ELECTROMAGNETIC EXCITATION OF SOUND IN A METALLIC PLATE

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We investigate the form and magnitude of the singularities that should be possessed by the surface impedance of a plate at frequencies corresponding to the formation of standing waves. These singularities are due to excitation of sound by an external electromagnetic wave as a result of interaction between the electrons and the lattice deformations. We consider the cases of strong external magnetic fields parallel and perpendicular to the plate. We show that the effects should be noticeable.

1. INTRODUCTION

THE deformation of a conducting medium is accompanied by the appearance of electromagnetic fields. The latter appear as a result of the fact that the electron system is taken out of equilibrium. In turn, the disturbance of the electrons by an external electromagnetic action leads to motion of the elastic medium. In the dynamic equations of elasticity theory, the interaction with electrons describe the well-known additional volume forces[1]. These equations, together with Maxwell's equations for the fields and the kinetic equation for the electron distribution function, form a coupled system that determines the spectrum of the corresponding excitations of the crystal. Usually the coupling between the fields and the motion of the medium is quite weak, and the branches of this spectrum can be classified as acoustic branches and various kinds of electromagnetic branches; the interaction of the excitations influences the phase velocities and the attenuations.

If electromagnetic waves are produced in a metallic sample, then their coupling with the sound oscillations causes transfer of a certain fraction of energy, generally speaking small, to excitation of the sound. However, this energy transfer can turn out to be appreciable in the case of a plate whose thickness is much smaller than the attenuation length of the sound. If the sound wave is in the required phase relation with the exciting electromagnetic waves at each reflection from the walls, then the amplitude of the sound increases by many times. It is obvious that the necessary phase conditions are satisfied for standing waves. The excitation of sound in the plate should therefore have a resonant character with respect to the frequency of the incident wave. Corresponding singularities should appear also in the dependence of the energy absorbed by the plate (or the surface impedance) on the frequency. The maximum values of these singularities may turn out to be of the order of the entire energy absorbed by the plate; one can then speak of effective sound generation.

We have been dealing so far with mutual transformation of weakly coupled branches of the spectrum in the system. In some cases, the interaction between the fields and the medium can be so strong that intersection of the spectral branches takes place. We have in mind the case when there exists in the metal a weakly damped helical wave, the length of which becomes comparable, at suitable values of the frequency ω and of the constant external magnetic field H_0, with the wavelength of the sound. For an infinite medium, such a problem was considered by Kaner and Skobov[2]. In the case of a plate and resonant excitation of standing waves outside the region of the branch intersection, the situation is analogous to that considered earlier. In the case of intersection of branches at the frequency of the standing wave, the corresponding singularity of the impedance should change strongly.

In a number of recent experimental investigations, resonance excitation of standing sound waves by an electromagnetic was observed in plates of bismuth (in a longitudinal field—parallel to the plate)[3] and aluminum (in a transverse field H_0)[4]. In our opinion, the physical nature of these effects is described by the foregoing mechanism. In the present paper we present a theoretical investigation of resonant (with respect to the standing wave) singularities of impedance, we analyze the shape of the resonance line as a function of the frequency ω and the field H_0, and estimate the coefficient of energy transfer to the sound wave. The sound wave produced in the plate is accompanied by an electromagnetic field characterized by an acoustic dispersion law. The amplitude of this field, which is weakly damped in the medium, may turn out to be noticeable under resonance conditions, particularly in the case when the main mechanism of the sound damping is electronic (in which case this field causes dissipation of the energy received by the sound). We shall estimate the amplitude of this field.

It should be noted that the mutual transformation of electromagnetic and sound waves in a half-space was investigated theoretically by Kontorovich and Tishchenko[5], who calculated the amplitudes of the excited waves. They, however, considered the case of a weak field H_0, which is of little interest (it does not influence the conductivity), and did not take into account the deformation force in the equation of motion of the medium. As will be shown below, the latter is usually more important in the case of a weak field than the induction force which was taken into account in[5].

It is noted in a recent paper by Quinn[6] that the experimental data of[4] can be explained with the aid of approximately the same picture as that described above, and the first steps were made there towards a theoretical calculation of the effect. Quinn, however, uses for the current an expression in which no account is taken of the induction part of the effective electric field $\dot{u} \times H_0/c$ ($\dot{u}$—rate of displacement of the medium), which plays in the case of a strong field a decisive role in the coupling between the electromagnetic and sound waves. In addition, the expression used in[6] for the force acting on the medium is actually suitable only under conditions of the normal skin effect. This can be readily seen by comparison with the expression rigorously derived by Kontorovich for the force[1].

2. SYSTEM OF EQUATIONS

The problem is described by the Maxwell equations

$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \qquad \mathrm{rot}\,\mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \tag{2.1}$$

by the kinetic equation for the electron distribution function[1]

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla + \Omega\frac{\partial}{\partial\varphi} + \hat{\nu}\right)\chi = e\mathbf{v}\hat{\mathbf{E}} - \Lambda_{ik}\dot{u}_{ik} \tag{2.2}$$

and by the equations of elasticity theory[1]

$$\rho\ddot{u}_i - \frac{\partial}{\partial x_k}c_{iklm}u_{lm} = \frac{1}{c}[\mathbf{j}\mathbf{H}_0]_i + \frac{\partial}{\partial x_k}\int d\tau_p\,\Lambda_{ik}\chi\frac{\partial f_0}{\partial\varepsilon} + \frac{m_0}{e}\frac{\partial j_i}{\partial t},$$
$$d\tau_p = 2h^{-3}d^3p. \tag{2.3*}$$

Here $\chi\partial f_0/\partial\epsilon$ is the non-equilibrium addition to the distribution function $f = f_0 + \chi\partial f_0/\partial\epsilon$, $\Omega = eH_0/mc$—cyclotron frequency, φ—phase angle; $\Lambda_{ik} = \lambda_{ik}(\mathbf{p}) - \overline{\lambda}_{ik}$, $\lambda_{ik}u_{ik}$—deformation addition to the electron dispersion law, $\overline{\lambda}_{ik}$—value of $\lambda(\mathbf{p})$ averaged over the Fermi surface; $\hat{\mathbf{E}}$—effective electric field:

$$\hat{\mathbf{E}} = \mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}_0] + \frac{1}{e}\nabla(\overline{\lambda}_{ik}u_{ik}). \tag{2.4}$$

It is assumed that it is possible to introduce a collision operator $\hat{\nu}$. In addition, it is assumed that the possible different carrier groups are described independently in analogous fashion.

In (2.3), c_{iklm} are the elastic moduli. For simplicity we confine ourselves to the case of a cubic crystal and the surface of a plate in the (100) plane. The right-side terms in (2.3) are the induction, deformation, and inertial forces (m_0—free electron mass).

The boundary conditions are: continuity of the tangential components of the electric and magnetic fields, continuity of the momentum flux density, and reflection conditions for the distribution function. An examination of the expression written out in[1] for the momentum flux density shows that the boundary conditions for the deformations are the same as customarily used in the theory of elasticity. The reflection conditions for χ at a plate thickness d exceeding the mean free path of the electrons l should not, as a rule, be of major significance. Assuming henceforth $d \gg l$, we shall disregard them completely for the sake of simplicity.

