INTERACTION OF ATOM WITH MONOCHROMATIC FIELD IN THE STRONG-COLLISION MODEL

A. B. KOL'CHENKO and S. G. RAUTIAN

Institute of Semiconductor Physics, Siberian Division, USSR Academy of Sciences

Submitted September 28, 1967

Zh. Eksp. Teor. Fiz. 54, 959–973 (March, 1968)

The problem of the behavior of atoms in the field of a plane traveling monochromatic wave is solved in the resonance approximation with allowance for the pressure effect in the strong-collision model.

1. INTRODUCTION

As is well known, the problem of the interaction between an atom and an electromagnetic field has no general exact solution even in the resonance approximation. Exact solutions are known at present only for two cases: 1) when the atom is stationary in a monochromatic field, and the relaxation processes are taken into account in the equation for the density matrix by linear terms with constant coefficients (see, e.g.,[1-3]); the case of an atom moving in the field of a plane traveling wave reduces to the same problem; 2) when the atom moves in the field of a plane standing monochromatic wave, and the frequency of the field coincides with the frequency of the transition and all the relaxation constants are the same[4,5]. On the basis of precisely these very simple cases, a picture was formulated of the nonlinear phenomena occurring in strong electromagnetic fields (the saturation effect, the change in the velocity distribution of the atoms, the kinetics of the transitions induced by the field, the spatial inhomogeneity of the medium, etc.). In all the remaining varied problems it is necessary to use approximate solutions, obtained with the aid of expansions in powers of various small parameters[6,7], and qualitatively interpret them in the spirit of the notions that are rigorously based for the exactly-solved problems.

It must be specially emphasized that all the foregoing pertains only to situations in which the relaxation is taken into account by the simplest method indicated above. At the same time, the experimental data, as well as theoretical considerations and calculations[8-15] show that in many gas systems of practical importance a major role is played by collisions accompanied by changes in the velocity of the atoms. At the same time, a radical change takes place in the relaxation kinetics[10,16], and the entire picture of the nonlinear phenomena should undergo a significant modification.

To describe collisions with change of velocity, one of the authors proposed[11] the following system of equations for the elements ρ_{ij} of the density matrix:

$$(\partial/\partial t + \mathbf{v}\nabla)\rho_{jj} = \pm 2\operatorname{Re}(iV_{nm}\rho_{mn}) + S_j + q_j \quad (j = m, n),$$
$$(\partial/\partial t + \mathbf{v}\nabla)\rho_{mn} = iV_{mn}(\rho_{mm} - \rho_{nn}) + S. \tag{1.1}$$

Here V_{mn} is the matrix element of the interaction, q_j is the number of acts of excitation per second at the level $j = m, n$ with velocity $\mathbf{v}$. The collision integrals S_j and S are given by the formulas

$$S_j = -\Gamma_j\rho_{jj} - \left\{\nu_j\rho_{jj} - \int A_j(\mathbf{v}', \mathbf{v})\rho_{jj}(\mathbf{v}')d\mathbf{v}'\right\},$$
$$S = -(\Gamma + i\Delta)\rho_{mn} - \left\{\nu\rho_{mn} - \int A(\mathbf{v}', \mathbf{v})\rho_{mn}(\mathbf{v}')d\mathbf{v}'\right\}. \tag{1.2}$$

The first terms in the expressions for S_j and S are determined by the quenching and by the level shifts of the atomic electron, occurring without a change in the velocity of the atom (processes of the first kind in the classification of[11]). These components of the collision integrals are "the ordinary" relaxation terms. The content of the curly brackets in (1.2) reflects the role of processes of a second type, in which both the state of the optical electron and $\mathbf{v}$ change. The kernels $A(\mathbf{v}', \mathbf{v})$ and $A_j(\mathbf{v}', \mathbf{v})$ ($\mathbf{v}'$, $\mathbf{v}$ - velocities of the atoms before and after collision) are determined by the model of the collisions.

The physical meaning and the region of applicability of (1.2) were discussed in detail in[11]. It is important to emphasize here, on the other hand, that the system (1.1) admits of an exact solution for arbitrarily large field amplitudes in the particular case of strong collisions, and for a monochromatic field, in the form of a traveling plane wave. The present paper is devoted to an analysis of this solution and to an investigation of the nonlinear phenomena corresponding to it.

2. GENERAL RELATIONS

We shall assume that the velocity of the atom $\mathbf{v}$ after the collision does not depend on its velocity $\mathbf{v}'$ prior to the collision (the strong-collision model.) From this assumption and from the natural requirement $S_j = S = 0$ in the equilibrium case it follows that

$$A_j(\mathbf{v}', \mathbf{v}) = \tilde{\nu}_j W(\mathbf{v}), \quad A(\mathbf{v}', \mathbf{v}) = \tilde{\nu} W(\mathbf{v}),$$
$$W(\mathbf{v}) = (\sqrt{\pi}\bar{v})^{-3}\exp\{-\mathbf{v}^2/\bar{v}^2\}, \tag{2.1}$$

i.e., the velocity distribution after the collision is Maxwellian. The differences

$$\Gamma_{j2} = \nu_j - \tilde{\nu}_j, \quad \Gamma_2 + i\Delta_2 = \nu - \tilde{\nu} \tag{2.2}$$

determine the quenching of the levels j and the broadening due to the interaction in collisions of the second type. We note that in the model (2.1) the departure frequencies (ν_j, ν) and the arrival frequencies ($\tilde{\nu}_j$, $\tilde{\nu}$) cannot depend on $\mathbf{v}$.

The sources of the populations q_j in (1.1) will be assumed to be independent of $\mathbf{r}$ and t, and the dependence on $\mathbf{v}$ will be assumed to be of the equilibrium type:

$$q_j = Q_j W(\mathbf{v}), \tag{2.3}$$

Q_j are the integral (with respect to $\mathbf{v}$) numbers of acts of excitation of the levels $j = m, n$ per unit time.

For a plane traveling monochromatic wave with frequency ω and wave vector $\mathbf{k}$, the matrix element $V_{mn}(\mathbf{r}, t)$ is given by

$$V_{mn}(\mathbf{r}, t) = Ge^{-i(\Omega t - \mathbf{kr})}, \quad \Omega = \omega - \omega_{mn}, \quad G = p_{mn}E/2\hbar, \tag{2.4}$$

where E is the amplitude of the wave, and p_{mn} and ω_{mn} are the matrix element of the dipole moment and the frequency for the transition $m \to n$.

