THE QUESTION OF NONCONSERVATION OF SPATIAL PARITY IN NUCLEAR ELECTROMAGNETIC TRANSITIONS

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The consequences of P-parity nonconservation in nuclear electromagnetic transitions for the polarization and resonance absorption of γ radiation are analyzed. Expressions are derived for the polarizations of individual Zeeman components of radiation which is a mixture of multipole types with different spatial parities. Formulas are given which describe the resonance absorption of such radiation. The cases of the mixtures E(1) – M(1), E(2) – M(2), E(2) – E(1), and M(2) – M(1) are analyzed in detail.

INTRODUCTION

RECENTLY a number of experimental groups have observed nonconservation of spatial parity in nuclear electromagnetic transitions.[1-3] In the present paper we analyze the consequences of nonconservation of the parity of nuclear states for the polarization and resonance absorption of γ radiation. Possibilities for observing parity nonconservation by means of γ-resonance experiments are discussed.

A consequence of the nonconservation of the P parity of nuclear states is the existence of nuclear γ radiation which is a mixture of multipole types of different parities, for example E(L) – M(L) or E(L) – E(L + 1).[4] Such mixtures are forbidden if the parity is an exact quantum number for nuclear states. The presence of mixtures of different multipole types (for definiteness, of two types) leads to interference effects, whose analysis make it possible in principle to draw conclusions about the character of the mixtures.

The interference effects for radiation of mixed parity appear not only in the individual Zeeman components, but also in the radiation from unpolarized nuclei. The radiation from unpolarized nuclei, however, will contain interference effects only for multipole mixtures of the type E(L) – M(L). The effects will manifest themselves as a circular polarization of such radiation. As for the individual Zeeman components, here there will be interference effects in the radiation of any mixture of multipole types of opposite parities [for example, E(L) – E(L + 1)], and the effects appear both in the angular distribution of intensity and in the polarization of the radiation. We shall examine the consequences of nonconservation of P parity for individual Zeeman components of the radiation, and from these results one can get the corresponding effects for radiation consisting of an arbitrary combination of Zeeman components.

It is well known that the interference of multipole types of different spatial parities leads to an asymmetry of the intensity of the radiation relative to the plane perpendicular to the axis of quantization (the direction of the field). The amount of asymmetry is directly connected with the mixing parameter δ, which is defined as the ratio of the amplitudes of the multipole types which are mixed.

The polarization of the mixed radiation also depends on δ. The polarization vector (or ellipse) of a γ-ray quantum of strictly defined multipole type can be represented in the form*

$$\hat{\mathbf{n}} = \frac{[\mathbf{k}\,[\hat{\mathbf{h}}\mathbf{k}]]}{|[\hat{\mathbf{h}}\mathbf{k}]|}\cos\alpha + i\frac{[\hat{\mathbf{h}}\mathbf{k}]}{|[\hat{\mathbf{h}}\mathbf{k}]|}\sin\alpha,$$

where $\mathbf{k}$ is the wave vector of the γ ray, $\hat{\mathbf{h}}$ is the unit vector in the direction of the axis of quantization (the magnetic field), and α is a known function of the cosine of the angle between $\hat{\mathbf{h}}$ and $\mathbf{k}$, which fixes the ratio of the axes of the polarization ellipse. When P invariance is violated the ratio of the axes of the ellipse is changed, i.e., the quantity α in the formula for $\hat{\mathbf{n}}$ changes, and the change of α contains terms linear in the mixing parameter δ. Therefore one can in principle detect a violation of P invariance by comparing the polarization of the given radiation with that of radiation known to be "pure." This method can scarcely be used in practice.

Another possibility is to compare the polarizations of two γ-ray quanta which are either obtained from each other by the space-inversion operation, or else have the same wave vector but are emitted in oppositely directed magnetic fields. For radiation characterized by a definite spatial parity the space-inversion operation does not change the polarization vector. This same operation on radiation which does not have a definite parity leads to a change of its polarization vector.

