HARD CERENKOV RADIATION AT MODERATE ENERGIES

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We consider the kinematics and determine the probability of hard Cerenkov radiation in the region of nonrelativistic and moderately relativistic energies of the radiating particle (the earlier analysis was carried out in the ultrarelativistic region). We consider the conditions under which the hard Cerenkov radiation could be observed and distinguished from bremsstrahlung. These conditions are apparently realizable in present day experiments.

THE emission of a Cerenkov photon by a fast particle passing through matter can be regarded as the occurrence of a photon elementary excitation in a medium—a dynamic system which consists of matter and of a transverse electromagnetic field interacting with the matter. In a transparent medium, the energy of the photon is connected with its momentum **q** by the relation $\omega = q/n(\omega)$, where $n(\omega)$ is the refractive index of the medium ($\hbar = c = 1$). The energy and momentum conservation laws impose a kinematic limitation $n(\omega) > 1$ on the occurrence of such a photon excitation of the medium under the influence of a fast particle. The condition $n(\omega) > 1$ is satisfied at frequencies $\omega < \omega_0$, where ω_0 is the upper limit of the ultraviolet region. This forbids the emission of one or several vacuum photons, for which $n(\omega) \le 1$, by a fast particle. However, such hard photons can be radiated by a fast particle with accompaniment of a photon with $\omega < \omega_0$ interacting with the matter. This process was considered in [1,2] and called there hard Cerenkov radiation. In fact, hard Cerenkov radiation is a form of bremsstrahlung, differing from the classical Bethe-Heitler bremsstrahlung only in the fact that the excess energy and momentum is taken then not by a separate nucleus, but by the entire medium in which the photon excitation is produced. The hard Cerenkov radiation is even closer in spirit to the bremsstrahlung accompanied not only by transfer of the excess momentum but also by excitation of the atoms of the medium [3]. It is furthermore very close to the radiation of a transverse vacuum photon, accompanied by excitation or annihilation in the medium of a longitudinal electromagnetic wave—a plasmon [4,5,2]. All these and other possible processes are considered from a unified point of view in [6]. Here we shall consider the hard Cerenkov radiation at lower energies E_0 of the fast particle than in [1,2]—in the moderately relativistic and nonrelativistic cases.

The expression for the probability per unit path, dw_{kq}, that a fast particle with initial velocity v_0 will emit a vacuum quantum with momentum **k** and energy $k = |\mathbf{k}|$, accompanied by a transverse photon interacting with the isotropic matter ("dressed" photon) with momentum **q** and energy $\omega < \omega_0$, was derived in [2]. It takes the form

$$dw_{\mathbf{kq}} = 2e^4 \frac{N_\omega + 1}{v_0} S_t \left(-\operatorname{Im} \frac{1}{\omega^2 \varepsilon_t(\omega) - q^2} \right) \frac{dk\, d\mathbf{q}}{(2\pi)^4 m^2 k}. \quad (1)$$

Formula (1) is suitable for both transparent and absorbing media. In this formula, $\epsilon_t = n^2(\omega)$ is the transverse dielectric constant of the medium, N_ω is the number of "dressed" photons with frequency ω per quantum state, and S_t is the sum over the spin states of the radiating particle with charge e (we shall assume this to be a Dirac particle), as given below. In the factor N + 1, the second term corresponds to spontaneous two-quantum Cerenkov radiation, and the first corresponds to stimulated emission of two quanta, the vacuum hard quantum and the dressed soft quantum, this radiation being stimulated by the soft quantum.

We shall henceforth assume the medium to be transparent. The presence of the small imaginary part of the dielectric constant in (1) leads in this case to the occurrence in (1) of a δ-like factor, expressing the energy conservation law:

$$-\operatorname{Im} \frac{1}{\omega^2 \varepsilon_t - \mathbf{q}^2 + i\delta} = \pi\delta(\omega^2 n^2 - q^2)$$
$$= \frac{\pi V}{2\omega n} \delta(\omega - \omega_q) = \frac{\pi V}{2\omega n} \delta(E_0 - E - k - \omega_q), \quad (2)$$

