DYNAMIC THERMAL EFFECT IN SUPERCONDUCTORS IN THE INTERMEDIATE STATE

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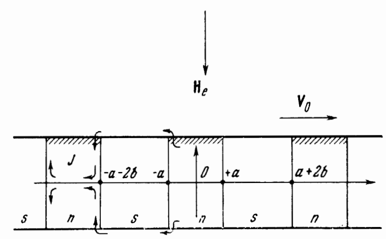
We discuss thermal effects that are linear in the current and arise when current flows through a superconductor in the intermediate state, in a direction perpendicular to the laminations. An expression is obtained for the rate of displacement of the superconducting regions, due to these thermal effects.

In recent experiments, Sharvin[1] observed motion of superconducting regions of superconductors in the intermediate state, produced by an external current flowing perpendicular to the direction of the laminations. One of the possible mechanisms of this motion, proposed by Sharvin, is connected with the presence in the normal layers of an intermediate state of Hall currents parallel to the phase boundaries. For superconductors with an uncompensated Hall effect at low temperatures ($T \ll T_c$), this mechanism is apparently the main one.

Besides motion due to the Hall currents, there exists a thermal motion mechanism which takes place in all superconductors, but which is manifest in pure form only for superconductors with a weakly pronounced Hall effect (compensated Hall effect in tin or lead, influence of impurities, etc.). The cause of the thermal motion is the Peltier heat released at the phase boundaries when current is made to flow perpendicular to the lamination lines. Being an effect linear in the current, the Peltier heat reverses sign on each succeeding phase boundary, heating some of them and cooling the others. The appearance of heat sources of opposite sign on the phase boundaries causes the critical magnetic field of the $\overrightarrow{sn}$ boundaries (the arrow indicates the direction of the current) to become smaller, and the critical magnetic field of the $\overrightarrow{ns}$ boundaries larger, than the average field in the normal layers (concerning the sign of the Peltier coefficient on the ns boundary see[2]). This lack of equilibrium is the cause of the boundary motion. We shall henceforth be interested in the steady-state uniform motion of the s-regions. The rate of this motion is determined not only by the strength of the Peltier-heat sources, but also by the magnitude of the induction currents produced when the boundaries move in the magnetic field, by the heat of the phase transition, by the thermal conductivity of the n and s layers, etc. In other words, the motion velocity V should be determined by a self-consistent solution of the electrodynamic and thermodynamic problems.

Concrete calculations of the indicated phenomenon were carried out for the simple case of a current-carrying superconducting plate placed in a magnetic field H_e perpendicular to the surface of the plate. The potential difference is applied in such a way that the current lines are orthogonal to the direction of plate laminations. The period of the laminations l and the relative magnitudes of the n and s layers at $j = 0$, an external field H_e, and



a temperature T_0 are assumed known (see[3], p. 236). The thickness of the plate is 2d, the density of the external current in the normal layers is j, the z axis is directed along the central line of the plate, the y axis is parallel to the stratification direction, the x axis is perpendicular to the surface of the plate, $H_e \parallel x$, $j \parallel z$, the thickness of the normal (superconducting layers) is 2a(2b). The origin coincides at any given instant of time with the center of the normal layer (see the figure).

1. The complete system of equations describing the distributions of T(x, y, z, t) and H(x, y, z, t) in the plate, consist of Maxwell's equations and the heat conduction equations

$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \quad \operatorname{rot} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad c_{n,s}\frac{\partial T_{n,s}}{\partial t} = -\operatorname{div} \mathbf{q}_{n,s},$$
$$\mathbf{j} = \sigma \mathbf{E} + \alpha \nabla T, \qquad \mathbf{q} = (\varphi + \alpha T)\mathbf{j} - \varkappa \nabla T; \tag{1}$$

here **q** is the energy flux (see[3], p. 142), φ the electric potential, α the thermoelectric coefficient, $\varkappa$ the coefficient of thermal conductivity, $c_{n,s}$ the specific heat of the normal and superconducting phases, σ the normal conductivity, and c the speed of light.