Under the foregoing conditions, the off-diagonal element $\rho_{mn}(\mathbf{r}, \mathbf{v}, t)$ must be sought in the form

$$\rho_{mn}(\mathbf{r}, \mathbf{v}, t) = \rho(\mathbf{v}) e^{-i(\Omega t - \mathbf{kr})}; \tag{2.5}$$

the populations[1)] ρ_{jj} do not depend on t or $\mathbf{r}$. Thus, the system (1.1) reduces to

$$(\Gamma_j + \nu_j)\rho_{jj} \mp 2\mathrm{Re}(iG\rho) - \tilde{\nu}_j W(\mathbf{v})\langle\rho_{jj}\rangle = Q_j W(\mathbf{v}),$$
$$(p + i\mathbf{kv})\rho - iG(\rho_{mm} - \rho_{nn}) - \tilde{\nu} W(\mathbf{v})\langle\rho\rangle = 0,$$
$$\langle\rho\rangle = \int \rho\, d\mathbf{v}, \quad \langle\rho_{jj}\rangle = \int \rho_{jj}\, d\mathbf{v}, \quad p = \Gamma + \nu - i(\Omega - \Delta). \tag{2.6}$$

It will be convenient in what follows to write (2.6) in matrix form. We introduce the matrices

$$\hat{\rho} = \begin{pmatrix} \rho_{mm} \\ \rho_{nn} \\ \rho_{mn} \\ \rho_{nm} \end{pmatrix}, \quad \hat{Q} = \begin{pmatrix} Q_m \\ Q_n \\ 0 \\ 0 \end{pmatrix}, \quad \hat{\tilde{\nu}} = \begin{pmatrix} \tilde{\nu}_m & 0 & 0 & 0 \\ 0 & \tilde{\nu}_n & 0 & 0 \\ 0 & 0 & \tilde{\nu} & 0 \\ 0 & 0 & 0 & \tilde{\nu}^* \end{pmatrix},$$

$$\hat{A}(\mathbf{v}) = \begin{pmatrix} \Gamma_m + \nu_m & 0 & -iG & iG \\ 0 & \Gamma_n + \nu_n & iG & -iG \\ -iG & iG & p + i\mathbf{kv} & 0 \\ iG & -iG & 0 & p^* - i\mathbf{kv} \end{pmatrix}. \tag{2.7}$$

Then the system (2.6) is given by

$$\hat{A}(\mathbf{v})\hat{\rho} - \hat{\tilde{\nu}} W(\mathbf{v})\langle\hat{\rho}\rangle = W(\mathbf{v})\hat{Q}. \tag{2.8}$$

In this form, Eq. (2.8) is perfectly analogous to the kinetic equation for the classical distribution function used in[10], and the method of its solution is obvious. Multiplying (2.8) from the left by $\hat{A}^{-1}$ and integrating with respect to $\mathbf{v}$, we get

$$\langle\hat{\rho}\rangle = [\hat{E} - \langle\hat{A}^{-1}\rangle\hat{\tilde{\nu}}]^{-1}\langle\hat{A}^{-1}\rangle\hat{Q}, \quad \langle\hat{A}^{-1}\rangle = \int W(\mathbf{v})\hat{A}^{-1}(\mathbf{v})\, d\mathbf{v}; \tag{2.9}$$

Here $\hat{E}$ is a unit matrix. Substituting (2.9) in (2.8) we can obtain after simple transformations

$$\hat{\rho} = \hat{A}^{-1}(\mathbf{v})[\hat{E} - \hat{\tilde{\nu}}\langle\hat{A}^{-1}\rangle]^{-1} W(\mathbf{v})\hat{Q}. \tag{2.10}$$

It is easy to verify that the matrices $\hat{A}(\mathbf{v})$, $\hat{E} - \hat{\tilde{\nu}}\langle\hat{A}^{-1}\rangle$, and $\hat{E} - \langle\hat{A}^{-1}\rangle\hat{\tilde{\nu}}$ are not singular, and therefore the transformation from (2.8) to (2.9) and (2.10) is legitimate.

Formulas (2.9) and (2.10) give the general solution of the problem, which will be investigated in detail in the succeeding sections. Here we consider some general properties of this solution. If the arrival term in the collision integral vanishes ($\tilde{\nu} = 0$; this means physically complete quenching in collisions of the second kind) then

$$\hat{\rho} = \hat{A}^{-1}(\mathbf{v}) W(\mathbf{v})\hat{Q}. \tag{2.11}$$

Thus, the matrix $\hat{A}^{-1}(\mathbf{v})$ gives the solution for the well-investigated case, when the system is described by three relaxation constants, including in our case the frequencies ν_m, ν_n, and ν. This is perfectly understandable physically, since each collision of the second kind leads to a complete quenching, and no change in velocity can arise in the case of the collision.

The drift of the atoms in velocity space is taken into account by the factors $[\hat{E} - \langle\hat{A}^{-1}\rangle\hat{\tilde{\nu}}]^{-1}$ and $[\hat{E} - \hat{\tilde{\nu}}\langle\hat{A}^{-1}\rangle]^{-1}$ respectively in formulas (2.9) and (2.10). The physical meaning of these factors is that they describe the increase (compared with the case $\hat{\tilde{\nu}} = 0$) of the time during which the atom can interact with the field. To illustrate this thought, let us consider the limiting case of vanishingly small G. We have here

$$\rho_{jj} = N_j W(\mathbf{v}), \tag{2.12}$$

$$N_j = \langle\rho_{jj}\rangle = \frac{Q_j}{\Gamma_j + \nu_j - \tilde{\nu}_j} = Q_j\left[\frac{1}{\Gamma_j + \nu_j} + \frac{\tilde{\nu}_j}{(\Gamma_j + \nu_j)(\Gamma_j + \nu_j - \tilde{\nu}_j)}\right],$$

$$\frac{\rho}{iNG} - \frac{W(\mathbf{v})}{p + i\mathbf{kv}}\frac{1}{1 - \tilde{\nu}X} = \frac{W(\mathbf{v})}{p + i\mathbf{kv}}\left[1 + \frac{\tilde{\nu}X}{1 - \tilde{\nu}X}\right],$$

$$\frac{\langle\rho\rangle}{iNG} = \frac{X}{1 - \tilde{\nu}X}, \quad p = \Gamma + \nu - i(\Omega - \Delta), \tag{2.13}$$

$$X = \langle[p + i\mathbf{kv}]^{-1}\rangle = \int \frac{W(\mathbf{v})\, d\mathbf{v}}{p + i\mathbf{kv}}, \quad N = N_m - N_n. \tag{2.14}$$

Formulas (2.12) and (2.13) can be easily interpreted in the sense of interest to us. The quantity $\tau_{1j} = (\Gamma_j + \nu_j)^{-1}$ is the mean lifetime of the atom at the level j with a constant velocity. The product $\tau_{1j}\tilde{\nu}_j$ is obviously the probability of arrival within a time τ_{1j} between two successive collisions of the second kind. Consequently the time

$$\tau_{2j} = \tau_{1j}\sum_{n=0}^{\infty}(\tau_{1j}\tilde{\nu}_j)^n = \frac{\tau_{1j}^2\tilde{\nu}_j}{1 - \tilde{\tau}_{1j}\nu_j} = \frac{\tilde{\nu}_j}{(\Gamma_j + \nu_j)(\Gamma_j + \nu_j - \tilde{\nu}_j)}, \tag{2.15}$$

which enters in (2.12) is the average lifetime after the first collision. On the other hand, the sum

$$\tau_j = \tau_{1j} + \tau_{2j} = \tau_{1j}[1 - \tilde{\nu}_j\tau_{1j}]^{-1} = (\Gamma_j + \nu_j - \tilde{\nu}_j)^{-1} \tag{2.16}$$

is the total average lifetime at the level j, increased compared with τ_{1j} by a factor $[1 - \tau_{1j}\tilde{\nu}_j]^{-1}$ as a result of the arrival term in the collision integral.

