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CURRENT FLUCTUATIONS IN STRONG ELECTRIC FIELDS

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The spectral intensity $S(\omega)$ of current fluctuations in a semiconductor near the stationary nonequilibrium state and located in a strong electric field **E** and a crossed magnetic field $\mathbf{H} \perp \mathbf{E}$ is calculated. The dominant mechanism of interaction between the electrons and the lattice is assumed to be the instantaneous spontaneous emission of optical phonons of energy $\hbar\omega_0$. As a result, scattering of electrons of energy $\epsilon = \hbar\omega_0$ is highly inelastic. The other, elastic scattering mechanisms are characterized by a relaxation time τ. When $H = 0$ the electron is accelerated over a time τ_E from $\epsilon = 0$ to $\epsilon = \hbar\omega_0$, emits a phonon, is stopped, and then begins to repeat the same motion periodically. Under these conditions the noise intensity $S(\omega)$ is a superposition of peaks at frequencies that are integer multiples of $2\pi/\tau_E$. The width of each peak is of the order of $1/\tau$ and the intensity proportional to the square of the electron velocity Fourier component. In contrast to $S(\omega)$, the differential conductivity $\sigma(\omega)$ is a small quantity and hence the Callen—Welton theorem does not hold at all. When H exceeds a certain critical value H_0', additional peaks at frequencies that are integer multiples of the cyclotron frequency ω_c appear. Those peaks whose frequencies are integer multiples of $2\pi/\tau_E$ vanish when a different critical field H_0 is exceeded, and at $H = H_0$ coincide jumpwise with the peaks at frequencies that are multiples of ω_c.

1. INTRODUCTION

CURRENT fluctuations in a system in a state of thermodynamic equilibrium are connected by well known universal relations[1] with the response of the system to a weak external electric field:

$$(\delta j_i \delta j_k)_\omega = \frac{kT}{\pi} \operatorname{Re} \sigma_{ik}(\omega). \tag{1.1}$$

On the left side of the equation is the intensity of the fluctuations of the current components at the frequency ω; T—thermodynamic temperature, $\sigma_{ik}(\omega)$—conductivity tensor at the same frequency ω. If we consider a system of electrons in a strong constant electric field **E**, then $\sigma_{ik}(\omega)$ can be interpreted as the differential conductivity, which determines the ac component of the current when an additional small alternating field of frequency ω acts on the system. However, inasmuch as such a system is non-equilibrium, relation (1.1) is not satisfied. Therefore a study of the current fluctuations gives additional information not contained in σ_{ik}.

The current fluctuations in a strong electric field ("hot" electrons) were investigated in a number of papers[2-7]. In all these papers, they considered almost-elastic scattering of the electrons when, as is well known[8], the electron momentum distribution function f(**p**) has small anisotropy. Then (with the exception of singular cases connected with electron runaway[9]) the symmetrical part of the distribution, i.e., the energy distribution $f_0(\epsilon)$, agrees qualitatively with the equilibrium distribution $\exp(-\epsilon/kT)$, if we replace in the latter the lattice temperature T by the electron temperature T*, which is connected with the average electron energy $\langle\epsilon\rangle$ by the relation $\langle\epsilon\rangle = (3/2)kT$. Therefore relation (1.1) remains valid in order of magnitude, if T is replaced in it by T*[3].

There are, however, known scattering mechanisms in which the deviation of the distribution f(**p**) from equilibrium is appreciable. Thus, in inelastic scattering of electrons by optical phonons, when the electron loses practically all its energy after emitting the optical phonon, the electron distribution has a sharp anisotropy[10-12]. It was shown earlier[13] that this situation is realized at low lattice temperatures in a certain electric-field interval:

$$kT \ll \hbar\omega_0, \quad E^- \ll E \ll E^+, \tag{1.2}$$

where $\hbar\omega_0$ is the energy of the optical phonon, and the characteristic fields are defined by the relations

$$eE^-\tau = p_0, \quad eE^+\tau_0 = p_0, \quad p_0 = \sqrt{2m\hbar\omega_0}, \tag{1.3}$$

in which τ_0 is the time of emission of the optical phonon and τ is the scattering time (by impurities and by acoustic phonons) in the energy interval $\epsilon < \hbar\omega_0$, where the emission of the optical phonon is impossible; it is assumed that $\tau \gg \tau_0$. It is natural to expect the current fluctuations in such scattering to have more distinct features than in elastic scattering. This question was touched upon by Price[2], who indicated that the fluctuation spectrum can have singularities and frequencies that are multiples of the reciprocal of the time $\tau_E = p_0/eE$ during which the electron accelerates from the energy $\epsilon = 0$ to the energy $\epsilon = \hbar\omega_0$.

In the present paper we calculate the spectrum of the current fluctuations in inelastic scattering of electrons under conditions (1.2) and in the presence of an external magnetic field H. The electron density n is assumed to be so small that the interaction between the electrons can be disregarded. We consider only long-wave fluctuations. This allows us to assume the spatial correlation to be absent and to disregard the fluctuations of the distribution in coordinate space, confining ourselves to fluctuations of the distribution in momentum space. It is assumed that the potential difference on the sample is determined by an external circuit: for long-wave fluctuations this means that the electric field in the sample does not fluctuate. We then find with the aid of the Poisson equation that the electron density likewise does not fluctuate.

