ENVELOPE SHOCK WAVES

L. A. OSTROVSKIĬ

Radiophysics Institute, Gor'kiy State University

Submitted November 15, 1967

Zh. Eksp. Teor. Fiz. 54, 1235—1243 (April, 1968)

It is shown that envelope shock waves. assuming the form of sharp transitions (jumps) between various values of the modulated wave amplitude and frequency, can exist in a nonlinear dispersive medium with a relaxing nonlinearity. Boundary conditions relating these quantities are found. The structure of the stationary shock wave, which may be either aperiodic or oscillatory, is determined. Its stability is investigated. The similarity and differences between the processes considered and shock waves of the ordinary type are discussed.

1. INTRODUCTION

A shock wave is a nonlinear process characteristic of weakly dispersed media, and admits of a strong distortion of the profile of the wave, up to formation of discontinuities. Such a situation is customary for motions of the continuous media considered in gasdynamics and magnetohydrodynamics; it is well known that electromagnetic shock waves are also possible. In media with strong dispersion, such as those studied in nonlinear optics and in many divisions of plasma dynamics, the nonlinearity is manifest principally in the space-time variations of slowly varying quantities—envelopes of the amplitude, of the frequency, etc., whereas the local structure of the wave (which is quasiharmonic in the case of small nonlinearity) remains unchanged.

The theory of such processes[1-8] leads to the conclusion that "envelope waves" experience cumulative distortions, which in many cases are analogous to the distortion of the instantaneous profile of the wave in a weakly-dispersive medium. In particular, simple (Riemann) envelope waves are possible[1,2,5] under certain conditions. It is natural to assume further that the propagation of such waves leads to the establishment of a sharp transition region (compared with the initial scale of the variation of the envelopes, but containing in general many periods of the field)—an envelope shock—and to generalize the methods of the theory of shock waves to strongly-dispersive media. Such a proposition was discussed earlier for quasiharmonic electromagnetic waves[1,7] and certain wave processes in continuous media[2]; boundary conditions for the corresponding discontinuities were written out.

However, the real existence of a stationary region of the shock type calls for the presence of definite dissipative mechanisms, otherwise the narrow field drop occurring in the simple wave will again broaden without limit as a result of dispersion, generating periodic modulation[5]. Therefore an informal theory of envelope shock waves, like that of ordinary shock waves, should take into account the effects of "viscous" type, which are not considered in the cited investigations, and in particular, compare the role of these effects and of dispersion spreading.

It should be noted that the suitable dissipative process should be quite distinct. A direct allowance for the linear viscosity, sufficient for the description of ordinary shock waves, does not lead to establishment of envelope shock waves and leads only to a general damping of the wave. A necessary condition is the presence of an inertial nonlinearity, which relaxes upon variation of the average intensity (or possibly frequency) of the wave, but has little influence in the absence of modulation. Allowance for such processes leads to an increase in the order of the average equations for the envelopes, necessary to obtain the sought transition region between two equilibrium states corresponding to unmodulated waves.

2. FUNDAMENTAL EQUATIONS

In the simplest cases (physical examples are considered below) the nonlinear relaxation is described by an equation of the type

$$\tau \partial\chi / \partial t + \chi = \beta m, \tag{1}$$

where $m = A^2$ is the intensity of the wave (A is its amplitude), χ is the nonlinear part of the polarizability of the medium, and τ and β are constants. In ordinary media $\tau > 0$ and $\beta > 0$, but other possibilities will be noted below.

Let us consider a plane traveling electromagnetic wave with slowly-varying amplitude and phase of the field

$$E = A(z, t) \exp \{i[\omega t - kz + \varphi(z, t)]\} + \text{к. с.}, \tag{2}$$

where ω and k are constants connected with the dispersion equation of the linear medium: $ck = \omega\sqrt{\epsilon(\omega)}$. The change occurring in A and φ in the case of small nonlinearity is described by a system of averaged equations[5], which can be written in the form [1)]:

$$\frac{\partial A}{\partial t} + v(1 + \varkappa\delta) \frac{\partial A}{\partial z} - \frac{\varkappa A}{2} \frac{\partial \delta}{\partial t} = 0, \tag{3}$$

$$\frac{\partial \delta}{\partial t} + v(1 + \varkappa\delta) \frac{\partial \delta}{\partial z} + \frac{\partial \chi}{\partial t} = -\frac{\varkappa}{2\omega^2} \frac{\partial}{\partial t}\left(\frac{1}{A} \frac{\partial^2 A}{\partial t^2}\right). \tag{4}$$

