

A translation of the Zhurnal Éksperimental'noi i Teoreticheskoi Fiziki

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Vol. 27, No. 5, pp. 679-864

(Russ. Orig. Vol. 54, No. 5, pp. 1273-1616)

November 1968

*PRODUCTION OF CHARGED PIONS IN COLLISIONS OF 600-MeV NEUTRONS WITH
NUCLEI OF VARIOUS ELEMENTS*

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Submitted June 22, 1967

Zh. Eksp. Teor. Fiz. 54, 1273-1291 (May, 1968)

We have used a magnetic spectrometer to study the production of charged pions in collisions of 600-MeV neutrons with nuclei of Be, C, Al, Cu, and Pb over a wide range of angles. For these nuclei we have obtained at five angles (16, 30, 60, 90, and 123°) the spectra of pions of both signs, on the basis of which we have found the cross sections, atomic-number dependences, and angular distributions of pion production. The influence of a number of effects on the characteristics of pion production in nuclei is discussed. The analysis performed permits us to obtain a qualitative picture of the basic regularities of pion production.

1. INTRODUCTION

ALONG with the study of pion production processes in nucleon-nucleon collisions, considerable interest is attached to the investigation of pion production in collisions of nucleons with nuclei. Study of pion production in nuclei permits us to obtain information on pion production in the elementary events and to check the consequences of the hypothesis of charge independence. In addition, pions produced in nuclei can serve as an effective means for investigation of nuclear structure, in particular of such characteristics as the spatial and momentum distributions of nucleons in nuclei.

A number of experimental studies have been devoted to pion production in complex nuclei on bombardment by protons. However, it must be noted that the existing results on production of charged pions have been obtained in a limited range of angles, which hinders a complete analysis of the characteristics of the pion production processes in complex nuclei.

Almost no experiments have been performed on the production of charged pions in the interaction of neutrons with nuclei. Furthermore, such experiments are not a simple repetition, from the point of view of charge symmetry, of experiments with protons, and allow us to obtain by comparing these results new information on the

processes occurring in the interaction of pions with nuclear matter.

In the present work we used a magnetic spectrometer¹⁾ to study the production of charged pions over a wide range of angles in collisions of neutrons with nuclei. The studies were made with the nuclei Be⁹, C¹², Al²⁷, Cu⁶⁴, and Pb²⁰⁷ in the neutron beam of the JINR synchrocyclotron. The experiments performed, in which energy spectra, angular distributions, and dependence of pion yield on atomic number were obtained under the same conditions and by the same technique, make it possible by comparison of the entire set of results to obtain a rather complete picture of the processes associated with pion production in complex nuclei.

2. EXPERIMENT

The measurements were made with an 18-channel magnetic spectrometer for which we have previously¹⁾ given a description and the calculation of the main characteristics. A neutron beam from the synchrocyclotron bombarded targets of the elements under study, which were placed on the rotational axis of the spectrometer.

¹⁾The experiment was performed in 1962, but for a number of reasons the analysis of the results was delayed and was carried out only recently.

Pions of both signs produced in the targets were analyzed by the spectrometer at five angles: 16, 30, 60, 90, and 123° with respect to the neutron beam. For each of the angles the magnetic field strength in the spectrometer was chosen so that pions with the maximum momenta possible in nucleon-nucleon collisions could be recorded. With this choice of field only an insignificant fraction on the high-energy side of the spectra of pions produced in complex nuclei were not detected by the spectrometer. The lower energy limit of the pions recorded was determined by the energy loss in passage of the pions through the target and the spectrometer counter and amounted to ~25 MeV.

The neutron spectrum has a broad energy distribution. The questions of effective neutron energy and the interpretation of results in studies with this beam have also been discussed in our earlier article.^[1] The weak dependence of the effective energy value on the excitation function for pion production allows us to assign the results to a definite effective neutron energy of ~600 MeV, as in the earlier paper.^[1]

The targets were prepared in the form of rectangular plates whose thicknesses were chosen so that all samples had closely the same ionization loss for pions.

The no-target background was insignificant, not exceeding several per cent in the very worst cases (measurement of π^+ mesons at small angles), and was due mainly to the system of mounting the targets in the beam.

A number of corrections were made to the experimental results, the most important of which was the correction for electron and positron contamination.

The β -particle contamination was determined by the same method as previously.^[1] Absolute normalization of the electron yield was performed experimentally. To determine the shape of the energy spectrum and angular distribution of the electrons we used data on γ -ray production in the interaction of protons with nuclei.^[2-4]

We have listed in Table I the calculated results for electron and positron contamination, averaged over all angles, as a percentage of the number of pions.

The electrons and positrons produced by γ rays from π^0 -meson decay are produced in practically equal numbers (the Compton effect in the nuclei gives a negligible contribution). The yield of π^- mesons from interaction of neutrons with the nuclei is several times greater than the yield of π^+ mesons. Therefore the positrons make a considerably greater contribution to the π^+ -meson spectrum than do electrons to the π^- -meson spectrum. As a result of this the accuracy of the spectrum measurements in the low-energy region turns out to be considerably better for π^- mesons.

As can be seen from Table I, the positron contamination is particularly high for the nuclei Cu and Pb. This leads to large errors in the π^+ -meson spectra, which practically exclude from consideration their low-energy parts, since the main part of the electron spectrum is superimposed on the low-energy region. In addition to

the corrections for electrons, we have applied to the results theoretical corrections for pion decay, for muon impurity, and for absorption of pions in the proton filler.

To determine the absolute cross-section values we compared the pion yield with the yield of recoil protons from elastic np scattering, for which the differential cross section has been measured by Kazarinov and Simonov.^[5] The yield of elastically scattered protons was determined in difference measurements with polyethylene and graphite at an angle of 60°.

The differential cross section for elastic np scattering obtained by Kazarinov and Simonov^[5] was measured by them in the same neutron beam with a low-energy cutoff of 450 MeV in the neutron spectrum. Therefore, for the method chosen to normalize the cross sections, it is necessary to correct for the fraction of pions which are produced by neutrons with energies below 450 MeV. The size of this correction for complex nuclei is considerably greater than the corresponding correction for production of pions in np collisions. This is due to the fact that the threshold for production of mesons in nuclei shifts toward lower energies. Furthermore, because of the difference in the mechanisms of production of π^+ and π^- mesons (π^+ mesons are produced in collisions of neutrons with protons of the nucleus, and a large fraction of π^- mesons are produced in nn collisions), the values of the corrections being discussed can also differ for π^+ and π^- mesons.