The interpretation of formula (2.13) for $\langle\rho\rangle$ is perfectly analogous. The quantity $X = \langle[p + i\mathbf{k}\cdot\mathbf{v}]^{-1}\rangle$ plays for $\langle\rho\rangle$ the same role as τ_{1j} for $\langle\rho_{jj}\rangle$ (cf. the equations for ρ_{jj} and ρ in (2.6)), i.e., it can be interpreted as the "mean lifetime of the off-diagonal element" between two collisions of the second type. The absence of quenching or of considerable phase shift in these collisions ($\tilde{\nu} \neq 0$) allows the atom to interact coherently with the field also after the collision. The arrival probability $\tilde{\nu}X$, which is analogous to $\nu_j\tau_{1j}$, can be naturally called the measure of the coherence of the radiation before and after the collision. On the other hand, the total average lifetime for $\langle\rho\rangle$

$$X + X\sum_{n=1}^{\infty}(\tilde{\nu}X)^n = X + \frac{\tilde{\nu}X}{1 - \tilde{\nu}X} = \frac{X}{1 - \tilde{\nu}X} \tag{2.17}$$

will be "increased by a factor $[1 - \tilde{\nu}X]^{-1}$ times" compared with the case $\tilde{\nu} = 0$. This circumstance, in par-

1)The quantities ρ_{jj} determine the numbers of atoms at the levels j in a unit velocity interval, i.e., the population density in velocity space. For brevity, we shall use the term "population" where it leads to no misunderstanding.

ticular, leads to a narrowing of the spectral line (the Dicke effect) which was considered in detail in [10].

In [10], where a formula physically identical with (2.13) was obtained for the first time, the interpretation was terminologically different but essentially the same as the one given here. Owing to the change of the velocity, the radiation of the atomic oscillator breaks up into trains with average duration $(\Gamma+\nu)^{-1}$, and within each train, the frequency is shifted by an amount of the Doppler shift $\mathbf{k}\cdot\mathbf{v}$. The first term in (2.17) would describe radiation of this kind, if the trains were perfectly incoherent, as is indeed the case when $\tilde{\nu}=0$. These trains, however, are in general coherent, and the n-th term of the series in (2.15) contributes to the radiation of those trains which were produced after the n-th collision, while formula (2.17) gives the result of the interference of all the possible trains. If the velocity were to remain unchanged during the collisions, then

$$X=\frac{1}{p+i\mathbf{kv}},\quad \frac{X}{1-\tilde{\nu}X}=\frac{1}{p-\tilde{\nu}+i\mathbf{kv}},$$

and the aggregate of all the trains would be equivalent to one train of longer duration $(p'-\tilde{\nu}')^{-1}$ $=(\Gamma+\nu-\nu')^{-1}$. Since the velocity of the atom changes upon collision with a distribution $W(\mathbf{v})$, such an equivalence does not take place, and one can speak only of an increase of the average coherence time in the sense indicated above. Thus, both interpretations are physically identical.

When account is taken of nonlinear effects, the interpretation of formulas (2.9) and (2.10) is less lucid, since the matter becomes more complicated by the mixing of the states m and n of the atom in the external field. Certain features connected with the arrival term in S can be traced also in the general case. If we separate Eq. (2.6) for ρ and $p+i\mathbf{k}\cdot\mathbf{v}$ and integrate with respect to $\mathbf{v}$, we get

$$\frac{\langle\rho\rangle}{iNG}=\frac{1}{1-\tilde{\nu}X}\int\frac{\rho_{mm}(\mathbf{v})-\rho_{nn}(\mathbf{v})}{N(p+i\mathbf{kv})}d\mathbf{v}. \qquad (2.18)$$

According to (2.18), the line shape[2)] is determined by two factors: by the "line narrowing" factor or factor of lengthening of the phase memory, $(1-\tilde{\nu}X)^{-1}$, and by the resonance factor $(p+i\mathbf{k}\cdot\mathbf{v})^{-1}$, averaged with the aid of the stationary distribution of the population difference with respect to their velocities. In such a formulation, all the nonlinear effects are "transferred" to $\rho_{mm}(\mathbf{v})-\rho_{nn}(\mathbf{v})$, and the factor $[(1-\nu X)(p+i\mathbf{k}\cdot\mathbf{v})]^{-1}$ remains the same as in the linear theory.

Formulas (2.9) and (2.10) give the solution of the stationary problem, i.e., the solution of Eq. (2.8), brought about by its right-hand side $QW(\mathbf{v})$. On the other hand, in the analysis of (2.12) and (2.13) we made extensive use of such concepts as lifetimes etc., which, properly speaking, are attributes of the problem of the temporal evolution of the system, i.e., the solution of the homogeneous equation corresponding to (2.8) with initial conditions. In view of the ergodicity of the conditions, this procedure is perfectly valid and, as we have seen, leads to a simple and lucid interpretation.

3. THE CASE $\hat{\tilde{\nu}}=0$

When $\hat{\tilde{\nu}}=0$, the density matrix is determined by the matrix $\hat{A}^{-1}$:

$$\hat{A}^{-1}=$$

$$\begin{vmatrix}
\frac{1}{\Gamma_m+\nu_m}\left[1-\frac{2G^2}{\Gamma_m+\nu_m}y'\right] & \frac{2G^2}{(\Gamma_m+\nu_m)(\Gamma_n+\nu_n)}y' & \frac{iG}{\Gamma_m+\nu_m}y & -\frac{iG}{\Gamma_m+\nu_m}y^* \\
\frac{2G^2}{(\Gamma_m+\nu_m)(\Gamma_n+\nu_n)}y' & \frac{1}{\Gamma_n+\nu_n}\left[1-\frac{2G^2}{\Gamma_n+\nu_n}y'\right] & -\frac{iG}{\Gamma_n+\nu_n}y & \frac{iG}{\Gamma_n+\nu_n}y^* \\
\frac{iG}{\Gamma_m+\nu_m}y & -\frac{iG}{\Gamma_n+\nu_n}y & y+\frac{\varkappa}{2}y' & \frac{\varkappa}{2}y' \\
-\frac{iG}{\Gamma_m+\nu_m}y^* & \frac{iG}{\Gamma_n+\nu_n}y^* & \frac{\varkappa}{2}y' & y^*+\frac{\varkappa}{2}y'
\end{vmatrix} \qquad (3.1)$$

We have introduced here the following notation:

$$y(\mathbf{v})=(p+i\mathbf{kv})^{-1}(1+2G^2\tau_1\,\mathrm{Re}[p+i\mathbf{kv}]^{-1})^{-1}$$

$$=\frac{\Gamma+\nu+i(\Omega-\Delta-\mathbf{kv})}{(\Gamma+\nu)^2(1+\varkappa)+(\Omega-\Delta-\mathbf{kv})^2},$$

$$\varkappa=2G^2\tau_1/p',\quad \tau_1=(\Gamma_m+\nu_m)^{-1}+(\Gamma_n+\nu_n)^{-1},$$

$$p=p'+ip''=\Gamma+\nu-i(\Omega-\Delta). \qquad (3.2)$$

Inasmuch as $\hat{A}^{-1}(\mathbf{v})$ depends on $\mathbf{v}$ only via $y(\mathbf{v})$, the matrix $\langle\hat{A}^{-1}\rangle$ is obtained from $\hat{A}^{-1}$ by replacing $y(\mathbf{v})$ by the function Y:

$$Y=\int y(\mathbf{v})W(\mathbf{v})d\mathbf{v}=\frac{1}{\sqrt{\pi}\,k\bar{v}}\int_{-\infty}^{+\infty}\frac{(p^*-ix)\exp[-x^2/(k\bar{v})^2]}{|p+ix|^2+p'^2\varkappa}dx, \qquad (3.3)$$

which can be expressed in terms of tabulated integrals [17]:

$$Y=\frac{\sqrt{\pi}}{k\bar{v}}\{(1+\varkappa)^{-1/2}w'(q)+iw''(q)\}, \qquad (3.4)$$

$$q=[\sqrt{1+\varkappa}(\Gamma+\nu)-i(\Omega-\Delta)]/k\bar{v},$$

$$w(q)=e^{q^2}[1-\Phi(q)],\quad \Phi(q)=\frac{2}{\sqrt{\pi}}\int_0^q e^{-u^2}du.$$