Thus a violation of P invariance for radiative processes has the consequence that γ rays emitted in exactly opposite directions have different polarization vectors. The corresponding consequence for the absorption of γ rays in transitions in which P invariance is violated is that the probability for absorption of a γ-ray quantum with polarization vector $\hat{\mathbf{n}}$ and wave vector $\mathbf{k}$ in a given transition is different from the absorption probability in the same transition for a γ-ray quantum with wave vector $-\mathbf{k}$ and the same polarization vector. An analogous manifestation of nonconservation of P parity for γ rays with a fixed wave vector $\mathbf{k}$ is found on comparing the emission (or absorption) processes occurring in

*$[\hat{\mathbf{h}}\mathbf{k}] \equiv \hat{\mathbf{h}} \times \mathbf{k}$.

oppositely directed magnetic fields.

An analysis of the effects associated with the polarization and angular distribution of intensity of mixed radiation can be made, for example, by means of resonance absorption of the γ radiation. It follows from [5,6] that the intensity of absorbed γ radiation (we are concerned with emission and subsequent absorption of individual Zeeman components) can be represented in the form

$$I\Sigma = II' |\hat{\mathbf{n}}^* \hat{\mathbf{n}}'|^2, \tag{1}$$

where I, $\hat{\mathbf{n}}$ are the intensity and polarization vector of the emitted radiation, and I′, $\hat{\mathbf{n}}'$ are the intensity and polarization vector of the radiation with the same wave vector emitted in the transition inverse to the absorbing transition. The expression (1) depends both on the angular distributions of intensity of the radiation and on its polarization properties.

Experimental observation of the effects described by Eq. (1) may be possible, for example, by the use of the Mössbauer effect; in studying the effects associated with violation of P invariance it is not necessary to split into their components the lines of both source and absorber—it suffices to split only one of the lines.

THE POLARIZATION OF RADIATION OF MIXED MULTIPOLE CHARACTER

Let us consider the process of emission of a γ-ray quantum. We confine ourselves at first to a γ transition of exactly defined multipole character. If as the result of a nuclear transition j_i, $m_i \to j_f$, m_f (where j, m are the nuclear spin and its projection in the direction $\hat{\mathbf{h}}$ of the magnetic field) a γ-ray quantum with wave vector **k** has been emitted, then in the general case the polarization of the radiation is characterized by a polarization ellipse with axes directed along the vectors

$$\hat{\mathbf{a}}_1 = \frac{[\hat{\mathbf{h}}\mathbf{k}]}{|[\hat{\mathbf{h}}\mathbf{k}]|}, \quad \hat{\mathbf{a}}_2 = \frac{[\mathbf{k}\,[\hat{\mathbf{h}}\mathbf{k}]]}{|[\hat{\mathbf{h}}\mathbf{k}]|}. \tag{2}$$

The ratio of the axes a and b of the polarization ellipse depends on the angle between the wave vector and the direction of the field. For individual Zeeman components and for special values of this angle the ellipse may degenerate into a straight line (or a circle), which corresponds to linear (circular) polarization. In the general case the ratio of the axes is given by the formula

$$a / b = E_2 / E_1. \tag{3}$$

The quantities E_1 and E_2 are determined by the following expressions:

$$E_i = \begin{pmatrix} j_f & L & j_i \\ m_f & M & -m_i \end{pmatrix} \chi(l, L)\, e_i. \tag{4}$$

Here L is the multipole character of the transition, M is the projection of the angular momentum carried away by the photon on the field direction, $\chi(l, L)$ is the reduced matrix element of the transition, and

$$e_1 = (2L+1)^{1/2}\Big\{\Big[\begin{pmatrix} l & 1 & L \\ M+1 & -1 & -M \end{pmatrix} Y_l^{-M-1}(\hat{\mathbf{k}}) - \begin{pmatrix} l & 1 & L \\ M-1 & 1 & -M \end{pmatrix} Y_l^{1-M}(\hat{\mathbf{k}})\Big]\cos\theta + \sqrt{2}\sin\theta \begin{pmatrix} l & 1 & L \\ M & 0 & -M \end{pmatrix} Y_l^{-M}(\hat{\mathbf{k}})\Big\},$$

$$e_2 = (2L+1)^{1/2}\Big[\begin{pmatrix} l & 1 & L \\ M+1 & -1 & -M \end{pmatrix} Y_l^{-M-1}(\hat{\mathbf{k}}) + \begin{pmatrix} l & 1 & L \\ M-1 & 1 & -M \end{pmatrix} Y_l^{1-M}(\hat{\mathbf{k}})\Big]. \tag{5}$$