where ω_q is the solution of the equation $\omega n(\omega) = q$, and $V = d\omega/dq$ is the group velocity. Between the energies and the momenta of the participating particle there exists also a kinematic connection expressed by the equation

$$\mathbf{p}_0 = \mathbf{p} + \mathbf{k} + \mathbf{q}, \quad (3)$$

whence

$$E_0 k (1 - v_0 \cos \vartheta) + E_0 \omega (1 - n v_0 \cos \theta) - k\omega (1 - n \cos \theta') + {}^1\!/_2 \omega^2 (n^2 - 1) = 0, \quad (4)$$

where θ' is the angle between **k** and **q**. Up to energies $E_0 \sim m^2/\omega$, i.e., up to $E_0 \sim 10^5$ m, and for ω in the optical band, it turns out (as will be shown later) that $k \ll p_0$ and $\theta \approx \theta'$, and (4) yields

$$\cos \theta \approx \frac{1}{v_0 n(\omega_q)} + k \frac{1 - v_0 \cos \vartheta}{v_0 \omega_q n(\omega_q)}. \quad (5)$$

We see that the angle θ in the case of hard Cerenkov radiation lies inside the single-quantum Cerenkov cone

$$\cos \theta_q \approx 1 / v_0 n(\omega_q). \quad (6)$$

Solving (5) with respect to k, we get

$$k = \omega_q \frac{n(\omega_q) v_0 \cos \theta - 1}{1 - v_0 \cos \vartheta}. \quad (7)$$

From this we get the maximum value $k = k_m$ (it corresponds to $\vartheta = \theta = 0$)

$$k_m = \omega_q \frac{n(\omega_q) v_0 - 1}{1 - v_0}. \quad (8)$$

k_m can greatly exceed ω. For example, when n = 2, the ratio k_m/ω is 3.0 at $E_0 - m$ = 350 keV, 5.4 at $E_0 - m$ = 500 keV, 9.8 at $E_0 - m$ = 750 keV, and 15.4 at $E_0 - m$ = 1 MeV. If ω is in the optical band, then k_m is in the x-ray band. For quanta with energy $k \sim k_m$, the refractive index is $n \leq 1$, and these quanta cannot be radiated as a result of the ordinary single-quantum Cerenkov effect, but they can be produced, as we shall show, via the two-quantum Cerenkov effect. In addition to the formation of hard quanta with n = 1, there can occur also the more trivial two-quantum emission of two dressed quanta[7]; in this case some can excite optical frequencies and others radial frequencies—stimulation of one frequency causes radiation not only at that frequency but also at a kinematically coupled frequency.

In the moderately ultrarelativistic case, when $m \ll E_0 \ll m^0/\omega$, the significant angles ϑ are small, $\vartheta \ll 1$, $v_0 = 1 - m^2/2E_0^2$, and (7) goes over into

$$k_m(\vartheta) = 2\frac{E_0^2}{m^2}\frac{n(\omega_q)\cos\theta - 1}{1 + E_0^2\vartheta^2/m^2}\omega_q, \qquad (9)$$

and we obtain for the upper limit of the spectrum

$$k_m = 2E_0^2\omega_q / m^2. \qquad (10)$$

This limit increases rapidly with increasing E_0, and for example when $\omega_q \sim 10^{-5}$ (optical band) and E_0 = 500 MeV = 10^3 m it amounts to k_m = 20 m = 10 MeV. At E_0 = 100 MeV and the same value of ω_q we get k_m = 250 keV. As noted earlier[1,2], the upper limit k_m approaches E_0 at $E_0 \gtrsim m^2/\omega_q$.

In the nonrelativistic and moderately relativistic cases $m \lesssim E_0$, the spin sum S_t can be written in the form

$$S_t \cong \frac{1}{2\varkappa_1^2\varkappa_2^2}\{-8(\varkappa_1+\varkappa_2)^2 v_{0\perp}^2 - 8\varkappa_1^2 k_\perp^2/E_0^2 + 16\varkappa_1(\varkappa_1+\varkappa_2)v_{0\perp}k_\perp/E_0 - 2(m/E_0)^2\varkappa_1\varkappa_2(\varkappa_1^2+\varkappa_2^2)\}; \qquad (11)$$

