ELECTRIC RESISTANCE OF THIN SINGLE-CRYSTAL ALUMINUM PLATES

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Highly sensitive equipment was used to measure the temperature dependence of the resistance of thin single-crystal plates of aluminum in the temperature interval 2–20°K. The results of the measurements offer evidence of an appreciable dependence of the electron-phonon mean free path on the sample size. The experimental data are analyzed within the framework of the Azbel' and Gurzhi theory of the electric conductivity of thin samples, based on the assumption that the efficiency of the normal processes of scattering of "glancing" electrons is increased by the influence of the boundaries of the sample. Use is made here of the temperature dependence of the resistance of the bulky sample and of the Pytte estimate of the contributions of the normal processes and of the Umklapp processes (U-processes) to the resistance of aluminum. Good agreement is observed between the experiment and theory. Observation of anomalies of the same character in the temperature dependence of the resistance of bulky and thin samples is interpreted as a manifestation of the exponential part of the contribution of the U-processes, due to the existence of a lower limit of the value of the wave vector of the phonons taking part in these processes.

INTRODUCTION

In recent experimental investigations connected with the study of the behavior of the electric conductivity of normal metals at low temperatures, the explanation of some of the results calls for a deep understanding of the role of different mechanisms of electron scattering in metals. These results include: observation of singularities in the temperature dependence of the electric resistance of bulky single crystals of high purity of elements of the third group (aluminum [1] and gallium [2]), the change in the exponent in the temperature dependence of the resistance of various samples of one and the same metal [3,4], violation of the Bloch-Gruneisen law for the electric conductivity (which, generally speaking, is more logical than satisfaction of this law, since the latter has been derived under rather crude assumptions) [3], unusual deviations from the Matthiesen rule [5], and a temperature dependence of the size effect in thin samples [4,6,7]. To explain these phenomena, use is made of theoretical investigations of the role of the processes with Umklapp [8,9], normal processes [9], interference effects due to oscillations of the impurity ion [10], dragging effect, etc.

By selecting the metal and the temperature interval it is possible to weaken greatly the role of the temperature-dependent mechanisms of dissipation of the electron quasi-momentum, whereas the probability of normal scattering processes remains quite high (at sufficiently low temperatures, the frequency of processes with Umklapp (U-processes) decreases exponentially with temperature, and the frequency of the normal processes is proportional to $(T/\Theta)^5$). The normal processes can then make a contribution to the resistance when the corresponding mean free path l_{ep}^{N} becomes comparable, roughly speaking, with some statistical parameter (the electron-impurity mean free path l_{ei}, the dimension of the sample)[9].

We have previously investigated bulky samples of aluminum [1], the temperature dependence of the resistivity of which reveals a singularity recalling that predicted by Gurzhi for the aforementioned condition [9]. In spite of the fact that the singularity for aluminum is not very large and is observed in a sufficiently narrow temperature interval, we have connected it with the role of the normal collisions in a weakly anisotropic metal. However, the recently published paper by Pytte [11], who calculated the electron phonon interaction in aluminum, admits, in our opinion, of another interpretation of the observed effect, as will be discussed below.

We present here the results of an investigation of thin single-crystal plates of aluminum of high purity at temperatures 2–20°K. The results of the measurements are compared with the data obtained with bulky single-crystal aluminum, with results of other authors, and with the theory.

