(\omega) &= -U(z) G(\mathbf{k}, \omega), \\ \{\omega - \varepsilon_0(\mathbf{k} + 2\mathbf{p}_{0z})\} \Phi_2(\omega) &= U(z) G(\mathbf{k}, \omega), \end{aligned} \quad (6)$$

where G, Φ_1 and Φ_2 are the Fourier forms of the Green's function

$$\langle a_{\mathbf{k}} | a_{\mathbf{k}}^+ \rangle, \quad \langle a_{\mathbf{k}-2\mathbf{p}_{0z}} | a_{\mathbf{k}}^+ \rangle \text{ and } \langle a_{\mathbf{k}+2\mathbf{p}_{0z}} | a_{\mathbf{k}}^+ \rangle$$

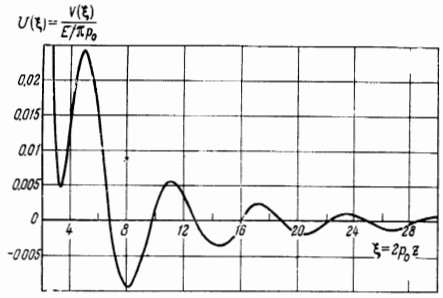
respectively.

The functions Φ_1 and Φ_2 differ from zero, since an inhomogeneous system is considered. In the chain of equations (6), functions of the type $\langle a_{\mathbf{k}+2n\mathbf{p}_{0Z}} | a_{\mathbf{k}}^+ \rangle$ with $n > 1$ are connected with band discontinuities of much higher order. By determining the poles of the G function, we find the spectrum

$$\varepsilon(\mathbf{k}) = {}^1/_2[\varepsilon_0(\mathbf{k}) + \varepsilon_0(\mathbf{k} - 2\mathbf{p}_{0z})] \mp \{{}^1/_4[\varepsilon_0(\mathbf{k} - 2\mathbf{p}_{0z}) - \varepsilon_0(\mathbf{k})]^2 + |U(z)|^2\}^{1/2}, \quad (7)$$

where the minus sign holds for $k_z < p_0$ and the plus sign for $k_z > p_0$. These formulas refer to the case $k_z > 0$; for $k_z < 0$, it is necessary to make the substitution $2p_{0z} \to -2p_{0z}$.

Let the direction of k_z be the polar axis. Then, it follows from Eq. (7) that if one moves along the direction θ, φ (θ is the polar angle, φ the azimuth), then at a distance $p_0/\cos\theta$ one has a gap of value $\Delta = 2|U(z)| = V_0(z)$. For motion in a given direction, the gap approaches the energy $\epsilon_0(p_0/\cos\theta)$, which is



Dependence of the potential U on ξ for $\kappa_D/2p_0 = 1$.

the same as the Fermi energy for $\theta = 0$, and goes to infinity as $\theta \to \pi/2$.

By assuming that all the considerations are valid for $z \gtrsim \bar{z}$, one can write

$$\Delta(z) = \frac{\Delta_m}{(1 + z/\bar{z})^2}, \quad (8)$$

where z is already different from zero and the maximum value of the gap is

$$\Delta_m = \frac{eE}{2p_0} \left(\frac{\varkappa_D}{2p_0}\right)^2 \frac{1}{(2p_0\bar{z})^2}. \quad (9)$$

Let us make some observations relative to the meaning and conditions of applicability of these results.

The dependence of the "gap" on the coordinates should be understood in the same sense as the "variable" forbidden band in the system of several contacting semiconductors with different forbidden bands. In other words, one can assume that the sample under study breaks up into parts which are large in comparison with the length of the oscillations, at which the potential differs but little from strictly periodic. Then, in each part, one can introduce a certain mean value of the forbidden band. The characteristic dimensions of the portion can be so chosen that, for example, the order of the mean free path (if the departure of the potential from periodic is insignificant) or less.