Let us write, finally, in explicit form the expressions for $\rho_{mm}-\rho_{nn}$ and ρ:

$$\rho_{mm}(\mathbf{v})-\rho_{nn}(\mathbf{v})=NW(\mathbf{v})[1+2G^2\tau_1\,\mathrm{Re}(p+i\mathbf{kv})^{-1}]^{-1}$$

$$=NW(\mathbf{v})[1-\varkappa(\Gamma+\nu)y'(\mathbf{v})]=$$

$$=NW(\mathbf{v})\left\{1-\frac{\varkappa(\Gamma+\nu)^2}{(1+\varkappa)(\Gamma+\nu)^2+(\Omega-\Delta-\mathbf{kv})^2}\right\}, \qquad (3.5)$$

$$\rho(\mathbf{v})=iGNW(\mathbf{v})y(\mathbf{v}),\quad \langle\rho\rangle=iGNY. \qquad (3.6)$$

In our case, namely $\hat{\tilde{\nu}}=0$, the decrease of the difference in the populations under the influence of the field is given by the factor $[1+2G^2\tau_1\mathrm{Re}(p+i\mathbf{k}\cdot\mathbf{v})^{-1}]^{-1}$ (cf. (3.5) and (2.12)), which includes the sum of the lifetimes at the levels m and n (τ_1) and the "lifetime" of the off-diagonal element. This is perfectly natural, since the field can interact with the atom only if both the polarizations and the difference of the diagonal elements differ from zero. Thus, new combinations of relaxation parameters are of importance for nonlinear effects. However, even here the interpretation, from the point of view of the evolution of the system, is physically perfectly clear and lucid.

In view of the dependence of $\mathrm{Re}(p+i\mathbf{k}\cdot\mathbf{v})^{-1}$ on the velocity, the changes induced by the field in $\rho_{mm}(\mathbf{v})$

---

2) In nonlinear problems it is possible to introduce several "line shape" concepts: the line shape of spontaneous emission, of stimulated emission, etc.; all are proportional to each other only in the linear approximation and differ when nonlinear effects are taken into account. By line shape we mean henceforth $\langle\rho\rangle/iNG$ as a function of the frequency.

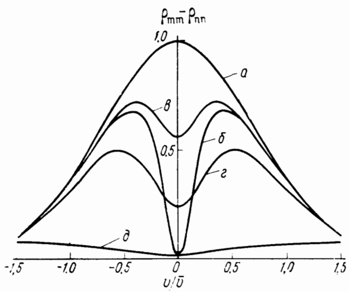

FIG. 1. Plot of $\rho_{mm}(v) - \rho_{nn}(v)$. $\Gamma_j = \Gamma, \nu_j = \nu, \tilde{\nu}_j = \tilde{\nu} = 0$; a – $\nu = 0$, $\kappa = 0$; b – $\nu = 0$, $\kappa = 0.83$; c – $\nu = 0.2\ k\bar{v}$, $\kappa = 0.83$; d – $\nu = 0.2\ k\bar{v}$, $\kappa = 3.3$; e – $\nu = 0.2\ k\bar{v}$, $\kappa = 83$

$- \rho_{nn}(v)$ are selective with respect to v. It is seen from (3.5) that the velocity distribution becomes nonequilibrium, since a characteristic resonant structure (the so-called "Bennett hole") appears on the Maxwellian curve near $\mathbf{k}\cdot\mathbf{v} = \Omega - \Delta$, with a width $(\Gamma + \nu)\sqrt{1 + \kappa}$ and with a relative depth $\kappa/(1 + \kappa)$ (Fig. 1).

In the analysis of the dependence of $\hat{\rho}$ on the pressure (i.e., on the collision frequencies ν_j and ν) it is possible to assume without loss of generality that Γ_j and Γ are constant. Otherwise it is possible to separate from Γ_j or Γ the part that depends on the pressure and to include it in ν_j, i.e., the matter reduces to a redistribution of the parameters.

It is seen from (3.2) that κ will be a decreasing function of the pressure. The relative depth of the "hole" which depends only on κ, will also decrease in this case, as is clearly seen from Fig. 1. We note that this change is connected, properly speaking, not with the diffusion of the atoms in velocity space, but simply with the lesser role of the field as a result of the decrease in the time of its interaction with the atom. On the other hand, the width of the hole

$$(\Gamma + \nu)\sqrt{1 + \varkappa} = \{(\Gamma + \nu)^2 + 2G^2(\Gamma + \nu)[(\Gamma_m + \nu_m)^{-1} + (\Gamma_n + \nu_n)^{-1}]\}^{1/2} \quad (3.7)$$

may vary in different fashion, depending on the value of κ and the relation between the relaxation parameters. At large values of ν and small κ, the principal role will be played by the term $(\Gamma + \nu)^2$, and the width will increase with increasing ν. On the other hand, if κ is sufficiently large and one of the Γ_j is smaller than Γ (we recall that $2\Gamma \geq \Gamma_m + \Gamma_n$), then the rate of decrease of the field term in (3.7) may be larger than the rate of increase of $(\Gamma + \nu)^2$, and the width on the whole be a decreasing function of the pressure. We note that in the widely used model $\Gamma = \Gamma_m = \Gamma_n$, $\nu = \nu_m = \nu_n$ (see, e.g.,[1]) the narrowing of the hole with increasing pressure is impossible, since here $(\Gamma + \nu)\sqrt{1 + \kappa} = \sqrt{(\Gamma + \nu)^2 + 4G^2}$.

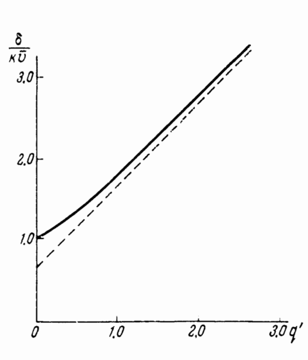

FIG. 2. Plot of the width of the function Y′ against q′.

We shall not analyze in detail the line shape, since it is practically the same as in the linear theory, the only difference being that $(\Gamma + \nu)$ must be replaced by $(\Gamma + \nu)\sqrt{1 + \kappa}$. We note only that the line width practically coincides with the Doppler width $k\bar{v}$ when $(\Gamma + \nu)\sqrt{1 + \kappa} \ll k\bar{v}$, increases monotonically with increasing $(\Gamma + \nu)\sqrt{1 + \kappa}$, and approaches $(\Gamma + \nu)\sqrt{1 + \kappa}$ if $(\Gamma + \nu)\sqrt{1 + \kappa} \gg k\bar{v}$ (Fig. 2). It should be clear from the foregoing that at large values of κ and when $2\Gamma > \Gamma_m + \Gamma_n$, there can exist a pressure region for which even quenching collisions produce line narrowing.