Here θ is the angle between $\hat{\mathbf{h}}$ and **k**. The spherical harmonics which appear in (5) are written in a coordinate system whose polar axis is along $\hat{\mathbf{h}}$ and whose azimuthal angles are measured from the vector **k** (i.e., the azimuthal angle of the vector $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$ is always zero). Therefore the spherical harmonics in (5) depend on only the one angle θ.

The polarization vector $\hat{\mathbf{n}}$ of the radiation can be written in the form

$$\hat{\mathbf{n}} = \hat{\mathbf{a}}_2 \cos\alpha + i\hat{\mathbf{a}}_1 \sin\alpha, \tag{6}$$

where $\operatorname{tg}\alpha = a/b = E_2/E_1$.

Let us now consider a transition in which there is present, besides the main multipolarity L_1 considered above, a small admixture of a multipolarity L_2 which has the opposite (forbidden) parity. The polarization of radiation with the same wave vector will now be different from the polarization of the radiation which does not contain the admixture. The change of the polarization vector consists of a change of the parameter α in (6). The value of $\tan\alpha'$, or, what is the same thing, the ratio of the axes a′, b′ of the new ellipse is:

$$\operatorname{tg}\alpha' = \frac{E_2(L_1) \pm E_2(L_2)}{E_1(L_1) \pm E_1(L_2)}. \tag{7}$$

The arguments of the E_i denote the multipole types. The two signs in (7) and subsequent formulas correspond to the two possible values (0 and π) of the relative phase of the matrix elements of the multipolarities which are mixed which are allowed by the requirement of T invariance.[7] In the approximation linear in δ we get from (7) for the change of the ratio of the axes of the polarization ellipse

$$\frac{a'}{b'} - \frac{a}{b} = \pm\frac{a}{b}\left[\frac{E_2(L_2)}{E_2(L_1)} - \frac{E_1(L_2)}{E_1(L_1)}\right]. \tag{8}$$

It follows from (4), (5) and (7) that under the space-inversion operation [which in (4) and (5) corresponds to replacement of **k** by −**k**], or under change of the sign of the magnetic field, there is a change of the polarization vector of radiation of mixed parity. The corresponding change of the ratio of the axes of the polarization ellipse is given by the expression

$$\operatorname{tg}\alpha(-\hat{\mathbf{k}}) - \operatorname{tg}\alpha(\hat{\mathbf{k}}) = \pm 2\,\frac{E_1(L_1)E_2(L_2) - E_2(L_1)E_1(L_2)}{E_1^2(L_1) - E_1^2(L_2)}. \tag{9}$$

The intensity of an individual component of the radiation with angular-momentum projection M can be written in the form

$$I(M) = \sum_{i, L_k} E_i^2(L_k) \pm 2\sum_i E_i(L_1)E_i(L_2). \tag{10}$$

RESONANCE ABSORPTION OF MIXED RADIATION

An analysis of the polarization and the angular distribution of the radiation can be made, for example, by means of resonance absorption of the γ rays. Let us consider the process of emission and subsequent absorption of a γ-ray quantum under the following conditions. The nucleus which emits the radiation is in a magnetic field **H**. The nucleus which absorbs the radiation is in a magnetic field **H**′. We shall assume that the magnetic fields **H** and **H**′ are such that the magnetic splitting of the nuclear levels is larger than the natural width of the lines, both in transition 1, with the emission of the γ-ray quantum, and in transition 2, with its absorption, so that in principle one can observe the emission and absorption of quite definite Zeeman lines. Therefore in what follows, when we speak of a transition, we shall always have in mind a transition between definite magnetic sublevels.

The general formula for the probability $I\Sigma$ that a γ-ray quantum will be emitted at angle θ with the direction of the magnetic field **H** in transition 1 and then be absorbed in transition 2 (with the wave vector **k** of the photon making angle θ' with the direction of the magnetic field **H**′) is given in [6].