The broadening of the levels as a consequence of collisions has the value

$$\delta E \sim 1/\tau \sim p_0/ml \sim \varepsilon_F/p_0 l,$$

where τ is the relaxation time, m the effective mass, and ϵ_F the Fermi energy. It is obvious that for the existence of a gap, satisfaction of the condition $\Delta \gg \delta E$ is necessary, i.e.

$$p_0 l \gg \varepsilon_F/\Delta. \quad (10)$$

In essence, our consideration is also limited by another condition

$$\Delta_m/\varepsilon_F \ll 1, \quad (11)$$

which appears as the result of the fact that we do not consider the effect of spectrum changes on $\Pi(k)$. Account of the opposite effect of appearance of the gap leads to the result that the pole on the real axis in $\Pi''(k)$ at the point $2p_0$ (because of which oscillations appear) shifts in the complex plane by an amount $\sim\Delta/v_F$. As a result, a damping factor $\exp(-z\Delta/v_F)$ appears in the oscillations, which eliminates them at distances $\sim v_F/\Delta$ much greater than the wavelength. A similar situation exists for oscillations near a charge impurity in a superconductor or in a normal metal at finite temperature.[5,6]

4. Oscillations of the penetrating field bring about a modulation of the electron charge density. We can therefore expect that a deformation of the lattice also takes place and is connected with the adjustment of the ion charge density to the electron charge density. We shall prove that this is actually the case.

From the equations of motion for the operators of the acoustic phonon field, we have

$$-i\frac{d\langle b_{\mathbf{k}}\rangle}{dt}=\omega_0(\mathbf{k})\langle b_{\mathbf{k}}\rangle+g\sqrt{\frac{\omega_0(\mathbf{k})}{2V}}\sum_{\mathbf{p}}\langle a^+_{\mathbf{p}-\mathbf{k}}a_{\mathbf{p}}\rangle. \tag{12}$$

The second term on the right side of (12) does not depend on the time, since the correlation functions in them have identical time arguments. It is clear that finite displacements in the lattice are obtained only from those $\langle b_{\mathbf{k}}\rangle$ in the equations for which the second term on the right side of (12) differs from zero. From the foregoing, it is then evident that such will be the case only for $\mathbf{k}=2\mathbf{p}_{0z}$. Setting

$$A=g\sqrt{\frac{\omega_0(2\mathbf{p}_{0z})}{2V}}\sum_{\mathbf{p}}\langle a^+_{\mathbf{p}-2\mathbf{p}_{0z}}a_{\mathbf{p}}\rangle \tag{13}$$

and solving (12) for $\mathbf{k}=2\mathbf{p}_{0z}$, we get

$$\langle b_{2\mathbf{p}_{0z}}(t)\rangle=C\exp\{-i\omega_0(2\mathbf{p}_{0z})t\}-A/\omega_0(2\mathbf{p}_{0z}), \tag{14}$$

where C is a constant of integration.

After averaging over a time interval that is large in comparison with $\omega_0^{-1}(2\mathbf{p}_{0z})$, we get for the total shift

$$\mathbf{q}(\mathbf{r})=\mathbf{q}_{2\mathbf{p}_{0z}}\cos(2\mathbf{p}_{0z}\mathbf{r}), \tag{15}$$

where the Fourier components of the vector $\mathbf{q}(\mathbf{r})$ are connected with the $b_{\mathbf{k}}$ by the relations

$$\mathbf{q}_{\mathbf{k}}=\mathbf{b}_{\mathbf{k}}/\sqrt{2\rho\omega_0(2\mathbf{p}_{0z})},$$

ρ is the density of the solid.

We now compute the amplitude of the displacement

$$\mathbf{q}_{2\mathbf{p}_{0z}}=\frac{\mathbf{p}_{0z}}{p_0}\frac{g}{2\omega_0(2\mathbf{p}_{0z})\rho^{1/2}}\sum_{\mathbf{p}}\langle a^+_{\mathbf{p}-2\mathbf{p}}\,a_{\mathbf{p}}\rangle. \tag{16}$$