EXPERIMENTAL PROCEDURE

To investigate both bulky and thin samples, we used apparatus having a voltage sensitivity 5×10^{-12} V, as mentioned earlier in [1]. The measurement scheme, compared with that proposed by Clark and used by Newbower and Neighbor [2], have higher input resistance (10^{-3} as against 10^{-8} ohm), although it has lower sensitivity. In addition, it is less sensitive to magnetic stray pickup. Its advantage is the possibility of working with high-resistance samples. The experimental setup is shown in Fig. 1. Its operating principle was borrowed from de Vrommen and Van Baarle [12] and makes use of a superconducting modulator. The modulator is a loop of tantalum wire 0.05 mm in diameter, with a resistance ≈0.3 ohm in the normal state. A distinguishing feature of the setup is the use of an electronic modulating-current shaper, in place of the electromechanical one used in [12]. (The waveform of the modulating signal was the same in both cases). This makes it possible to increase the frequency of the modulator superconductivity destruction without greatly increasing the parasitic pickup, thus improving the efficiency of the transformers, the overall transfer coefficient of which is $\sim 10^5$. Two transformers are used. One is in helium (shown in Figs. 1 and 2) and the other is at room temperature (it is contained in the SA block of Fig. 1). The measurements were performed by a null method. The standards were pieces of copper or brass wire, the resistance of which at the working temperature was calibrated with an RZ08 potentiometer ($R_N \approx 10^{-6}-10^{-5}$ ohm).

The measuring current through the samples was 0.5–1 A. During the course of the experiment, we measured simultaneously the currents through the sample (I) and through the standard resistance (I_N) with the resistance such that the conditions $R\Delta I \ll \Delta V$ at $R_N\Delta I_N \ll \Delta V$ were always satisfied (ΔV is the maximum voltage density of the sample, and R and R_N are respectively the resistances of the investigated sample and of the standard. The use of an RZ08 instrument with sensitivity 2×10^{-8} V as the current-meter on standard resistance coils (not shown in Fig. 1), connected in series in the circuits of I and I_N. The absolute error in the measurement of the resistance $\Delta\delta_T$ of the bulky samples (where $\delta_T = R_T/R_{300}$ is the ratio of the resistance measured at the temperature of the experiment to the resistance measured at room temperature $T \approx 300°K$) was $\approx 10^{-6}$ ($\sim$1%) in the helium temperature region, and $\approx 1 \times 10^{-5}$ ($\sim$4%) above 15°K. For thin samples $\Delta\delta_{4.2} = (0.01-0.04) \times 10^{-4}$ (0.1–0.4%) and $\Delta\delta_{15} \approx (0.01-0.1) \times 10^{-4}$ (0.1–0.5%). We took into account here also the temperature measurement error ($\sim$0.1°).

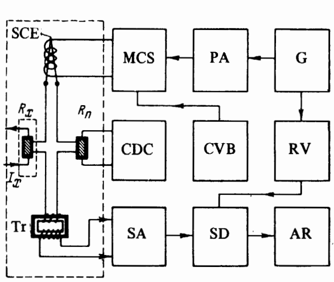


FIG 1.

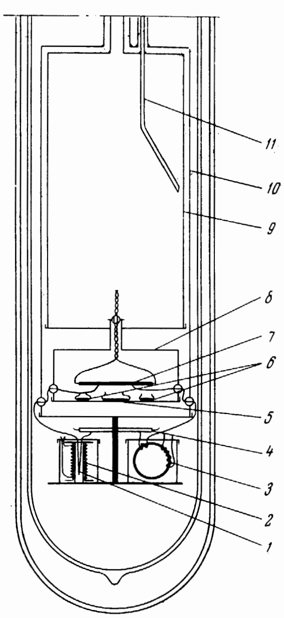


FIG 2.

FIG. 1. Block diagram of measuring setup: SCE–superconducting element, Tr–transformer, MCS–modulation current shaper, PA–power amplifier, CVB–block of constant reference voltages, G–generator, RV–reference voltage block, SA–selective amplifier, 540 Hz, SD–synchronous detector, AR–automatic recorder, CDC–calibration direct current block.

FIG. 2. Cryostat for obtaining intermediate temperatures: 1–modulator, 2–solenoid, 3–transformer, 4–standard resistance, 5–heater, 6–temperature pickups, 7–sample, 8–ampoule, 9–internal chamber, 10–external chamber, 11–overflow tube of internal chamber.

The recording apparatus comprised a low-noise preamplifier with noise voltage 5×10^{-8} V and commercial instruments (amplifier, synchronous detector, automatic recorder).