Here $\delta = (\omega^{-1}\partial\varphi/\partial t) \ll 1$ is the variable part of the fre-

[1)] In (2) and (3) we discard the nonlinear terms of second order of smallness, which are responsible for certain specific effects [1,5], but which are of no importance for what follows; when $\tau = 0$, the remaining system is equivalent to a one-dimensional parabolic equation for a complex field amplitude [8].

quency, $v(\omega) = (dk/d\omega)^{-1}$ is the group velocity, and $\kappa = (\omega/v)dv/d\omega$ is the dispersion parameter. When χ is suitably defined (for a monochromatic wave we have $\chi = vk'/\omega$, where $k'(\omega, A)$ is the nonlinear increment to the wave number), these equations are valid for a wave of any nature.

In the system (1), (3), and (4) we can separate two independent time scales[2]: τ and $\tau_g = \omega^{-1}(2A^2_{max}|\beta/\kappa|)^{-1/2}$, where A_{max} is the maximum value of the amplitude. The course of the wave process depends on the relations between the characteristic duration T_0 of the initial perturbation and the indicated parameters[3].

Let us assume first that $T_0 \gg \tau$, τ_g, and then the terms with higher derivatives in (1) and (4) can be discarded. Then Eqs. (3) and (4) constitute a second-order quasilinear system, the properties of which depend significantly on the sign of the product $\kappa\beta$[5]. When $\kappa\beta < 0$, the system is hyperbolic and has two families of real characteristics, and when $\kappa\beta > 0$ it is elliptic. We consider here essentially the case $\kappa\beta < 0$, which admits of shock waves[4]. In this case equations (3) and (4) have two families of characteristics $(dz/dt) = u_\pm$, along which small disturbances δ' and m' of the constant values δ and m propagate. The corresponding quantities are given by the relations

$$u_\pm = v(1 + \varkappa\delta \pm \sqrt{pm}), \quad \varkappa\delta' = \pm\sqrt{p/m}\; m', \tag{5a}$$

where $p = -\kappa\beta > 0$. These two families correspond to two types of simple waves in the form[5]

$$\varkappa\delta = \varkappa\delta_0 \pm 2\sqrt{pm}; \quad m = F_\pm[t - z/u_\pm(m)];$$
$$u_\pm = v(1 + \varkappa\delta_0 \pm 3\sqrt{pm}). \tag{5b}$$

Here δ_0 is a constant and $F_\pm$ are arbitrary functions. The quantities $I_\pm = \delta \mp 2\sqrt{pm}$, which are conserved in simple waves, are Riemann invariants for the given problem. The wave F_+ (fast) or F_- (slow) is deformed in such a way, that the sections with $\partial m/\partial t > 0$ cancel out for F_+ and the sections with $\partial m/\partial t < 0$ for F_-. We note that the function $n\delta$, together with the group velocity $v_{gr}(\delta)$ at the instantaneous frequency $\omega(1 + \delta)$, which is equal to $v(1 + \kappa\delta)$, always increases in the cancelling sections of both waves, in analogy with the density or pressure of a simple compression wave in gasdynamics.

3. BOUNDARY CONDITIONS ON THE DISCONTINUITY

The solutions (5) are no longer valid after the wave deformation leads to non-unique values of m and δ. If we disregard further the discarded terms in the initial equations, then we must admit the appearance of discontinuous solutions—shock waves. Integrating Eqs. (3) and (4) with respect to z in the narrow region containing the discontinuity at the given instant of time, we obtain the boundary conditions relating the values of m and δ on both sides of the discontinuity:

$$b(m_2 - m_1) - \varkappa(m_2\delta_2 - m_1\delta_1) = 0, \tag{6}$$
$$b(\delta_2 - \delta_1) - {}^1/_2\varkappa(\delta_2^2 - \delta_1^2) + \beta(m_2 - m_1) = 0. \tag{7}$$