To determine these corrections we plotted the excitation functions for production of charged pions and the corresponding effective neutron spectra on the basis of existing data on the interaction of protons with nuclei.^[6] The corrections found amount to 9% for the π^+ -meson cross section and 12% for π^- mesons.

3. EXPERIMENTAL RESULTS

The experimental pion spectra at two angles, after introduction of all corrections, are shown in Figs. 1 and 2. Figures 1a and b show π^+ spectra for 30 and 90°, and Figs. 2a and b show π^- spectra for the same angles.

The magnetic field strengths chosen allowed us to measure at each angle the overwhelming fraction of the pion spectra, except for the "tails" in the high energy region. In order to investigate the shape of the spectra

	Be	C	Al	Cu	Pb
N_{e^+}/N_{π^+}	22%	23%	25%	33%	70%
N_{e^-}/N_{π^-}	3,5%	3,8%	4,7%	6,0%	10%

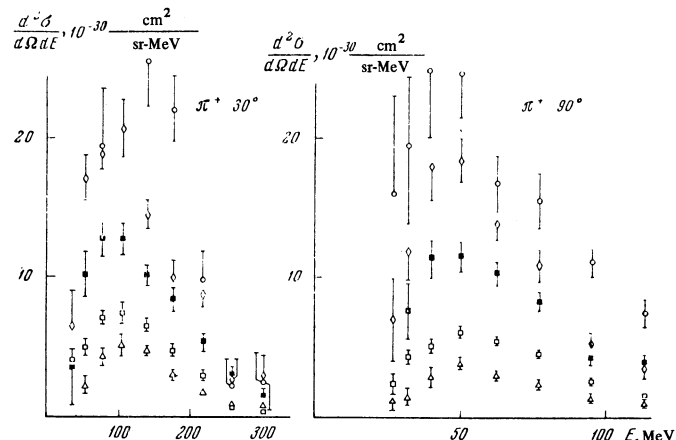


FIG. 1. π^+ -meson spectra: a - 30°, b - 90°. Δ - Be, \square - C, \blacksquare - Al, \diamond - Cu, \circ - Pb.

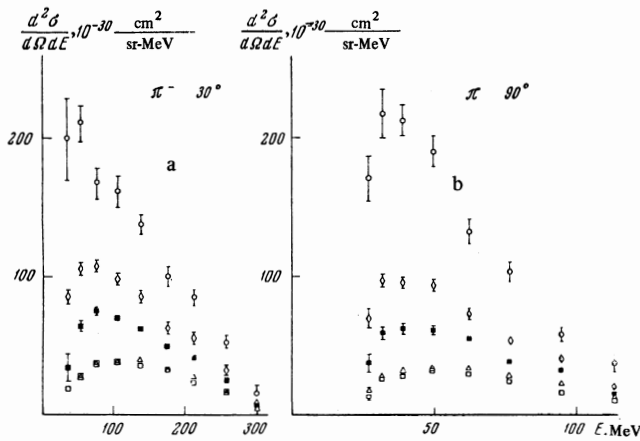


FIG. 2. π^- -meson spectra: a - 30° , b - 90° . Δ - Be, \square - C, \blacksquare - Al, \diamond - Cu, \circ - Pb.

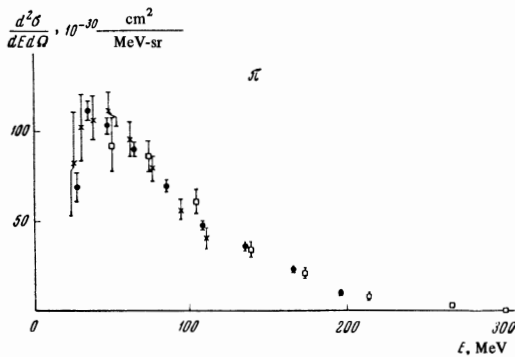


FIG. 3. π^- -meson spectra from copper at 60° for different magnetic field values: \times - 9000 G, \bullet - 12,000 G, \square - 15,000 G.

up to the maximum pion energies, we made measurements with higher values of magnetic field than in the main measurements. These measurements not only clarified the shape of the spectra but also made it possible to monitor the operation of the various channels of the spectrometer. In Fig. 3 we have plotted the results of such measurements of the π^- spectrum from copper at 60° for three magnetic-field values. It is evident

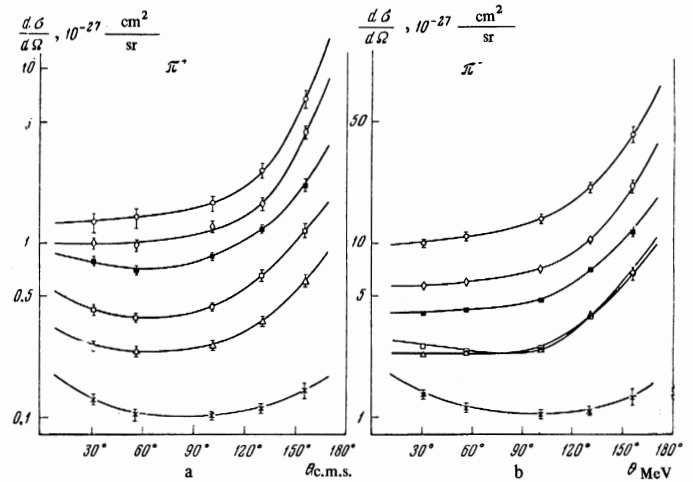


FIG. 4. Angular distributions of π^+ mesons (a) and π^- mesons (b) in the c.m.s. \times - H, Δ - Be, \square - C, \blacksquare - Al, \diamond - Cu, \circ - Pb.

from this figure that the π^- spectrum extends almost to 300 MeV.

Table II lists the differential cross sections for production of π^+ mesons, and Table III those for π^- mesons, found by integration over the pion energy spectra for five angles. For comparison we have shown also the data for hydrogen.^[1] The errors listed in Tables II and III are the errors in the relative measurements.

In normalization of the absolute differential cross section values, we have taken into account mesons produced by neutrons with energies below 450 MeV. The small relative contribution of these pions ($\sim 10\%$) does not appreciably affect the shape of the spectra and leads only to a small increase in the number of low-energy pions.