The value of $\langle a^+_{\mathbf{p}-2\rho_{0Z}}a_{\mathbf{p}}\rangle$ is identical, except for sign, with the function Φ_1 as $t'\to t+\delta$. By determining Φ_1 from the set of equations (6) and substituting the result in (16), we find

$$\sum_{\mathbf{p}}\langle a^+_{\mathbf{p}-2\mathbf{p}_{0z}}a_{\mathbf{p}}\rangle=2\pi|U|\sum_{\mathbf{p}}\int_{-\infty}^{+\infty}\frac{d\omega}{[\omega-\varepsilon_1(\mathbf{p})-i\delta][\omega-\varepsilon_2(\mathbf{p})+i\delta]}$$
$$=\frac{|U|}{2\pi}\sum_{\mathbf{p}}\frac{1}{\{[\varepsilon_0(\mathbf{p})-\varepsilon_0(\mathbf{p}-2\mathbf{p}_{0z})]^2+4|U|^2\}^{1/2}}\approx\frac{|U|}{2\pi}\frac{mp_0}{\pi^2}\Pi(2\mathbf{p}_{0z}), \tag{17}$$

where $\epsilon_{1,2}(\mathbf{p})$ are given by Eq. (7) and $\Pi(\mathbf{p})$ is given by (2); here we neglect the term $4|U|^2$ under the radical. Account of it leads to corrections $\sim(\Delta_m/\epsilon_F)\ln(\Delta_m/\epsilon_F)$. We finally get

$$q_{2p_{0z}}\approx\frac{1}{5}\zeta^{1/2}\sqrt{\frac{1}{\pi}\frac{m}{M}}\frac{|U|}{\omega_0(2\mathbf{p}_{0z})}p_0^{1/2}a_0^{3/2}, \tag{18}$$

where ζ is a dimensionless constant of the electron-phonon interaction, m the effective mass of the electron and M the mass of the ion.

Strictly speaking, it is necessary to take into account also the inverse effect of the displacements in the lattice on the electron spectrum. This would be done correctly by including the electron-phonon interaction in the initial Hamiltonian with the external field. However, it is clear that this leads to corrections to Δ that are proportional to ζ.

5. We consider the problem of the absorption of electromagnetic waves in the system under study with a one-dimensional gap. We shall be interested in the imaginary part of the dielectric tensor $\epsilon''_{\alpha\beta}(\omega)$. As a result of the appearance of the external constant field, the system becomes not only inhomogeneous but also anisotropic. It is quite evident that in our case, when the static field penetrates in the z direction, the tensor $\epsilon_{\alpha\beta}$ has the form

$$\varepsilon_{\alpha\beta}=\varepsilon_{\alpha\alpha}\delta_{\alpha\beta}\ \text{and}\ \varepsilon_{xx}=\varepsilon_{yy}\neq\varepsilon_{zz},$$

in which the values ϵ_{xx} and ϵ_{yy} are identical with the values in the isotropic case. This is connected with the fact that the electrons move in the field of the light wave in the directions of the x and y axes in the same way as in the absence of the static field. The motion along the static field is appreciably changed as the result of the appearance of the gap in the energy spectrum.

We can represent the value of $\epsilon''_{zz}(\omega)$ in the form:[7]

$$\varepsilon_{zz}''(\omega)=\frac{2}{(2\pi)^2}\frac{4\pi e^2}{q^2}\int d^3p\,f(\mathbf{p},\mathbf{q})\,\delta(\omega+\varepsilon_1(\mathbf{p}-\mathbf{q})-\varepsilon_2(\mathbf{p}+\mathbf{K})), \tag{19}$$

where the function $f(\mathbf{p},\mathbf{q})$ appears in the calculation of the matrix element in terms of plane electron waves that are distorted by Bragg reflections, $|\mathbf{K}|=2\mathbf{p}_{0Z}$. In the limit $\mathbf{q}\to 0$, if we introduce the variables $x=\cos(\mathbf{p},\mathbf{K})$ and $y=2\mathbf{p}/|\mathbf{K}|$, the function $f(\mathbf{p},\mathbf{q})/q^2$ takes the form

$$f(x,y)=\left(\frac{\varepsilon_F}{\Delta}\right)^2\left[1+\left(\frac{2\varepsilon_F}{\Delta}\right)^2(1-yx)^2\right]^{-2}\cos^2(\mathbf{qp})p_0^{-2}.$$