2. CALCULATION PROCEDURE

To calculate the fluctuation spectrum we use a method developed in[3,6]. It is convenient, however, to rewrite the formulas of these investigations in such a form that the quantities entering here do not depend on the normalization volume. We introduce the quantity

$$\gamma_i(\mathbf{p})_\omega = \frac{1}{2\pi}\int_0^\infty dt\, e^{i\omega t}\,\overline{\delta f(\mathbf{p},t)\delta j_i(0)}. \tag{2.1}$$

Here $\delta_j(t)$ is the fluctuation of the i-th component of the current density at the instant t, and $\delta f(\mathbf{p}, t)$ is the fluctuation of the distribution function at the point $\mathbf{p}$ at the instant t. The bar denotes averaging over the ensemble or over the initial instant of time. The distribution is normalized to the concentration n. With the aid of the Wigner-Khinchine theorem we get

$$(\delta j_i \delta j_k)_\omega = \int (d\mathbf{p})[ev_i(\mathbf{p})\gamma_k(\mathbf{p})_\omega + ev_k(\mathbf{p})\gamma_i(\mathbf{p})_{-\omega}], \tag{2.2}$$

where $v_i(\mathbf{p})$ are the electron-velocity components. The equal-time correlator of the distribution function can be obtained by assuming that the concentration does not fluctuate:

$$\overline{\delta f(\mathbf{p},0)\delta f(\mathbf{p}',0)} = \delta(\mathbf{p}-\mathbf{p}')\bar{f}(\mathbf{p}) - \bar{f}(\mathbf{p})\bar{f}(\mathbf{p}')/\bar{n}, \tag{2.3}$$

where $\bar{f}$ is the stationary distribution and $\bar{n} = n$ is stationary concentration. The equation for γ is obtained[3,6] by using (2.3) together with the Onsager hypothesis, i.e., by assuming that the fluctuation of the distribution evolves in time in the same manner as the distribution itself:

$$\frac{\partial}{\partial t}\delta f = L\delta f, \tag{2.4}$$

where L is the usual kinetic operator, consisting of the field and collision terms. Then the equation for γ takes the form

$$(L + i\omega)\gamma_i(\mathbf{p})_\omega = -\frac{1}{2\pi}e[v_i(\mathbf{p}) - u_i]\bar{f}(\mathbf{p}), \tag{2.5}$$

where $\mathbf{u}$ is the average electron velocity in the stationary state.

Let us consider first the formal solution of Eq. (2.5), assuming that the function γ can be expanded in the eigenfunctions of the operator L, which are determined by the equation

$$[L + i\omega(\xi)]\varphi(\xi|\mathbf{p}) = 0. \tag{2.6}$$

Here ξ numbers the eigenvalues and the functions. Since L is a not-self-adjoint operator, it is necessary to introduce the eigenfunctions of the adjoint operator $\psi(\xi|\mathbf{p})$[14], which form together with $\varphi(\xi|\mathbf{p})$ a biorthogonal system. The value $\xi = 0$ will be referred to the eigenvalue $\omega(0) = 0$, corresponding to the eigenfunction $\varphi(0|\mathbf{p}) = \bar{f}(\mathbf{p})/n$, which coincides, apart from normalization, with the stationary distribution. Therefore all the $\psi(\xi|\mathbf{p})$ with $\xi \neq 0$ are orthogonal to $\bar{f}(\mathbf{p})$. Bearing these remarks in mind, we can easily obtain a formal solution of (2.5); substituting it in (2.2), we get

$$(\delta j_i \delta j_k)_\omega = \frac{c_{ik}}{\omega} + \frac{1}{2\pi i}\sum_\xi{}' \left[\frac{a_k^*(\xi)b_i(\xi)}{\omega(\xi)-\omega} + \frac{a_i^*(\xi)b_k(\xi)}{\omega(\xi)+\omega}\right], \tag{2.7}$$

where

$$a_i(\xi) = \int (d\mathbf{p})\bar{f}(\mathbf{p})ev_i(\mathbf{p})\psi(\xi|\mathbf{p}), \tag{2.8}$$

$$b_i(\xi) = \int (d\mathbf{p})ev_i(\mathbf{p})\varphi(\xi|\mathbf{p}), \tag{2.9}$$

$$c_{ik} = \frac{e}{2\pi i n}[a_i^*(0)u_k - a_k^*(0)u_i]. \tag{2.10}$$

The prime at the summation sign in (2.7) indicates that the term $\xi = 0$ has been left out of the "summation" and its contribution is written separately. The corresponding low-frequency noise $1/\omega$ will be of no interest to us, and this term will be disregarded.

The main equation (2.5) will be solved by using the methods employed in[15] to find $\bar{f}$. In this connection, we recall certain concepts and symbols. It is assumed that the optical phonon is emitted instantaneously ($\tau_0 = 0$), as soon as the electron reaches the equal-energy surface Σ defined by the condition $\epsilon(\mathbf{p}) = \hbar\omega_0$. Then all of the electrons are in the passive region Ω (bounded by the surface Σ) where $\epsilon(\mathbf{p}) < \hbar\omega_0$. It is convenient to solve (2.5) in a coordinate system connected with the trajectories in momentum space: S—length of arc along the trajectory, and α—parameters determining the trajectory. A special role is played by the principal trajectory $\hat{\alpha}$, which passes through the point $\mathbf{p} = 0$ corresponding to $s = \hat{s}$. The remaining trajectories are called secondary. In the presence of a magnetic field, there can exist trajectories that are closed in Ω; the region occupied by them is denoted Ω_c. Similarly Ω_a denotes the region occupied by trajectories that are open in Ω; these trajectories begin and end on the surface Σ, on its sections Σ^- and Σ^+ respectively. The start and end of the trajectory correspond to values $s = s_-$ and $s = s_+$. A special position is occupied by the invariant trajectories—the closed ones and the section of the open principal trajectory from $\hat{s}$ to s_+. Fast processes—motion under the influence of the field and emission of optical phonons—do not change the number of the electrons on these trajectories.