Inside the plate, all the quantities depend only on the coordinate z, reckoned from the inward normal from its surface. We shall consider a steady-state oscillation mode, when the time dependence of the fields and of the deformations is determined by the external perturbation ($\sim e^{-i\omega t}$, ω—frequency of the electromagnetic wave incident on the plate).

We shall solve the system of equations with the aid of a Fourier cosine transformation

$$f(z) = \frac{1}{d}\sum_{n=-\infty}^{\infty}\cos kz f^k; \qquad k = \frac{n\pi}{d}; \qquad n = 0, \pm 1, \ldots,$$
$$f^k = \int_0^d \cos kz f(z)\,dz. \tag{2.5}$$

To this end, we transform the kinetic equation into a second-order equation, introducing, just as in[7], the function $\psi = \chi(\mathbf{v}) - \chi(-\mathbf{v})$. From (2.2) we get

$$\left(\frac{\partial^2}{\partial z^2} - \hat{L}^2\right)\psi = -2\hat{L}\left(\frac{1}{v_z}e\mathbf{v}\hat{\mathbf{E}}\right) - \frac{2}{v_z}\Lambda_{iz}\frac{\partial^2\dot{u}_i}{\partial z^2},$$
$$\hat{L} = \frac{\Omega}{v_z}\frac{\partial}{\partial\varphi} + \frac{\hat{\nu} - i\omega}{v_z}. \tag{2.6}$$

With the aid of the function ψ, Eq. (2.3) also reduces to a second-order equation, since the deformation force is expressed in terms of an integral with respect to $d\tau_p$ of

$$\frac{\partial}{\partial z}(\chi(\mathbf{v}, z) + \chi(-\mathbf{v}, z)) = -\hat{L}\psi + \frac{2e\mathbf{v}\hat{\mathbf{E}}}{v_z}$$

(we used (2.2) in the derivation).

The use of the cosine transformation introduces explicitly the boundary values of the derivatives of the corresponding functions into the equations for the Fourier transforms. We shall consider the case of an unstressed plate: in this case $\partial u_i/\partial z|_{z=0,d} = 0$. As already mentioned, we omit the boundary values of $\partial\psi/\partial z$. As a result we get from (2.6), by the same method as in[7], a solution periodic in φ for the Fourier transform ψ:

$$\psi^k = -2\int_{-\infty}^{\varphi}d\varphi'\frac{1}{\Omega}\Big[-e\mathbf{v}'\hat{\mathbf{E}}^k\cos\Big(\int_{\varphi}^{\varphi'}\frac{kv_z''}{\Omega}d\varphi''\Big)$$
$$- k\dot{u}_j^k\Lambda_{jz}'\sin\Big(\int_{\varphi}^{\varphi'}\frac{kv_z''}{\Omega}d\varphi''\Big)\Big]\exp\Big(\int_{\varphi}^{\varphi'}\gamma\,d\varphi''\Big),$$
$$\hat{\gamma} = (\hat{\nu} - i\omega)/\Omega. \tag{2.7}$$

The expression for the Fourier transform of the current density j^k is

$$j_i^k = -\frac{e}{2}\int d\tau_p\psi^k\frac{\partial f_0}{\partial\varepsilon}v_i = \sigma_{ij}^k\hat{E}_j^k + p_{i,zj}^k\dot{u}_j^k,$$
$$\sigma_{ij}^k = -e^2\int d\tau_p\frac{\partial f_0}{\partial\varepsilon}v_i$$
$$\times\int_{-\infty}^{\varphi}d\varphi'\frac{1}{\Omega}v_j'\cos\Big(\int_{\varphi}^{\varphi'}\frac{kv_z''}{\Omega}d\varphi''\Big)\exp\Big(\int_{\varphi}^{\varphi'}\gamma\,d\varphi''\Big),$$
$$p_{i,jz}^k = -ek\int d\tau_p\frac{\partial f_0}{\partial\varepsilon}v_i\int_{-\infty}^{\varphi}d\varphi'\frac{1}{\Omega}\Lambda_{jz}'\sin\Big(\int_{\varphi}^{\varphi'}\frac{kv_z''}{\Omega}d\varphi''\Big)\exp\Big(\int_{\varphi}^{\varphi'}\gamma\,d\varphi''\Big). \tag{2.8}$$

We shall henceforth omit the indices k for simplicity. Equations (2.3) reduce to the form

$$(-\rho\omega^2 + k^2c_{izzi})u_i = \frac{1}{c}[\mathbf{j}\mathbf{H}_0]_i$$
$$+ p_{iz,j}\hat{E}_j - S_{izjz}\dot{u}_j - \frac{m_0 i\omega}{e}j_i. \tag{2.9}$$

The expressions for $p_{iz,j}$ differ from $p_{j,zi}$ in that v_i

*$[\mathbf{j}\mathbf{H}_0] \equiv \mathbf{j} \times \mathbf{H}_0$.

and Λ_{iz} are interchanged:

$$S_{izjz} = -k^2 \int d\tau_p \frac{\partial f_0}{\partial \varepsilon} \Lambda_{iz} \int_{-\infty}^{\varphi} d\varphi' \frac{1}{\Omega} \Lambda_{jz}' \cos\left(\int_{\varphi}^{\varphi'} \frac{k v_z''}{\Omega} d\varphi'' \right) \exp\left(\int_{\varphi}^{\varphi'} \gamma \, d\varphi'' \right). \quad (2.10)$$

After eliminating the field H and using the cosine transformation, Eqs. (2.1) yield

$$k^2 E_\alpha - \frac{4\pi i \omega}{c^2} j_\alpha = E_\alpha'(d) \cos kd - E_\alpha'(0);$$
$$j_z = 0; \quad \alpha = x,\ y; \quad E'(0,\ d) = \partial E / \partial z|_{z=0,d}. \quad (2.11)$$

The use of the neutrality condition $j_z = 0$ makes it possible to eliminate with the aid of (2.8) the field $\hat{E}_z^k$ from Eqs. (2.9) and (2.11). Then j, σ, p, and S are replaced by the "renormalized" quantities, which we designate by the symbol $\sim$:

$$\tilde{j}_\alpha = \tilde{\sigma}_{\alpha\beta} \hat{E}_\beta + \tilde{p}_{\alpha,\, zj} \dot{u}_j; \quad \tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma_{\alpha z}\sigma_{z\beta} / \sigma_{zz};$$
$$\tilde{p}_{\alpha,\, zj} = p_{\alpha,\, zj} - \frac{\sigma_{\alpha z} p_{z,\, zj}}{\sigma_{zz}}; \quad \tilde{S}_{izjz} = S_{izjz} + \frac{p_{iz,\, z} p_{z,\, zj}}{\sigma_{zz}}. \quad (2.12)$$

The subsequent analysis will apply to the cases when H_0 is parallel and perpendicular to the surface of the plate.