In the long-wave limit, we thus obtain

$$\varepsilon_{zz}''(\omega)=\frac{8}{3a_0p_0}\left(\frac{\varepsilon_F}{\Delta}\right)^2\frac{1}{\sqrt{1-4\Delta^2/\omega^2}}\int_0^1 y^2dy\int_{-1}^{+1}dx$$
$$\times\delta\left[yx-1+\frac{\sqrt{\omega^2-4\Delta^2}}{4\varepsilon_F}\right]\left[1+\frac{4\varepsilon_F^2}{\Delta^2}(1-yx)^2\right]^{-2},$$

where $a_0=1/me^2$ is the Bohr radius. As the result of integration, we obtain

$$\varepsilon_{zz}''(\omega)=\begin{cases}0, & \omega<2\Delta(z),\\ \dfrac{2}{3a_0p_0}\left(\dfrac{\varepsilon_F}{2\Delta}\right)\left(\dfrac{2\Delta}{\omega}\right)^3\left[2-\dfrac{\sqrt{\omega^2-4\Delta^2}}{4\varepsilon_F}\right], & \omega>2\Delta(z).\end{cases} \tag{20}$$

It is then seen that $\epsilon''_{zz}(\omega)$ has a finite jump at the point $\omega=2\Delta$ in contrast with the system with a three-dimensional gap in the spectrum, where the jump is infinite ($\epsilon''(\omega)\sim 1/\sqrt{\omega^2-4\Delta^2}$). As the distance from resonance increases, $\epsilon''_{zz}(\omega)$ falls off as ω^{-3} to a value associated with the residual scattering in the isotropic case.

If the absorption coefficient is a function of z, then the intensity of the transmitted light is characterized by the expression

$$I=I_0\exp\{-\bar{k}l_x\},\qquad \bar{k}=\frac{1}{l_z}\int_0^{l_z}k(z)\,dz,$$

where $l_{X,Z}$ is the thickness of the sample in the directions x or z. It then follows that in our case $\Delta=\Delta(z)$, one must use the value

$$\bar{\varepsilon}_{zz}''(\omega)=\frac{1}{l_z}\int_{z_0}^{l_z}\varepsilon_{zz}''(\omega,z)\,dz=\frac{8}{9a_0p_0}\frac{1}{(2p_0l_z)}\frac{1}{(2p_0z_0)^3}\frac{\varepsilon_F}{\omega}\left(\frac{\Delta_m}{\omega}\right)^2, \tag{21}$$

where z_0 is determined from the condition $\omega=\Delta(z_0)$ and $l_Z\gg z_0$.

6. We now discuss the features of the electron spectrum for penetration of the electric field into the superconductor. First, we note that the character of the penetration of the field in the superconductor remains as before. This is connected with the fact that the polarization operator in the static limit $\Pi(k)$ does not change (with accuracy to within corrections of the order Δ/ϵ_F) in the transition to the superconducting state. The set of Gor'kov equations in our case has the form

$$\begin{aligned}
&\{\omega - \varepsilon_0(\mathbf{p}) + \mu\} G(\mathbf{p}, \omega) = \frac{1}{2\pi} - i\lambda F(0+) F^+(\mathbf{p}, \omega) \\
&\qquad + |U|[\Phi_1(\mathbf{p}, \omega) - \Phi_2(\mathbf{p}, \omega)], \\
&\{\omega - \varepsilon_0(\mathbf{p} - 2\mathbf{p}_{0z}) + \mu\}\Phi_1 = |U| G(\mathbf{p}, \omega), \\
&\{\omega - \varepsilon_0(\mathbf{p} + 2\mathbf{p}_{0z}) + \mu\}\Phi_2 = -|U| G(\mathbf{p}, \omega), \\
&\{\omega + \varepsilon_0(\mathbf{p}) - \mu\} F^+(\mathbf{p}, \omega) + i\lambda F^+(0+) G(\mathbf{p}, \omega) = 0.
\end{aligned} \tag{22}$$

Here λ is the constant of effective electron-electron interaction. For the function $G(p, \omega)$ we get the equation

$$\left[\omega - \xi(\mathbf{p}) - \frac{\lambda^2 |F(0+)|^2}{\omega + \xi(\mathbf{p})} - \frac{|U|^2}{\omega - \xi(\mathbf{p} - 2\mathbf{p}_{0z})} - \frac{|U|^2}{\omega - \xi(\mathbf{p} + 2\mathbf{p}_{0z})}\right] G(\mathbf{p}, \omega) = \frac{1}{2\pi},$$

where, as usual, $\xi(p) = p^2/2m - \mu \approx v_0(|p| - p_0)$. The spectrum close to one of the zone discontinuities, for example, when $p \to p_{0Z}$, has the form

$$\omega = [\xi^2 + \lambda^2 |F(0+)|^2 + |U|^2]^{1/2}. \tag{23}$$

Thus, in the direction along the electric field, the squares of the superconducting and dielectric gaps are combined. A similar situation holds for consideration of one-dimensional metal-like systems.[8]