The indices 1 and 2 pertain here to the quantities in before and behind the jump, respectively, $b = (v_y/v) - 1 \ll 1$, where v_y is the velocity of the jump. We note that without loss of generality we can put $\delta_1 = 0$, inasmuch as the choice of ω in (2) is arbitrary. We then get from (6) and (7)

$$b^2 = 2pm_2^2 / (m_1 + m_2), \tag{8}$$
$$\varkappa\delta_2 = b(1 - m_1/m_2) = \pm\sqrt{2p}(m_2 - m_1)/\sqrt{m_1 + m_2}. \tag{9}$$

It is obvious that the two different states 1 and 2, which are connected by conditions (8) and (9), exist only when $p > 0$. Specification of the states (m_1, $\omega_1 = \omega$) on one side of the jump and of its velocity (b) determines completely the state (m_2, $\omega_2 = \omega + \omega\delta_2$) on the other side, and for an arbitrary velocity, (9) gives the relation between δ_2 and m_2. According to (9), two branches of $\delta_2(m_2)$ curves emerge from the given point (m_1, $\delta_1 = 0$) on the (m, δ) plane (Fig. 1). Each of them can be regarded as the analog of the Hugoniot adiabat in gasdynamics, and the $\delta(m)$ dependence from (5b) can be set in correspondence with the Poisson adiabat. Comparing (5b) and (9), we can show that these curves have at a specified common point a tangency of second order, i.e., at a given variation of one of the quantities (say m), the others change in identical fashion accurate to quantities of order $(m_2 - m_1)^2$ inclusive. In particular, the Riemann invariants $I_\pm = \kappa\delta \pm 2(\sqrt{pm} - \sqrt{pm_1})$ in a weak shock wave are small quantities of the order $(m_2 - m_1)^3$. All this corresponds to the well known properties of ordinary shock waves.

4. STRUCTURE OF ENVELOPE SHOCK WAVES

The next step is to forego the discontinuity idealization and to analyze Eqs. (1), (3), and (4) in their complete form. We seek a solution in the form of a stationary wave that depends on one variable $\xi = t - z/v_y$, v_y = const; this makes it possible to go over to ordinary differential equations. With this, Eqs. (3) and (4) have first integrals in the form

$$b(m - m_1) - \varkappa\delta m = 0, \tag{10}$$
$$\delta\left(b - \frac{\varkappa\delta}{2}\right) + \chi - \chi_1 = \frac{\varkappa}{2\omega^2 A}\frac{d^2A}{d\xi^2} \tag{11}$$

(here, as before, $\delta_1 = 0$). Equations (10) and (11) together with (1) form a third-order system. Even a qualitative investigation of this system is difficult, and we shall therefore consider the case $\tau_g \ll \tau$. Then the

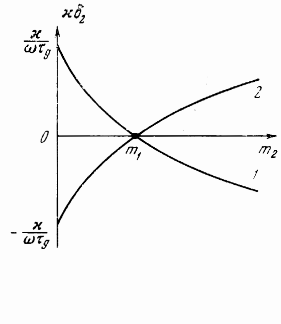

FIG. 1. Shock adiabat relating the change in the intensity and frequency of the envelope discontinuity. Curve 1 corresponds to a slow wave (b < 0) and curve 2 to a fast wave (b > 0). In stable discontinuities $\kappa\delta_2 > 0$.

[2] $\tau_g^{-1} = \Omega_m$ is the characteristic frequency at which equilibrium is attained between nonlinear distortion and dispersion spreading [5].

[3] The influence of the parameters τ_g and τ is similar to the influence of dispersion and viscosity in the equations for the instantaneous values of the field

[4] When $\kappa\beta > 0$, the most characteristic effect is self-modulation (temporal self-focusing) [5].

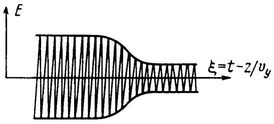

FIG. 2. Form of stationary envelope shock wave at $\tau \gg \tau_g$.

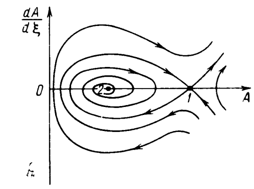

FIG. 3. Form of the phase plane of Eq. (13) (at $\tau \ll \tau_g$); the picture is symmetrical with respect to the axis A = 0. The shock wave corresponds to the separatrix joining the points 1 and 2.