3. PLATE IN FIELD PARALLEL TO THE SURFACE

We direct H_0 along the x axis. Then it follows from symmetry considerations that the tensors σ_{ik}, $p_{i,kl}$, $p_{ik,l}$, and $S_{ik,lm}$, in which only one of the indices equals x, vanish; on the other hand, if only one of the indices equals y, the tensors reverse sign upon permutation (in p and S this is permutation through the comma); in all other cases the tensors are symmetrical. The equations then break up into two independent systems—two equations for E_x and u_x and three equations for E_y, u_z, and u_y. The first system eventually transforms into

$$\left(k^2 - \frac{4\pi i \omega}{c^2} \sigma_{xx}\right) E_x - \frac{4\pi \omega^2 \sigma_{xx}}{c^3} \sqrt{4\pi\rho s_t^2}\, \alpha_{tx} u_x = E_x'(d) \cos kd - E_x'(0),$$
$$\rho s_t^2 \left(k^2 - \frac{\omega^2}{s_t^2} - i\omega D_{tx}\right) u_x + \frac{\sigma_{xx}}{c} \sqrt{4\pi\rho s_t^2}\, \alpha_{tx}' E_x = 0. \quad (3.1)$$

Here s_t is the velocity of the transverse sound ($\rho s_t^2 = c_{xzzx} = c_{yzzy}$);

$$\alpha_{tx} = \frac{c}{\tilde{\sigma}_{xx}} \frac{\tilde{p}_{x,\, zx}}{\sqrt{4\pi\rho s_t^2}}, \quad \alpha_{tx}' = \alpha_{tx} - \frac{i\omega c m_0}{e s_t \sqrt{4\pi\rho}}, \quad D_{tx} = \frac{1}{\rho s_t^2} \tilde{S}_{xzxz}. \quad (3.2)$$

The second system consists of the following equations:

$$\left(k^2 - \frac{4\pi i \omega}{c^2} \tilde{\sigma}_{yy}\right) E_y - \frac{4\pi \omega^2 \tilde{\sigma}_{yy}}{c^3} \sqrt{4\pi\rho}\, (s_l \alpha_l u_z + s_t \alpha_{ty} u_y) = E_y'(d) \cos kd - E_y'(0),$$
$$\rho s_l^2 \left[k^2 - \frac{\omega^2}{s_l^2} - i\omega \left(D_l + \frac{4\pi \tilde{\sigma}_{yy}}{c^2} \alpha_l^2 \right) \right] u_z + i\omega S u_y + \frac{\tilde{\sigma}_{yy}}{c} \sqrt{4\pi\rho s_l^2}\, \alpha_l E_y = 0,$$
$$\rho s_t^2 \left(k^2 - \frac{\omega^2}{s_t^2} - i\omega D_{ty}\right) u_y - i\omega S u_z - \frac{\tilde{\sigma}_{yy}}{c} \sqrt{4\pi\rho s_t^2}\, \alpha_{ty}' E_y = 0. \quad (3.3)$$

Here s_l is the velocity of the longitudinal sound ($\rho s_l^2 = c_{zzzz}$),

$$\alpha_l = \frac{V_a}{s_l} \left[1 - \frac{c}{H_0} \frac{\tilde{p}_{y,\, zz}}{\tilde{\sigma}_{yy}} \right]; \quad D_l = \frac{1}{\rho s_l^2} \tilde{S}_{zzzz} - \frac{4\pi \tilde{\sigma}_{yy}}{c^2} \left(\alpha_l - \frac{V_a}{s_l} \right)^2,$$
$$S = \tilde{S}_{zy,\, zz} - \frac{H_0}{c} \tilde{p}_{y,\, zy} - \frac{i\omega m_0 H_0}{ec} \tilde{\sigma}_{yy}; \quad V_a^2 = \frac{H_0^2}{4\pi\rho}, \quad (3.4)$$

α_{ty} and D_{ty} are determined by formulas (3.2) with the index s replaced by y. In (3.4), V_a is the Alfvén velocity.

Let us turn to the analysis of the system (3.3). The predominant excitation of the longitudinal (u_z) or transverse (u_y) oscillations is determined by the ratio of the parameters α_l and α_t. These parameters can be estimated by using the expression for the deformation potential in the form:

$$\Lambda_{ik} = \frac{\lambda}{v^2} \left(v_i v_k - \frac{1}{3} v^2 \delta_{ik} \right), \quad (3.5)$$

which is suitable in the case of cubic symmetry[8] ($\lambda \sim \epsilon_0$—Fermi energy). Using (2.8) and (2.11), we can express $\tilde{p}$ and $\tilde{S}$ in terms of the values of σ. We present certain expressions obtained for the Fermi sphere:

$$\tilde{p}_{\alpha,\, z\alpha} = \frac{\lambda}{2\varepsilon_0} en \left(\frac{\tilde{\sigma}_{\alpha\alpha}}{\sigma_0} - 1 \right); \quad \tilde{p}_{y,\, zz} = \frac{\lambda}{2\varepsilon_0} en \left(\frac{\tilde{\sigma}_{yy}}{\gamma\sigma_0} + \frac{\sigma_{yz}}{\sigma_{zz}} \right);$$
$$D_l = \left(\frac{\lambda}{2\varepsilon_0} \right)^2 \frac{mn\Omega\gamma}{\rho s_l^2} \left[\frac{\sigma_0 \sigma_{yy}}{\sigma_{zz}\sigma_{yy}} - 1 - \frac{1}{3} k^2 l^2 \right]. \quad (3.6)$$

Here

$$\sigma_0 = \frac{e^2 n l}{m v_0}, \quad l = \frac{v_0 \tau}{1 - i\omega\tau},$$

l—mean free path, v_0—velocity on the Fermi surface, and $\tau = \hat{\nu}^{-1}$—relaxation time. With the aid of (3.6) and the known asymptotic values of σ (see, for example,[9]) it is easy to verify that in the strong magnetic field, i.e., at $\gamma = (1 - i\omega\tau)/\Omega\tau \ll 1$, the ratio $\alpha_l/\alpha_t \sim 1/\gamma^3$ at $kR \ll 1$ and $\sim 1/\gamma\sqrt{kR}$ at $kR > 1$ ($R = v_0/\Omega$—radius of the Larmor orbit). Thus, the transverse waves can be disregarded when $\gamma \ll 1$ and, in the case of $kR > 1$, when $\gamma^2 kR < 1$. To the contrary, in a weak magnetic field, when $\gamma \gg 1$ we have $\alpha_l/\alpha_t \sim 1/\gamma$ when $kl \ll 1$ and $\sim 1/\gamma kl$ when $kl > 1$. In this case the longitudinal waves do not play any role and can be left out from (3.3). Then the equations containing E_y and u_y practically coincide with (3.1) when $\gamma \gg 1$.