It can, however, be seen already from the structure of $I\Sigma$ [Eq. (1)] that the expression for $I\Sigma$ depends on the polarization vectors and on the angular distributions of intensity. Accordingly, this quantity can be used for the observation of effects associated with nonconservation of spatial parity. Having in mind the study of violations of P invariance, we shall examine cases in which at least one of the transitions consists of a mixture of multipole types of different spatial parities.

The expression for $I\Sigma$ can be written in the form

$$I\Sigma(j_i, j_f, m_i, m_f, M, \mathbf{h}; \mathbf{k}; j_i', j_f', m_i', m_f', M', \hat{\mathbf{h}}') = \sum_{i=1}^{4} (A_i^2 + B_i^2)$$
$$+ 2[(A_1A_2 + A_3A_4 + B_1B_2 + B_3B_4)\cos\eta + (A_1A_3 + A_2A_4 + B_1B_3$$
$$+ B_2B_4)\cos\eta' + (A_1A_4 + B_1B_4)\cos(\eta - \eta')$$
$$+ (A_2A_3 + B_2B_3)\cos(\eta + \eta') + (A_2B_1 + A_4B_3 - A_1B_2 - A_3B_4)\sin\eta$$
$$+ (A_1B_3 + A_2B_4 - A_3B_1 - A_4B_2)\sin\eta' + (A_4B_1 - A_1B_4)\sin(\eta - \eta')$$
$$+ (A_2B_3 - A_3B_2)\sin(\eta + \eta')], \tag{11}$$

where $M = m_i - m_f$, $-M' = m_i' - m_f'$, and η and η' are the relative phases of the matrix elements of the multipolarities which are mixed in the source nucleus and in the absorber nucleus; also

$$A_1 = I_{11}\sum_i E_i(L_1)E_i(L_1'), \quad A_2 = I_{11}\sum_i E_i(L_2)E_i(L_1'),$$
$$A_3 = I_{11}\sum_i E_i(L_1)E_i(L_2'), \quad A_4 = I_{11}\sum_i E_i(L_2)E_i(L_2'),$$

$$B_1 = -I_{12}\sum_{i\neq m} E_i(L_1)E_m(L_1'), \quad B_2 = -I_{12}\sum_{i\neq m} E_i(L_2)E_m(L_1'),$$
$$B_3 = -I_{12}\sum_{i\neq m} E_i(L_1)E_m(L_2'), \quad B_4 = -I_{12}\sum_{i\neq m} E_i(L_2)E_m(L_2'), \tag{12}$$

$$I_{11} = \hat{\mathbf{a}}_1\hat{\mathbf{a}}_1' = \hat{\mathbf{a}}_2\hat{\mathbf{a}}_2' = \cos(\varphi_h - \varphi_{h'}),$$
$$I_{12} = \hat{\mathbf{a}}_1\hat{\mathbf{a}}_2' = -\hat{\mathbf{a}}_2\hat{\mathbf{a}}_1' = \sin(\varphi_h - \varphi_{h'}), \tag{13}$$

where φ_h, $\varphi_{h'}$ are the polar angles of the vectors $\hat{\mathbf{h}}$, $\hat{\mathbf{h}}'$ relative to the axis $\hat{\mathbf{k}}$.

The quantities $\hat{\mathbf{a}}_i$ and E_i are determined from Eqs. (2), (4), and (5), with the modification that in obtaining the quantities E_i for transition 2 [in (12) their arguments are distinguished by primes] one must interchange the roles of initial and final nuclear states, i.e., in Eq. (4) one must set

$$j_f = j_i', \quad m_f = m_i', \quad j_i = j_f', \quad m_i = m_f', \quad M = M'. \tag{14}$$

Let us consider in more detail the case in which both transitions (1 and 2) contain admixtures of forbidden parity. It is easy to get from it the practically most interesting case, in which transitions 1 and 2 are the same, i.e., the same nucleus is both source and absorber of the radiation. Taking into account the restrictions imposed by the requirement of T invariance ($\eta, \eta' = 0, \pi$), we get from (11)

$$I\Sigma = \sum_{i=1}^{4} (A_i^2 + B_i^2) + 2[(A_1A_4 + B_1B_4)\cos(\eta - \eta')$$
$$+ (A_2A_3 + B_2B_3)\cos(\eta + \eta') + (A_1A_2 + A_3A_4$$
$$+ B_1B_2 + B_3B_4)\cos\eta + (A_1A_3 + A_2A_4 + B_1B_3 + B_2B_4)\cos\eta']. \tag{15}$$