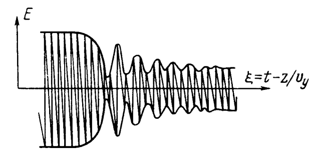

FIG. 4. Structure of shock waves corresponding to Fig. 3.

term in the right side of (11) can be described, as before, and after certain transformations we arrive at a first-order equation

$$\tau\frac{dm}{d\xi}+\frac{m}{2m_1{}^2m_2{}^2}[m(m_1+m_2)+m_1m_2](m_1-m)(m-m_2)=0. \quad (12)$$

Here $m_{1,2}$, as above, are the values of m before and behind the shock front (as $\xi \to \mp\infty$). Equations (12) can be integrated, but the corresponding analytic expression is rather cumbersome. It is obvious that the amplitude changes monotonically in the interval between the equilibrium points m_1 and m_2, and the sign of the derivative $dm/d\xi$ in this interval depends only on the sign of τ. If $\tau > 0$, then only a shock drop with a decreasing amplitude is possible, i.e., with $m_2 < m_1$ (Fig. 2). The characteristic duration of the shock front T is of the order of $\tau/(1 - m_2/m_1)$, i.e., $T \gtrsim \tau$.

Let us note the singularity arising when $m_2 \to 0$ (we have in mind the case $\tau > 0$). It is easy to see that when $m_2 \ll m_1$, the field decreases rapidly over an integral on the order of $\tau(m_2/m_1)^2$, to a value comparable with m_2, after which it approaches m_2 asymptotically during a time on the order of τ. With this, $b \to 0$, and δ changes from zero to $\delta_2 = 1/\omega\tau_g$. In the limit, when $m_2 = 0$, the shock front degenerates in accordance with (12) into a jump; if, however, $\tau(m_2/m_1)^2$ becomes of the order of τ_g, then Eq. (12) is no longer valid.

Let us consider now the case $\tau \ll \tau_g$. In this case we can assume that (1) contains a small parameter τ in the derivative, and we can break up the process into a fast part, occurring within the time on the order of τ, and one that is slow compared with τ. As is clear from (1), when $\tau > 0$, the trajectories of the fast motions in phase space (χ, A, $dA/d\xi$) of the initial system lead within a time on the order of τ to a cylindrical surface $\chi = \beta A^2$, in the vicinity of which occur all the slow motions. For the latter, $\tau d\chi/d\xi \ll \beta m \approx \chi$, and putting $d\chi/d\xi = \beta m/d\xi$ (it is impossible to discard the derivative of χ completely, for then the system becomes conservative and has no solutions corresponding to a shock wave), we arrive at the equation

$$\tau_g{}^2\frac{m_1}{A}\frac{d^2A}{d\xi^2}+\tau\frac{dm}{d\xi}+\frac{m(m_1+m_2)+m_1m_2}{m^2(m_1+m_2)}(m_1-m)(m-m_2)=0, \quad (13)$$

where, as usual, $m = A^2$, and in τ_g we put $A^2_{max} = m_1$.

It is easy to investigate the solutions of (13) on the phase plane (corresponding to the indicated surface $\chi = \beta A^2$). As above, the sign of the resultant change of the field depends only on the sign of τ; if $\tau > 0$, then $m_2 < m_1$. However, the change of m now contains oscillations. Indeed, as follows from (13), one equilibrium point is a saddle point, and the other is a focus, with the value of m in the first of them always larger than that in the second. Therefore when $\tau > 0$, the approach to m_2 is accompanied by damped oscillations of the intensity (Figs. 3 and 4).

The duration of the aperiodic section T_1 is of the order $\tau_g/\sqrt{1 - m_2/m_1}$; the period of oscillations T_1' is of the same order. The attenuation time of the latter, T_2, is of the order of $(\tau_g^2/\tau)(m_1/m_2)$, and the total duration of the shock front T is close to T_2 (Eq. (13) is valid for $T_2 \gg T_1'$, when the process is essentially oscillatory). If $m_2 \to 0$, then $T \to \infty$ is a result of the oscillating section (there is no influence of the nonlinear relaxation when $m \to 0$); to be sure, this entire section lies in the region of vanishingly small m, and only the initial aperiodic drop from m_1 to $m \sim m_2$ with duration τ_g can be regarded as the shock wave. We note that the straight line A = 0 on the phase plane (Fig. 3) is singular. This singularity is retained also when $\tau = 0$, when the shock waves are impossible, for all stationary solutions with $\delta \neq$ const [5].