We shall consider the case of strong fields ($\gamma \ll 1$ and $\gamma^2 kR < 1$). For the Fourier components of E_y and u_z we get from the first two equations of (3.3), where the terms with u_y have been discarded (we omit the index l):

$$E_y = [E_y'(d) \cos kd - E_y'(0)] f_k / \Delta_k,$$
$$u_z = -[E_y'(d) \cos kd - E_y'(0)] 4\pi \tilde{\sigma}_{yy} \alpha / sc\Delta_k \sqrt{4\pi\rho}. \quad (3.7)$$

Here

$$\Delta_k = \left(k^2 - \frac{4\pi i \omega \tilde{\sigma}_{yy}}{c^2} \right) \left(k^2 - \frac{\omega^2}{s^2} - i\omega D \right) - \frac{4\pi i \omega \tilde{\sigma}_{yy}}{c^2} k^2 \alpha^2,$$
$$f_k = k^2 - \frac{\omega^2}{s^2} - i\omega \left(D + \frac{4\pi \tilde{\sigma}_{yy}}{c^2} \alpha^2 \right). \quad (3.8)$$

The equation $\Delta_k = 0$ is the dispersion equation for the case in question, and its roots determine the spectrum of the waves. The last term of (3.8) contains the parameter α^2, which is small in the actually realized cases ($\sim H_0^2/4\pi\rho s^2$ when $kR \ll 1$ and $\sim H_0^2 kR/4\pi\rho s^2$ when $kR > 1$ but $\gamma^2 kR < 1$). By virtue of the smallness of α^2, the roots of $\Delta_k = 0$ are close to the roots of the equations

$$k^2 = \frac{4\pi i \omega \tilde{\sigma}_{yy}(k)}{c^2} \left(1 + \frac{\alpha^2 k_e^2}{k_e^2 - \omega^2/s^2 - i\omega D} \right), \quad (3.9)$$

$$k^2 = \frac{\omega^2}{s^2} + i\omega\left[D + \alpha^2 \frac{4\pi\tilde{\sigma}_{yy}k_s^2}{c^2k_s^2 - 4\pi i\omega\tilde{\sigma}_{yy}}\right]. \qquad (3.10)$$

On the right side in (3.10), in the terms $\sim \alpha^2$, it is necessary to replace k throughout by k_e—the root of the equation $k^2c^2 = 4\pi i\omega\tilde{\sigma}_{yy}(k)$, and in (3.10) by $k_s = \omega/s$.

4. TRANSFER OF ENERGY TO THE SOUND OSCILLATIONS IN A STRONG LONGITUDINAL FIELD. SINGULARITIES OF SURFACE IMPEDANCE

Assume that a wave with E parallel to y is normally incident on the plate. The plate is located in an antinode of the magnetic field, i.e., $H(0) = H(d)$ and $E'(0) = E'(d)$ ($E' = -i\omega H/c$) (a consideration of an arbitrary phase shift between the waves on the surfaces $z = 0$ and $z = d$ entails no difficulties, and can lead only to a change in the resonant conditions for the excitation).

In the steady state, the energy W_s accumulated by the sound does not change. The energy $\dot{W}_s$ acquired per unit time from the electromagnetic field is dissipated as a result of the damping of the sound. It is easiest to determine $\dot{W}_s$ by distinctly separating the exciting forces in the equation of motion of the medium. Eliminating E_y from this equation, we get

$$\rho s^2\left[k^2 - \frac{\omega^2}{s^2} - i\omega\left(D + \alpha^2 \frac{4\pi\tilde{\sigma}_{yy}k^2}{c^2k^2 - 4\pi i\omega\tilde{\sigma}_{yy}}\right)\right]u_z = G_k. \qquad (4.1)$$

Here

$$G_k = E_0'(1 - \cos kd)\frac{\alpha c\tilde{\sigma}_{yy}\sqrt{4\pi\rho s^2}}{k^2c^2 - 4\pi i\omega\tilde{\sigma}_{yy}} \qquad (4.2)$$

is the Fourier component of the exciting force. The average energy transferred to the sound per unit time and per unit volume is determined by the equation

$$\dot{W}_s = \frac{1}{2d}\operatorname{Re}\int_0^d dz\,\dot{u}(z)G^*(z), \qquad (4.3)$$

which can be reduced with the aid of (2.5), (3.7), and (4.2) to the form

$$\dot{W}_s = -\frac{4\pi\omega}{d^2c^2}|E_0'|^2\operatorname{Re}\left\{i\sum_{n=-\infty}^{\infty}\alpha^2\tilde{\sigma}_{yy}^2\frac{1-\cos kd}{\Delta_k(k^2 + 4\pi i\omega\tilde{\sigma}_{yy}/c^2)}\right\},$$
$$k = n\pi/d \qquad (4.4)$$

Here and throughout we confine ourselves to the case when $\omega \ll \hat{\nu} = 1/\tau$—collision frequency; then σ and ρ can be regarded as real.

We shall use the Poisson summation formula (see, for example,[10]), according to which

$$\frac{1}{d}\sum_{n=-\infty}^{\infty}g\left(\frac{n\pi}{d}\right)(1-(-1)^n) = \sum_{m=-\infty}^{\infty}\{G(2md) - G[(2m+1)d]\},$$

$$G(x) = \frac{1}{\pi}\int_{-\infty}^{\infty}e^{ikx}g(k)\,dk. \qquad (4.5)$$

The values of G(x) can be obtained with the aid of the residue theory. In determining $\dot{W}_s$ we can omit the contribution of the poles of (3.9) corresponding to waves damped in the skin layer, since this value will ultimately contain the small parameter α^2. On the other hand, the fraction of $\dot{W}_s$ due to the weakly damped (damping length $\gg$ d) acoustic solutions of (3.10) will contain, besides α^2, also a resonant factor. By suitably closing the integration contour in (4.5), we obtain for the sought part of G, accurate to terms $\sim \alpha^2$:

$$G_s(x) = \frac{isc^4}{(4\pi)^2\omega^3}\frac{\alpha^2}{1+\beta^2}e^{ik_s|x|}. \qquad (4.6)$$

Here k_s is the root of the equation (3.10) with Im $k_s > 0$:

$$k_s = \frac{\omega}{s}\left(1 - q + \frac{i}{Q}\right), \quad q = \frac{\alpha^2}{2(1+\beta^2)},$$
$$\frac{1}{Q} = \frac{s^2D}{2\omega} + \frac{\alpha^2\beta}{2(1+\beta^2)}. \qquad (4.7)$$

By β we denote

$$\beta = \frac{\omega^2}{s^2}\frac{c^2}{4\pi\omega\tilde{\sigma}_{yy}(\omega/s)}. \qquad (4.8)$$

We present the values of the parameters α, β, and Q for the limiting cases $kR \ll 1$ and $kR > 1$, which can be readily obtained with the aid of (3.4), (3.6), and the asymptotic expressions for σ^{ik}[9]:

$$kR \ll 1: \quad \alpha = \frac{V_a}{s}, \quad \beta = \frac{\beta_0}{\gamma^2 + \sigma_{yz}^2/\sigma_0\sigma_{zz}},$$
$$Q^{-1} = \frac{\alpha^2}{2}\left[\frac{\beta}{1+\beta^2} + \left(\frac{\lambda}{2\varepsilon_0}\right)^2\frac{k_s^2R^2}{15\beta_0}\right],$$
$$kR > 1: \quad \alpha^2 = \frac{2}{9\pi}\left(\frac{\lambda}{2\varepsilon_0}\right)^2\frac{V_a^2}{s^2}kR(1 - \sin 4k_sR),$$
$$\beta = \frac{4}{3}k_sR\beta_0,$$
$$Q^{-1} = \frac{\alpha^2\beta}{2(1+\beta^2)} + \frac{1}{12}\frac{V_a^2}{s^2}\left(\frac{\lambda}{2\varepsilon_0}\right)^2\frac{k_sR}{\beta_0} \qquad (4.9)$$

(when $kR \ll 1$ in an ordinary metal $\sigma_{yz}^2/\sigma_0\sigma_{zz} = 1$, and in a metal with carrier compensation $\sigma_{yz} = 0$). Here

$$\beta_0 = \frac{\omega^2}{s^2}\delta_0^2, \quad \delta_0^2 = \frac{c^2}{4\pi\omega\sigma_0} \qquad (4.10)$$

δ_0—depth of skin layer in the normal skin effect without a magnetic field.