The boundary conditions call for continuity of the temperature [1)], current density **j**, and energy flux density **q** on all the phase boundaries (the phase boundaries are assumed to be geometrically flat, i.e., δ/a and $\delta/b \ll 1$, δ—depth of penetration of the magnetic field in the bulky superconductor). In addition ([3], p. 228), on all the phase boundaries the magnetic field is equal to the critical value, the electric field tangential to the ns boundaries is $E_t = -V \times H_c/c$ (V—velocity of the boundary), and the total flux of the external magnetic field

[1)]We neglect the temperature jumps on the ns and sn boundaries, due to the thermal resistance of the boundary [4], in view of the smallness of these jumps in the region $T \sim T_c$ which is of significance for the thermal motion.

through each n layer is conserved. The connection between the critical magnetic field H_c and the temperature is assumed in the form

$$H_c(T) = H_c(0)(1-\theta^2), \quad \theta = T/T_c(0). \tag{2}$$

From the foregoing equations and boundary conditions it is seen that the vectors j and q have in the general case all three components different from zero. The currents j_z in the normal layers are connected with the external field E_z[2]. On crossing the normal phase boundaries, the current lines j_z in the superconducting layers become bent in a definite manner (the Meissner effect), such as to ensure the continuity of j_z from one normal layer to the other while remaining within the depth δ of the given s layer. The continuity of the current j_z on the ns and sn boundaries, with allowance for the continuity of T and q_z, leads to the appearance of Peltier heat on the phase boundaries ([3], p. 144):

$$Q_{\substack{ns\\sn}} = \pm \Pi j_z, \quad \Pi \equiv \Pi_{ns} = T\alpha_n(T), \quad \alpha_s \equiv 0. \tag{3}$$

The Peltier coefficient Π_{ns} can in general depend on the velocity of motion of the boundary. We shall assume below that this dependence, for small values of V, is analytic, i.e.,

$$\Pi_{ns}(V) = \Pi_{ns}(0) + \frac{\partial \Pi_{ns}}{\partial V}\Big|_0 V + \ldots.$$

The Peltier heat causes motion of the boundaries along the z axis. As a result of this, a field $E_y = -VH_c/c$ is induced in the normal layers (V—velocity of motion of the ns and sn boundaries), i.e., $j_y \neq 0$ and $q_y \neq 0$. The existence of normal $j_y \neq 0$ in a plate with boundaries $y = \pm L$ is possible because a superconducting current, due to the inequality of the critical magnetic field on the sn and ns boundaries, flows in a direction opposite to the normal currents j_y. Thus, the current lines j_y in the normal layers are closed by the neighboring superconducting layers. From the thermal point of view, Peltier heat $Q_{\pm L} = \pm \Pi_{ns} j_y$ should also be released on the boundaries $y = \pm L$, where the currents j_y go into the superconducting layers. These heat sources, however, unlike Q_{ns} and Q_{sn} from (3), are located on the outer surface of the plate, i.e., they are in direct thermal contact with the thermostat. Their influence on the volume of the plate is therefore small (to the degree that b/L is small).

In addition to the heat sources that are linear in the current and are located on the ns, sn, and $y = \pm L$ boundaries, there are volume heat sources. All are of the order of j^2. The thermal fluxes due to such sources are of the order of $\kappa(T_0 - \overline{T})/t$, where $\overline{T}$ is the average temperature of the plate and d are the characteristic dimensions of the plate.

2. Let the density of the external current be small, so that all the quantities change insignificantly upon application of this current as compared with their equilibrium values. In this approximation, the volume heat sources of order j^2 are insignificant. It is also to neglect the Peltier-heat sources on the $y = \pm L$ boundaries, since usually $b/L \ll 1$. As a result of these simplifications, the temperature gradients along the axes x and y drop out. But when $\nabla_x T = \nabla_y T \equiv 0$, the phase boundaries are flat and parallel to each other (with the exception of the surface layers near $x = \pm d$ and $y = \pm L$, the influence of which can be neglected by virtue of the small parameters $b/d \ll 1$, $b/L \ll 1$). Taking into account the fact that each s-layer is equipotential, we can conclude therefore that E_z = const, the currents $j_z = \sigma E_z + \alpha_n \nabla_z T$ have a uniform density, and consequently the sources Q_{ns} and Q_{sn} from (3) do not depend on x or y. The boundary conditions for H then also cease to depend on x and y.

Thus, in the linear approximation the entire system of equations depends only on z and t. Assuming also that the regime of motion with constant velocity V, of interest to us actually exists, we write out the one-dimensional system of equations in explicit form

$$D_{ns}\frac{d^2T_{ns}}{d\eta^2} = -V\frac{dT_{ns}}{d\eta}, \quad D_H\frac{d^2H}{d\eta^2} = -V\frac{dH}{d\eta},$$
$$D_{ns} = \frac{\varkappa_{ns}}{c_{ns}}, \quad D_H = \frac{c^2}{4\pi\sigma}, \tag{4}$$

$\eta = z - Vt$; T_n and T_s are the temperature distributions on the z axis for the n and s regions, respectively. The diffusion equation for H is usually obtained under the assumption that $j = \sigma E$. In our case $j = \sigma\mathbf{E} + \alpha\nabla T$, but the temperature addition does not enter in the equation for H in view of the fact that curl $\nabla \equiv 0$. The magnetic field of the current j, which is of first order of smallness, likewise does not enter explicitly in (4), as can be readily verified. It is taken into account by the boundary conditions.