Mention must be made of the case $\kappa\beta > 0$, in which, as already indicated, there exist no two simultaneous different nonzero equilibrium positions, and when $\tau \neq 0$ there exist no bounded stationary solutions with $b \neq 0$. However, when b = 0 and $\delta = 0$ there is possible here one singular type of stationary wave with $m_1 \neq 0$ and $m_2 = 0$ (when $\tau > 0$). The structure of such a wave contains growing oscillations ahead of the front.

Let us dwell briefly on the question of energy dissipation on the shock front. As is well known, dissipation (growth of the entropy) is inevitable in an ordinary shock wave. This holds true also for envelope shock waves, inasmuch as the relaxation process is irreversible. However, it is difficult to write out the corresponding expression for the power P dissipated on the front in our case, first because it is necessary to know for this purpose the expressions for b and δ_2 with greater accuracy than in (8) and (9), and second because no correct phenomenological expression for the average energy density in a nonlinear dispersive medium is known at all. However, the order of magnitude of P can be estimated by considering the change of the energy in a modulated wave, with allowance for (1). As a result we find that the power loss density is of the order of $(\tau/\beta)(\partial\chi/\partial t)^2$; taking into account also the estimates of the duration of the shock front in the cases (12) and (13), we get $P \approx v\beta m_1(m_1 - m_2)$. [6]

[5] A similar singularity when $A \to 0$ should be possessed also by two-dimensional stationary structures considered in the analysis of self-focusing [9], if the phase is not constant in the transverse direction.

[6] Actually X has also a small imaginary component [10], which is not taken into account in (1) and which leads, generally speaking, to a comparable dissipation. This dissipation, however, occurs also beyond the limits of the shock front and is not a characteristic of the process under consideration.

5. STABILITY

The real existence of shock waves close to stationary ones is possible only if the latter are stable. Let us consider first the usual approximation, when the shock wave is regarded as a jump, and let us investigate its stability with respect to perturbations with $T_0 \gg \tau, \tau_g$. Such perturbations propagate over characteristics corresponding to (5a). The well known prescription for separating stable discontinuities[11] consists of requiring that the number of independent quantities describing the perturbations (including the perturbation of the velocity of the jump) be equal to the number of the boundary conditions relating these quantities (in our case these conditions are obtained from (6) and (7)). It is easy to see that in order to satisfy this requirement the perturbation of the stable jump should generate only one wave traveling away from it, which in turn is possible if one of the following relations is satisfied between the velocity of the jump v_y and the velocities of the perturbations $u_\pm$ on both sides of the jump

$$(u_\pm)_1,\ (u_-)_2 < v_y < (u_+)_2; \tag{14a}$$

$$(u_-)_1 < v_y < (u_+)_1,\ (u_\pm)_2, \tag{14b}$$

where the subscripts 1 and 2 distinguish, as before, between the quantities in the front of the jump and behind it. Using the corresponding expressions ((5a) for $u_\pm$ and (8) and (9) for v_y), we find that condition (14a) is satisfied when

$$m_2 > m_1, \quad b > 0, \quad \varkappa\delta_2 > 0, \tag{15a}$$

and (14b) is satisfied

$$m_2 < m_1, \quad b < 0, \quad \varkappa\delta_2 > 0. \tag{15b}$$

Consequently, only fast jumps with increasing intensity and slow ones with decreasing intensity are stable; for either of them, the instantaneous group velocity $v_{gr} = v(1 + \kappa\delta)$ increases. As expected, each of the cases (15a) or (15b) corresponds to one of the simple waves (5b), the evolution of which leads to the given type of jump [7]; however, as follows from the preceding section, actually only the jump (15b) is possible when $\tau > 0$ and the jump (15a) when $\tau < 0$.

As is well known, in the analysis of the stability of a shock wave it is necessary to take into account the fact that even a monochromatic wave is unstable in a relaxing nonlinear medium (state of equilibrium at the beginning or the end of the shock front). Indeed, linearizing the initial expressions (1), (3), and (4) in full form relative to the constant values A and δ, and seeking perturbations proportional to $\exp i(\Omega t - Kz)$, we obtain [8]

$$vK = \Omega \pm \Omega\left(\frac{\varkappa^2\Omega^2}{4\omega^2} + \frac{pm}{1 + i\Omega\tau}\right)^{1/2}. \tag{16a}$$