$$\varkappa_1 \cong -2(E_0\omega_q/m^2)\gamma_1, \quad \varkappa_2 = 2(E_0k/m^2)\gamma_2,$$
$$\varkappa_1 + \varkappa_2 \cong -2(k\omega_q/m^2)\gamma_3; \qquad (12)$$

$$\gamma_1 = n(\omega_q)v_0\cos\theta - 1, \quad \gamma_2 = 1 - v_0\cos\vartheta,$$
$$\gamma_3 = n(\omega_q)\cos\theta' - 1, \quad v_{0\perp}^2 = v_0^2\sin^2\theta, \quad k_\perp^2 = k^2\sin^2\theta',$$
$$v_{0\perp}k_\perp = v_0k(\cos\vartheta - \cos\theta\cos\theta') \qquad (13)$$

In the derivation of (11)—(13) from the general expressions of[2], we used the smallness of $\omega_q \ll m$. The quantities γ_1 and γ_3 which enter into these formulas are of the order of 1 and $\gamma_2 \sim (m/E_0)^2$, since only angles $\vartheta \lesssim m/E_0$ are significant. We get $\kappa_1 \sim \kappa_2 \sim m\omega/E_0^2$ and $\kappa_1 + \kappa_2 \sim (E_0\omega/m^2)^2$, inasmuch as in the nonrelativistic and moderately relativistic cases $k \sim k_m$ is, in accordance with (8) and (10), of the order of $k \sim E_0\omega/m$. Each of the first three terms in the curly brackets of (11) is of order of magnitude $(E_0\omega/m^2)^2$, and the order of the remaining two is $(m/E_0)^2(E_0\omega/m^2)^4$ and is smaller than each of the first three terms at $E_0 \gg m$. However, the sum of the first three terms has, as a result of mutual cancellations, the same order of magnitude $(m/E_0)^2(E_0\omega/m^2)^2$ as the last two terms. As a result, the spin sum (11) is of the order of $S_t \sim (m/E_0)^2$. In the moderately ultrarelativistic case, when $m \ll E_0 \ll E_0 m/\omega$, and the angle $\vartheta \lesssim m/E_0$ is small, the sum S_t takes the form

$$S_t = \left(\frac{m}{E_0}\right)^2 + \left(\frac{m}{E_0}\right)^2\left(1 - \frac{4(m\vartheta/E_0)^2}{[(m/E_0)^2+\vartheta^2]^2}\right) + 4\left(\frac{m}{E_0}\right)^2 \times \frac{n^2-1}{(n\cos\theta-1)^2}\sin^2\theta\left(1 - 4\frac{(m\vartheta/E_0)^2}{[(m/E_0)^2+\vartheta^2]^2}\cos^2\varphi\right) \sim \left(\frac{m}{E_0}\right)^2, \qquad (14)$$

where φ is the angle between the planes ($\mathbf{p}_0$, $\mathbf{k}$) and ($\mathbf{p}_0$, $\mathbf{q}$). It is seen from (14) that when the angle θ approaches the Cerenkov cone, S_t increases in proportion to $\gamma_1^{-2} = (n\cos\theta - 1)^{-2}$.

Substituting (2) and (11) or (14) in (9), we obtain for the probability of the hard Cerenkov radiation the expression

$$dw_{\mathbf{kq}} = \frac{e^4V(N_\omega+1)S_t}{2(2\pi)^3v_0n\omega m^2k}\delta(\omega-\omega_q)\,d\mathbf{k}\,d\mathbf{q}. \qquad (15)$$

Let us discuss now certain possibilities of experimentally observing the hard Cerenkov radiation. The main difficulty lies apparently in the presence of a strong background of "normal" bremsstrahlung quanta, which can be confused with the Cerenkov hard quanta. There are different ways of getting around this difficulty. First, it is possible to intensity the hard Cerenkov radiation, stimulating it with a sufficiently strong beam of soft quanta with frequency ω_1 and momentum $\mathbf{q}_1$. We shall assume this beam to be strictly monochromatic, and then

$$N_\omega = 2^{-1}(2\pi)^3N_1\delta(\mathbf{q}-\mathbf{q}_1), \qquad (16)$$

where N_1 is the number of quanta with momentum $\mathbf{q}_1$ per unit volume. The function in (16) can be rewritten in the form

$$\delta(E_0 - E - k - \omega_1) \cong (1/v_0k)\,\delta[\cos\vartheta - \cos\vartheta(k)],$$

where $\cos\vartheta(k)$ is expressed in terms of k with the aid of formula (7),

$$\cos\vartheta(k) = v_0^{-1}[1 - (\omega_1/k)(n_1v_0\cos\theta_1 - 1)]. \qquad (17)$$