The boundary conditions of the problem, calculated schematically above, assume in the linear approximation the form

$$T_n\big|_{\eta=\pm a} = T_s\big|_{\eta=\pm a}, \quad T_n\big|_{\eta=\mp a} = T_s\big|_{\eta=\pm a\pm 2b}, \tag{5}$$

$$\pm\varkappa_n\frac{dT_n}{d\eta}\Big|_{\pm a} \mp \varkappa_s\frac{dT_s}{d\eta}\Big|_{\pm a} = \mp q(T_0)V(T_0) \pm Q(T_0),$$
$$q(T) = -T\frac{H_c}{4\pi}\frac{dH_c}{dT}. \tag{6}$$

In view of the periodicity of the general solution, the boundary conditions are written for one complete cycle of alternating n and s boundaries, and should then repeat periodically. The jump $dT/d\eta$ (6) is determined by the difference between the strength of the Peltier-heat sources (3) and the heat q(T)V consumed in the displacement of the boundaries (q(T) is the heat of the phase transition). It should be noted that in the general case the right side of (6) should have the form $\mp q^{(\pm)}V^{\pm} \pm Q^{\pm}$, where the indices ±, such as for the velocity $V^{\pm}$, indicate that the corresponding quantities on the boundary with coordinate +a do not coincide with the values on the boundary

[2]We note that in the case of a plate with boundaries $y = \pm L$, the total external current J_z is not equal to Sj_z, where S is the area of the cross section of the plate in the y direction, and j_z is the current density in the normal layer. This is connected with the fact that part of the current flows through superconducting bridges of thickness ~ b, joining the superconducting layers of the plate on the boundaries $y = \pm L$. However, the current through these bridges is a bounded quantity, and therefore, for a sufficiently large total current j_z and for $b/L \ll 1$, the contribution of the currents through the bridges to the total current j_z becomes insignificant. Then $j_z \approx Sj_z$, and the density of the normal currents can still be sufficiently small, so that the linear approximation used below is valid.

$-a$: $V^+ \neq V^-$, $q^+ \neq q^-$, $Q^+ \neq Q^-$. The quantities $q(T_0)$, $V(T_0)$, and $Q(T_0)$, or simply q_0, V_0, Q_0 in (6) are the first terms of the expansions of $q^\pm$, $V^\pm$, and $Q^\pm$ in powers of $(T^\pm - T_0)/T_0 \ll 1$, which corresponds to the linear approximation. For example,

$$V^\pm = V_0\left(1 \pm O\left(\frac{\Delta T}{T_0}\right) + \ldots\right).$$

The Peltier coefficient in the linear approximation coincides with Π_{ns} for the stationary boundary.

The boundary conditions for $H(\eta)$ are written in the form

$$H(\eta)|_{\pm a} = H_c^\pm - H_j, \quad H_c^\pm \equiv H_c(T^\pm); \tag{7}$$

$$\left.\frac{dH}{d\eta}\right|_{\pm a} = -\frac{4\pi\sigma}{c^2} H_c(T_0) V_0; \tag{8}$$

$$\int_{-a}^{+a} H(\eta)\, d\eta = 2H_c(a+b),$$
$$2(a+b) = l(T_0, \tilde{H}_c), \quad \tilde{\mathbf{H}}_c = \mathbf{H}_c + \mathbf{H}_j; \tag{9}$$

here $H(\eta) \equiv H_x(\eta)$ and H_j is the magnetic field of the current j averaged over the thickness of the plate.

The system of equations (2)—(9) is complete. In the general case of small currents, it has no stationary solution (stationary in the sense of the dependence of all the quantities only on $z - Vt$). This is connected with the fact that the stationarity condition $V^+ = V^-$ does not coincide with any of the boundary conditions given above, and is thus redundant. However, the nonstationary $V^+ \neq V^-$ cannot be a monotonic function of the time. The noncoincidence of the boundary velocities implies a corresponding decrease (increase) of the density of the magnetic field in the normal layers of the metal (condition (9) with account of $a + b = \text{const}$), preventing further changes in the dimensions of the normal regions. The competition of these two factors has a periodic character with period $2a^2/D_H$, and leads to additional velocity oscillations, superimposed on the main uniform motion of the superconducting regions along the current lines, described by the linear theory.