Consequently, real Ω correspond to complex $K = K' + iK''$, and, as can be obtained from (16a), the imaginary part of K satisfies the relation

$$(vK'')^2 = \Omega^2\left\{\frac{\sqrt{f^2 + \Omega^2\tau^2} - f}{2(1 + \Omega^2\tau^2)}\right\}, \tag{16b}$$

where $f = pm + (\kappa/2\omega)^2\Omega^2(1 + \Omega^2\tau^2)$, and the root is taken algebraically. As already indicated, shock waves correspond to the case $p > 0$. According to (16b), even a shock wave which is stable in the sense of (15) cannot exist for an infinitely long time, since oscillations destroying the stationary structure are excited near its front when $\tau \neq 0$. One can speak, however, of a real existence of a quasi stationary shock front if $vK''_{max}T \ll 1$, where T is the front duration, and K''_{max} is the maximum increment of the instability. The existence of a maximum of (16b) when $p > 0$ was investigated separately for two cases.

A. If $\tau \ll \tau_g$, then

$$\Omega_1 \approx \left(\frac{\tau}{\tau_g}\right)^{2/3}, \quad vK_{max}'' = \frac{\sqrt{pm_1}}{2\tau}\left(\frac{\tau}{\tau_g}\right)^{1/6}, \tag{17a}$$

where Ω_1 is the value of Ω at which $K'' = K''_{max}$. If $(\tau/\tau_g)^{1/6} \sim 1$, i.e., $(\tau/\tau_g) \lesssim 10^5–10^6$, then $vK''_{max}T \approx \sqrt{pm_1}/(1 - m_2/m_1)$; this expression is always small, except in the case of very weak drops, when $(1 - m_2/m_1) \lesssim \sqrt{pm_1}$. In all other cases the instability described by formula (16b) is practically insignificant for shock waves.

B. If $\tau \ll \tau_g$, then

$$\Omega_1 \approx \frac{1}{\tau}, \quad vK_{max}'' \approx \frac{\sqrt{pm_1}}{4\tau_g} \approx \frac{\omega\beta m_1}{2}. \tag{17b}$$

(This result is approximately valid up to $\tau \sim \tau_g$.) With this, $vK''T \lesssim (\kappa/8\omega\tau) \ll 1$, i.e., the instability is insignificant relative to the scale of the shock-wave front in this case, too.

We note that in both cases the quantity $1/\Omega_1$ is smaller than or of the order of the minimum scale of variation of the quantities in the stationary front (τ if $\tau \gg \tau_g$ and τ_g if $\tau \ll \tau_g$), therefore the "quasistationary" analysis, when the increment is obtained from (16b) at constant m, is justified also for the entire shock transition.

We note that for ordinary shock waves it is possible to make rather general statements concerning a one-to-one correspondence between the stable jumps in the "discontinuity" approximation and the stationary solutions with dissipation taken into account[14]. These statements are connected, however, with the requirement that the field be stable away from the jump, and are not suitable in our case. Thus, although according to (15) there are two possible types of stable signals, the stationary solutions of (12) and (13) correspond, for a given sign of τ, to only one of these type.

6. NONLINEAR RELAXATION MECHANISMS

Mechanisms of this type are being intensely investigated at the present time in connection with problems of nonlinear optics. An example of an inertial nonlinearity described by expressions (1) is the relaxation of the Kerr effect in liquids. Typical values for this case are $\tau \sim 10^{-11}$ sec and $\beta \sim 10^{-11}$ cgs esu ($\tau > 0$, $\beta > 0$). Recognizing that in the wavelength region 0.5–1 μ (the range of emission of high-power solid-state lasers) we have $\kappa \sim -10^{-1}–10^{-2}$ [15], we can state, first, that envelope shock waves are possible ($\kappa\beta < 0$) and, second, that

---

7) The conclusion that the frequency increases on an evolution jump[2] is in general incorrect.