The measurements were made in the special cryostat in which intermediate temperatures could be obtained (Fig. 2). A copper ampoule with the sample, pickups for temperature reading and stabilization, and heater to equalize the temperature gradients was filled with gaseous helium and was soldered to the lower flange of an internal chamber, which was separated from the volume with the liquid by means of a vacuum cavity. When working at helium temperatures, the internal chamber was filled with liquid helium, the vapor of which was pumped off. To obtain higher temperatures, a heater was used. The temperature was stabilized with an electronic stabilizer capable of maintaining the temperature, provided a suitable pickup is available, accurate to 10^{-4} deg. We used a carbon pickup with an annealing temperature 790°C[1)]. The temperature was read in the entire working interval 2–20°K with a GaAs semiconducting pickup with almost linear characteristic (sensitivity 1 mV/deg and reading accuracy 0.1 deg). The measurement data near 20°K, read with the aid of the pickup were verified directly in liquid hydrogen. The external and internal chambers of the cryostat were dismountable, with indium seals between them. The modulator, solenoid, transformer, and standard resistance were in the external volume at a fixed temperature ($\approx$4°K), maintained by pumping and stabilizing the helium vapor pressure.

SAMPLES

The initial material was a large single-crystal aluminum block obtained by zone melting. Bulk samples measuring $5 \times 5 \times 30$ mm were cut from it by the electric-spark method and were etched. The samples had $R_{300}/R_{4.2} = 15{,}600$ (the electron-impurity mean free path $l_{ei} \sim 0.5$ mm); annealing for a day at T = 500°C did not increase this ratio.

The stock pieces for the preparation of thin samples, having approximately the same orientation as the bulk samples, were cut from the same block. By etching and chemical polishing they were reduced to the required dimensions: plate width 1–1.5 mm, length 10–15 mm, thickness 15, 18, 24, 35, and 80 μ. The thickness was determined optically with approximate accuracy 20%. The electrical contacts (copper wires of diameter smaller than 0.01 mm) were welded with the aid of a ruby laser. Measures were taken to eliminate the stresses that could appear at low temperatures. This is between contacts with several millimeters (up to

1)The pickup was kindly supplied by N. N. Mikhailov (Institute of Physics Problems, USSR Academy of Sciences), to whom the authors are most grateful.

5 mm). The contacts were welded to the bulky sample by a spark method.

MEASUREMENT RESULTS

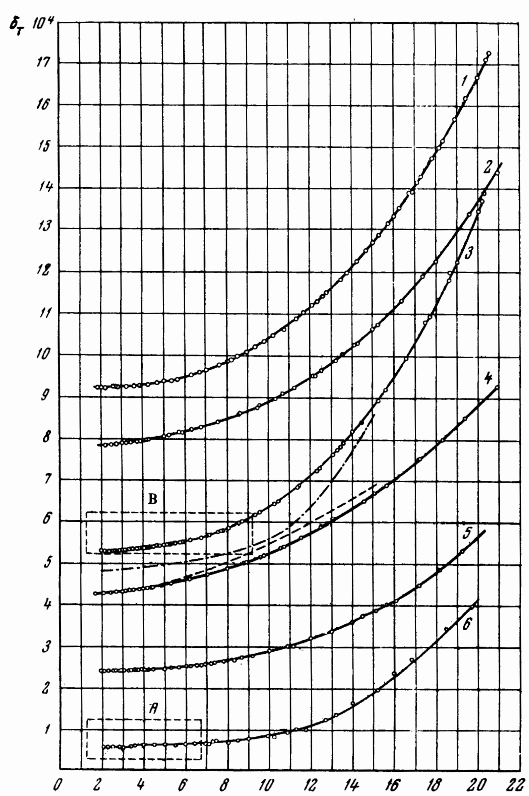
Figure 3 shows the results of an investigation of the temperature dependence of the electric resistance of 10 single-crystal aluminum plates in the temperature region 2–20°K. The ordinates represent the ratio of the sample of resistance at the experimental temperature to the resistance at room temperature (°K): $\delta_T = R_T/R_{300} = \rho_T/\rho_{300}$ (ρ—resistivity). We see that curve 3, corresponding to a sample d = 24 μ thick, falls somewhat outside the general picture—it is deeper than the remaining curves. The reasons for this are not quite clear, although such a difference recalls the deviation from the Matthiesen rule observed by other workers in a number of metals (for aluminum, for example, in[5]). In addition, structural defects of various types can appear in the samples during their preparation. The deformation of the sample is demonstrated by the calculated values of the mean free path corresponding to the residual resistance, for different samples. It can be assumed that the sample 24 μ thick is the most perfect (it was actually shorter than all other samples, making it most convenient to manipulate it). It is precisely on the resistance curve of this sample that the singularity is distinctly observed in the region of temperatures below 5°K (Fig. 3B).