3. The general solution of (4) is of the form

$$T(\eta) = c_1 + c_2 e^{-V\eta/D}, \quad H(\eta) = c_3 + c_4 e^{-V\eta/D_H}.$$

Noting that in the linear approximation $Va/D \ll 1$ and $Va/D_H \ll 1$, expanding the exponentials in powers of $V\eta/D$ and confining ourselves to the linear terms, we obtain

$$H = A\eta + H_c, \quad A = (H^+ - H^-)/2a < 0;$$
$$T = T_0 + T^*;$$
$$T_n^* = \frac{b}{a}\frac{Q_0 - q_0 V_0}{\varkappa_s + \varkappa_n b/a}\eta, \quad -a \leqslant \eta \leqslant +a;$$
$$T_s^* = \frac{Q_0 - q_0 V_0}{\varkappa_s + \varkappa_n b/a}(a + b - \eta), \quad +a \leqslant \eta \leqslant a + 2b, \tag{10}$$

and

$$V_0 = \frac{\Lambda_0 \Pi_{ns} j}{4\pi\sigma c^{-2} H_c + q_0 \Lambda_0}, \tag{11}$$

where

$$\Lambda_0 = \frac{2H_c(0) T_0 b/a}{T_c^2(0)(\varkappa_s + \varkappa_n b/a)}.$$

For the Peltier coefficient it is necessary to take either the experimental values (see [5], p. 90), or use the estimate $\Pi_{ns} = H_c^2(0)\theta^2/3\pi eN$ ([5] pp. 87, 90 and [6], p. 29), where N is the number of electrons per cm^3.

Expression (11) allows us to make a number of qualitative remarks concerning the behavior of V_0 as a function of the parameters determining V_0. As a function of the temperature, V_0 decreases when θ tends to zero. In the opposite limiting case, when $\theta \to 1$, V_0 ceases to depend on θ. The velocity depends essentially on the ratio b/a. If $b/a \ll 1$, then V_0 tends to zero like b/a, and in the case $b/a \gg 1$ it ceases to depend on b/a. The ratio b/a determines the contribution made to the velocity by the thermal conductivity coefficients κ_s and κ_n. When $b/a \ll 1$, the principal role is played by κ_s, and when $b/a \gg 1$ by the coefficient κ_n. The possibility of such a separation may turn out to be convenient for the study of the singularities of the thermal conductivity of a superconductor in the intermediate state, which were investigated by Andreev[7].

In conclusion, let us estimate V_0. Putting $b/a \sim 1$, $\theta \sim 1$, and $\kappa_s < \kappa_n$, using for κ_n the expression $\kappa_n = (1/3)\pi^2\sigma(k/e)^2 T$ (the Wiedemann-Franz law, k—Boltzmann's constant), and recognizing that the inequality $4\pi\sigma c^{-2} H_c > q_0\Lambda_0$ is satisfied at the customary values $\sigma \sim 10^{18}$—10^{20} sec^{-1}, $H_c(0) \sim 10^2$—10^3 G, and $T_c(0) \sim 1$—$10°$K, we neglect $q_0\Lambda_0$ in the denominator of (11) and rewrite V_0 in the form

$$V_0 = \frac{V_d}{\pi^4}\frac{H_c(0)}{H_c(T)}\left(\frac{H_c(0)\theta ce}{\sigma T_c(0) k}\right)^2 \approx (10^{-1} - 10^{-4}) V_d,$$

where $V_d = j/eN$ is the carrier drift velocity. For comparison we note that motion of the superconducting regions due to the uncompensated Hall effect occurs at a velocity $V_H \sim \alpha V_d$, $\alpha < 1$[1], with V_H reaching a maximum value at $T = 0$.

The drift velocities corresponding to the linear approximation are of the order of $V_d \sim 10^{-3}$—10^{-4} cm/sec, so that $V_0 \sim 10^{-4}$—10^{-6} cm/sec. The current j at $V_d \sim 10^{-3}$ cm/sec has a value ~ 1 A/cm^2. The magnetic field H_j at such currents and at a plate thickness $d \sim 1$ cm is $H_j \sim 1$ G, i.e., $H_j \ll H_c$, as it should be in the linear approximation.

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