Figure 4 shows the angular distributions of pions from the nuclei studied, in the center of mass of the two colliding nucleons. In these figures we have also shown the angular distributions for π^+ and π^- mesons produced in np collisions.^[1] The differential cross sections for π^- mesons from np collisions in Fig. 4b have been increased by a factor of ten. The curves are drawn from

Table II. Differential cross sections for π^+ -meson production

Nucleus	16°		30°		60°		90°		123°	
	10^{-27} cm ² /sr	Rel. un.*	10^{-27} cm ² /sr	Rel. un.*	10^{-27} cm ² /sr	Rel. un.*	10^{-27} cm ² /sr	Rel. un.*	10^{-27} cm ² /sr	Rel. un.*
H	0.45 ± 0.03	0.295 ± 0.019	0.29 ± 0.02	0.256 ± 0.020	0.135 ± 0.012	0.227 ± 0.020	0.06 ± 0.003	0.158 ± 0.009	0.032 ± 0.003	0.091 ± 0.009
Be	0.99 ± 0.05	0.63 ± 0.03	0.74 ± 0.04	0.64 ± 0.04	0.37 ± 0.02	0.62 ± 0.03	0.22 ± 0.02	0.55 ± 0.035	0.18 ± 0.01	0.52 ± 0.03
C	1.57 ± 0.07	1.00 ± 0.04	1.15 ± 0.05	1.00 ± 0.04	0.60 ± 0.02	1.00 ± 0.03	0.40 ± 0.025	1.00 ± 0.03	0.35 ± 0.03	1.00 ± 0.08
Al	2.97 ± 0.16	1.89 ± 0.20	2.12 ± 0.08	1.84 ± 0.07	1.19 ± 0.05	1.98 ± 0.11	0.75 ± 0.04	1.88 ± 0.11	0.64 ± 0.05	1.53 ± 0.14
Cu	3.76 ± 0.24	2.39 ± 0.15	2.98 ± 0.15	2.59 ± 0.13	1.73 ± 0.12	2.88 ± 0.20	1.05 ± 0.08	2.63 ± 0.20	1.30 ± 0.10	3.71 ± 0.30
Pb	4.90 ± 0.70	3.12 ± 0.40	4.32 ± 0.50	3.74 ± 0.40	2.36 ± 0.23	3.94 ± 0.30	1.66 ± 0.15	4.16 ± 0.41	2.02 ± 0.20	5.76 ± 0.60

*The cross section for carbon is taken as unity.

Table III. Differential cross sections for π^- -meson production

Nucleus	16°		30°		60°		90°		123°	
	10^{-27} cm ² /sr	Rel. un.	10^{-27} cm ² /sr	Rel. un.	10^{-27} cm ² /sr	Rel. un.	10^{-27} cm ² /sr	Rel. un.	10^{-27} cm ² /sr	Rel. un.
H	0.49 ± 0.03	0.051 ± 0.003	0.33 ± 0.02	0.044 ± 0.003	0.138 ± 0.008	0.038 ± 0.003	0.058 ± 0.003	0.024 ± 0.001	0.030 ± 0.003	0.015 ± 0.002
Be	8.70 ± 0.20	0.90 ± 0.02	7.13 ± 0.20	0.97 ± 0.03	3.45 ± 0.10	0.97 ± 0.03	2.46 ± 0.03	1.02 ± 0.03	2.05 ± 0.07	1.01 ± 0.04
C	9.66 ± 0.20	1.00 ± 0.02	7.32 ± 0.12	1.00 ± 0.02	3.56 ± 0.14	1.00 ± 0.04	2.40 ± 0.15	1.00 ± 0.05	2.02 ± 0.16	1.00 ± 0.08
Al	15.1 ± 0.3	1.56 ± 0.04	12.7 ± 0.3	1.74 ± 0.04	6.55 ± 0.14	1.83 ± 0.04	4.30 ± 0.10	1.83 ± 0.03	3.52 ± 0.10	1.74 ± 0.05
Cu	21.1 ± 0.4	2.18 ± 0.04	18.2 ± 0.4	2.49 ± 0.05	10.0 ± 0.03	2.80 ± 0.09	6.55 ± 0.20	2.83 ± 0.07	6.30 ± 0.20	3.12 ± 0.20
Pb	37.3 ± 4.0	3.86 ± 0.41	33.7 ± 1.8	4.60 ± 0.20	19.6 ± 1.0	5.50 ± 0.28	13.1 ± 0.7	5.46 ± 0.25	12.4 ± 1.2	6.14 ± 0.60

Table IV

Nu- cleus	σ_i^+		σ_i^-		$(A/19)^{2/3}$
	10^{-27} cm^2	Rel. un.	10^{-27} cm^2	Rel. un.	
H	1.3 ± 0.2	0.19 ± 0.03	1.3 ± 0.2	0.032 ± 0.005	—
Be	4.01 ± 0.23	0.58 ± 0.03	40.2 ± 1.6	0.98 ± 0.04	0.82
C	6.9 ± 0.4	1.00 ± 0.06	41.0 ± 2.0	1.00 ± 0.05	1.00
Al	13.1 ± 0.6	1.90 ± 0.09	73.0 ± 2.5	1.78 ± 0.06	1.71
Cu	21.0 ± 0.15	3.04 ± 0.22	115 ± 4	2.81 ± 0.10	3.01
Pb	29.7 ± 3.0	4.31 ± 0.43	220 ± 23	5.36 ± 0.56	6.66

the experimental points.

From integration of the angular distributions obtained, we have determined values of the total cross sections for production of charged pions from nuclei, which are listed in Table IV.

Figure V shows the total cross section for pion production as a function of atomic number A.

4. DISCUSSION OF RESULTS

Accurate calculations of the effects discussed in the present work involve a large quantity of laborious computations, the quantity of which is aggravated by the broad energy distribution of the incident neutron beam. Therefore the discussions given below at this point are intended to provide mainly a qualitative picture of the processes which determine the features of pion production in nuclei.

Nucleon energies of several hundred MeV correspond to wavelengths less than the size of the lightest nuclei. Therefore the starting point for all discussions of collisions of high energy nucleons with nuclei is the assumption that the primary nucleon interacts with the individual nucleons of the nucleus.

a) **Cross sections.** We will compare the cross sections obtained with those for production of charged pions in collisions of protons with nuclei.

From the point of view of charge symmetry, for nuclei with the same number of protons and neutrons the characteristics of production of pions of one sign in interaction of neutrons with the nuclei, if we do not take into account the Coulomb effect, should be the same as for production of pions of the opposite sign in interaction of protons with the same nuclei.