8) Formulas of the type (16a) were investigated earlier in the cases $\tau \to 0$ [5,8] and $\tau_g \to 0$ [12,13], including also for non-plane (quasiplane) perturbations.

the intensity of the field at which $\tau \approx \tau_g$ does not exceed 10^4 W/cm^2. Inasmuch as the intensity of laser pulses used in nonlinear optics is much higher (10^6-10^7 W/cm^2 and higher), we have for them $\tau \gg \tau_g$, and the limit to which the duration of a certain section of the wave can decrease, which is equal to the duration of the stationary shock wave, is of the order of τ [9)]. As to the distance necessary for the "toppling" of the simple wave and the occurrence of a shock wave ($z_p \approx vT_0/2\sqrt{\kappa\beta m}$, where T_0 is the initial duration of the pulse and m is its maximum intensity), in the case of a self-focusing filament ($\beta m \gtrsim 0.1$), z_p amounts to several centimeters at $T_0 \sim 10^{-10}$ sec. Consequently, the formation of shock discontinuities is possible in focused ultrashort pulses (although an appreciable broadening of the spectrum of the focusing pulse due to the nonlinear deformations occurs also at much larger durations[15,16]).

A unique influence can be exerted by nonlinear processes connected with spatial dispersion. These are striction effects, for which the changes of the polarizability are described by the equation of the sound waves; the latter does not coincide with (1), but for the stationary plane waves it leads to the form (1). With this, the signs of the parameters τ and β coincide with the sign of the difference $v_{ac}^2 - v_y^2$, where v_{ac} is the speed of sound, and τ and β are usually negative for an electromagnetic wave [10)]; physically this is connected with the singularities of the excitation of sound by a source moving with supersonic velocity. For this case, we obtain from the foregoing analysis formally a shock transition with $m_2 > m_1$, but the question of its stability calls for a special investigation. Similar mechanisms can play a noticeable role, as is well known, both in nonlinear optics and in the propagation of electromagnetic waves in a plasma.

Envelope shock waves can apparently be realized in the radio band by using ferrites or semiconductor diodes; the difficulties arising in this case are of the same order as in the observation of effects of self-modulation of a wave in systems with $\kappa\beta > 0$[17].

9)An important exception is the case $m_2 \to 0$ discussed above, when it is possible to separate on the front a local drop with duration of the order of τ_g.

10)For non-one-dimensional waves, but those not modulated in time, the striction leads to a positive nonlinear polarizability ($\beta > 0$) [12]. In the general case, the sign of τ or β depends on the relation between the transverse and longitudinal gradients of the field.

1 L. A. Ostrovskiĭ, Zh. Tekh. Fiz. **33**, 905 (1963) [Sov. Phys.-Tech. Phys. **8**, 679 (1964)].

2 G. B. Whitham, Proc. Roy. Soc. **283**, 238 (1965).

3 L. A. Ostrovskiĭ, Radiotekhnika i élektronika **10**, 1176 (1965).

4 G. W. Whitham, J. Fluid Mech. **22**, 273 (1965).

5 L. A. Ostrovskiĭ, Zh. Eksp. Teor. Fiz. **51**, 1189 (1966) [Sov. Phys.-JETP **24**, 797 (1967)].

6 M. J. Lighthill, J. Inst. Math. Appl. **1**, 269 (1965).

7 L. A. Ostrovsky, in Proc. of a Symposium held at Delft, Netherlands, 1965, Pergamon, 1967, p. 581.

8 A. G. Litvak and V. I. Talanov, Izv. Vuzov, Radiofizika **10**, 539 (1967).

9 V. I. Talanov, ibid. **7**, 564 (1964).

10 Ya. I. Frenkel', Sobranie izbrannykh trudov (Selected Works), v. 3, AN SSSR, 1959.

11 L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred, Gostekhizdat, 1953 [Fluid Mechanics, Addison-Wesley, 1959].

12 V. I. Bespalov, A. G. Litvak, and V. I. Talanov, Paper at Second All-union Symposium on Nonlinear Optics, Novosibirsk, 1966.

13 S. A. Akhmanov and A. P. Sukhorukov, ZhETF Pis. Red. **5**, 108 (1967) [JETP Lett. **5**, 87 (1967)].

14 G. I. Freidman, Izv. Vuzov, Radiofizika **6**, 536 (1963).

15 T. K. Gustafson, P. L. Kelley, F. de Martini, and C. H. Townes, Paper at Third All-union Symposium on Nonlinear Optics, Erevan, 1967.

16 L. A. Ostrovskiĭ, ZhETF Pis. Red. **6**, 807 (1967) [JETP Lett. **6**, 260 (1967)].

17 L. I. Zagryadskaya and L. A. Ostrovskiĭ, Izv. Vuzov Radiofizika **11**, No. 6 (1968).

Translated by J. G. Adashko
143