4. ABSENCE OF "PHASE MEMORY" ($\tilde{\nu} = 0$)

Of great importance for applications is the case when large phase shifts are produced in collisions of the second kind, so that the phase relations are destroyed practically completely in each collision. In this case $\tilde{\nu} = 0$, but for the diagonal elements it is possible to have $\tilde{\nu}_j \neq 0$ and even $\tilde{\nu}_j = \nu_j$ (frequently adiabatic collisions).

For $\nu = 0$ it follows from (2.9), (2.10), and (3.1) that

$$\rho_{mm}(v) - \rho_{nn}(v) = \frac{NW(v)}{1 + 2G^2\tau_2 Y'}\{1 - \varkappa(\Gamma + \nu)y'(v)\} = NW(v)\left\{1 - \frac{2G^2\tau_2 Y' + 2G^2\tau_1 y'(v)}{1 + 2G^2\tau_2 Y'}\right\}, \quad (4.1)$$

$$\rho / iGN = W(v)y(v) / [1 + 2G^2\tau_2 Y'], \quad \langle\rho\rangle / iGN = Y / [1 + 2G^2\tau_2 Y'], \quad (4.2)$$

$$\tau_2 = \tau - \tau_1 = \sum_{j=m,n} \tilde{\nu}_j(\Gamma_j + \nu_j)^{-1}(\Gamma_j + \nu_j - \tilde{\nu}_j)^{-1},$$

$$\tau = \sum_{j=m,n} (\Gamma_j + \nu_j - \tilde{\nu}_j)^{-1}. \quad (4.3)$$

From a comparison of (4.1), (4.2) with (3.5), (3.6) we see that the changes connected with $\nu_j \neq 0$ reduce to an additional factor $[1 + 2G^2\tau_2 Y']^{-1}$, which does not depend on v. The change in the difference of the populations contains, besides the "Bennett hole," also a part which is nonselective in v, or a "band" with Maxwellian distribution (see the second line in (4.1)). The ratio of the areas of the "band" and the "hole" is equal to τ_2/τ_1.

The singularities noted above, which are illustrated in Fig. 3, can be naturally interpreted from the tem-

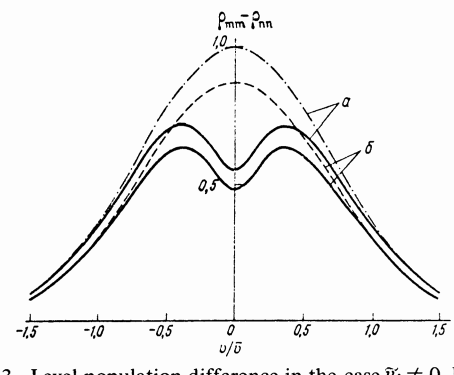

FIG. 3. Level population difference in the case $\tilde{\nu}_j \neq 0$. $\Gamma_j = \Gamma = 0.02\ k\bar{v}$, $\nu_j = \nu = 0.2\ k\bar{v}$, $\tilde{\nu} = 0$, $\kappa = 0.83$; a – $\tilde{\nu}_j = 0$; b – $\tilde{\nu}_j = 0.5\ \nu_j$. The dashed and dash-dot lines denote Maxwellian distributions.

poral point of view assumed by us. The parameter κ, which enters in $y(v)$ and determines the magnitude of the selective changes in $\rho_{mm} - \rho_{nn}$, contains the time τ_1, i.e., it characterizes the interaction of the field with the atom during the time elapsed up to the first collision of the second type. On the other hand, the parameter τ_2, which is the difference between the sum of the total lifetimes τ and τ_1, is all the remaining times during which the external field can appear. It can therefore be assumed that the factor $[1 + 2G^2\tau_2 Y']^{-1}$ describes the additional saturation due to the interaction during the time τ_2. It is perfectly natural that this additional saturation is not selective in v. In this respect, it is analogous to the saturation in the so-called "homogeneous broadening." This circumstance is a direct consequence of the assumed model.

The collisions lead to the appearance in velocity space of a particle flux proportional to $\tilde{\nu}_j$ and to the area $2G^2\tau_{1j}Y'$ of the non-equilibrium part. During the time $(\Gamma_j + \nu_j - \tilde{\nu}_j)^{-1}$ this flux leads to a departure of $\tilde{\nu}_j(\Gamma_j + \nu_j - \tilde{\nu}_j)^{-1}2G^2\tau_{1j}Y'$ atoms, and over the lifetime at both levels, to a departure of

$$\sum_{j=m,n} \tilde{\nu}_j(\Gamma_j + \nu_j - \tilde{\nu}_j)^{-1}\, 2G^2\tau_{1j}Y' = 2G^2\tau_2 Y'$$

atoms. On the other hand, by virtue of the main property of the model, the role of the atoms with velocity v is determined simply by their number, and everything reduces, consequently, to a proportional decrease of the velocity distribution, in full correspondence with formula (4.1).

Separation of the selective part in (4.1) is meaningful if the width of the function $y(v)$ is much smaller than $k\bar{v}$, i.e., $(\Gamma + \nu)\sqrt{1+\kappa} \ll k\bar{v}$. With this

$$1 + 2G^2\tau_2 Y' \cong 1 + 2\sqrt{\pi}\, G^2\tau_2 (k\bar{v})^{-1} \exp\left\{-\left(\frac{\Omega - \Delta}{k\bar{v}}\right)^2\right\} \tag{4.4}$$

and the additional saturation depends on the pressure only via τ_2. Let us consider τ_{2j} - one of the terms of τ_2 in (4.3). It is easy to show that τ_{2j} increases with pressure until $\nu_j \le \Gamma_j\sqrt{\nu_j/(\nu_j - \tilde{\nu}_j)}$, and decreases at larger ν_j. The maximum value of τ_{2j} is

$$\max \tau_{2j} = \frac{1}{\Gamma_j} \frac{\tilde{\nu}_j}{[\sqrt{\nu_j} + \sqrt{\nu_j - \tilde{\nu}_j}]^2} \tag{4.5}$$

and increases with decreasing $\nu_j - \tilde{\nu}_j$. Consequently, if $\nu_j - \tilde{\nu} \ll \nu_j$ and $\nu_j \gg \Gamma_j$, then it may happen that $\kappa_{2j} \gg \tau_{1j}$. The same pertains to κ_2 and τ_1.

According to the foregoing, the ratio of the integrals of the selective and nonselective parts, which is equal to τ_2/τ_1, can be very large. For the distribution in itself, however, the ratio of the two terms in (4.1) is

$$\frac{\tau_2}{\tau_1}\frac{Y'}{y'} \cong \sqrt{\pi}\frac{\tau_2}{\tau_1}\frac{\Gamma + \nu}{k\bar{v}}(1 + \varkappa) \qquad (\Omega = \Delta) \tag{4.6}$$

and may be not small when $\tau_2 \gg \tau_1$ even if $\Gamma + \nu \ll k\bar{v}$. Consequently the nonselective part can be significant not only in the integral sense (with respect to v), but also for the distribution with respect to v[3].