From the experimental point of view, the simplest nucleus with the same number of neutrons and protons is carbon. Unfortunately, data on pion production in

collisions of protons with nuclei, including carbon, have been obtained for the most part in a limited range of angles and therefore the existing information on total cross sections is very meager. The total cross section for production of π^+ mesons in collisions of protons with carbon at 660 MeV has been found by Meshkovskii et al.^[7]:

$$\sigma_{pC\pi^+} = (46.7 \pm 5.1) \cdot 10^{-27} \text{ cm}^2 \quad (1)$$

The cross section for π^- mesons measured by us and assigned to an energy of 660 MeV is

$$\sigma_{nC\pi^-} = (50.0 \pm 3.0) \cdot 10^{-27} \text{ cm}^2 \quad (2)$$

which is in good agreement with (1).

It is well known that the cross sections for production of charged and neutral pions in interaction of nucleons with nuclei which have isotopic spin $T = 0$ should be related by the expression^[8]

$$\sigma^+ + \sigma^- = 2\sigma^0. \quad (3)$$

The cross-section values obtained by us for production of charged pions from carbon, together with the results of Dunaitsev and Prokoshkin^[4] on production of π^0 mesons in collisions of protons with carbon, allow us to verify this relation. Substituting the appropriate cross sections into expression (3), we obtain for the left side the value $(47.9 \pm 2.0) \times 10^{-27} \text{ cm}^2$, and for the right side $(44.8 \pm 2.8) \times 10^{-27} \text{ cm}^2$. Thus, the results obtained are in completely satisfactory agreement with the prediction of the hypothesis of charge independence of nuclear forces.

Relation (3) is valid also for the differential cross sections. We will apply it to the existing data for 90° in the laboratory system. We have previously^[9] obtained the cross sections for production of π^0 mesons from nuclei for the same angle in the same neutron beam.

The corresponding differential cross sections are compared in Table V. It is evident from this table that, as for the total cross sections, relation (3) is satisfied for carbon, and for the remaining elements (except copper) the discrepancy exceeds the experimental errors. The comparison which we have made of cross sections obtained with the same neutron beam is a further confirmation of the principle of charge independence.

b) **Dependence on atomic number.** Most experiments show that the dependence of cross sections on atomic number (the A dependence) is close to the function $A^{2/3}$, i.e., they indicate a preferential role for surface production of pions.

The pion spectra analyzed in the present work are characterized by a broad distribution in energy. For the angles of 16 and 30° the main part of the spectra lies in the region of greatest change of the cross section of

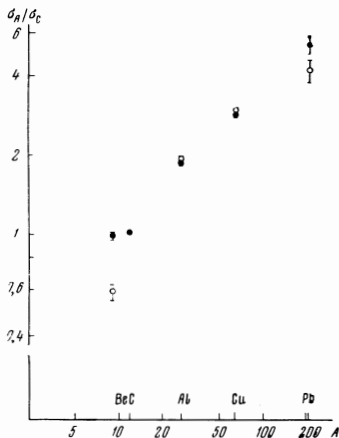


FIG. 5. Total cross sections for pion production in relative units, as a function of atomic number: \circ — total cross sections for π^+ meson production, \bullet — π^- mesons.

Table V

Nu- cleus	$(d\sigma/d\Omega)_{90^\circ}^{\pi^+} + (d\sigma/d\Omega)_{90^\circ}^{\pi^-}$, $10^{-27} \text{cm}^2/\text{sr}$	$2(d\sigma/d\Omega)_{90^\circ}^{\pi^0}$, $10^{-27} \text{cm}^2/\text{sr}$
Be	2.68 ± 0.009	2.04 ± 0.30
C	2.80 ± 0.45	2.62 ± 0.38
Al	5.45 ± 0.12	4.20 ± 0.60
Cu	7.60 ± 0.2	7.30 ± 1.0
Pb	14.80 ± 0.7	11.00 ± 1.6

interaction of pions with matter. The total cross sections for interaction of pions with nucleons increase by more than a factor of ten between pion energies of 50 and 200 MeV. Accordingly, the mean free paths for pions in nuclear matter should also change substantially in this energy interval. For an energy of 50 MeV the pion mean free path has a value comparable with the nuclear radius and therefore the internal nucleons in the nucleus should provide a definite contribution to the pion yield from the nucleus. For energies close to the resonance, because of the strong absorption of pions, the role of the internal nucleons should be insignificant. This difference should appear as a steeper A dependence for the low-energy parts of the spectra obtained than for pions with energies of 200 MeV.

In Table VI we have given in relative units the A dependences for π^- mesons with energies of 50 and 190 MeV for angles of 16 and 30°. A similar comparison can also be made for π^+ mesons. The data presented show a substantial difference in the behavior of the A dependence for different pion energy values. The A dependence for 50-MeV pions rises noticeably faster than $A^{2/3}$. On the other hand for 190-MeV pions the A dependence is expressed by a function considerably flatter than $A^{2/3}$.

The dependence of the A function on pion energy indicates that pion production originates over the entire volume of the nucleus. Therefore the form of the A dependence is determined to a considerable degree by the magnitude of pion absorption in nuclear matter. In the general case of a broad spectrum of the pions produced, the different portions of the spectrum give different contributions to the combined A dependence and the resulting function is determined to a large extent by the specific form of the pion spectrum.

From the spectra obtained for the five angles we can trace the change in the A dependence with angle for fixed values of pion energy. A comparison made in this way for several energy values does not reveal a noticeable difference between the dependences for the different angles. These results show that the dominant role in the variation of the A dependence with angle is played by the softening of the generated pion spectrum with increasing angle.

A number of features of nuclear structure appear in

Table VI

Nu- cleus	$(\frac{A}{12})^{2/3}$	$\left(\frac{d^2\sigma}{d\Omega dE} \right)^{\pi^-} / \left(\frac{d^2\sigma}{d\Omega dE} \right)_C^{\pi^-};$			
		$E=50 \text{ MeV}$		$E=190 \text{ MeV}$	
		16°	30°	16°	30°
Be	0.82	0.85 ± 0.08	1.00 ± 0.06	0.94 ± 0.06	1.04 ± 0.06
C	1.00	1.00 ± 0.08	1.00 ± 0.06	1.00 ± 0.07	1.00 ± 0.05
Al	1.71	2.42 ± 0.20	2.37 ± 0.12	1.37 ± 0.08	1.58 ± 0.09
Cu	3.01	4.20 ± 0.30	4.20 ± 0.17	2.03 ± 0.10	2.08 ± 0.11
Pb	6.66	7.90 ± 0.70	8.64 ± 0.85	2.63 ± 0.29	3.22 ± 0.25

comparison of the A dependence for different signs of pions. The difference in the relative change of the π^+ and π^- yields is due first of all to the fact that the numbers of protons and neutrons increase in a different way with change of atomic number. In order to exclude to a first approximation effects associated with this difference, we can compare the cross sections per proton for production of π^+ mesons and per neutron for π^- mesons. For this purpose it is necessary to divide the cross sections obtained (see Tables II and III) for π^+ mesons by Z, and for π^- mesons by the expression

$$F_n^- = (A - Z) + (\sigma_{np^-} / \sigma_{nn^-})Z, \quad (4)$$

whose second term allows us to exclude the relatively small fraction of π^- mesons produced from protons.