Carrying out the summation in (4.5), we obtain for the resonant part of $\dot{W}_s$:

$$\dot{W}_s = -|H(0)|^2\frac{s}{4\pi d}\frac{\alpha^2}{1+\beta^2}\operatorname{Re}\left(i\,\mathrm{tg}\frac{dk_s}{2}\right). \qquad (4.11)$$

In the vicinity of the resonance, i.e., when

$$\omega = \omega_n + \Delta\omega, \quad \frac{\omega_n d}{2s}(1-q) = \left(n + \frac{1}{2}\right)\pi, \quad \frac{\Delta\omega}{\omega} \ll 1, \quad (4.12)$$

we have

$$\dot{W}_s = \frac{|H(0)|^2\omega\alpha^2Q}{8\pi(1+\beta^2)\left(n+\frac{1}{2}\right)^2\pi^2[1 + (Q\Delta\omega/\omega_n)^2]}. \qquad (4.13)$$

Under the experimental conditions, the effect of excitation of sound by an electromagnetic field can be revealed by the singularities of the dependence of the energy absorbed by the crystal (or of the surface impedance) as a function of the frequency. Let us analyze these singularities. The time-averaged density of the electromagnetic-energy flux through the surface of the plate, absorbed per unit volume of the crystal, is

$$\dot{W} = -\frac{c}{8\pi d}\operatorname{Re}H^*(0)[E(0) - E(d)]. \qquad (4.14)$$

Expressing E(0) and E(d) in terms of (3.7) and (2.5), we get

$$\dot{W} = \frac{c}{8\pi d}|H(0)|^2\operatorname{Re}\zeta, \qquad (4.15)$$

where ζ—surface impedance of the plate:

$$\zeta=-\frac{2i\omega}{cd}\sum_{n=-\infty}^{\infty}\frac{f_k}{\Delta_k}[1-(-1)^n]$$
$$=-\frac{2i\omega}{c}\sum_{m=-\infty}^{\infty}\{F(2md)-F[(2m+1)d]\},$$
$$F(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}dke^{ikx}\frac{f_k}{\Delta_k}. \quad (4.16)$$

In determining F(x), the corrections $\sim\alpha^2$ for the poles of (3.9) can be omitted. The parts of F corresponding to these poles then coincide with the usual solution of the electromagnetic problem in the plate. We denote the contribution of these parts by ζ_e:

$$\operatorname{Re}\zeta_e\approx\frac{\omega}{c}\delta_H, \quad (4.17)$$

where δ_H is the depth of the skin layer in the magnetic field H_0. After calculations similar to those performed above, we obtain for the second part of ζ, connected with the excitation of the sound:

$$\zeta_s=2i\frac{s}{c}\alpha^2\frac{1}{(\beta-i)^2}\operatorname{tg}\frac{dk_s}{2}. \quad (4.18)$$

In the vicinity of the resonance (4.12) we have

$$\operatorname{Re}\zeta_s=2\frac{s}{c}\frac{\alpha^2Q}{(\beta^2+1)^2\pi(n+{}^1/_2)}\frac{1-\beta^2+2\beta Q\Delta\omega/\omega_n}{1+(Q\Delta\omega/\omega_n)^2}. \quad (4.19)$$

The function Re ζ_s vanishes when $\Delta\omega/\omega_n = -(1-\beta^2)/2\beta Q$; the curve is asymmetrical on the left and on the right of this point and reaches a maximum

$$\operatorname{Re}\zeta_{s\,\max}=2\frac{s}{c}\frac{\alpha^2Q}{(\beta^2+1)^2\pi\left(n+\frac{1}{2}\right)}\quad\text{at}\quad\frac{\Delta\omega}{\omega_n}=\frac{\beta}{Q}, \quad (4.20)$$

and a minimum

$$\operatorname{Re}\zeta_{s\,\min}=-\beta^2\operatorname{Re}\zeta_{s\,\max}\quad\text{at}\quad\frac{\Delta\omega}{\omega_n}=-\frac{1}{Q\beta}. \quad (4.21)$$

Far from resonance ζ_s is a small quantity $\sim\alpha^2$.

5. PLATE IN TRANSVERSE FIELD ($H_0 \parallel z$)

The symmetry conditions leave nonvanishing values for only those components of the tensors σ, p, and S in which either all the indices are equal to z, or two indices differ from z. In the latter case, the components with different indices (x and y) reverse sign upon permutation, and all the others are equal to one another. As a result, only E_x, E_y, u_x, and u_y are coupled with external field. The corresponding equations (2.9) and (2.11) are best written for circularly polarized fields and displacements. After simple transformations we obtain for a plate placed, as above, in an antinode of the magnetic field:

$$\left(k^2\mp\frac{4\pi\omega}{c^2}\sigma_\pm\right)E_\pm+\frac{4\pi\omega^2}{c^3}\sigma_\pm\sqrt{4\pi\rho s^2}\,a_\pm u_\pm$$
$$=-E_\pm'(0)(1-\cos kd),$$
$$\rho s^2\left[k^2-\frac{\omega^2}{s^2}\pm\omega\left(D_\pm-\frac{4\pi\sigma_\pm}{c^2}a_\pm a_\pm'\right)\right]u_\pm$$
$$+\frac{\sigma_\pm}{c}\sqrt{4\pi\rho s^2}\,a_\pm'E_\pm=0. \quad (5.1)$$

We have put here

$$E_\pm=E_x\pm iE_y,\quad u_\pm=u_x\pm iu_y,\quad \sigma_\pm=\sigma_{xy}\pm i\sigma_{xx},$$
$$a_\pm=\frac{V_a}{s}\left(1\pm i\frac{c}{H_0}\frac{p_\pm}{\sigma_\pm}\right),\quad a_\pm'=a_\pm\pm\frac{\omega V_a}{\Omega_0 s},$$
$$p_\pm=p_{x,zy}\pm ip_{x,zx},$$
$$D_\pm=\frac{S_\pm}{\rho s^2}+\frac{4\pi\sigma_\pm}{c^2}\left(a_\pm-\frac{V_a}{s}\right)^2,$$
$$S_\pm=S_{xzyz}\pm iS_{xzxz}; \quad (5.2)$$

$s = s_t$—transverse velocity of sound; $V_a = H_0/\sqrt{4\pi\rho}$, $\Omega_0 = eH_0/m_0c$—cyclotron frequency of the free electron.