An important factor is that the angle of emission of the hard quanta is uniquely connected with their energy (a certain smearing is the result of the small angular and energy scatter of the beam of soft quanta). Integrating now (14) with respect to $\cos\vartheta$, and also with respect to $d\mathbf{q}$, using (5) and (17), we obtain

$$dw_k = \frac{e^4V_1N_1S_{t1}}{4m^2v_0^2n_1\omega_1}dk\Delta\varphi, \qquad (18)$$

where n_1 and V_1 are the refractive index and the group velocity at the frequency ω_1; $\Delta\varphi$ is the registered interval of the azimuthal angles; S_{t1} is expressed in terms of $\cos\vartheta(k)$, which is defined by formula (17), and in terms of $\omega_q = \omega_1$ with the aid of (11)—(14). We call attention to the fact that the probability (18) does not depend on the registered interval $\Delta\vartheta$ of the values of the angle ϑ, since ϑ is connected with k by formula (17). This independence remains in force so long as $\Delta\vartheta$ is larger than the scatter of the angles ϑ, connected with the scatter of k within the interval dk and with the angular and energy scatters of the soft-photon beam.

Let us compare the probability (18) with the probability of the usual Bethe-Heitler bremsstrahlung, which has in the moderately relativistic case the order of magnitude

$$dw^{\mathrm{br}} \sim \frac{Z^2e^6N_a}{m^2v_0^2}\frac{dk}{k}\frac{\Delta\varphi\Delta\vartheta}{4\pi} \qquad (19)$$

where N_a is the number of nuclei per unit volume and eZ is their charge. The order of magnitude of the ratio of (18) to (19) is

$$R = \frac{dw_k}{dw_k^{\mathrm{br}}} \sim \frac{1}{e^2}\frac{N_1}{Z^2N_a}\frac{k}{\gamma_1^2\omega_1}\frac{(m/E_0)^4}{\vartheta\Delta\vartheta} \sim 100\,\frac{N_1}{Z^2N_a}\frac{1}{\gamma_1}\frac{(m/E_0)^2}{\vartheta\Delta\vartheta}. \qquad (20)$$

It can amount to 100% and more at reasonable values of the quantities which enter in it. Indeed, the ratio $k/\omega_1 \sim \gamma_1(E_0/m)^2$ can be, for example, ~ 10 when $m/E_0 \sim \gamma_1 \sim 10^{-1}$. At small $Z^2 \sim 10-100$ $N_a \sim 10^{22}$ cm^{-1}, and $\vartheta \sim 10^{-2}$ we obtain $R = 1$, taking $N_1 = 10^{16}-10^{17}$ kV/cm^2 (this corresponds to an optical-quantum beam intensity 10^8-10^9 W/cm^2) and $\Delta\vartheta = 10^{-4}$.

The use of a coincidence circuit (for the optical and hard bremsstrahlung quanta) should greatly facilitate the observation conditions, making possible, although with difficulty, even the observation of spontaneous hard Cerenkov radiation (see the brief discussion in[2]). An appreciable attenuation of the background of bremsstrahlung quanta could also be obtained by passing the charged particles through a channel in matter, with a diameter smaller than the wavelength of the soft quanta. In this case the Cerenkov radiation will be produced as before, but not bremsstrahlung.

Observation of hard Cerenkov radiation would be of interest for a confirmation of the correctness of the usually employed quantum description of photons in matter (single-quantum Cerenkov radiation arises already in the classical theory, while the quantum corrections are small and are difficult to observe). The dynamics of formation of a dressed photon in matter, which was not taken into account by us or by other workers, could affect also the probability of the hard Cerenkov radiation. Stimulation of the emission of x-ray photons by optical photons is also of interest.

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