Comparison of the cross sections obtained in this way shows that at all angles the relative dependence of the π^- yield on atomic number lags behind the dependence of the π^+ yield. An exception is the element beryllium, for which the π^- yield exceeds the π^+ yield.

In order to explain the features of the A dependence found for pions of different sign we will consider two effects: nonuniformity of the distribution of protons and neutrons in the surface of the nucleus, and charge exchange of pions in the nucleus.

Until recently the more widely held point of view was that the radii of the distributions of protons and neutrons for light and medium nuclei coincide and for heavy nuclei the neutron distribution is only insignificantly more extended (by 0.1–0.2 F) than the proton distribution.^[10] However, a number of studies, for example, the recently published work of Burhop^[11] and some earlier work^[6,12], indicate a more substantial excess of neutrons in the nuclear surface, particularly for heavy nuclei.

Among the nuclei studied, we can expect a nonuniformity of the neutron and proton distributions to appear for Be and Pb. The properties of the Be nucleus are explained by use of the model described by Allison,^[13] according to which the Be⁹ nucleus consists of two α particles and a neutron. The α particles are distributed in space with the maximum density of nuclear matter, and the neutron, as is also the case in the deuteron, occurs practically outside the radius of action of nuclear forces. According to this model we should expect for beryllium a higher π^- -meson yield per neutron than for carbon. The ratio found of the total cross sections for production of π^- mesons in carbon and beryllium does not differ from unity:

$$\sigma_C^{\pi^-} / \sigma_{Be}^{\pi^-} = 1.02 \pm 0.05, \quad (5)$$

in spite of the fact that there is one more neutron in carbon.

The ratio of the π^+ -meson yields

$$\sigma_C^{\pi^+} / \sigma_{Be}^{\pi^+} = 1.72 \pm 0.14 \quad (6)$$

turns out to be larger than the ratio of the numbers of protons in these nuclei (1.5). This fact can be interpreted in the following way, that the "excess" neutron not only is responsible for a relative increase in the production of π^- mesons in the nucleus Be⁹ but also acts as a shell, hindering to a definite degree the emission from the nucleus of π^+ mesons formed from the protons in the nucleus. It is evident that for the lead nucleus the excess of neutrons in the surface should result in a large

absorption of π^+ mesons. However, in the case of bombardment of nuclei by neutrons, a second important process connected with pion charge exchange competes with the above effect.

Charge-exchange processes lead to a relative increase in the number of π^+ mesons. This is due to the fact that the number of additional π^+ mesons produced as the result of charge exchange of π^0 mesons exceeds the loss of original π^+ mesons from charge exchange of the latter, while for π^- mesons the analogous processes are considerably less important and even can lead to a net decrease in the π^- -meson yield. With increasing atomic number the charge-exchange effect begins to play an increasing role, leading to an observable lag in the relative π^- -meson yield. This lag is particularly noticeable for the nuclei Al and Cu. For lead the presence of the neutron shell should lead to the reverse effect—suppression of the yield of π^+ mesons, which are produced mainly from the protons of the nucleus. The observed decrease in the relative difference of π^+ and π^- yields in lead is apparently the result of the combined action of the effects discussed.

The validity of the picture discussed is confirmed by comparison of the results with the data on production of pions in collisions of protons with nuclei, expressed in a similar way per proton or per neutron. In the case of bombardment by protons, both of the effects discussed should act in the same direction, namely to result in a relatively larger yield of π^- mesons from the nucleus. Actually, experiments^[6,14] show that in bombardment by protons the π^- -meson yield outstrips the π^+ -meson yield with increasing atomic number. For nuclei from C to Cu, for which the neutron-shell effect is unimportant, it is observed that the A dependence is nearly the same for π^+ mesons produced in collisions of protons with nuclei and for π^- mesons produced from nuclei by neutrons.

A significant result is the disagreement for lead of the relative yields of π^+ mesons produced by protons and π^- mesons produced by neutrons, which in agreement with the picture discussed indicates the opposing action of the neutron-shell effect (which plays a large role for the case of lead).

c) Spectra. Table VII lists the average energies E_{av} in MeV for all the pion spectra studied, together with the data for hydrogen^[11] obtained in the same neutron beam as used in the present measurements.

A general regularity is the "softening" of the spectra with increasing atomic number of the nucleus. This feature appears more noticeably for the π^- -meson spectra. One of the factors determining this regularity apparently is the energy dependence of the cross section for interaction of pions with nuclear matter. The shape of the pion spectra in nucleon-nucleon collisions at 600 MeV^[11] is such that at practically all angles the high energy pions have high interaction cross sections. This circumstance leads to the fact that, of the fraction of pions produced in the internal nucleons of the nucleus, the low-energy pions give a greater contribution to the observed spectrum. The angular dependence of the effect discussed is an argument in favor of such a mechanism, which leads to softening of the spectra with increasing atomic number. The softening of the spectra should appear more strongly for small angles, since

Table VII

	Angle	H	Be	C	Al	Cu	Pb
π^+ mesons	16°	163±4	163±4	150±4	162±5	144±6	146±15
	30°	141±4	127±4	117±4	124±4	117±5	118±10
	60°	83±3	90±3	82±3	81±3	78±3	79±6
	90°	48±3	62±3	61±3	59±3	56±3	57±5
	123°	29±4	56±2	54±3	60±3	49±3	48±5
π^- mesons	16°	155±4	162±4	162±5	142±5	132±5	126±20
	30°	135±4	138±4	137±4	129±4	118±4	100±7
	60°	81±3	86±3	86±3	78±3	72±3	68±4
	90°	48±3	62±3	63±3	58±3	55±3	52±3
	123°	27±4	52±2	50±2	53±3	47±3	43±3

the "elementary" pion spectra at these angles encompass the energy region with the greatest changes in the pion interaction cross section. The data in Table VII show that for the π^- spectra at 16 and 30° a relatively greater softening of the spectra is observed.