The values of the relative resistivity δ_T at 20°K in bulky samples from different blocks differ from one another and from the data of other authors by approximately 30%. This discrepancy is evidence of deviation from the Matthiesen rule. Thus, the bulky sample should come from the same batch as the thin samples, for in the analysis of the results one makes use of information on the temperature dependence of the bulky-sample resistance. The corresponding data for the latter are shown in Fig. 3 (curve 6) and in Fig. 3A.

DISCUSSION OF MEASUREMENT RESULTS

Bulky sample. As noted above, the singularity observed at helium temperatures was discussed by us earlier[1] within the framework of the Gurzhi theory. However, the quantitative theory of Pytte[11] of electron-phonon interaction in aluminum makes it possible, in our opinion, to propose a different interpretation, which affords a possibility of simultaneously discussing the experimental results for the thin samples too. Generally speaking, the experimental data on the temperature dependence of the electric resistance of very pure bulky single crystals of aluminum at the very lowest temperatures (below 20°K) are practically nonexistent. In[13] the temperature dependence above 4.2°K was only estimated (especially above 8°K, where there are few experimental points). The reconstruction of the behavior of an ideal resistance from measurements on thin samples, as was done in[14], is not justified, since the mean free paths and their behavior in bulky and thin samples are essentially different, both from theoretical premises and from experimental results (see below).

As is well known, the lattice resistivity ρ_L (disregarding electron-electron interaction) was calculated for a normal metal with a closed Fermi surface at low temperatures (the Bloch-Gruneisen T^5 law) with allowance for only normal scattering processes, assuming the Debye model for the phonons. Using the experimental data on the phonon spectrum and its temperature dependence in aluminum, and also making several assumptions concerning the distortion of the Fermi sphere near the boundaries of the Brillouin zone, Pytte obtained the concrete form of the matrix elements of the electron-phonon interaction. It turns



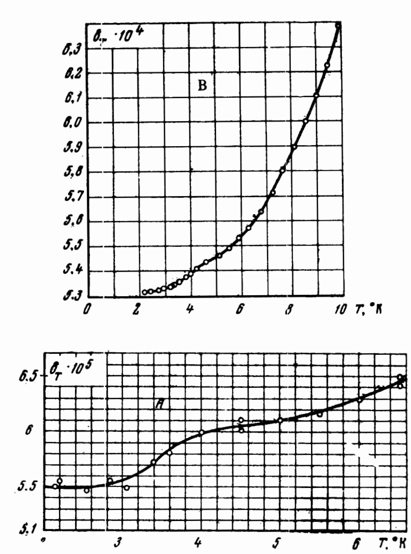


FIG. 3. Temperature dependence of the electric resistance of single crystals of aluminum: 1–d = 15μ, 2–d = 18μ, 3–d = 24μ, 4–d = 35μ, 5–d = 80μ, 6–bulky sample. A and B–sections of curves 6 and 3 in enlarged scale. For the d = 35μ sample, the figure shows the theoretical curves calculated in accordance with the theory of Azbel' and Gurzhi (dash) and under the assumption that $l^d = l^\infty$ (dash-dot).

out that in aluminum the contribution of the normal processes to the resistance in the temperature region where $T/\Theta \le 0.15$ (Θ—characteristic temperature), should be less than 1%. The greater part of the resistance at low temperatures, in a sufficiently wide interval, is due to Umklapp processes, which give $\rho_U \propto T^4$. It is interesting that this takes place even for a spherical Fermi surface, provided the latter is sufficiently large (as in the case of aluminum). Thus, the Peierls limitation[15], which is connected with the existence of a certain smallest value of the wave vector of the phonon capable of taking part in the Umklapp process, turns out to be not so stringent even at sufficiently low temperature.