Let us consider now emission line shape determined by formula (4.2). It is easy to see that the denominator $1 + 2G^2\tau_2 Y'$ leads to an increase of the line width and to a decrease of its value at the maximum compared with (3.6), but does not disturb the symmetry with respect to the frequency $\Omega = \Delta$ (Fig. 4). As to the relative role of the selective and nonselective saturations, everything said concerning $\rho_{mm} - \rho_{nn}$ holds also for $\langle\rho\rangle$.

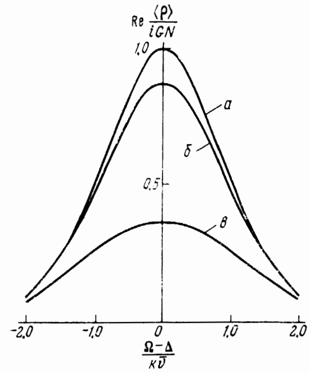

FIG. 4. Emission line contour for $\tilde{\nu}_j \neq 0$, $\Gamma_j = \Gamma = 0.02\, k\bar{v}$, $\nu_j = \nu$, $\tilde{\nu} = 0$, $\kappa = 0.83$; a – $\tilde{\nu}_j = 0$; b – $\tilde{\nu}_j = 0.5\,\nu_j$; c – $\tilde{\nu}_j = \nu_j$

5. THE CASE $\tilde{\nu}_j$, $\tilde{\nu} \neq 0$

Before we analyze in detail the rather complicated case when the arrival terms differ from zero in all three collision integrals, let us consider the general expression for $\rho_{mm} - \rho_{nn}$:

$$\rho_{mm}(v) - \rho_{nn}(v) = N\left\{W(v) - 2G^2\tau_2 W(v)\,\mathrm{Re}\left[\frac{\langle\rho\rangle}{iGN}\right] - 2G^2\tau_1\,\mathrm{Re}\left[\frac{\rho(v)}{iGN}\right]\right\}, \tag{5.1}$$

which is obtained from Eq. (2.6) for ρ_{jj} by eliminating $\langle\rho_{jj}\rangle$. It follows from (5.1) that in the general case the change in the population difference in the field contains nonselective and selective parts (the second and third terms of (5.1)), and the ratio of their integrals, just as in Sec. 4, is equal to τ_2/τ_1. However, the absolute values of the integrals and the dependence of the selective term on v change upon introduction of $\tilde{\nu} \neq 0$. To illustrate this, let us consider the relatively simple case $\tilde{\nu}'' = 0$, $\Omega = \Delta$:

$$\frac{\rho}{iGN} = \frac{W(v)y(v)/(1 - \tilde{\nu}Y')}{1 + 2G^2\tau_2 Y'/(1 - \tilde{\nu}Y')}, \quad \frac{\langle\rho\rangle}{iGN} = \frac{Y'/(1 - \tilde{\nu}Y')}{1 + 2G^2\tau_2 Y'(1 - \tilde{\nu}Y')},$$

$$\rho_{mm}(v) - \rho_{nn}(v) = NW(v)\,[1 + 2G^2\tau_2 Y'/(1 - \tilde{\nu}Y')]^{-1} \cdot \{1 - \varkappa(\Gamma + \nu)y'(v)/(1 - \tilde{\nu}Y')\}. \tag{5.2}$$

Comparing (5.2) with (4.1) and (4.2) we note that the functions $y(v)$ and Y are now replaced by $y(v)/(1 - \tilde{\nu}Y')$ and $Y/(1 - \tilde{\nu}Y')$. It is easy to see that this leads to an increase in line intensity, in the relative depth of the hole, and in the saturation effect as a whole; the form of the "hole" remains unchanged, just as the ratio of the areas of the "band" and the "hole" (τ_2/τ_1).

Thus, both the adiabaticity of the collisions $(\tilde{\nu}_j \neq 0)$ and the phase memory $(\tilde{\nu} \neq 0)$ lead to an intensification of the saturation effect. However, the physical causes of this are different. The role of the arrival terms for diagonal elements consists, first of all, of eliminating the non-equilibrium velocity distribution of the atoms. Inasmuch as the non-equilibrium character is due to the field, the parameter τ_2, which contains $\tilde{\nu}_j$, is encountered only in the form of a factor of G^2. On the other hand, the increase of the "average lifetime" ρ directly affects the line shape and this is

[3] This circumstance was apparently first pointed out by Kazantsev and Surdutovich [15].

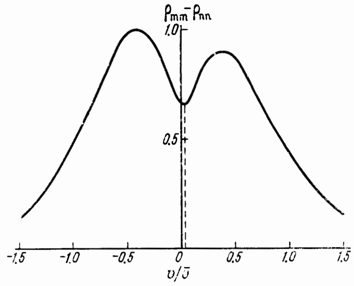

FIG. 5. Plot of $\rho_{mm}(v) - \rho_{nn}(v)$ for $\tilde{\nu} \neq 0$. $\Gamma_j = \Gamma = 0.02\,k\bar{v}$, $\tilde{\nu}_j = \nu_j = \nu = 0.2\,k\bar{v}$, $\Omega - \Delta = 0$, $\kappa = 0.83$, $\tilde{\nu}' = \tilde{\nu}'' = 0.5\,\nu$

the only reason why it influences the nonlinear effects. Accordingly, the parameter $\tilde{\nu}$ enters also in the linear theory.

The specific behavior of the arrival in the collision integral for ρ is manifest in the change of the shape of the "hole." Combining Eqs. (2.6) for ρ_{jj} and ρ, we can transform (5.1) into

$$\rho_{mm} - \rho_{nn} = NW(v)\left\{\left[1 - 2G^2\tau_2 \operatorname{Re}\frac{\langle\rho\rangle}{iGN}\right][1 - \varkappa(\Gamma + \nu)y'(v)] - \varkappa(\Gamma + \nu)\operatorname{Re}\left[\tilde{\nu}y(v)\frac{\langle\rho\rangle}{iGN}\right]\right\}. \quad (5.3)$$

The first term in (5.3) depends on $\mathbf{v}$ just as in the case $\tilde{\nu} = 0$ (see formula (4.1) and is an even function of $\Omega - \Delta - \mathbf{k}\cdot\mathbf{v}$. From the definition (3.2) of the function $y(v)$, and from the fact that $\tilde{\nu}$ and $\langle\rho\rangle$ are complex, it follows that the second term in (5.3) will contain both an even function and an odd function of $\Omega - \Delta - \mathbf{k}\cdot\mathbf{v}$, i.e., the "hole" is asymmetrical (Fig. 5). Thus, the "phase memory" leads to distinct peculiarities in the non-equilibrium part of the velocity distribution. peculiarities closely connected with the dispersion properties of $\langle\rho\rangle$. Incidentally, it must be borne in mind that in those cases when the selectivity of $y(v)$ is appreciable, $(\Gamma + \nu)\sqrt{1+\kappa} \ll k\bar{v}$, the second term in (5.3) will be small, amounting to a fraction on the order of $|\bar{\nu}|/k\bar{v}$ of the first.