Another cause leading to a relatively larger softening of the spectra at small angles may be the scattering of pions in the nuclei, as the result of which low-energy pions produced at large angles are added to the pion spectra at small angles.

A further feature of the behavior of the pion spectra from nuclei shows up when they are compared with the spectra from hydrogen for various angles. It is evident from Table VII that up to 60° the pion spectra from hydrogen are "harder" than most of the pion spectra from complex nuclei, in correspondence with the action of the processes discussed above. For 90° and particularly 120° the reverse picture exists. The average energies of the spectra from nuclei turn out to be shifted toward higher energies in comparison with those from hydrogen.

Scattering of pions in the nucleus may be the main cause of this phenomenon. We can estimate the role of scattering by comparing the cross sections for production^[11] and scattering of pions in free collisions.^[15] Such an estimate shows that under conditions where the pion ranges are comparable with or less than the nuclear dimensions, the greater part of the pions emitted from the nucleus at large angles may be pions which have undergone a scattering in the nucleus.

Another important factor determining a number of features of pion spectra from nuclei is the intranuclear motion of the nucleons.

As a result of the intranuclear motion the pion spectra turn out to extend to energies considerably larger than in nucleon-nucleon collisions. The π^- -meson spectrum from copper at 60°, shown in Fig. 3 as an example, exhibits distinctly a meson "tail" extending up to 300 MeV, whereas the maximum energy of a pion produced in a collision with a 600-MeV neutron with a nucleon at rest is 156 MeV. A kinematic calculation shows that the entire observed pion tail can be explained by the intranuclear motion.

Let us now consider separately the features of the π^+ and π^- spectra obtained. It is obvious that the relative difference in the π^+ and π^- spectra should be due in the first place to the difference in the primary mechanism of pion production and, secondly, to nuclear structure effects which can affect the π^+ and π^- spectra in different ways.

For light nuclei (Be, C) where nuclear structure effects do not play a large role, the features of the primary mechanism appear in the difference of the π^+ and π^-

spectra. The shape of the π^- spectra obtained from Be and C in the energy region above the peak turns out to be similar to the shape of the π^- spectrum from pd collisions^[16] and correspondingly the observed π^+ spectra in this region are close to the π^+ spectra from pd collisions.

The spectra of π^+ mesons from carbon (the nucleus most suitable for comparison, because of the identical number of protons and neutrons) turn out (Table VII) to be somewhat softer than the π^- -meson spectra, in correspondence with the features of pion production in nn and np collisions.^[1] However, this difference is not great and we should therefore expect that the relative role of effects due to nuclear structure will appear to a large degree, in the difference of π^+ and π^- spectra, particularly for nuclei with large atomic numbers. The experimental results confirm this suggestion. It is evident from Table VII that for the nuclei Al, Cu, and Pb the π^- -meson spectra turn out to be somewhat softer than the π^+ -meson spectra.

Consideration of a number of "nuclear" effects associated with such phenomena as an excess of neutrons in the nuclear surface, production of pions by nucleons which have undergone elastic scattering in the nucleus, difference in the momentum distributions of protons and neutrons, and the influence of the Pauli principle, shows that these effects should facilitate the relative softening of the π^+ spectra in n + nucleus collisions, and act in the opposite direction in p + nucleus collisions. Therefore these effects cannot explain the observed softer spectra of π^- mesons in heavy nuclei.

Let us consider now the effect due to charge exchange of pions. The spectra of pions which have undergone charge exchange obviously should differ from the spectra of the primary pions. The change of the primary pion spectrum in charge exchange depends on its shape and its position with respect to the excitation function for charge exchange. Therefore, depending on the incident nucleon energy and the angle of measurement, the spectrum of charge-exchange pions can be shifted in different directions.

For the present measurements we can use our earlier data^[1] to estimate the deformation of the spectra used as initial spectra. The results of this estimate show that the greatest change in the spectra should be expected for angles of 30 and 60°. According to this estimate the energies corresponding to the peaks of the spectra of charge-exchanged pions for these angles turn out to be shifted toward higher energies by 40–45 MeV. For angles of 16 and 90° this shift is smaller, and for 123° it is practically absent. We have seen that the charge-exchange effect plays an important role in production of π^+ mesons. Charge-exchanged pions should comprise an appreciable part of the yield. Thus, charge exchange should lead in the present measurements to a relatively large content of high energy pions in the π^+ -meson spectra.

A second factor, which also acts in the direction of the observed softening of the π^- spectra relative to the π^+ spectra, may be associated with the Coulomb field of the nuclei. The small magnitude of the Coulomb field of nuclei, relative to the kinetic energy of the pions, and the comparatively slow nature of its variation, relative to other parameters, allow us to estimate the action of

the Coulomb field, proceeding from simple classical considerations.^[17] It is assumed that the action of the Coulomb field reduces to acceleration of π^+ mesons emitted from the nucleus and slowing down of π^- mesons. To a rough approximation this effect is quantitatively represented in the following way, that a π^+ meson produced in a nucleus with an initial energy E_0 has an experimentally observed energy, on emission from the nucleus, $E = E_0 + V_{\text{Coul}}$, and a π^- meson correspondingly $E = E_0 - V_{\text{Coul}}$, where E_{Coul} is the energy of the Coulomb barrier. As a result the shift between the π^+ and π^- spectra can reach an appreciable value for heavy nuclei (~ 36 MeV for lead). A quantitative estimate of the magnitude of the Coulomb effect on the basis of comparison of the spectra obtained is hindered by the simultaneous effect on the spectra of the factors enumerated above.

On the high-energy side the Coulomb effect should appear more distinctly in our measurements. This circumstance is due, first of all, to the high accuracy of the picture discussed of the shift of the spectra for high energy pions. In the second place, the action of most of the nuclear effects appears mainly in the deformation of the low-energy parts of the spectra.

Comparison of the spectra shows a systematic difference of the upper limits of the π^+ and π^- spectra for Al, Cu, and Pb. For practically all angles the π^+ spectra extend further toward high energies than the π^- spectra. This phenomenon has two features: 1) an increase of the relative shift of the spectra with increasing atomic number, and 2) constancy within experimental error of the absolute value of this shift for all angles. Both of these features correspond to a spectral shift effect resulting from the action of the Coulomb potential. For a quantitative evaluation of the Coulomb effect we have shown in Fig. 6 in relative units the π^+ and π^- spectra from lead for the angles 90 and 123°, displaced in different directions along the energy axis by 18 MeV. The figure shows that the upper limits of the π^+ and π^- spectra agree within experimental error after correction for the Coulomb effect.