In the expression (15) let us sum over the magnetic quantum numbers of the initial (final) transition; this corresponds to not resolving the emission (absorption) line and assuming that the magnetic sublevels are equally populated. Then for mixtures of the type E(L) – E(L + 1) [M(L) – M(L + 1)] the result can be represented as a product of intensities:

$$\sum_{m_i, m_f, M} I\Sigma = {}^1/_2 I_0 I'(\theta'), \quad I_0 = \frac{1}{2j_i + 1}\sum_{m_i, m_f} I_{m_i\, f}, \tag{16}$$

where I_0 is the intensity of the radiation from the unpolarized nucleus and I and I′ are the same quantities as in (1). Their explicit form can be found, for example, by means of (10). For mixtures of the type E(L) – M(L)

$$\sum_{m_i, m_f, M} I\Sigma = \frac{1}{2} I_0 I'(\theta') - \frac{I_0\delta\cos\eta}{1 + \delta^2}\{E_1(L_1')E_2(L_1')$$
$$+ E_1(L_2')E_2(L_2') + 2[E_1(L_1')E_2(L_2') + E_2(L_1')E_1(L_2')]\cos\eta'\}. \tag{17}$$

The difference between the structures of the expressions (16) and (17) is due to the fact that for the mixture E(L + 1) – E(L) the radiation from unpolarized nuclei is unpolarized, but for the mixture E(L) – M(L) it is partially circularly polarized. Therefore in spite of the fact that the absorption depends on the polarization the result of averaging in the former case, Eq. (16), is simply proportional to the product of the intensities, but in the expression (17), besides the product of intensities $I_0 I'(\theta')$, there is a term from the dependence of the absorption on the polarization of the radiation.

THE CASE OF DIPOLE AND QUADRUPOLE MIXTURES

In Eq. (5) it is convenient to include the dependence of the sign of E_i on M entirely in the factor e_i. Table I gives the explicit forms of the e_i, taking the dependence of E_i on the sign of M fully into account, for dipole and quadrupole radiations. By means of (4) and (7), we get for the angular dependence of the parameter α, which characterizes the polarization of the γ-ray quantum, the result

$$\operatorname{tg}\alpha = \frac{ae_2(L_1) + be_2(L_2)\cos\eta}{ae_1(L_1) + be_1(L_2)\cos\eta'}, \tag{18}$$

where e_1, e_2 are given in Table I, and a and b are deter-

Table I. $e_i(M)$ for dipole and quadrupole radiations

$e_i(M)$	$E(1)$	$E(2)$	$M(1)$	$M(2)$
$e_1(0)$	$-\sin\theta$	$^1/_2\sin 2\theta$	0	0
$e_1(\pm 1)$	$\mp\cos\theta$	$\cos 2\theta$	-1	$\pm\cos\theta$
$e_1(\pm 2)$		$-^1/_2\sin 2\theta$		$\mp\sin\theta$
$e_2(0)$	0	0	$\sin\theta$	$-^1/_2\sin 2\theta$
$e_2(\pm 1)$	1	$\mp\cos\theta$	$\pm\cos\theta$	$-\cos 2\theta$
$e_2(\pm 2)$		$\pm\sin\theta$		$^1/_2\sin 2\theta$

Table II. $\tilde{A}$ for mixtures E(1) − M(1)

$M - M'$	$0-0$	$0-\pm 1$	$1-\pm 1$
$\tilde{A}_1$	$\sin\theta\sin\theta'$	$\pm\sin\theta\cos\theta'$	$1\pm\cos\theta\cos\theta'$
$\tilde{A}_2$	0	$\sin\theta$	$\cos\theta\pm\cos\theta'$

Note. The quantities a, b, a', b' which connect $\tilde{A}$ with A_i, B_i [cf. (20)] are given in Eq. (21).