The quantity $\langle\rho\rangle$, which determines the line shape and enters in (5.3), can be written in the form:

$$\frac{\langle\rho\rangle}{iGN} = \frac{Y(1 - \tilde{\nu}^*Y^*) - \tilde{\nu}^*\varkappa Y'^2}{|1 - \tilde{\nu}Y|^2 + 2G^2\tau_2\{Y' - \tilde{\nu}'[|Y|^2 + \varkappa Y'^2]\} - \varkappa[\tilde{\nu}' - |\tilde{\nu}|^2Y']Y'}. \quad (5.4)$$

It is possible to obtain from (5.4) relatively simple results only in the limiting cases of small and large values of $(\Gamma + \nu)\sqrt{1+\kappa}/k\bar{v}$. In the former case, for the region near the center of the line, it follows from (5.4) that

$$\operatorname{Re}\frac{\langle\rho\rangle}{iGN} = \frac{\sqrt{\pi}}{k\bar{v}}\frac{1}{\sqrt{1+\varkappa}}\left\{1 - \left(\frac{\Omega - \Delta + 2\tilde{\nu}''}{k\bar{v}}\right)^2 + \frac{\sqrt{\pi}\,\tilde{\nu}'}{k\bar{v}\sqrt{1+\varkappa}} - \frac{2}{\sqrt{\pi}}\frac{(1+\varkappa)(\Gamma+\nu) + \pi G^2\tau^2}{k\bar{v}\sqrt{1+\varkappa}}\right\}$$

$$|(\Gamma + \nu)\sqrt{1+\varkappa} - i(\Omega - \Delta)| \ll k\bar{v}, \quad 2G^2\tau_2 \ll k\bar{v}\sqrt{1+\varkappa}. \quad (5.5)$$

If $\tilde{\nu}'' \neq 0$, then the maximum of the line occurs at the frequency

$$\Omega_2 = \Delta - 2\tilde{\nu}''. \quad (5.6)$$

The coefficient 2 at $\tilde{\nu}''$ in (5.6) is connected, as shown in[10], with the simultaneity of the change in the velocity of the atom and in the phase shift of the atomic oscillator in collisions of the second type. We note that in the approximation (5.5), the frequency Ω_2 does not depend on κ, i.e., it coincides with the frequency in the linear theory[10].

The only positive term in (5.5) (containing $\tilde{\nu}'$) reflects the increase of the intensity and the narrowing of the line as a result of the phase memory (the Dicke effect). The presence of $\kappa \neq 0$ detones a decreased role of $\tilde{\nu}'$. To the contrary, negative terms in (5.5) increase with increasing κ. Consequently, the nonlinear effects lead only to line broadening.

In the limiting case opposite to (5.5), it is possible to use the asymptotic value of $w(q)$:

$$w(q) \cong \frac{1}{q} - \frac{(k\bar{v})^2}{2q^3}, \quad |q| \gg 1. \quad (5.7)$$

With the aid of (5.7) we obtain for the central region of the line in lieu of (5.4)

$$\frac{\langle\rho\rangle}{iGN} = \frac{\tilde{\Gamma} + \gamma_d + i(\Omega - \Delta + \tilde{\nu}'')}{(\tilde{\Gamma} + \gamma_d)^2[1 + 2G^2\tau/(\tilde{\Gamma} + \gamma_d)] + (\Omega - \Delta + \tilde{\nu}'')^2},$$

$$\tilde{\Gamma} = \Gamma + \nu - \tilde{\nu}', \quad \gamma_d = \tilde{\nu}'(k\bar{v})^2/2(\Gamma + \nu)^2(1 + \varkappa),$$

$$k\bar{v}, \ \Omega - \Delta + \tilde{\nu}'' \ll (\Gamma + \nu)\sqrt{1+\varkappa}. \quad (5.8)$$

Formula (5.8) defines a line having a dispersion form with a saturation parameter $2G^2\tau/(\tilde{\Gamma} + \gamma_d)$, a diffusion width γ_d, and a total impact width $\tilde{\Gamma}$. The maximum of the line is shifted by an amount equal to the impact shift $\Delta - \tilde{\nu}''$ (see (5.6)) in accordance with the fact that in the diffusion approximation (5.7) the statistical dependence of the Doppler and impact broadening mechanisms is insignificant[10].

We call attention to the fact that γ_d contains a factor $(1+\kappa)^{-1}$ besides $(\Gamma + \nu)^{-2}$, i.e., increasing either the collision frequency or the field decreases the diffusion width. However, the physical mechanisms that lead to the appearance of these factors are quite different. Collisions decrease the velocity of the Brownian motion of the atom, as a result of which the phase difference due to the Doppler effect increases more slowly, the lifetime of the off-diagonal element ρ increases, and the line width decreases. On the other hand, an increase of the external field causes the atoms whose velocities differ from the resonant value $\mathbf{k}\cdot\mathbf{v}_{res} = \Omega - \Delta$ to interact more effectively with the field, and at sufficiently large values of κ the Doppler frequency shift ceases to play any role. Accordingly, formula (5.7) remains in force if the collision frequencies are relatively small, but $1 + \kappa \gg [k\bar{v}/(\Gamma + \nu)]^2$. From this point of view, the dependence of the total line width on the field in (5.8) is instructive:

$$\gamma = [(\tilde{\Gamma} + \gamma_d)^2 + 2G^2\tau(\tilde{\Gamma} + \gamma_d)]^{1/2}.$$

The diffusion width γ_d decreases with increasing field, and the factor G^2 increases. It is easy to show that by virtue of $\tau = \tau_1 + \tau_2 > \tau_1$ and $(\Gamma + \nu)\sqrt{1+\kappa} \gg k\bar{v}$, the latter factor is stonger, and γ is a monotonically increasing function of G^2.

The foregoing considerations regarding the difference between the physical causes of the levelling of the Doppler effect as the result of collisions and nonlinear effects explain also the deviation of formula (5.8) from the results of a theory based on the diffusion equations of Andreeva and Kuznetsova[13]. These equations are given by (in our notation)

$$\left(\frac{\partial}{\partial t}+\Gamma_j+\nu_j-\tilde{\nu}_j\right)\langle\rho_{jj}\rangle-D_j\nabla^2\langle\rho_{jj}\rangle\mp 2\,\mathrm{Re}[iV_{nm}\langle\rho_{mn}\rangle]=Q_j$$

$$(j=m,n),$$

$$\left[\frac{\partial}{\partial t}+\tilde{\Gamma}+i(\Delta-\tilde{\nu}'')\right]\langle\rho_{mn}\rangle-D\nabla^2\langle\rho_{mn}\rangle-iV_{mn}\langle\rho_{mm}-\rho_{nn}\rangle=0. \tag{5.9}$$

For the field (2.4), Eqs. (5.9) yield the expression

$$\frac{\langle\rho\rangle}{iGN}=\frac{\tilde{\Gamma}+Dk^2+i(\Omega-\Delta+\tilde{\nu}'')}{(\tilde{\Gamma}+Dk^2)^2[1+2G^2\tau/(\tilde{\Gamma}+Dk^2)]+(\Omega-\Delta+\tilde{\nu}'')^2}. \tag{5.10}$$

Consequently, the role of the diffusion width is played here by the combination Dk^2, which does not contain the factor $(1+\kappa)^{-1}$. This is perfectly natural, since in Eqs. (5.9) no account is taken of events with characteristic times $(\Gamma+\nu)^{-1}$ or $(\Gamma_j+\nu_j)^{-1}$, and they are valid only if $\Gamma+\nu \gg k\bar{v}$. If at the same time $\kappa\sim 1$, then

$$\frac{2G^2\tau}{\tilde{\Gamma}+\gamma_d}=\varkappa\frac{\tau}{\tau_1}\frac{\Gamma+\nu}{\tilde{\Gamma}+\gamma_d}\gg 1.$$

Near the center of the line, formulas (5.10) and (5.8) coincide,

$$\frac{\langle\rho\rangle}{iGN}\cong\frac{1}{2G^2\tau},$$

and do not contain the parameter κ. Therefore (5.10) does not differ essentially from (5.8) in the region of its validity. However, for (5.8) to be valid less stringent conditions are sufficient, $(\Gamma+\nu)\sqrt{1+\kappa} \gg k\bar{v}$, and this reflects the second, field-connected cause of the levelling of the Doppler effect contained in (5.8).