In concluding this section we will attempt to characterize in the following way the distinguishing features of the spectra of π^+ and π^- mesons produced in collisions of protons and neutrons of energy 600–700 MeV with nuclei.

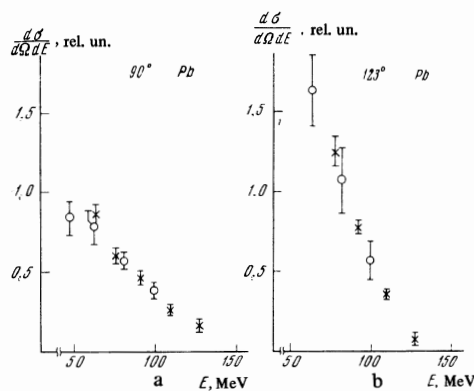


FIG. 6. High-energy portions of pion spectra for 90° (a) and 123° (b) from Pb in relative units after shifting by the value of the Coulomb potential. \circ — π^+ mesons, \times — π^- mesons.

For light nuclei we can assume that the pion spectra are close to the spectra of pions produced in nucleon-nucleon collisions. With increase of atomic number the nature of the spectra is determined mainly by nuclear structure features.

Analysis of the role of various nuclear effects shows that the spectra of π^- mesons produced in p + nucleus collisions for heavy nuclei should be the lowest in energy. The spectra of π^- mesons from n + nucleus collisions also should be characterized by a shift toward low energies which increases with atomic number. However, this property should appear to a lesser degree than for π^- mesons from p + nucleus collisions. Softening of the π^+ spectra from n + nucleus collisions should appear still less. Finally, the spectra of π^+ mesons from p + nucleus collisions should have the highest energies and should depend only weakly on atomic number.

For small angles we should expect a substantially greater difference between the π^+ and π^- spectra from p + nucleus collisions, compared to the difference of the π^+ and π^- spectra in n + nucleus collisions.

d) Angular distributions. The pion angular distributions in the c.m.s. are shown in Fig. 4. Comparison with the angular distributions for hydrogen^[1] which are shown in the same figure shows a substantial role for nuclear structure even for the very lightest nuclei studied (Be and C), for which a significant asymmetry of the pion angular distribution with respect to 90° is observed. This result is evidence that the angular distributions are a characteristic very sensitive to secondary processes in the nucleus.

For a quantitative evaluation of the asymmetry of the angular distribution we will introduce the coefficient η , used also by Mal'tsev and Prokoshkin,^[18]

$$\eta = [f(170^\circ) - f(10^\circ)] / f(90^\circ), \quad (7)$$

where $f(10^\circ)$, $f(90^\circ)$, and $f(170^\circ)$ are the values of the angular distribution function of the pions for the corresponding angles in the c.m.s. In Table VIII we have listed the coefficients η for all nuclei studied, together with the values for hydrogen.

The η values presented show a significant amount of angular asymmetry, increasing with increasing atomic number of the nucleus.

The observed asymmetry is due to the action of a group of secondary processes in the nucleus. The most important role here is played by pion scattering. This leads to an increase in the number of pions emitted at large angles in the laboratory system, which correspondingly should lead to a dominance of pion emission in the backward hemisphere in the c.m.s.

The increase in asymmetry with increasing atomic number is produced not only by pion scattering processes but also by absorption of nucleons in the nucleus.

Comparison of the asymmetry of the angular distributions of pions of different sign permits us, as in the preceding sections, to observe certain features of the mani-

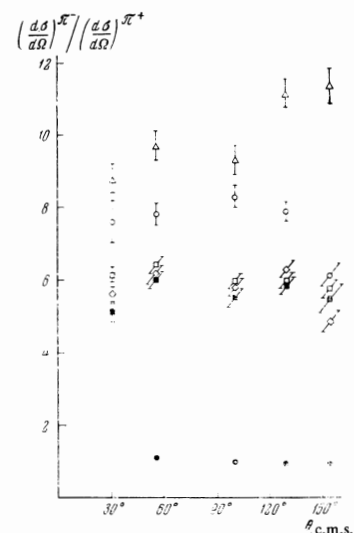


FIG. 7. Ratio of yields of π^- and π^+ mesons as a function of angle in the c.m.s. ● — H, Δ — Be, \square — C, \blacksquare — Al, \diamond — Cu, \circ — Pb.

festation of the nuclear structure.

It is evident from Table VIII that for Be, the lightest of the nuclei studied, the asymmetry of π^+ mesons is less than that of π^- mesons. This difference can be explained by the difference in the initial mechanism of π^+ and π^- production.

For the nuclei with large atomic weights the data in the table show the reverse picture. The angular distributions of π^+ mesons in Cu and Pb are characterized by a larger asymmetry than the angular distributions of π^- mesons. For heavy nuclei the features of the initial mechanism of pion production are equalized by secondary nuclear processes. One of these, as we have discussed above, is pion charge exchange, which affects π^+ -meson production in an important way. The process of scattering with charge exchange, like ordinary scattering, should lead to appearance of an asymmetry with respect to 90° c.m.s. Furthermore, the angular distribution of charge-exchanged pions in the energy region from 40 to 170 MeV is itself characterized by a large asymmetry.^[19]

e) Ratio of yields of pions of different sign. Some features of pion production processes in nuclei appear more clearly on comparison of the ratios of yields of pions of different sign.

The ratios of the differential cross section $(d\sigma/d\Omega)^{\pi^-}$ to $(d\sigma/d\Omega)^{\pi^+}$ for the nuclei studied are shown in Fig. 7.