In terms of the new variables, the volume element is $(d\mathbf{p}) = g(\alpha s)(d\alpha)ds$, and the density of states near the trajectory is $g(\alpha) = \int ds g(\alpha s)$. We introduce also the velocity of motion along the trajectory $\dot{s}(\alpha s)$ and the time of motion along the trajectory from the point s_0 to the point s:

$$t(\alpha|s_0, s) = \int_{s_0}^{s}\frac{ds'}{\dot{s}(\alpha s')} = t(\alpha|s). \tag{2.11}$$

The second form of the symbol pertains to those cases when the value of s_0 is immaterial. The total time of motion along the trajectory is

$$\tau_F(\alpha) = \int\frac{ds}{\dot{s}(\alpha s)}. \tag{2.12}$$

It is useful to bear in mind the relation

$$\frac{\partial}{\partial s}[g(\alpha s)\dot{s}(\alpha s)] = 0, \tag{2.13}$$

from which it follows that

$$g(\alpha s)\dot{s}(\alpha s)\tau_F(\alpha) = g(\alpha). \tag{2.14}$$

The most significant is the period of revolution over the closed trajectory and the period of the cycle of acceleration along the principal open trajectory

$$\tau_F(\alpha)=\oint\frac{ds}{\dot{s}(\alpha s)}\qquad(\alpha\in\Omega_c),\qquad \tau_F(\hat{\alpha})=\int\limits_{\hat{s}}^{s_+}\frac{ds}{\dot{s}(\hat{\alpha}s)}.\qquad(2.15)$$

In terms of these coordinates, the operator L assumes the form

$$Lf(\alpha s)=Df(\alpha s)+S(f|\alpha s).\qquad(2.16)$$

The first term describes here the fast processes—motion under the influence of the field and emission of optical phonons:

$$Df(\alpha s)=-\dot{s}(\alpha s)\frac{\partial}{\partial s}f(\alpha s)+\frac{1}{g(\alpha s)}\delta(\alpha-\hat{\alpha})\delta(s-\hat{s})I(f),\qquad(2.17)$$

$$I(f)=\int\limits_{\Omega_a}(d\alpha)g(\alpha s_+)\dot{s}(\alpha s_+)f(\alpha s_+)=\int\limits_{\Omega_a}(d\alpha)g(\alpha)\frac{f(\alpha s_+)}{\tau_F(\alpha)}.\qquad(2.18)$$

The integral I is the flux of electrons that reach the surface Σ^+ under the influence of the field. The second term in (2.16) describes slow processes—the scattering inside Ω (by impurities or acoustic phonons, and also the absorption of optical phonons with prompt emission—compound scattering[16]):

$$S(f|\alpha s)=-\frac{f(\alpha s)}{\tau(\alpha s)}+B(f|\alpha s),\qquad(2.19)$$

$$B(f|\alpha s)=\int(d\alpha')\int ds' g(\alpha's')W(\alpha's',\alpha s)f(\alpha's').\qquad(2.20)$$

Here W is the probability of the corresponding scattering and τ the lifetime relative to this scattering. In addition, it is necessary to satisfy the following condition

$$f(\alpha s_-)=0.\qquad(2.21)$$

3. ABSENCE OF SCATTERING IN THE PASSIVE REGION

We shall solve first Eq. (2.5) assuming that there is no scattering inside $\Omega(\tau=\infty)$. The stationary distribution $\bar{f}$ was obtained in this approximation in[15], and by using (2.14) it can be written in the form

$$\bar{f}(\alpha s)=\frac{1}{g(\alpha)}\delta(\alpha-\hat{\alpha})\,\theta(s-\hat{s})\hat{n},\quad \alpha\in\Omega_a,\qquad(3.1a)$$

$$\bar{f}(\alpha s)=\frac{1}{g(\alpha)}[\delta(\alpha-\hat{\alpha})\,\hat{n}+\tilde{n}(\alpha)],\quad \alpha\in\Omega_c.\qquad(3.1b)$$

Here $\theta(s)$ is the Heaviside step function, $\hat{n}$ is the number of electrons on the principal trajectory and $\tilde{n}(\alpha)$ the number of electrons on the closed secondary trajectory α. The distribution $\bar{f}$ differs from zero only on the invariant trajectories. We now find the eigenfunctions of the operator L in the same approximation. Integrating the expression

$$D\varphi(\alpha s)+i\omega\varphi(\alpha s)=0\qquad(3.2)$$

with respect to s and satisfying the boundary condition (2.21), we get

$$\varphi(\alpha s)=\frac{\tau_F(\hat{\alpha})}{g(\hat{\alpha})}I(\varphi)\,\delta(\alpha-\hat{\alpha})\theta(s-\hat{s})\exp\{i\omega t(\hat{\alpha}|\hat{s},s)\},\quad \alpha\in\Omega_a,\qquad(3.3a)$$