The expressions for the Fourier transforms of the fields and displacements are:

$$E_\pm=-\frac{E_\pm'(0)(1-\cos kd)f_\pm}{\Delta_\pm},$$
$$u_\pm=\frac{E_\pm'(0)(1-\cos kd)4\pi\sigma_\pm a_\pm'}{\sqrt{4\pi\rho s^2}\,c\Delta_\pm} \quad (5.3)$$

where

$$\Delta_\pm=\left(k^2\mp\frac{4\pi\omega}{c^2}\sigma_\pm\right)\left(k^2-\frac{\omega^2}{s^2}\pm\omega D_\pm\right)\mp\frac{4\pi\omega\sigma_\pm}{c^2}a_\pm a_\pm'k^2,$$
$$f_\pm=k^2-\frac{\omega^2}{s^2}\pm\omega\left(D_\pm-\frac{4\pi\sigma_\pm}{c^2}a_\pm a_\pm'\right). \quad (5.4)$$

Further calculations are analogous to those made in Sec. 4. We write out the expression for the density of the energy absorbed by the plate per unit time:

$$\dot W=\frac{c}{8\pi d}\operatorname{Re}[\mathbf{EH}^*]_z|_0^d=\frac{c}{8\pi d}|H_+(0)|^2\operatorname{Re}\zeta,$$
$$\zeta=\zeta_++\zeta_-,\quad \zeta_\pm=-\frac{i\omega}{cd}\sum_{n=-\infty}^{\infty}[1-(-1)^n]\frac{f_\pm}{\Delta_\pm}$$
$$=-\frac{i\omega}{c}\sum_{m=-\infty}^{\infty}\{F_\pm(2md)-F_\pm[(2m+1)d]\};$$
$$F_\pm(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}dk\,e^{ikx}\frac{f_\pm}{\Delta_\pm}. \quad (5.5)$$

A decisive role is played in the calculation of the surface impedance by the pole singularities $f_\pm/\Delta_\pm$ corresponding to the roots of the equations $\Delta_\pm = 0$. Therefore, in determining F(x), we shall disregard the contribution due to the branch points of the integrand. The latter are connected with the fact that the quantity $\sigma_\pm$ for a Fermi sphere is equal to[9]:

$$\sigma_\pm=-\frac{3}{2}\sigma_0\frac{1}{kl}\left[a_\pm-\frac{1-a_\pm^2}{2}\ln\frac{a_\pm-1}{a_\pm+1}\right];$$
$$a_\pm=\frac{1\pm i\gamma}{kR}. \quad (5.6)$$

Analogous singularities are possessed by $p_\pm$ and $S_\pm$, which enter in $f_\pm$ and $\Delta_\pm$; it is thus easy to show with the aid of (2.8), (2.10), (3.5), and (5.2), that

$$p_\pm=\mp i\frac{\lambda}{2\varepsilon_0}en\left(1+\frac{1\pm i\gamma}{\gamma}\frac{\sigma_\pm}{\sigma_0}\right);$$
$$S_\pm=\mp i\frac{\lambda}{2\varepsilon_0}\frac{m\Omega}{e}(1\pm i\gamma)p_\pm.$$

Dispersion equations analogous to $\Delta_\pm = 0$ were investigated by Skobov and Kaner[2]. For "minus" polarization in a strong field H_0 ($\gamma \ll 1$), at a definite value of ω, it is possible that the branch of the weakly damped helical electromagnetic waves, which exists when $kR \ll 1$, will intersect the acoustic branch. In this case, in order to find the corresponding parts of F and ζ (5.5) it is necessary to use the exact expressions for the roots of $\Delta_- = 0$ in the region where the branches intersect. Let us consider this case.

When $kR \ll 1$ it is possible to disregard in f_- and Δ_- (5.4) the small contributions due to the deformation and inertial forces from (2.3), i.e., omit D_-, and use $\alpha = V_a/s$ in lieu of α_- and α'_-. Near the branch intersection

$$\frac{\omega^2}{s^2} = k_0^2(1+\alpha^2) + \frac{\omega^2}{s^2}\varepsilon, \quad \varepsilon \ll 1;$$
$$k_0^2 = \frac{4\pi\omega\sigma_0\gamma}{c^2} \tag{5.7}$$

(ϵ characterizes the "resonance detuning"). For the roots of $\Delta_- = 1$ we have

$$(k^-)^2_{1,2} = \frac{\omega^2}{s^2}\left\{1 + \frac{i\gamma - \varepsilon}{2} \pm \sqrt{\alpha^2 + \frac{(i\gamma-\varepsilon)^2}{4}}\right\}. \tag{5.8}$$

Calculations similar to those made in Sec. 4 yield for the considered part of the impedance ζ_-:

$$\zeta_- = -\frac{i\omega}{2c}\left\{\left[1 + \frac{i\gamma - \varepsilon - 2\alpha^2}{\sqrt{(i\gamma-\varepsilon)^2 + 4\alpha^2}}\right]\frac{1}{k_1^-}\operatorname{tg}\frac{dk_1^-}{2} + \left[1 - \frac{i\gamma - \varepsilon - 2\alpha^2}{\sqrt{(i\gamma-\varepsilon)^2 + 4\alpha^2}}\right]\frac{1}{k_2^-}\operatorname{tg}\frac{dk_2^-}{2}\right\}. \tag{5.9}$$

Here $k^-_{1,2}$ are the roots of (5.8) with $\operatorname{Im} k > 0$.

The detailed form of the dependence of ζ_- on ω near the establishment of the standing wave (i.e., at frequencies satisfying simultaneously the condition (5.7) with $\epsilon \ll 1$ and the resonant condition analogous to (4.12) for k_1^- or k_2^-) depends on the ratio of the small parameters α, γ, and ϵ, and is difficult to visualize in the general case. It is clear, however, that the height of the resonance is determined here essentially by the quality factor Q corresponding to the damping of the wave $k^-_{1,2}$, and not by the product $\alpha^2 Q$, as in the case of weak coupling between the sound and the field.

In the region of frequencies and fields far from the intersection, and also for waves with "plus" polarization, the contribution to the surface impedance from the electromagnetic and sound waves can be calculated independently, as was done in Sec. 4:

$$\zeta = \zeta_e + \zeta_s. \tag{5.10}$$

Here ζ_e is the electromagnetic part of the impedance. For waves damped in the skin layer (with polarization "minus"—in metals with equal electron and hole densities or in ordinary metals with $kR > 1$, and also for waves with "plus" polarization) this quantity is $\sim \omega/ck_e^\pm$, where $k_e^\pm$ are the roots of the equation

$$k^2 = \pm\frac{4\pi i\omega}{c^2}\sigma_\pm(k).$$

If a weakly damped helical wave can exist, then its contribution to ζ_e is

$$\zeta_e^- = -\frac{i\omega}{ck_e^-}\operatorname{tg}\frac{dk_e^-}{2}, \quad k_e^- = k_0\left(1 + \frac{i\gamma}{2}\right). \tag{5.11}$$

In the vicinity of the resonance corresponding to the condition

$$\frac{dk_0}{2} = \left(n + \frac{1}{2}\right)\pi + x, \quad x \ll 1, \tag{5.12}$$

we have

$$\zeta_e^- = \frac{\omega d(1+iy)}{c\gamma\left(n+\frac{1}{2}\right)^2\pi^2(1+y^2)}, \quad y = \frac{2x}{(n+{}^1/_2)\pi\gamma}. \tag{5.13}$$