Thus, the Pytte result reduces to the following. The lattice resistance of the bulky sample of aluminum takes into account both the contribution ρ_N from the normal electron scattering processes, and the contribution ρ_U from processes with Umklapp. The character of the temperature dependence of the resistance at low temperatures is determined essentially by the latter. At not too low temperatures (when the Peierls limitation is of no significance), $\rho_U \propto T^3$, and at very low temperatures it is possible to have $\rho_U \propto \exp(-\Theta/\beta T)$[15] and $\rho_U \propto T^4$ in the intermediate temperature region. A deviation from this picture can be caused by the presence of a term $\rho_N \propto T^5$.

Our experimental results for a bulky sample are in fair agreement with these concepts. In the region 4–15 K they can be described by the following empirical relation

$$\delta_T = \delta_0 + \alpha T^n, \tag{1}$$

where $\delta_0 = 6.319 \times 10^{-5}$, $\alpha = 2.294 \times 10^{-9}$ [$\deg^{-n}$], and $n = 4.05$.

The steeper section for $T < 5^\circ$K, can be attributed to the existence of a lower limit for q (wave vector of the phonon) in the U-processes[2)]. Pytte did not estimate the temperature region where this should take place, since this requires detailed information on the band structure and on the deformation of the Fermi surface near the boundaries of the Brillouin zone.

In the temperature region above 15°K, the section of the experimental curve is not described by the empirical relation (1); the exponent decreases here to 3.27. A similar exponent (smaller than 4) was observed also in this temperature interval in[3,4]. This agrees qualitatively with Pytte's calculations.

Thin samples. Olsen[6] called attention to the dependence of the size effect on the temperature (a similar phenomenon was subsequently observed also by others[7,4,14]. He was the first to advance the idea of the influence of the surface on the efficiency of the electron-phonon collisions. A clarification of the premise calls for measurements of the electric resistance of thin samples in a wide range of temperatures, especially below 20°K. It is desirable here to have thin and bulky samples with the same history, a factor taken into account in performing the measurements.

2) Since the temperature interval corresponding to the steep part of the curve is small, it is difficult to distinguish the exponential curve from the T^5 curve.

An examination of the curves of Fig. 3 reveals the following. The residual resistance is reached at lower temperatures in thin samples than in bulky ones. Obviously, this is possible if the corresponding volume mean free paths differ at comparable temperatures, the mean free path being smaller in thin samples. It is natural to assume that its temperature dependence is smoother than that in bulky samples. When speaking of the volume mean free path in thin samples, we imply at all times, of course, a certain effective mean path. To compare the indicated free path lengths, it is convenient to subdivide them in accordance with the sample size, using the Fuchs theory[16], which is sufficiently well confirmed by a large number of experimental investigations[6,7,14], including ours, for the temperature region in which the electron-phonon interaction can be disregarded, i.e., for the residual-resistance region (Fig. 4). The Fuchs formula should be used in this case in a somewhat different form than customarily:

$$\delta_T{}^d = \frac{A}{d} k f(k), \tag{2}$$

where $A = \delta_T^\infty \bar{l}_T^\infty$ ($\bar{l}_T$ is the mean free path at the temperature of the experiment; the indices d and ∞ pertain respectively to thin and bulky samples); $k = d/\bar{l}_T^d$, $k \ll 1$;

$$f(k) = \left[1 + \frac{3}{4}k\left(1 - \frac{k^2}{12}\right)\int_k^\infty \frac{e^{-\xi}}{\xi}d\xi - \frac{3}{8k}(1 - e^{-k}) - \frac{e^{-k}}{8}\left(5 + \frac{k}{2} - \frac{k^2}{2}\right)\right]^{-1}. \tag{3}$$

The scattering is assumed to be completely diffuse.