$$\varphi(\alpha s)=C(\alpha)\exp\{i\omega t(\alpha|s)\},\quad \alpha\in\Omega_c,\qquad(3.3b)$$

where C is an integration constant. Substituting (3.3a) in (2.18), we get

$$I(\varphi)=I(\varphi)\exp\{i\omega\tau_F(\hat{\alpha})\}.\qquad(3.4a)$$

Stipulating that $\varphi(\alpha s)$ must be periodic in s on the closed trajectories, we get

$$C(\alpha)=C(\alpha)\exp\{i\omega\tau_F(\alpha)\}.\qquad(3.4b)$$

Equation (3.4) should be satisfied by choosing a certain $\omega\neq 0$, and this yields the eigenvalue. Obviously, this can be done only if $\varphi(\alpha s)$ differs from zero on only one of the invariant trajectories β. Then ω should be chosen to be a multiple of the frequency of revolution on this invariant trajectory. We thus find ultimately the eigenfunctions and the eigenvalues. For the principal unclosed trajectory $\beta=\hat{\alpha}$

$$\varphi(l\hat{\alpha}|\alpha s)=\delta(\alpha-\hat{\alpha})g(\hat{\alpha})^{-1/2}\theta(s-\hat{s})\exp\{il\omega(\hat{\alpha})t(\hat{\alpha}|\hat{s},s)\},\ \hat{\alpha}\in\Omega_a,$$
$$\omega(\hat{\alpha})\equiv\hat{\omega}=2\pi/\tau_F(\hat{\alpha}),\quad l=\pm1,\pm2,\ldots\qquad(3.5a)$$

For a closed trajectory

$$\varphi(l\beta|\alpha s)=\delta(\alpha-\beta)g(\beta)^{-1/2}\exp\{il\omega(\beta)t(\beta|s)\},$$
$$\omega(\beta)=2\pi/\tau_F(\beta),\quad l=\pm1,\pm2,\ldots\qquad(3.5b)$$

The obtained functions form an orthonormal system

$$\int(d\alpha)\int ds g(\alpha s)\varphi^*(l\beta|\alpha s)\varphi(l'\beta'|\alpha s)=\delta_{ll'}\delta(\beta-\beta'),\qquad(3.6)$$

from which it follows that in the approximation under consideration $\psi(\xi)=\varphi(\xi)$. This agrees with the fact that the eigenvalues of the operator iL are real in this approximation.

Calculating the coefficients (2.8) and (2.9), we get

$$a_i(l\beta)=g(\beta)^{-1/2}n(\beta)ev_i^l(\beta),\qquad(3.7)$$

$$b_i(l\beta)=g(\beta)^{1/2}ev_i^l(\beta),\qquad(3.8)$$

where $v_i^l(\beta)$ is the l-th Fourier component of the i-th velocity component on the trajectory β:

$$v_i^l(\beta)\equiv\hat{v}_i^l=\frac{1}{\tau_F(\hat{\alpha})}\int\limits_{\hat{s}}^{s_+}dt(\hat{\alpha}|s)v_i(\beta s)\exp\{il\omega(\hat{\alpha})t(\hat{\alpha}|s)\},\quad \beta=\hat{\alpha}\in\Omega_a,\qquad(3.9a)$$

$$v_i^l(\beta)=\frac{1}{\tau_F(\beta)}\oint dt(\beta|s)v_i(\beta s)\exp\{il\omega(\beta)t(\beta|s)\},\quad \beta\in\Omega_c.\qquad(3.9b)$$

Substituting (3.7) and (3.8) in (2.7), we can find the noise spectrum. For a correct calculation of the singularities it is necessary to replace $\omega(\xi)$ by $\omega(\xi)-i\nu$, where $\nu>0$, and then let $\nu\to 0$. The basis of this procedure is the fact that when τ is finite all the states of the system relax to $\bar{f}$, and therefore in such a case $\omega(\xi)$ should contain an imaginary part, with $\operatorname{Im}\omega(\xi)<0$. It is important that the foregoing does not pertain to $\xi=0$, where $\omega(0)=0$ is an exact eigenvalue also when τ is finite. Discarding the low-frequency background, we get

$$(\delta j_i\,\delta j_k)_\omega=\frac{1}{2\pi i}\int(d\beta)n(\beta)$$
$$\times\sum_{l=1}^{\infty}\left\{ev_i^l(\beta)^*ev_k^l(\beta)\left[\frac{1}{l\omega(\beta)+\omega-i\nu}+\frac{1}{-l\omega(\beta)-\omega-i\nu}\right]\right.$$
$$\left.+ev_k^l(\beta)^*ev_i^l(\beta)\left[\frac{1}{l\omega(\beta)-\omega-i\nu}+\frac{1}{-l\omega(\beta)+\omega-i\nu}\right]\right\}.\qquad(3.10)$$

Using the well known representation of the δ-functions and letting $\nu \to 0$, we ultimately get

$$(\delta j_i \delta j_k)_\omega = \int (d\beta) n(\beta) \sum_{l=1}^{\infty} [ev_i^l(\beta)^* ev_k^l(\beta)\delta(\omega + l\omega(\beta)) + ev_k^l(\beta)^* ev_i^l(\beta)\delta(\omega - l\omega(\beta))]. \quad (3.11)$$

This result admits of a very simple interpretation. Each electron located on one of the invariant trajectories "makes noise" at frequencies that are multiples of the frequency of its revolution on this trajectory. The noise intensity at each harmonic is determined by the corresponding Fourier component of the velocities. We note that the result (3.11) can be obtained also by direct integration of (2.5) along each of the invariant trajectories. If desired, it is possible to expand on each trajectory in the eigenfunctions (3.5), which constitute a complete system for each of the invariant trajectories. This can be regarded also as a justification of the formal solution.