mined from (21) and (22). We denote by b the analog of a for the admixed multipolarity. With the same notations we get for the intensities of the individual Zeeman components of the radiation the expression

$$I=\sum_i [a^2 e_i^2(L_1)+b^2 e_i^2(L_2)+2ab e_i(L_1) e_i(L_2)\cos\eta]. \qquad (19)$$

To determine $I\Sigma$ we must use Eqs. (11) and (15) and the expressions (12) for A_i and B_i. The values of A_i and B_i for dipole and quadrupole mixtures in cases in which the transitions 1 and 2 are the same mixture can be obtained by means of Tables II–IV. Tables II and III give the quantities $\tilde{A}_1$ and $\tilde{A}_2$ for mixtures E(1) − M(1) and E(2) − M(2). The quantities A_i and B_i are connected with $\tilde{A}_1$ and $\tilde{A}_2$ in the following way:

$$\frac{A_1}{I_{11}}=aa'\tilde{A}_1,\quad \frac{A_2}{I_{11}}=a'b\tilde{A}_2,\quad \frac{A_3}{I_{11}}=ab'\tilde{A}_2,\quad \frac{A_4}{I_{11}}=bb'\tilde{A}_1$$
$$\frac{B_1}{I_{12}}=aa'\tilde{A}_2,\quad \frac{B_2}{I_{12}}=a'b\tilde{A}_1,\quad \frac{B_3}{I_{12}}=ab'\tilde{A}_1,\quad \frac{B_4}{I_{12}}=bb'\tilde{A}_2, \qquad (20)$$

Table 4 gives A_i, B_i for E(2) − E(1) [M(2) − M(1)] mixtures. The quantities a(|M|) and b(|M|) contained in A_i, B_i can be determined by means of the following relations.

Multipole character E(1) [M(1)]:

$$a(0)=\sqrt{\frac{3}{2}}\begin{pmatrix} j_f & 1 & j_i \\ m_f & 0 & -m_i\end{pmatrix}|\chi|,$$

$$a(1)=\frac{\sqrt{3}}{2}\begin{pmatrix} j_f & 1 & j_i \\ m_f & 1 & -m_i\end{pmatrix}|\chi|. \qquad (21)$$

Multipole character E(2) [M(2)]:

$$a(0)=\sqrt{\frac{15}{2}}\begin{pmatrix} j_f & 2 & j_i \\ m_f & 0 & -m_i\end{pmatrix}|\chi|,$$

$$a(1)=\frac{\sqrt{5}}{2}\begin{pmatrix} j_f & 2 & j_i \\ m_f & 0 & -m_i\end{pmatrix}|\chi|,\quad a(2)=\frac{\sqrt{5}}{2}\begin{pmatrix} j_f & 2 & j_i \\ m_f & 2 & -m_i\end{pmatrix}|\chi| \qquad (22)$$

The quantities a'(|M'|) and b'(|M'|) are also given by (21) and (22), but with the substitutions (14). Strictly speaking, the quantities χ in Eqs. (21) and (22) are different from the reduced matrix elements, and are connected with I_0 [cf. (16)] by the relation

$$\int I_0 d\Omega=\frac{4\pi}{2j_i+1}\left(|\chi(L_1)|^2+|\chi(L_2)|^2\right).$$

Tables 2–4 do not give the values of A_i and B_i for all transitions, but we can obtain the values of $I\Sigma$ for all transitions if we use the facts that the expression (15) satisfies the relation

$$I\Sigma(M,\hat{\mathbf{h}};\mathbf{k};M',\hat{\mathbf{h}}')=I\Sigma(M',\hat{\mathbf{h}}';\mathbf{k};M,\hat{\mathbf{h}}), \qquad (23)$$

and the change from $I\Sigma(M,\hat{\mathbf{h}};\mathbf{k};M',\hat{\mathbf{h}}')$ to $I\Sigma(-M,\hat{\mathbf{h}};\mathbf{k};-M',\hat{\mathbf{h}}')$ leads in (15) to a change of sign of the coefficients of $\cos\eta$ and $\cos\eta$. We note that simultaneous change of the signs of $\hat{\mathbf{h}}$ and $\hat{\mathbf{h}}'$ also leads to a change of sign of the coefficients of $\cos\eta$ and $\cos\eta$ in $I\Sigma$.