In the direction perpendicular to the electric field, there remains only the superconducting gap. The anisotropy of the gap, which is brought about by the external field, must manifest itself primarily in the dependence of the absorption of light and sound on the frequency and the polarization.

7. Like the Kohn effect, the character of the penetration of the electric field depends materially on the form of the Fermi surface. In particular, the presence of plane pieces of the latter strongly intensifies the singularity in the polarization operator.[9] Calculation from Eq. (1) in this case leads to a slow damping of the field in the bulk of the specimen ($V \sim 1/z$) in place of $V \sim 1/z^2$ for a spherical Fermi surface.

If there are two sets of particles (for example, electrons and holes in a semimetal or semiconductor) of appreciably different masses and Fermi momenta, two sets of oscillations—of charge density and displacements—should be produced in the lattice. Correspondingly, two gaps should appear in the spectrum of excitations, one for electrons, the other for holes.

In the case of an anisotropic Fermi surface, an oscillation of charge and potential corresponds to each direction, associated with the Fermi momentum in that direction. It is possible that x-ray and optical measurements of the corresponding periodic displacements in the lattice will permit the determination of the shape of the Fermi surface in this way.

In a quantized magnetic field, the polarization operator has the form[10]

$$\Pi(k) = -\frac{2m\omega_L}{(2\pi)^2} \sum_{\nu,\nu'} \int_{-\infty}^{+\infty} dp_z \Lambda_{\nu\nu'}\left(\frac{k_t}{\sqrt{2m\omega_L}}\right) \frac{n_{\nu p_z} - n_{\nu', p_z - k_z}}{\varepsilon_\nu(p_z) - \varepsilon_{\nu'}(p_z - k_z)},$$

where k_t is the transverse component of the field, $\omega_L = eH/mc$, $\Lambda_{\nu\nu'}(x)$ is which goes over to the symbol $\delta_{\nu\nu'}$ for small values of its argument and which corresponds to the transition to strong fields.

It is then seen that the electron gas, in strong quantized magnetic fields, transforms into a system of degenerate electron gases with different Fermi momenta $p_{0\nu} = \{2m[\epsilon_F - \omega_L(\nu + 1/2)]\}^{1/2}$, where $\nu = 0, 1, 2, 3, \ldots$. Each of these gases behaves, in the direction of the magnetic field, as a one-dimensional system with a logarithmic singularity in $\Pi(k)$ as $k_Z \to 2p_{0Z}$. In this case, the picture of the penetration of the electric field into the sample along the magnetic field should consist of a collection of oscillations, each of which has its own wavelength $1/2p_{0\nu}$ and which falls off in the bulk of the sample as $1/z$. The problem of the gap in the spectrum in this case is very complicated, and is connected with the quasi-one-dimensionality of the system.[8]

8. There also exists a set of phenomena in which the effect of spatial oscillations of the potential can appear. We shall only point them out without a detailed discussion.

A. Penetration of an external electric field into a degenerate ferroelectric with a first-order phase transition of near but above the Curie point, can bring about its stratification into alternate paraelectric and ferroelectric regions. This effect is a simple consequence of the shift of the Curie point in the electric field and the presence of an electric field that is oscillating in space. Obviously, it is necessary, in order that the effect exist, that the thickness of the transition layer be less than the wavelength of the oscillation, $1/2p_0$. It is well known that for 180° domain walls, the thickness of the transition layer amounts to $\sim 10^{-8}$ cm. In such a case, the required condition can be satisfied.

B. Inasmuch as the cause of the external electric field is immaterial, the effects of oscillation of the potential should take place in contact phenomena for systems in which at least one of the contacting samples in degenerate. In particular, if there is contact of a semimetal with a solid having a different work function, then a gap should be formed in the semimetal as a consequence of the effect of the field arising from the contact potential difference. The volt-ampere characteristic can then be non-ohmic when such a contact is introduced in a circuit.