It is convenient to break up the noise intensity into two components corresponding to the contributions of the principal and secondary trajectories:

$$(\delta j_i \delta j_k)_\omega \equiv S_{ik}(\omega) = \hat{S}_{ik}(\omega) + \tilde{S}_{ik}(\omega), \quad (3.12)$$

where the contribution of the principal trajectory is

$$\hat{S}_{ik}(\omega) = \hat{n} \sum_{l=1}^{\infty} [\hat{ev}_i^{l*} \hat{ev}_k^l \delta(\omega + l\hat{\omega}) + \hat{ev}_k^{l*} \hat{ev}_i^l \delta(\omega - l\hat{\omega})] \quad (3.13)$$

and the contribution of the secondary trajectories $\tilde{S}_{ik}(\omega)$ is expressed by a formula similar to (3.11), except that $n(\beta)$ is replaced by $\tilde{n}(\beta)$. The function $\hat{S}_{ik}(\omega)$ has δ-like singularities at the points $\omega = l\hat{\omega}$, whereas in $\tilde{S}_{ik}(\omega)$ the singularities become smeared out after integration with respect to β. However, if the spread of the revolution frequencies $\omega(\beta)$ for different closed trajectories in the region Ω_c is small ($\Delta\omega \ll \omega_c$, where ω_c is a certain average revolution frequency), then the function $\tilde{S}_{ik}(\omega)$ consists of a set of peaks of width $\Delta\omega$ at the points $\omega = l\omega_c$.

The general form of the noise spectrum $S_{ik}(\omega)$ depends on the topology of the trajectories in the region Ω[15]. In sufficiently weak magnetic fields H, when there are no closed trajectories, $\tilde{S}_{ik}(\omega) = 0$, and the spectrum $\hat{S}_{ik}(\omega)$ represents a "comb" of equidistant δ-like peaks with pitch $\hat{\omega}$. With increasing H, the pitch $\hat{\omega}$ decreases. In a certain field H_0', closed trajectories appear; owing to the contribution of $\tilde{S}_{ik}(\omega)$, a second "comb" appears in the spectrum, having peaks of width $\Delta\omega$ with a smaller pitch $\omega_c < \hat{\omega}$. With further increase of H, the pitch of the first comb continues to decrease, and that of the second increases. In a certain field H_0, the principal trajectory becomes closed. At this value of the field, the frequency $\hat{\omega}$ becomes jumpwise equal to ω_c and both combs coalesce. The jumplike change of the noise spectrum $S_{ik}(\omega)$ at $H = H_0$ has the same physical nature as the jumps of the dissipative current obtained in[13,15].

Let us discuss now the connection between the noise spectrum and the differential conductivity or, in other words, the question of the degree of satisfaction of (1.1). We confine ourselves for simplicity to the case $H = 0$. Then only the Fourier components of the velocity parallel to E ∥ z differ from zero in the approximation under consideration, and consequently only $S_{zz}(\omega)$ differs from zero. We have (we omit the indices z)

$$S(\omega) = n \sum_{l=1}^{\infty} |ev^l|^2 [\delta(\omega - l\hat{\omega}) + \delta(\omega + l\hat{\omega})]. \quad (3.14)$$

We now calculate $\sigma_{zz}(\omega) \equiv \sigma(\omega)$. To this end we write

$$E = \bar{E} + E' e^{-i\omega t}, \quad f = \bar{f} + f' e^{-i\omega t}, \quad (3.15)$$

where $\bar{E}$ and $\bar{f}$ are the stationary values, and E′ and f′ are the high frequency small additions. When H = 0 we have $\hat{s} = eE$, so that the operator D can be represented in the form $\bar{D} + D' \exp[-i\omega t]$, where $\bar{D}$ and D′ correspond to the fields $\bar{E}$ and E′. The kinetic equation for the correction f′ takes the form

$$(\bar{D} + i\omega) f' = -D' \bar{f}. \quad (3.16)$$

The equation for f′ differs from the equation for γ only in the form of the right-hand side, as should indeed be the case[3]. The distribution $\bar{f}$ is given by formula (3.1a), and it is convenient to choose as the coordinates $\alpha = (p_x, p_y)$ and $s = p_z$. Then

$$\bar{f}(\mathbf{p}) = \frac{n}{p_0} \delta(p_x)\delta(p_y)\theta(p_z), \quad (3.17)$$

where p_0 is the length of the principal trajectory. Further, substituting (3.17) in (2.17), we get

$$D'\bar{f}(\mathbf{p}) = -eE' \frac{\partial}{\partial p_z} \bar{f}(\mathbf{p}) + \delta(\mathbf{p}) eE' \int\int dp_x dp_y \bar{f}(\mathbf{p}) = 0. \quad (3.18)$$

Solving (3.16) by the method as (2.5), we find that f′ = 0 and consequently

$$\sigma(\omega) = 0. \quad (3.19)$$

Comparison of (3.19) and (3.14) shows that in the inelastic-scattering mechanism under consideration the relation (1.1) is violated most strongly. Therefore the determination of the "noise" temperature T_n with the aid of relation $kT_n = \pi S(\omega)/(\omega)$ is not advantageous in this case.