Using the fact that when the arguments are the same $I\Sigma$ is the square of the intensity, we get a different expression for the intensity:

$$I(M)=\frac{1}{I_{11}}(A_1+A_4)+\frac{2}{I_{11}}A_2\cos\eta, \qquad (24)$$

in which the A_i are given by Tables 2–4 with $\theta=\theta'$, $a=a'$, $b=b'$, $M=M'$.

Table III. $\tilde{A}$ for mixtures E(2) − M(2)

$M - M'$	$0-0$	$0-\pm 1$	$0-\pm 2$	$1-\pm 1$	$1-\pm 2$	$2-\pm 2$
$\tilde{A}_1$	$^1/_4\sin 2\theta\sin 2\theta'$	$^1/_2\sin 2\theta\cos 2\theta'$	$-^1/_4\sin 2\theta\sin 2\theta'$	$\cos 2\theta\cos 2\theta'\pm$ $\pm\cos\theta\cos\theta'$	$-^1/_2\cos 2\theta\sin 2\theta'\mp\cos\theta\sin\theta'$	$^1/_4\sin 2\theta\sin 2\theta'\pm\sin\theta\sin\theta'$
$\tilde{A}_2$	0	$\pm ^1/_2\sin 2\theta\cos\theta'$	$\mp ^1/_2\sin 2\theta\sin\theta'$	$\cos\theta\cos 2\theta'\pm$ $\pm\cos 2\theta\cos\theta'$	$-^1/_2\cos\theta\sin 2\theta'\mp\cos 2\theta\sin\theta'$	$^1/_2(\sin\theta\sin 2\theta'\pm\sin 2\theta\sin\theta')$

Note. The quantities a, b, a', b' which connect $\tilde{A}$ with A_i, B_i [cf. (20)] are given in Eq. (22).

Table IV. A_i, B_i for multipole mixture E(2) − E(1) [M(2) − M(1)]

$M - M'$	$0-0$	$0-\pm 1$	$0-\pm 2$	$1-\pm 1$	$1-\pm 2$	$2-\pm 2$
$\frac{A_1}{aa'I_{11}}$	$^1/_4\sin 2\theta\sin 2\theta'$	$^1/_2\sin 2\theta\cos 2\theta'$	$-^1/_4\sin 2\theta\sin 2\theta'$	$\cos 2\theta\cos 2\theta'\pm$ $\pm\cos\theta\cos\theta'$	$-^1/_2\cos 2\theta\sin 2\theta'\mp$ $\mp\cos\theta\sin\theta'$	$^1/_4\sin 2\theta\sin 2\theta'\pm\sin\theta\sin\theta'$
$\frac{A_2}{ba'I_{11}}$	$-^1/_2\sin\theta\sin 2\theta'$	$-\sin\theta\cos 2\theta'$	$^1/_2\sin\theta\sin 2\theta'$	$-\cos\theta\cos 2\theta'\mp$ $\mp\cos\theta'$	$^1/_2\cos\theta\sin 2\theta'\pm\sin\theta'$	0
$\frac{A_3}{ab'I_{11}}$	$-^1/_2\sin 2\theta\sin\theta'$	$\mp ^1/_2\sin 2\theta\cos\theta'$	0	$\mp\cos 2\theta\cos\theta'-$ $-\cos\theta$	0	0
$\frac{A_4}{bb'I_{11}}$	$\sin\theta\sin\theta'$	$\pm\sin\theta\cos\theta'$	0	$1\pm\cos\theta\cos\theta'$	0	0
$\frac{B_1}{aa'I_{12}}$	0	$\pm ^1/_2\sin 2\theta\cos\theta'$	$\mp ^1/_2\sin 2\theta\sin\theta'$	$\cos\theta\cos 2\theta'\pm$ $\pm\cos 2\theta\cos\theta'$	$\mp\cos 2\theta\sin\theta'-^1/_2\cos\theta\sin 2\theta'$	$^1/_2(\sin\theta\sin 2\theta'\pm\sin 2\theta\sin\theta')$
$\frac{B_2}{ba'I_{12}}$	0	$\mp\sin\theta\cos\theta'$	$\pm\sin\theta\sin\theta'$	$-\cos 2\theta'\mp$ $\mp\cos\theta\cos\theta'$	$\pm\cos\theta\sin\theta'+^1/_2\sin 2\theta'$	0
$\frac{B_3}{ab'I_{12}}$	0	$-^1/_2\sin 2\theta$	0	$-\cos 2\theta\mp$ $\mp\cos\theta\cos\theta'$	0	0
$\frac{B_4}{bb'I_{12}}$	0	$\sin\theta$	0	$\cos\theta\pm\cos\theta'$	0	0