Charge symmetry permits us to compare directly the ratios found for carbon in the present work with those obtained by proton bombardment of carbon. The results of such a comparison are shown in Fig. 8. It can be seen from this figure that in the angular region up to 60° the experimental results for protons and neutrons agree within experimental error. A theoretical calculation employing the Monte Carlo method^[18] predicts a decrease in the pion yield ratio in carbon at large angles. Figure 8 shows that within the accuracy of the

Table VIII

Nu- cleus	H	Be	C	Al	Cu	Pb
η^{π^+}	$0,08 \pm 0,20$	$2,5 \pm 0,4$	$3,2 \pm 0,5$	$4,0 \pm 0,6$	$7,9 \pm 1,1$	$9,8 \pm 1,1$
η^{π^-}	$-0,19 \pm 0,20$	$3,5 \pm 0,5$	$3,1 \pm 0,5$	$3,3 \pm 0,3$	$4,7 \pm 0,6$	$5,1 \pm 0,6$

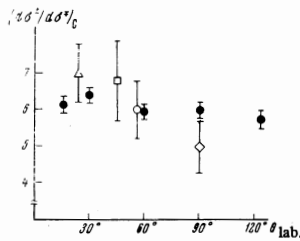


FIG. 8. Ratio of yields of charged pions from carbon as a function of angle in the laboratory system. ● — $d\sigma^-/d\sigma^+$ — neutrons, present work; Δ — $d\sigma^+/d\sigma^-$ — protons, 660 MeV [21]; \square — $d\sigma^+/d\sigma^-$ — protons, 660 MeV [20]; \circ — $d\sigma^+/d\sigma^-$ — protons, 660 MeV [16]; \diamond — $d\sigma^+/d\sigma^-$ — protons, 660 MeV [22].

present measurements the ratio $(d\sigma/d\Omega)\pi^-/(d\sigma/d\Omega)\pi^+$ in carbon does not change with angle. Qualitatively, however, the prediction of a decrease in ratio with increasing angle finds confirmation in our measurements for heavy elements, where the role of secondary processes should increase. Thus, for copper and especially for lead there is a noticeable decrease in the ratio $(d\sigma/d\Omega)\pi^-/(d\sigma/d\Omega)\pi^+$ for an angle of 150° (Fig. 7).

Another feature of the results obtained, which is noticeable in Fig. 7, is the fact that for all the nuclei studied a systematic reduction of the ratio $(d\sigma/d\Omega)\pi^-/(d\sigma/d\Omega)\pi^+$ is observed at 30° . This reduction apparently is the consequence of the production of pions by scattered nucleons. In the case of bombardment by neutrons, pion production by scattered nucleons increases the relative fraction of π^+ mesons. This occurs as a result of the fact that neutrons which have undergone charge exchange in the scattering process or recoil protons produce π^+ mesons, in contrast to the initial neutrons, in a considerably more intense reaction. At small angles two factors act to increase this effect. The first is associated with the fact that the probability of neutron charge exchange increases with decreasing angle. The second factor is the increase of the pion production cross section with increasing nucleon energy at small angles.

Comparison of the yield ratios of pions produced in nuclei with an excess of neutrons, on bombardment by protons and neutrons, indicates a preferential concentration of the excess neutrons at the surface of the nucleus. In fact, the neutron-shell effect should affect the pion yield ratios in opposite directions for bombardment by protons and by neutrons. This pattern appears distinctly in comparison of the ratios per nucleon for the nuclei Be, C, and Pb. Thus, according to the data of several authors,^[16,14,20,21] the ratio $(d\sigma/d\Omega)\pi^+/(d\sigma/d\Omega)\pi^-$ obtained on bombardment by protons decreases by a factor of 1.1–1.2 on transition from carbon to beryllium, while for our measurements with neutrons $\sigma^{\pi^+}/\sigma^{\pi^-}$, on the other hand, increases by a factor of 1.38 ± 0.006 . In the transition from carbon to lead, in both cases the yield ratio of pions of corresponding signs should decrease as the result of charge exchange, but in the case of bombardment by neutrons this decrease should be substantially smaller because of the neutron “shell.” Comparison of our results with experimental data for proton bombardment confirms this fact.

Another manifestation of the neutron-shell effect in our measurements may be the occurrence of some in-

crease in the σ^-/σ^+ ratio (per neutron) in transition from copper to lead.

5. CONCLUSION

All of the data obtained are interpreted on the assumption of the primary, elementary nature of the interactions of the nucleons and generated pions in the nucleus. The characteristics of the pion production processes in light nuclei clearly display the features of the initial mechanism of pion production. With an increase of atomic number, the characteristics are determined mainly by nuclear structure features.

Comparison of the total and differential cross sections obtained in the present work for production of pions on bombardment of nuclei by neutrons with data for the corresponding processes in proton beams provides evidence for the validity of the principle of charge independence of nuclear forces for the pion production processes studied.

A decisive role for the pion production processes is played by absorption and scattering of the pions in nuclear matter. The importance of this factor appears primarily in the dependence of the pion yield on atomic number. Analysis of the atomic number dependences found shows that the changes in this characteristic for different incident nucleon energies and different angles of observation can be explained qualitatively by the energy dependence of the cross section for interaction of pions with nuclear matter. The same cause leads to a softening of the pion spectra with increasing atomic number.

Analysis of the pion spectra and angular distributions allows us to conclude that scattering is important even in the lightest nuclei. Pion scattering processes are the main cause of the observed asymmetry in the angular distributions. The asymmetry increases with increasing atomic number. This circumstance is assisted by the absorption of nucleons in the nucleus. Analysis of the pion spectra provides evidence that the intranuclear motion of the nucleons substantially affects the shape of the spectra.

The measurements performed, in which production of pions of both signs was studied under the same conditions, allow us, by comparing the characteristics for π^+ and π^- mesons, to study the role of such effects as charge exchange of pions and nucleons, the effect of the nuclear Coulomb potential, and the difference in the distributions of neutrons and protons within the nucleus. Pion charge exchange has a considerable influence on the relatively low-intensity process of π^+ -meson production, and leads to a noticeable difference in the dependence of π^+ and π^- yield on atomic number. The charge-exchange effect is responsible for: the difference in the π^+ and π^- spectra, the difference in the asymmetries for π^+ and π^- mesons, the decrease in the π^- and π^+ yield ratios at large angles, and the difference in the yield ratios of pions of different sign produced in nuclei from the corresponding ratios for the elementary cross sections. The relative shift of the upper limits of the π^+ and π^- spectra for heavy nuclei can be explained by the action of the nuclear Coulomb barrier.

Analysis of certain features appearing in the relative difference of the π^+ and π^- yield dependence on atomic

number and measurement angle permits us to conclude that there is a preferential concentration of the excess neutrons at the nuclear surface.

The author takes this occasion to express his deep gratitude to V. P. Dzhelepov, corresponding member of the Academy of Sciences of the U.S.S.R., without whose support and constant attention the present work could not have been completed. The author also thanks V. S. Kiselev and V. B. Flyagin for assistance in the measurements and discussion.

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Translated by C. S. Robinson

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