Similar phenomena should also be produced by the adsorption of ions on the surface of semimetals and metals. The same sort of effects should also appear near charged dislocations.

C. We have considered a system in the form of a plane condenser. Evidently the effect should be different in spherical and cylindrical capacitors, where the gap will no longer be spatially one-dimensional.

D. The oscillating course of the potential will actually be connected with the degeneracy of the electron gas. Degeneracies can be obtained in the system by using different means of "pumping" of the electrons. Then there will exist all the effects considered, during the lifetime of the non-equilibrium electrons in the conduction band.

9. We shall now make a series of estimates for the effects and quantities considered. First of all, we estimate the amplitude of the potential in the region $z > \bar{z}$. According to (3b),

$$V_0 \approx \frac{eE}{2p_0}\left(\frac{\varkappa_D}{2p_0}\right)^2 (2p_0\bar{z})^{-2}.$$

Usually $(\kappa_D/2p_0) \approx 1$. Then we take for the estimate $V_0 \sim (eE/2p_0) \times (2p_0 z)^{-2}$. Hence, for a semimetal and for $E = 10^4$–10^6 V/cm, $2p_0\bar{z} \approx 10$ and $p_0^{-1} \approx 10^{-5}$ cm, we get $V_0\ 10^{-3} - 10^{-2}$ eV. For a metal, for $E \approx 10^6$ V/cm, $(2p_0\bar{z})^2 \approx 10$ and $p_0^{-1} \approx 10^{-8}$ cm, we get $V_0 \approx 10^{-3}$ eV, i.e., for a metal the conditions of observation of the effect are "at the limit." Since, in accord with (7), $\Delta \approx V_0$, the estimates for Δ are identical with the foregoing

Let us verify the feasibility of the condition for the existence of the gap: $\epsilon_F/\Delta \ll p_0 l$. For a metal, setting $\epsilon_F/\Delta \approx 10^4$ and $p_0^{-1} \approx 10^{-8}$ cm, we have $l \gg 10^{-4}$ cm. Such a condition is frequently realized for low temperatures for small excitations above the Fermi surface. For a semimetal, it can also be fulfilled.

With respect to metals, it is however necessary to note the following: inasmuch as the oscillations take place at atomic distances, the surface of the metal should be atomically smooth for the formation of a clear picture of the oscillations and of the energy gap. To obtain such a surface is difficult and perhaps impossible. Nevertheless, even for a rough surface of the metal, the penetration depth of the electric field can be large in comparison with the Debye radius.

We now estimate the displacement in the lattice under the effect of modulation of the electron density. According to (16), the amplitude of the displacement is a quantity

$$q \approx \frac{1}{5}\zeta^{1/2}\left(\frac{1}{\pi}\frac{m}{M}\right)^{1/2}\frac{\Delta}{\omega_0(2p_{0z})}p_0^{1/2}a_0^{3/2}.$$

For a metal with $\zeta \approx 1$, $m/M \approx 10^{-5}$, $\Delta \approx 10^{-3}$ eV, $\omega_0(2p_{0Z}) \sim \omega_D \approx 10^{-2}$ eV, $p_0^{-1} \approx 10^{-8}$ cm, we have: $q \approx 3 \times 10^{-5} p_0^{-1} \approx a_0$, where a_0 is the lattice constant.

For a semimetal,

$$\zeta = g_M^2 \frac{1}{a_0^2 \varepsilon p_0^2}\frac{mp_0}{2\pi^2},$$

where g_m is the dimensional constant of electron-phonon interaction, ϵ is the static dielectric constant ($\epsilon \approx 100$ for Bi). Then, setting $\Delta \approx 10^{-3}$ eV, $\omega_0(2p_{0Z}) \approx 10^{-4}$ eV and $p_0 \approx 10^6$ cm^{-1}, we get $q \approx 10^{-4} a_0$. This estimate shows that for such displacements in the lattice, it is already necessary to take into account the effects of anharmonicity.

For the observation of the optical absorption at the gap, we must clearly use the possibility of modulation of the gap width by means of the penetration of the variable electric field. By tuning the detector to the modulation frequency, we can split off the absorption constant, for example, into free carriers.

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