4. ALLOWANCE FOR COLLISIONS IN THE PASSIVE REGION

It is physically obvious that the main effect of the influence of the elastic collisions inside Ω will be the broadening of the peaks in the noise spectrum—each δ-function in (3.11) is replaced by a peak of finite width Γ, and in order of magnitude $\Gamma \approx 1/\tau$. It follows from (1.2) that $\tau \gg \tau_F$, i.e., $\Gamma \ll \hat{\omega}, \omega_c$, and therefore the peaks remain well resolved. A more detailed estimate of the width of each peak can be obtained on the basis of a formal expansion of (2.7), from which it is seen that the width of the peak Γ connected with the eigenvalue ξ is determined by the imaginary part of the eigenvalue $\Gamma(\xi) = -\mathrm{Im}\,\omega(\xi)$.

The imaginary correction to the eigenvalues connected with the closed principal trajectory can be obtained by a perturbation method. To this end we write down the eigenfunction and the eigenvalue in the form $\varphi = \varphi^0 + \varphi'$ and $\omega = \omega^0 + \omega'$, where φ^0 is defined by (3.5a) and $\omega^0 = l\hat{\omega}$. Now, assuming that S(f) is small compared with Df, we get

$$(D + i\omega^0)\varphi'(\alpha s) = -S(\varphi^0|\alpha s) - i\omega'\varphi^0(\alpha s). \quad (4.1)$$

This equation can be rewritten in the form:

$$D(\exp\{i\omega^0 t(\alpha|s,s_+)\}\varphi'(\alpha s)) = -\exp\{i\omega^0 t(\alpha|s,s_+)\}[S(\varphi^0|\alpha s)+i\omega'\varphi^0(\alpha s)]. \quad (4.2)$$

This can be easily verified by using the explicit form of the operator D, in accordance with (2.17). Since the fast processes conserve the number of electrons in the region Ω_c, it can be readily verified that

$$\int_{\Omega_a}(d\alpha)\int dsg(\alpha s)Df(\alpha s)=0 \quad (4.3)$$

for any function $f(\alpha s)$ satisfying the boundary condition (2.21). Integrating (4.2) with respect to Ω_a, using (4.3), and substituting the explicit form of φ^0 in (2.20), we find the correction to the eigenvalue $\omega^0 = l\hat{\omega}$:

$$\omega' = -i\frac{1}{\tau(\hat{\alpha})}[1-e^{i\delta_l}], \quad (4.4)$$

where $\tau(\alpha)$ is the average lifetime on the trajectory α,

$$\frac{1}{\tau(\alpha)}=\int\frac{dt(\alpha|s)}{\tau_F(\alpha)}\frac{1}{\tau(\alpha s)}, \quad (4.5)$$

and

$$e^{i\delta_l}=\tau(\alpha)\int_{\Omega_a}(d\alpha)\int dsg(\alpha s)\int\frac{dt(\hat{\alpha}|s')}{\tau_F(\hat{\alpha})}W(\hat{\alpha}s',\alpha s) \times\exp\{il\omega(\hat{\alpha})[t(\hat{\alpha}|\hat{s},s')+t(\alpha|s,s_+)]\}. \quad (4.6)$$

The first term in (4.4) is connected with the departure of the electron from the trajectory $\hat{\alpha}$, and according to (4.5) it retains its meaning for any invariant trajectory. The second term is characteristic of the principal trajectory and is connected with the possible return of the electron to this trajectory. Its meaning can be understood as follows. The electron can be scattered at any point s′ of the principal trajectory and fall into the point s of the trajectory α. If the trajectory $\alpha \in \Omega_a$, then the electron, moving on this trajectory, reaches after a time $t(\alpha|s, s_+)$ the region Σ^+, emits an optical phonon, falls in the point $p = 0$, and now moving on the principal trajectory, returns after a time $t(\hat{\alpha}|\hat{s}, s')$ to the point from which it left the principal trajectory. Thus, the argument of the exponential in the integral is the loss of phase as a result of the scattering, and δ_l is the result of its averaging over all possible scattering acts. The width of the peak at the frequency $l\omega$ is found to be

$$\Gamma_l=\frac{1}{\tau(\hat{\alpha})}[1-\cos\delta_l]. \quad (4.7)$$

The employed method, based on perturbation theory, is not applicable if the principal trajectory is closed, for in this case there exist eigenvalues that are arbitrarily close to each other. The same pertains to the peaks connected with closed secondary trajectories. In this case it is possible to employ a method similar to the Wigner-Weisskopf method[17], investigating the "decay" of the states (3.5) under the influence of the scattering inside Ω. Let us consider the nonstationary problem for the distribution

$$\frac{\partial f}{\partial t}=Df+B(f)-\frac{f}{\tau} \quad (4.8)$$

with initial condition

$$f(\alpha s,0)=\varphi(l\beta|\alpha s). \quad (4.9)$$

We seek the solution in the form

$$f(\alpha s,t)=e^{-\lambda t}u(\alpha s)+\chi(\alpha s,t), \quad u(\alpha s)=\delta(\alpha-\beta)v(s),\quad \chi(\alpha s,0)=0, \quad (4.10)$$

where the term $e^{-\lambda t}u$ describes the electrons that remain on the trajectory β, and the term χ describes the electrons going off to other trajectories. If the solution of the form (4.10) exists, then it is assumed that λ corresponds to the eigenvalue $\lambda = i\omega'$ of the stationary problem of the non-hermitian operator L.