As is well known, in the first approximation expression (3) takes the form

$$f(k) = \frac{4}{3}\frac{1}{k \ln k^{-1}}, \tag{4}$$

but we used in the calculation the exact values of f(k), calculated by Sondheimer[17] from formula (3). (If we put $l_T^\infty = \bar{l}_T^d$, then it follows from (2) that $f(k) = \delta_T^d/\delta_T^\infty$). In calculating the constant A in formula (2), the mean free path in the bulky sample at $T = 4.2^\circ$K was assumed equal to 0.5 mm (on the basis of different experimental values of $\bar{l}_{4.2}{}^\infty$ in aluminum of similar purity; the error may amount to 20%[18,19].

The data obtained for the temperature dependence of the volume free path (or the parameter k from expression (2)) in thin samples are presented in the form of the solid curves in Fig. 5 on a logarithmic scale. (The values of $1/l_0$, corresponding to the region of the

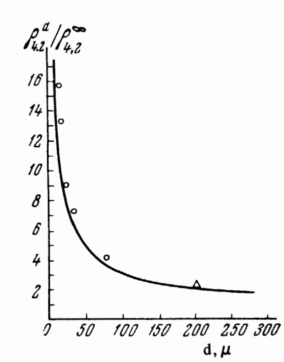


FIG 4. Comparison of experimental data in the region of the residual resistance with the Fuchs theory (formulas (2) and (3)): O—present work, Δ—recalculated from Aleksandrov's data [3], curve—theoretical.

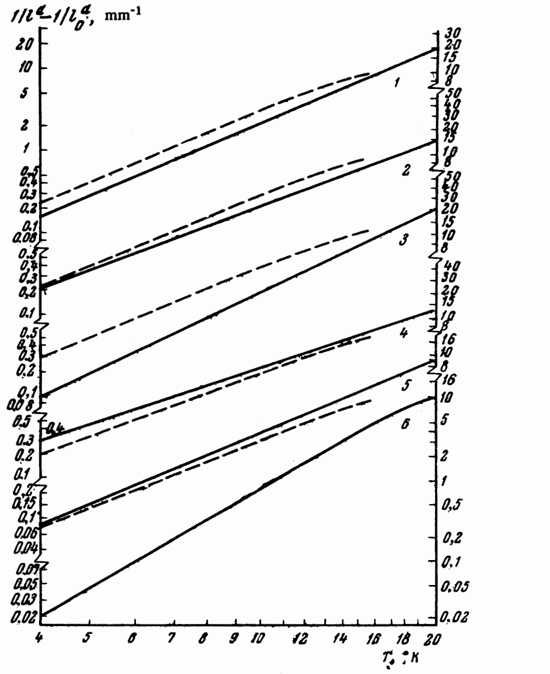


FIG. 5. Temperature dependence of the parameter k (in units of $k/d = 1/l^d$) of different samples: 1—d = 50μ, 2—d = 18μ, 3—d = 24μ, 4—d = 35μ, 5—d = 80μ, 6—bulky sample. Solid curves—obtained from the experimental data for thin samples and from formula (2); dashed curves—from formula (9) and experimental data for the bulky sample.

residual resistance, were subtracted and amount to 5.03, 4.467, 2.35, 2.78, and 2.52 mm^{-1} for curves 2, 3, 4, and 5 respectively). We see that they are close to straight lines, i.e., they can be described by the power-law dependence

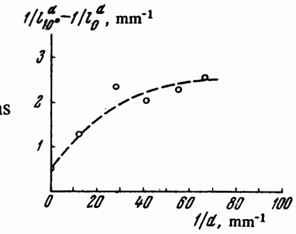
$$1/l_T - 1/l_0 = aT^n, \tag{5}$$

and their slope is determined by the value of the exponent. Curve 6 on this figure is plotted analogously for the bulky sample, and was calculated using the temperature dependence of δ_T^∞ and the relation δ_T^∞ = const. $1/l_T^\infty$ ($1/l_0^\infty$ = 1.98 mm^{-1}, see formula (7)).