In (5.10) we put

$$\zeta_s = \zeta_s^+ + \zeta_s^-, \quad \zeta_s^\pm = -i\frac{s}{c}\frac{\alpha_\pm^2}{(1+\beta_\pm)^2}\operatorname{tg}\frac{dk_s^\pm}{2};$$
$$\beta_\pm = \mp\frac{\omega^2c^2}{s^2 4\pi i\omega\sigma_\pm(\omega/s)}. \tag{5.14}$$

Here $\operatorname{Im} k_s^\pm > 0$,

$$(k_s^\pm)^2 = \frac{\omega^2}{s^2}\left[1 - \frac{\alpha_\pm\alpha_\pm'}{1+\beta_\pm} \mp \frac{s^2 D_\pm}{\omega}\right]. \tag{5.15}$$

The limiting values of the parameters $\alpha_\pm$ and $\beta_\pm$, and of $Q = \operatorname{Re} k_s^\pm \operatorname{Im} k_s^\pm$ are:

$$kR \ll 1: \quad \alpha = \frac{V_a}{s}, \quad \beta_\pm \approx \begin{cases} \pm\beta_0/\gamma, \\ i\beta_0/\gamma^2, \end{cases}$$
$$Q_\pm^{-1} = \frac{\alpha^2}{2}\begin{cases} \dfrac{\beta_0\gamma^2}{(\beta_0\pm\gamma)^2 + \gamma^2\beta_0^2} + \left(\dfrac{\lambda}{2\varepsilon_0}\right)^2\dfrac{k_s^2R^2\gamma^2}{5\beta_0}, \\ \dfrac{\beta_0\gamma^2}{\beta_0^2+\gamma^4}; \end{cases}$$
$$kR > 1: \quad \alpha = -\frac{4i}{3\pi}\frac{V_a}{s}\frac{\lambda}{2\varepsilon_0}k_sR, \quad \beta_\pm = \pm\frac{4i}{3\pi}\beta_0 k_s l,$$
$$Q_\pm^{-1} = \frac{2}{3\pi}\frac{V_a^2}{s^2}\left(\frac{\lambda}{2\varepsilon_0}\right)^2\frac{\gamma kR}{\beta_0}. \tag{5.16}$$

When $kR \ll 1$, the presented upper values in (5.16) correspond to a normal metal, and the lower ones to a metal with carrier compensation.

It follows from (5.14) and (5.16) that in the vicinity of the resonance, i.e., at frequencies ω satisfying the condition (4.12), the dependence of ζ_s on $\Delta\omega$ is different when $k_sR \ll 1$ for a normal metal and for a metal with compensation. For a normal metal, where $\beta_\pm$ is real, we have in this case the usual Lorentz curve:

$$\zeta_s^\pm \approx \frac{s}{c}\frac{V_a^2}{s^2}Q\left\{\left(n+\frac{1}{2}\right)\pi\left(1\pm\frac{\beta_0}{\gamma}\right)^2\left[1+\left(\frac{Q\Delta\omega}{\omega_n}\right)^2\right]\right\}^{-1} \tag{5.17}$$

(we recall that we are now considering a region far from the intersection of the branches, i.e., $|\beta_0/\gamma - 1| \sim \max(\beta_0/\gamma, 1)$).

For a metal with compensation, where $\beta_\pm = i\beta_0/\gamma^2$, the curve is analogous to that obtained in Sec. 4:

$$\operatorname{Re}\zeta_{s\,\max}^\pm \approx \frac{s}{c}\frac{\alpha^2 Q}{\left(n+\frac{1}{2}\right)\pi(1+\beta_0^2/\gamma^4)^2} \quad \text{at} \quad \frac{\Delta\omega}{\omega_n} = -\frac{\beta_0}{\gamma^2 Q},$$
$$\operatorname{Re}\zeta_{s\,\min}^\pm = -\frac{\beta_0^2}{\gamma^4}\operatorname{Re}\zeta_{s\,\max}^\pm \quad \text{at} \quad \frac{\Delta\omega}{\omega_n} = \frac{\gamma^2}{\beta_0 Q} \tag{5.18}$$

A dependence on $\Delta\omega$ similar to (5.18) characterizes also the singularities of the impedance at shorter wavelengths ($k_sR > 1$).

6. ESTIMATE OF THE MAGNITUDE OF THE SOUND EXCITATION EFFECT. ELECTROMAGNETIC FIELD INSIDE A PLATE UNDER RESONANCE CONDITIONS

A. Longitudinal field. The magnitude of the effect of sound excitation can be estimated by the ratio of $\zeta_{s\,\max}$ (4.20) to ζ_e (4.17) or of the resonant value $\dot{W}_s$ (4.13) to $\dot{W}$ (4.15) off resonance:

$$p_n = \frac{\zeta_{s\,\max}}{\zeta_+} = \frac{\alpha^2 Q}{(1+\beta^2)^2(n+{}^1/_2)^2\pi^2}\frac{d}{\delta_H},\quad \frac{\dot{W}_s}{\dot{W}} = p_n(1+\beta^2). \quad (6.1)$$

Here n is the number of the resonance in (4.12). The quantity p_n can be called the transfer coefficient.

It is obvious that the most favorable values of the parameter are $\beta \lesssim 1$. With the aid of (4.9) we can readily verify that in an ordinary metal (without carrier compensation) we have $\beta \ll 1$, and only at high frequencies ($\omega \sim 10^9$ Hz) can it approach unity. We recall that the results obtained for ζ are valid under the conditions used in Secs. 3 and 4:

$$\gamma = (\Omega\tau)^{-1} \ll 1, \quad \omega\tau \ll 1. \quad (6.2)$$

In the case of a metal with compensation and for $k_s R \ll 1$ we have $\sigma_{yz} = 0$ and β turns out to be larger by a factor γ^{-2}. In bismuth it increases even more, also as a result of the small concentrations and masses ($n \approx 3 \times 10^{17}$ cm^{-3} $m \approx 3 \times 10^{-29}$ g). Under real experimental conditions, when the plate dimension $d \approx 1-10^{-1}$ cm corresponds to first resonances with $\omega \approx 1-10$ MHz, the value of β (4.9) in bismuth at $k_s R \ll 1$ can be of the order of unity. To this end it is necessary to have the smallest of the possible values of H_0 satisfying the strong-field condition $\gamma \ll 1$, and large mean free paths l. With increasing field, β increases like H_0^2. The value of β can be of the order of unity also for larger frequencies. Then $k_s R > 1$ and resonances with $n \gg 1$ take place.