We consider first the "decay" of the principal unclosed trajectory $\beta = \alpha \in \Omega_a$, and show that in this case we obtain the same result as when the perturbation method is used. Substituting (4.10) in (4.8), we separate the terms containing the singularity $\delta(\alpha - \hat{\alpha})$ from those regular in α. As is customary in the Wigner-Weisskopf method, we neglect the term $S(\chi)$, which describes the redistribution among the states in which the decay takes place. We then obtain the system of equations

$$e^{-\lambda t}\left[D+\lambda-\frac{1}{\tau(\hat{\alpha}s)}\right]v(s)+\frac{1}{g(\hat{\alpha}s)}\delta(s-\hat{s})I(\chi)=0, \quad (4.11)$$

$$\left[\frac{\partial}{\partial t}+\dot{s}(\alpha s)\frac{\partial}{\partial s}\right]\chi(\alpha s,t)=e^{-\lambda t}B(u|\alpha s), \quad (4.12)$$

$$B(u|\alpha s)=\int_{\hat{s}}^{s_+}ds'g(\hat{\alpha}s')W(\hat{\alpha}s',\alpha s)v(s'). \quad (4.13)$$

In order to calculate $I(\chi)$ it is necessary to know χ for $\alpha \in \Omega_a$. The solution of (4.12) for such α is

$$\chi(\alpha s,t)=\int_{s_-}^{s}dt(\alpha|s')\theta(t-t(\alpha|s',s))\exp\{-\lambda(t-t(\alpha|s',s))\}B(u|\alpha s'). \quad (4.14)$$

Substituting (4.13) in (4.14), putting $s = s_+$ and substituting in (2.18), we get

$$I(\chi)=\int_{\hat{s}}^{s_+}ds'g(\hat{\alpha}s')v(s')\int_{\Omega_a}(d\alpha)\int_{s_-}^{s_+}dsg(\alpha s)W(\hat{\alpha}s',\alpha s)\theta(t-t(\alpha|s,s_+)) \times\exp\{-\lambda(t-t(\alpha|s,s_+))\}. \quad (4.15)$$

As is customary in the Wigner-Weisskopf method, we consider times that are longer compared with the period of the rapid motion, $t \gg \tau_F$. It is seen from (4.15) that the factor θ is then insignificant, $I(\chi)$ depends on the time like $e^{-\lambda t}$, and consequently Eq. (4.11) can be satisfied. It assumes the following form:

$$\left[D+\lambda-\frac{1}{\tau(\hat{\alpha}s)}\right]v(s)+\frac{1}{g(\hat{\alpha}s)}\delta(s-\hat{s})\int_{\Omega_a}(d\alpha)\int dsg(\alpha s)\int ds'g(\hat{\alpha}s') \times v(s')W(\hat{\alpha}s',\alpha s)\exp\{\lambda t(\alpha|s,s_+)\}=0. \quad (4.16)$$

This equation is solved for v(s) and λ by successive approximations in $1/\tau$. Putting $v = v^0 + v'$ and $\lambda = \lambda^0 + \lambda'$, where the primes denote small terms proportional to $1/\tau$, we get

$$v^0(s)=C\theta(s-\hat{s})\exp\{il\omega(\hat{\alpha})t(\hat{\alpha}|\hat{s},s)\}, \quad \lambda^0=il\hat{\omega}. \quad (4.17)$$

From a comparison of (4.17) and (3.5) we see that, setting the integration constant $C = g(\alpha)^{-1/2}$, we satisfy the initial condition (4.9), accurate to small terms. We then substitute $v = v^0$ and $\lambda = \lambda^0$ in the small terms of (4.16). Subsequently, using (2.14), we reduce the integral to $\exp(i\delta_l)$. From the condition for the solvability of the equation for $v'(s)$, just as for Eq. (4.1), we get $\lambda' = i\omega'$, where ω' is defined by (4.4).

Exactly the same method can be used also in the case when the principal trajectory is closed. The solution is then given by formula (4.4). For a secondary closed trajectory β, this method yields

$$\omega' = -\frac{i}{\tau(\beta)}, \quad \Gamma(\beta) = \frac{1}{\tau(\beta)}. \tag{4.18}$$

The broadening is the same for all peaks; the absence of the term $\exp(i\delta_l)$ is natural, since the electrons do not return to the trajectory β. We note that if the spread of the revolution frequencies over the closed secondary trajectories is large, $\Delta\omega\tau \gg 1$, then the collision broadening for the corresponding peaks is insignificant.

Another effect of the influence of the elastic collisions in Ω is the occurrence of a weak background without pronounced singularities in the noise spectrum. This background is connected with the fact that at finite τ the distribution $\bar{f}$ differs from zero also on the non-invariant trajectory[15]. However, on these trajectories the electrons do not execute periodic motion and therefore cannot produce singularities in the noise spectrum. The appearance of the background is connected mathematically with the fact that at such value of f the solution of (2.5) cannot be expanded in terms of the functions (3.5), which all vanish on the non-invariant trajectories.