We see that at equal temperatures the free path in thin samples is much shorter than in bulky ones, as is illustrated in Fig. 6 for the temperature 10°K. The curves on Fig. 5 have different slopes for different samples, with curve 6 (bulky sample) being the steepest, corresponding to the largest exponent. This offers evidence that in thin samples the character of the electron scattering changes; in particular, the influence of the boundaries causes the role of the normal processes to be more noticeable than in bulky samples. This effect can be analyzed by using the theory of electric conductivity of thin samples, developed by Azbel' and Gurzhi [20] who showed that the increasing role of normal processes in plates is connected with their thickness.

It is possible to separate the contributions corresponding to the normal processes and the processes with Umklapp by using experimental data on the temperature dependence of the free path in the bulky samples and Pytte's conclusions. To this end, we used the temperature region of curve 6 in Fig. 3, in which the

FIG. 6. Electron-phonon free path length in samples of different dimensions at temperature 10° K.



anomaly was still insignificant. In other words, the reciprocal of the free path (which is proportional to the collision frequency) is represented in the temperature interval above 5°K in the form

$$1/l^\infty = 1/l_0 + 1/l_{ep}{}^U + 1/l_{ep}{}^N, \tag{6}$$

where $1/l_{ep}^U = \alpha T^4$, $1/l_{ep}^N = \beta T^5$, and l_{ep}^N and l_{ep}^U are the electron phonon free paths under normal collisions and collisions with Umklapp, respectively. The quadratic term connected with the electron-electron interaction is disregarded, but this hardly influences the final result. The coefficients α and β were obtained by the method least squares with an electronic computer. For the interval 5–15°K we obtained

$$1/l^\infty = 1{,}98 + 7{,}345\cdot 10^{-5}[\mathrm{mm}^{-1}\cdot \mathrm{deg}^{-4}]T^4 + 6{,}273\cdot 10^{-7}[\mathrm{mm}^{-1}\cdot \mathrm{deg}^{-5}]T^5. \tag{7}$$

The error with which this polynomial describes the experimental curve does not exceed 3%.

Azbel' and Gurzhi [20] proposed the following expression for the parameter determining the temperature dependence of the electric conductivity of thin plates (k in formula (2)):

$$k = \frac{d}{l^\infty} + \frac{d/l_{ep}{}^\infty}{(T/\Theta)^2 + (d/l_0{}^\infty)^2 + (d/l_{ep}{}^\infty)^{2/3}}, \tag{8}$$

where l_0^∞ is the electron-impurity free path, which is responsible for the residual resistance in the bulky sample. The role of the U-processes is not reflected in (8). They should obviously be taken into account in the same manner as the other large-angle scattering mechanism, namely the electron-impurity mechanism. Bearing also in mind the fact that l_{ep}^∞ in (8) stands for a quantity connected with normal processes, expression (8) should be used in a somewhat different form:

$$k = \frac{d}{l^\infty} + \frac{d/l_{ep}{}^N}{(T/\Theta)^2 + [d(1/l_0{}^\infty + 1/l_{ep}{}^U)]^2 + (d/l_{ep}{}^N)^{2/3}}. \tag{9}$$

The terms connected with the electron-electron interaction were omitted, as before.

In determining the quantity 1/k, which was substituted in the formula for the electric conductivity (for example in (4)), it is necessary to take into account the approximate character of expression (9) by introducing a correction on the order of unity, which may turn out to be important for large k (see [20]). The quantity $1/k' \equiv 1 + 1/k$ is thus analogous to 1/k (it determines the temperature dependence of the electric conductivity). The quantity d/k', which recalls l_T^d, describes a certain effective frequency of electron-phonon collisions in thin samples, although in itself it has no rigorous physical meaning.

From a comparison of the free-path values $1/l_0^2$

corresponding to the region of the residual resistance of different samples (see above) we see that the difference between them can reach 250%. Generally speaking, some discrepancy between them and the corresponding value in the bulky sample would not be surprising, but a discrepancy by a factor of almost 2.5 is apparently evidence of a certain damage to the plates during the course of their preparation. Therefore we assumed that $1/l_0^\infty$ in (9) is equal to the value of $1/l_0^d$ of the corresponding sample, starting from the premise that in the region of the residual resistance we should have $l_0^\infty = l_0^d$. Finally, the values of $1/l_{ep}^U$ and $1/l_{ep}^N$ for different temperatures were taken from (7).