Equation (6.1) contains the value of the acoustic quality factor of the system Q. In the preceding analysis, we took into account only the electronic sound damping. Generally speaking, it is necessary to bear in mind also other damping mechanisms. They can be taken into account phenomenologically, by replacing in the obtained formulas $1/Q$ by $1/Q + 1/Q'$, where Q' is the "nonelectronic" quality factor, and Q is given by formulas (4.9). We consider first the case when $Q \gg Q'$ and it is necessary to introduce in (6.1) the "nonelectronic" quality factor Q', which does not depend on the field H_0. Then we have in accord with (6.1) and (4.9)

$$p_n \approx \frac{Q' H_0^2}{4\pi\rho s^2(1+\beta^2)^2(n+{}^1/_2)^2\pi^2} \times \begin{cases} d/\delta_H, & k_s R \ll 1, \\ \left(\dfrac{\lambda}{2\varepsilon_0}\right)^2 \dfrac{4}{9\pi}\dfrac{R}{\delta_H}\left(n+\dfrac{1}{2}\right)\pi(1-\sin 4k_s R), & k_s R > 1. \end{cases} \quad (6.3)$$

If Q' is such that p_n reaches values $\gtrsim 1$, then the effect of sound excitation is quite appreciable. In ordinary metals, as already noted, it is possible to omit β from (6.2). The dependence of p_n on H_0 is different for $k_s R \ll 1$ and $k_s R > 1$; it is also different for the cases of normal ($\delta_H > R$) and anomalous ($\delta_H < R$) skin effect. The common tendency for all the cases is the growth of p_n with increasing H_0, and the decrease of p_n with increasing frequency (increasing number n). The temperature dependence of p_n in ordinary metals should apparently be determined essentially by the variations of Q'.

In bismuth, the effect can be limited by the large value of β. Therefore, the initial growth of p_n with increasing field should subsequently give way to a decrease, when $\beta \sim H_0^2$ becomes much larger than unity. In the case of standing waves with large n, when $k_s R > 1$, there can appear, in accordance with (6.3), oscillations of the type of geometric resonance: the value of p_n depends on the ratio of the length of the sound wave (or thickness of the plate) and the dimension of the electron orbit.

If electronic damping of sound prevails, then it is necessary to use in (6.1) the value of Q from (4.9). We present tentative values of the transfer coefficient p_n for the case $k_s R \ll 1$, when the second deformation term predominates in Q^{-1} (4.9) in normal metals in not too strong fields, and the first (induction) term predominates in bismuth. For ordinary metals we have in this case

$$p_n \approx \frac{30}{(n+{}^1/_2)^2\pi^2}\left(\frac{2\varepsilon_0}{\lambda}\right)^2\frac{\delta_0^2 d}{R^2\delta_H} \quad (6.4)$$

(δ_0 and δ_H—depth of penetration in normal skin effect without a magnetic field and in the skin effect in a magnetic field). For the normal effect ($R \gg \delta$) we have $\delta_0 = \delta_H$ and $p_n \sim \delta_0 d/R^2 \gg 1$; for the anomalous skin effect ($R > \delta$) we have $\delta_0^2 \approx \delta_H^3/R$ and $p_n \sim \delta_H^2 d/R^3$. In the case of bismuth

$$p_n \approx 2d \,/\, (1+\beta^2)(n+{}^1/_2)^2\pi^2\beta\delta_H \quad (6.5)$$

the effect depends essentially on the value of β.

It was noted in the introduction that the electromagnetic wave accompanying the standing sound wave, which attenuates in space just as weakly as the sound, can possess a noticeable amplitude under resonance condition. Let us estimate this amplitude, using formulas (3.7) and (2.5) for this purpose. Calculations analogous to those made in Sec. 4 yield

$$E(z) = E_e(z) + E_s(z), \quad E_e(z) = \frac{iE_0'}{k_e}[e^{ik_e z} - e^{ik_e(d-z)}],$$
$$E_s(z) = E_0'\frac{\alpha^2}{k_s(\beta-i)^2}\frac{\sin k_s(z-d/2)}{\cos(k_s d/2)}. \quad (6.6)$$

Here E_e is the usual field that attenuates in the skin layer ($\mathrm{Im}\,k_e \sim 1/\delta_H$), and E_s is the sought field accompanying the sound wave. Under resonance conditions (4.12) we have

$$E_s(z) \approx -\frac{E_0' d\alpha^2 Q(-1)^n}{2(\beta-i)^2(n+{}^1/_2)^2\pi^2}\frac{\sin k_s(z-d/2)}{i+Q\Delta\omega/\omega_n}. \quad (6.7)$$

The ratio of the amplitudes E_s and E_e is in this case $\sim p_n$ (6.1). Therefore when an intense standing sound wave is excited there should exist in the bulk of the metal an electric field with amplitude comparable with the amplitude of the field in the skin layer (and even exceeding it when electronic damping predominates). The ratio of the corresponding amplitudes of the magnetic fields $\sim p_n k_s/k_e \approx p_n \beta^{1/2}$ can also turn out to be appreciable.

It is difficult for the time being to compare the results obtained above with the experimental preliminary data of Gantmakher and Dolgopolov[3], who observed excitation of sound in a bismuth plate in a longitudinal magnetic field. Apparently the acoustic Q of the samples was determined in[3] by the "nonelectronic" damping, since the value of Q estimated by the authors is much lower than the pure "electronic" Q under the same conditions. The general tendency of the line intensity to increase with the field H_0, followed by a decrease, agrees with the theory. The phenomenon of the locking of the generator frequency in the case of intensely developed sound can be attributed to the fact that the energy density stored in the acoustic system can exceed in the case of resonance the density of the energy W fed from the generator to the sample. Actually, calculation of W_S shows that under conditions (4.12) we have

$$W_s = \frac{1}{2d}\int_0^d dz\rho|\dot{u}|^2 \approx \frac{|H(0)|^2\alpha^2 Q^2}{16\pi(1+\beta^2)(n+1/2)^2\pi^2}. \quad (6.8)$$

After the generator is tuned away from the resonator frequency, the sample itself operates during the time of the transient ($t \sim W_S/\dot{W}_S \approx Q/\omega$) like a generator of appreciable intensity at the resonant frequency.

We note in conclusion that in a weak magnetic field, when $\gamma_> = (\Omega_< \tau)^{-1} \gg 1$, the effect of sound excitation, as shown by an analysis of the equations presented in Sec. 3, decrease in general by at least a factor $\gamma_<^{-2}$ ($\gamma_< = (\Omega_> \tau)^{-1} \ll 1$). Therefore the absence of the effect in In, which was noted in[3] is probably connected with the fact that the experiments were performed in fields that were weak for In.

B. Transverse Field. In this case the analysis for the resonances away from the branch-intersection region is in general similar to that made above (with the values of β replaced in (6.1) in accordance with the results obtained in Sec. 5). It is of interest to compare the height of the resonance lines for a standing helical wave (5.13) and a standing sound wave (5.17). If electronic damping predominates, then the ratio of these quantities is $\sim \beta_0/\gamma$, which usually is much less than unity and can become close to unity only in very strong fields and at high frequencies. Consequently, under conditions when electronic damping predominates, the acoustic resonances should be much more strongly pronounced. On the other hand, if the "nonelectronic" damping predominates in the acoustic system, then the relative magnitude of the peaks depends on the value of $Q\gamma\alpha^2$. If the lengths of the helical and sound waves are equal and a standing wave is established, the effect, as already noted, increases appreciably and the character of the resonance line can be analyzed by means of formula (5.9).

Experimental data on the excitation of the sound in the transverse field[4] have for the time being mainly a qualitative character and a detailed comparison with them is premature.

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