We note that the broadening of the peaks and the occurrence of the background can be connected with the fact that the optical phonon is emitted not instantaneously ($\tau_0 \neq 0$). Then the electrons penetrate during their acceleration the active region $\epsilon(\mathbf{p}) > \hbar\omega_0$, and after emission of the optical phonon they return not exactly to $\mathbf{p} = 0$. Therefore the principal trajectory becomes "smeared out" and with it the critical fields H_0' and H_0, at which the re-alignment of the noise spectrum takes place, become "smeared out." Concrete estimates depend on the dispersion of $\epsilon(\mathbf{p})$ and are given in the next section for a parabolic band.

5. PARABOLIC DISPERSION LAW

By way of illustration, let us consider a parabolic dispersion law with an effective-mass tensor m, assuming $\mathbf{H} \perp \mathbf{E}$. The analysis is interesting also because this dispersion law has certain features. Let us assume the following notation: m_d—mass of the state density, m_h—cyclotron mass, m_e—ohmic mass[15], $m_0 = m_h^2/m_e$. All these masses depend only on the orientation of the fields E and H, which are specified by the unit vectors e and h. We introduce

$$v_0 = (2\hbar\omega_0/m_0)^{1/2}, \quad p_0 = (2m_e\hbar\omega_0)^{1/2} = m_h v_0,$$
$$v_D = c\frac{E}{H}, \quad \varkappa = \frac{v_0}{2v_D} = \frac{H}{H_0}, \quad H_0 = \frac{2c}{v_0}E. \tag{5.1}$$

We then obtain the frequency of revolution over the closed trajectory ω_c, which is the same for all trajectories, and the frequency of acceleration over the principal unclosed trajectory

$$\omega_c = \frac{eH}{m_h c}, \quad \hat{\omega} = \frac{2\pi}{\tau_E}\frac{\varkappa}{\arcsin\varkappa}, \quad \tau_E = \frac{p_0}{eE}. \tag{5.2}$$

The Fourier components of the velocity for the principal unclosed trajectory are

$$\hat{\mathbf{v}}^l = \tfrac{1}{2}v_0[\mathbf{S}A^l(\varkappa) + \mathbf{S}^*A^l(-\varkappa)], \tag{5.3}$$

where the dimensionless vector, which depends only on the orientation of the fields, is

$$\mathbf{S} = m_h m^{-1}\mathbf{e} - i\frac{m_h^2}{m_d^3}[\mathbf{e}, m\mathbf{h}] \tag{5.4*}$$

and

$$A^l(\varkappa) = \frac{-\varkappa + i\sqrt{1-\varkappa^2}}{2\pi l - 2\arcsin\varkappa}. \tag{5.5}$$

The Fourier components for the closed trajectory α are

$$\mathbf{v}^l(\alpha) = \frac{1}{2}\left[\frac{\varepsilon - \mathbf{p}\mathbf{v}(\alpha) + \tfrac{1}{2}(\mathbf{v}(\alpha)m\mathbf{v}(\alpha))}{\hbar\omega_0}\right]^{1/2}\mathbf{S}, \quad l = 1,$$
$$\mathbf{v}^l(\alpha) = 0 \qquad l \neq 0, 1. \tag{5.6}$$

Here $\mathbf{v}(\alpha)$ is the average velocity on the trajectory α, defined in accordance with (4.6)[15]. The fraction in the expression for $\mathbf{v}^1$ is an integral of the motion and is of the order of unity.

As shown in[15], there are no closed trajectories when $\kappa < 1/2$. The noise spectrum contains peaks at frequencies $l\hat{\omega}$, with intensities that decrease like $1/l^2$, in accord with (5.5). When $\kappa = 1/2$ there appear closed trajectories ($H_0' = H_0/2$), with $\omega_c = \hat{\omega}/6$. According to (5.6), this leads to the appearance not of a "comb," as in the general case, but of only one peak at the frequency ω_c. The intensity of this peak is proportional to the number of electrons in Ω_c, and therefore, in accordance with (4.11a)[15], it increases like $(\kappa - 1)^{5/2}$. When $\kappa = 1$, the principal trajectory becomes closed, and $\omega_c = \hat{\omega}/2$. Therefore, when H approaches H_0 from the low side, we have a "comb" at frequencies $l\hat{\omega} = 2l\omega_c$, and one additional peak at $\hat{\omega}/2 = \omega_c$. As soon as H becomes larger than H_0, the "comb" vanishes and there remains the one peak at the frequency ω_c.

It is possible to take into account the finite time of emission of the optical phonon in the case of $H = 0$ and for an isotropic effective mass. Then, as follows from (2.19)[13], the electrons penetrate into the active region $\epsilon(\mathbf{p}) > \hbar\omega_0$ to a depth $\Delta p \approx p_0(\tau_0/\tau_E)^{2/3}$. The solution of the eigenvalue equation (2.6) gives the width of the peak

$$\Gamma_l = a\hat{\omega}(\tau_0/\tau_E)^{2/3}l^2, \quad a \approx 1. \tag{5.7}$$

The ratio of the times is connected here with the depth of penetration, and peaks of the higher harmonics broaden more, since in their case more "fluctuation waves" of length p_0/l are contained in the length Δp, and the phase collapses more strongly. The smearing of the critical fields is determined from the estimate $\Delta\kappa \approx (\tau_0/\tau_E)^{1/2}$.

We note that the results of this investigation can be generalized in obvious fashion to the case of a many-valley semiconductor, for which the question of the invariance of the trajectories was considered in[18].

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*$[\mathbf{e}, m\mathbf{h}] \equiv \mathbf{e} \times m\mathbf{h}$.

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