The values of k′/d calculated in this manner are represented in the form of the dashed curves in the same Fig. 5. Taking into account the same assumptions which were made in the calculation of the theoretical k′/d curves, and also the error in the determination of the thickness of the samples, the agreement with the theory of Azbel′ and Gurzhi should be regarded as good.

For the sample 35 μ thick we calculated the resistance curves from the Fuchs formula with allowance for the Azbel' and Gurzhi correction (the second term on the the right side of (9)) and without this correction (the dashed and dash-dot curves on Fig. 3, respectively). Similar curves are obtained also for the remaining samples.

Notice must also be taken of the following. If we take account in (9) of the detailed course of the function $1/l^\infty$, which has a singularity that affects the temperature dependence of the resistance of the bulky sample, then we can naturally expect an analogous singularity in thin samples, provided it is connected with U-processes. However, it is seen from the very same expression (9) that the normal processes smooth out the temperature dependence of the resistance of thin samples, so that the indicated anomaly will be less pronounced. It seems to us that the singularity on the resistance curve of the 24-μ sample is of the aforementioned nature.

CONCLUSIONS

In the investigation of thin samples, their thicknesses were chosen such as to come closest to the conditions under which, in accordance with the Gurzhi theory[9], the hydrodynamic situation accompanied by the appearance of a minimum of the resistance in the thin samples can arise. To this end, satisfaction of the following inequality is required:

$$l_{ep}^N \ll d \ll (l_{ep}^N l_{ep}^U)^{1/2},\ l_{ei}$$

(the notation is the same as before). Using the data for the bulky sample and the usual assumption that the main contribution to the resistance is made by the normal processes, we obtain at 6°K the ratios $d/l_{ep}^N \approx 2$ and $d/(l_{ep}^N l_{ep}^U)^{1/2} \approx 0.5$, and at 10°K we have $d/l_{ep}^N \approx 5$ and $d/(l_{ep}^N l_{ep}^U)^{1/2} \approx 1/3$. We see that the inequalities are quite weak, so that we can hardly expect the appearance of the indicated mechanism, if it exists at all, in aluminum of such purity as used by us. To broaden the limits of the inequality it is obviously necessary either to increase the purity very greatly (say by two orders of magnitude), or to choose a material in which the probability of the U-processes is quite small within a reasonable range of temperatures (apparently, alkali metals). On the other hand, bearing in mind Pytte's ideas, these inequalities cannot be satisfied at all for aluminum at temperatures at which the relations $l_{ep}^N > l_{ep}^U$ and $l_{ep}^N > l_{ei}$ hold true, i.e., for our temperature interval. No minimum was observed on the experimental resistance curves.

The results obtained in this investigation can be summarized as follows. We measured and investigated the temperature dependence of the resistance of thin single-crystal plates and of bulky aluminum, having a common history. We observed that the volume free path and its behavior in the investigated temperature region depend on the dimension of the sample. The experimental results were analyzed by using the theories of Fuchs and of Azbel' and Gurzhi for the electric conductivity of thin samples, as well as Pytte's conclusions, all of which explain the experimental results sufficiently well.

A singularity, qualitatively analogous to that observed in the bulky sample, was observed on the temperature dependence of the resistance of one of the thin samples. It is proposed that its nature is connected with the exponential decrease of the probability of electron-phonon U-processes in aluminum.

In conclusion, it should be noted that it is assumed in both the Bloch-Gruneisen theory and in Pytte's investigation that the phonons are in equilibrium, i.e., no account is taken of the dragging of the phonons by electrons at low temperatures. In addition, Pytte has made a number of assumptions. In view of all this, it is impossible to evaluate rigorously the results obtained at the lowest temperatures (say, at helium temperatures).

We are deeply grateful to B. I. Verkin for interest in the work and support, R. N. Gurzhiĭ for a discussion of the results of the experiment and valuable remarks, and V. V. Andrievskiĭ for help with the measurements.

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