Note. For mixtures E(1) − E(2) [M(1) − M(2)] the quantities a, a' are given by (22) and the quantities b, b' by (21).

CONCLUSION

The main result of the present paper is the derivation of formulas which describe the absorption of γ radiation which is a mixture of multipole types of opposite parities, for the case in which the lines are split into components both in the source and in the absorber. In particular, the formulas obtained make it possible to bring out specific features of possible Mössbauer experiments for the purpose of studying nonconservation of spatial parity.

Whereas ordinarily a method for detecting violations of P invariance are based either on an analysis of the asymmetry of the angular distribution of intensity of individual components of the radiation,[1] or else on an analysis of the polarization of the radiation,[2,3] Mössbauer experiments are simultaneously sensitive to effects of a violation of P invariance in both the polarization and the intensity of the radiation. Therefore, for example, the difference in the Mössbauer absorption with two opposite directions of the magnetic field applied to the source (or the absorber)

$$\frac{I\Sigma(\hat{h}, k, \hat{h}') - I\Sigma(-\hat{h}, k, \hat{h}')}{I\Sigma(\hat{h}, k, \hat{h}') + I\Sigma(-\hat{h}, k, \hat{h}')}, \qquad (25)$$

as found from (15) and (17), is not the same as the corresponding asymmetry in the angular distribution of the intensity (cf.[8]). Another feature of Mössbauer experiments is that with them one can detect both multipole mixtures of the types E(L) – E(L + 1) and M(L) – M(L + 1), and mixtures of the type E(L) – M(L). It is impossible, for example, to detect mixtures of the former type by means of a method analogous to[2,3], since the corresponding radiation is not circularly polarized.

We shall give numerical estimates of the effects of P-parity nonconservation to be expected in Mössbauer experiments. The quantity determined experimentally in such experiments is a ratio of the type of (25). As in the case of measurements of circular polarization or of the asymmetry of the angular distribution of radiation, the effect is proportional to the mixing parameter δ. In the Mössbauer experiments, however, the maximum value of the proportionality coefficient, for optimal conditions for the observation of the effect, is about twice as large [see Eqs. (15)–(17)]. We can write the mixing parameter for γ transitions from nuclear states containing an admixture of the "forbidden" parity in the form $\delta = FR$, where F is the amplitude of the admixed state with the "forbidden" parity in the wave function of the nucleus and R is the ratio of the matrix element for the "forbidden" process to that for the allowed process.

The breaking of the parity symmetry of nuclear states associated with the breaking of parity symmetry in the weak interactions is characterized, according to calculations with the theory of weak currents,[4] by a value of F which lies in the range 10^{-6}–10^{-7}. This means that the effect can be observed experimentally only for transitions in which the allowed process is strongly impeded in comparison with the forbidden process, i.e., the ratio R is large. For the most widely known Mössbauer transition at 14 keV in ^{57}Fe the admixture amplitude is $F \sim 4 \cdot 10^{-7}$, and $R \sim 30$,[9] and consequently the expected effect is of the order of 10^{-5}, which is about an order of magnitude smaller than the accuracy which is achievable experimentally.[10] It must be pointed out, however, that the value $R \sim 30$ is by no means the maximum value of R. For example, a transition at 123 keV in ^{173}Lu is known, for which $R \sim 10^3$.[4] Therefore the negative result of experiments with ^{57}Fe [10] does not exclude the possibility of detecting the effect with other Mössbauer transitions, even with the existing experimental technique.

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