It is seen from (5.3) and (3.2) that in the approximation (5.8) the velocity distribution is Maxwellian, and is smaller compared with $G=0$ by a factor

$$1+\frac{2G^2\tau}{\Gamma+\gamma_d}\frac{(\tilde{\Gamma}+\gamma_d)^2}{(\Gamma+\gamma_d)^2+(\Omega-\Delta+\tilde{\nu}'')^2}$$

This is to be expected, since the line has a dispersion shape, and the saturation should be of the same type as in the case of homogeneous broadening.

6. CONCLUSION

In the considered exact solution of the kinetic equation in the strong-collision model, it is possible to separate, in our opinion, aspects which are typical of the model from results which essentially do not depend on the model. The former include the conservation of the shape of the "hole," which is determined by the function $y(v)$, and the appearance on $\rho_{mm}(v)-\rho_{nn}(v)$ of a "band" whose area is related to the area of the "hole" like τ_2/τ_1. If the changes of velocity of the order of v were to accumulate in the course of many collisions (the Chandrasekhar weak-interaction model[18,10]), then it is natural to expect the "hole" to become broadened by an amount equal to the drift of the atom in velocity space, which occurs within the lifetime of the atom levels m and n[11]. In this case no "band" can occur, since the drift has a "local" character (integral of collisions of differential and not integral type). For the same reasons, the change in the entire picture with changing pressure can have an entirely different form.

At the same time, features untypical of the model are, apparently, the establishment of the difference in the role of arrival for the diagonal and off-diagonal elements of ρ (see the discussion of formula (5.2)), although both factors should lead to an intensification of the nonlinear effects. We emphasize, finally, that the temporal interpretation presented consistently in the present article, based on concepts connected with the evolution of the system, is also characteristic of nonlinear phenomena, since an important role should be played by the times of interaction between the atom and the field and by the sum of the effects occurring within the lifetime at both levels.

Our entire analysis was carried out for equilibrium functions $q_j(v)$. We can easily get rid of this limitation by finding in lieu of (2.9) and (2.10)

$$\langle\hat{\rho}\rangle=[\hat{E}-\langle\hat{A}^{-1}\rangle\hat{\tilde{\nu}}]^{-1}\int\hat{A}^{-1}(\mathbf{v})\hat{q}(\mathbf{v})d\mathbf{v},$$

$$\hat{\rho}(\mathbf{v})=\hat{A}^{-1}(\mathbf{v})\left\{\hat{q}(\mathbf{v})+W(\mathbf{v})\hat{\tilde{\nu}}[\hat{E}-\langle\hat{A}^{-1}\rangle\hat{\tilde{\nu}}]^{-1}\int\hat{A}^{-1}(\mathbf{v})\hat{q}(\mathbf{v})d\mathbf{v}\right\}. \tag{6.1}$$

Formulas (6.1) clearly reflect the existence of two periods of evolution of the atom (before and after the collision of the second type), during which the velocity distributions are different ($q(v)$ and $W(v)$). This is seen, in particular, from the fact that $\hat{\rho}(v)$ contains two terms that cannot be unified (see (2.10)), and two types of averaging of $\hat{A}^{-1}(v)$ are encountered, with weights $\hat{\nu}W(v)$ and $\hat{q}(v)$. An analysis of (6.1) leads essentially to the same main conclusions as obtained with equilibrium q_j: the transitions induced by the field cause the appearance of a "band" and a "hole" on the $\rho_{mm}-\rho_{nn}$ curve, their areas are related like τ_2/τ_1, the adiabaticity of the collisions and the phase memory intensify the saturation effect, etc. A more detailed analysis calls for concretization of $q_j(v)$, which is beyond the scope of the present article. We therefore confine ourselves to the foregoing remarks, hoping to return to this problem in another place.

[1] A. Vuylsteke, Elements of Maser Theory, Van Nostrand, 1960.

[2] V. M. Faĭn and Ya. I. Khanin, Kvantovaya radiofizika (Quantum Radiophysics), Soviet Radio, 1965.

[3] S. G. Rautian, Fiz. Tverd. Tela 6, 1857 (1964) [Sov. Phys.-Solid State 6, 1462 (1964)].

[4] S. G. Rautian and I. I. Sobel'man, Zh. Eksp. Teor. Fiz. 44, 832 (1963) [Sov. Phys.-JETP 17, 664 (1963)].

[5] S. G. Rautian, Dissertation, Physics Institute, Academy of Sciences, 1966.

[6] W. Lamb, in: Kvantovaya optika i kavantovaya-radiofizika (Quantum Optics and Quantum Radiophysics) (Russ. Transl.), Mir, 1966, p. 281.

[7] T. A. Germogenova and S. G. Rautian, Zh. Eksp. Teor. Fiz. 46, 745 (1964) [Sov. Phys.-JETP 19, 507 (1964)].

[8] W. R. Bennet, Jr., S. F. Jacobs, J. T. LaTourette, and P. Rabinowitz, Appl. Phys. Lett. 5, 56 (1964).

[9] R. Cordover, J. Parks, A. Javan, and A. Szoke, Physics of Quantum Electronics, McGraw Hill, 1966, p. 591.

[10] S. G. Rautian and I. I. Sobel'man, Usp. Fiz. Nauk 90, 209 (1966) [Sov. Phys.-Usp. 9, 701 (1967)].

[11] S. G. Rautian, Zh. Eksp. Teor. Fiz. 51, 1176 (1966) [Sov. Phys.-JETP 24, 788 (1967)].

[12] B. L. Gyorffy and W. E. Lamb, Jr., Physics of Quantum Electronics, McGraw Hill, 1966, p. 602.

[13] T. D. Andreeva and T. I. Kuznetsova, Preprint, Physics Institute, Academy of Sciences No. 17, 1967.

[14]A. P. Kazantsev, Zh. Eksp. Teor. Fiz. 51, 1751 (1966) [Sov. Phys.-JETP 24, 1183 (1967)].

[15]A. P. Kazantsev and G. I. Surdutovich, Transactions, Second Symposium on Nonlinear Optics, Novosibirsk, 1966.

[16]A. I. Burshtein, Zh. Eksp. Teor. Fiz. 48, 850 (1965) [Sov. Phys.-JETP 21, 567 (1965)].

[17]V. N. Faddeeva and N. M. Terent'ev, Tablitsy Znacheniĭ integrala veroyatnosteĭ ot kompleksnogo argumenta (Tables of the Values of the Probability Integral of Complex Argument), Gostekhizdat, 1954.

[18]S. Chandrasekhar, Revs. Modern Phys. 15, 1 (1943).

Translated by J. G. Adashko
110