SPIN WAVES AND FERROMAGNETIC RESONANCE IN A FERRIMAGNET

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Submitted January 5, 1968

Zh. Eksp. Teor. Fiz. **54**, 1851–1864 (June, 1968)

We consider spin waves and correlation functions in a two-sublattice Heisenberg ferromagnet at temperatures below the transition point T_c. We obtain the damping of the spin waves and show that at large wavelengths the damping is small $T < T_c$, if the magnetomechanical ratios of the sublattices are equal ($\mu_1 = \mu_2$). When $\mu_1 \neq \mu_2$, the damping of the thin wave with wave vector $\mathbf{k} = 0$ is not equal to zero and is proportional to $(\mu_1 - \mu_2)^2$. The width of the ferromagnetic resonance in a ferrimagnet is analyzed qualitatively and estimated quantitatively.

1. INTRODUCTION

As is well known, in a ferrimagnet, a model of which consists of two spin sublattices between which there is antiferromagnetic ordering, there exist two branches of collective excitations. One of them is analogous to spin waves in a ferromagnet, and the other corresponds to optical oscillations in the spin system. In the usually considered region of low temperatures ($T \ll T_c$, where T_c—transition temperature) the occupation numbers of the optical excitations are exponentially small and therefore the influence of the latter on the kinetics of the spin waves can be neglected. In this case the relaxation processes in ferrimagnets and ferromagnets do not differ at all qualitatively, provided the magnetomechanical ratios for the spins of the different sublattices are equal ($\mu_1 = \mu_2$) or provided there is no external magnetic field. Calculations of spin-wave damping at low temperatures, performed for a ferromagnet but qualitatively suitable also for a ferrimagnet, are given, for example, in[1-4].

The damping of a spin wave with wave vector $\mathbf{k} = 0$ determines the line width of ferromagnetic resonance absorption. It should be noted that the width of the resonance curves in ferrite single crystals is connected with certain processes[3,5]. Different causes of the line broadening of ferromagnetic resonance (FMR) are discussed in numerous theoretical papers. The role of defects in an inhomogeneous magnetic structure, which cause the coupling between the homogeneous precession excited at resonance ($\mathbf{k} = 0$) with the inhomogeneous spin waves ($\mathbf{k} \neq 0$) has been explained in sufficient detail. Clogston et al.[6] considered the processes of scattering of homogeneous precession by microscopic magnetic fluctuations caused by the disordered distribution of the magnetic ions over the crystal-lattice sites of ferrite-spinels. Kittel et al.[7] investigated relaxation processes with participation of impurity magnetic ions having a large spin-lattice relaxation frequency and exchange-coupled with the spin system. A strong broadening of the resonance curves in samples with rough surface can be attributed[8] to the excitation of spin waves with $\mathbf{k}$ on the order of the reciprocal dimensions of the roughnesses.

The processes listed above make an appreciable contribution to the line width, but their influence can be weakened to a considerable degree[9,10] by performing the investigations on yttrium iron garnet single crystals grown with an insignificant impurity content (less than 10^{-7}) and with well polished surfaces. The nature of the apparently "true" line width observed in this case has not yet been sufficiently well studied. The processes of interaction between the homogeneous precession and the spin waves occur also in an ideal ferrodielectric (spin-spin relaxation)[1,2]. However, the results obtained by considering the mechanisms of spin-spin relaxation relaxation at low temperature[1,2] and extrapolated to room temperatures give a line width which is smaller by several orders of magnitude than necessary to explain the experiment[11].

In the vicinity of the Curie point, the line width increases sharply[5,9]. A hypothesis was advanced[7] that such an increase is due to relaxation processes connected with thermal fluctuations of the spontaneous moment. The contribution of the fluctuations of magnetization to the line width was taken into account phenomenologically[12,13] and it was shown under certain assumptions that the corresponding broadening is proportional to $(T_c - T)^{-1/2}$. A microscopic approach makes it possible to explain the role of the spin waves and magnetization fluctuations in the damping of the homogeneous precession, and to obtain for an ideal ferromagnet near T_c an analytic expression for the line width as a function of the temperature and the demagnetizing factors of the line shape[14].

In the present paper we investigate the influence of a two-sublattice structure of an ideal ferrodielectric on the kinetics of the spin waves in a broad temperature interval. It is clear from physical considerations that the structure of the ferrite cell, which contains several magnetic atoms with uncompensated spin does not change the concluded existence of long-wave excitations in the system at all temperatures below critical[15]. However, the temperature and frequency dependence of the damping of the spin waves in the ferrite can differ qualitatively from the corresponding dependence in a ferromagnet. This difference is connected with the following: 1) the existence of optical excitations, scattering from which, just as scattering from spin waves, is quite significant at $T \sim T_c$; 2) the spontaneous-moment inhomogeneity, which is proportional to the difference of the spontaneous moments of the sublattices and which influences the fluctuation scattering; 3) the inequality of the magnetomechanical ratios for the spins of the different sublattices ($\mu_1 \neq \mu_2$), which leads to an additional damping of the spin waves in an external magnetic field H. When $\mu_1 \neq \mu_2$, the frequencies of the inhomogeneous spin precession in the sublattices are different. But the exchange coupling prevents rotation of spins with different frequencies. As a result, the unique mutual "friction" of the sublattices in the system can lead to a finite damping of the spin wave even at $\mathbf{k} = 0$ in the absence of magnetic-anisotropy interactions between the sublattice spins. The exchange interaction V_0 should greatly decrease these broadening effects. For the reasons listed above, the relative role of the process of spin-wave scattering by the fluctuations of the spontaneous moment and by the collective excitations is different in different temperature and magnetic-field regions. The main purpose of the present paper is to consider these relaxation processes and to express the corresponding broadenings in terms of the microscopic parameters of the two-sublattice system.

The analysis is carried out with the aid of the temperature diagram technique developed in[15,16], for temperatures $T < T_c$ and magnetic fields $uH \ll V_0$, when antiferromagnetic ordering is present in the system and the concept of spin excitations is meaningful.

2. DIAGRAM TECHNIQUE FOR A TWO-SUBLATTICE SPIN SYSTEM

We consider a two-sublattice system, confining ourselves to the case of a magnetically-isotropic crystal. We denote by $\mathbf{r}_1$ and $\mathbf{r}_2$ the sites of the first and second sublattices and assume that antiferromagnetic interaction exists between the sublattices. We consider the case when the interaction inside the sublattices is small, as is the case, for example, in an yttrium iron garnet. The Hamiltonian of such a system, placed in an external magnetic field H, is

$$\mathcal{H} = \sum_{\mathbf{r}_1\mathbf{r}_2} V(\mathbf{r}_1-\mathbf{r}_2)\mathbf{S}_{\mathbf{r}_1}\mathbf{S}_{\mathbf{r}_2} - \sum_{\mathbf{r}_1}\mu_1\mathbf{H}\mathbf{S}_{\mathbf{r}_1} - \sum_{\mathbf{r}_2}\mu_2\mathbf{H}\mathbf{S}_{\mathbf{r}_2}, \quad (1)$$

where $\mathbf{S}_{\mathbf{r}_1}$ and $\mathbf{S}_{\mathbf{r}_2}$ are the spin operators at the sites $\mathbf{r}_1$ and $\mathbf{r}_2$; $\mu_1 S_1$ and $\mu_2 S_2$ are the magnetic moments, and $V(\mathbf{r}_1 - \mathbf{r}_2) > 0$.

We rewrite $\mathcal{H}$ in the form $\mathcal{H}_0 + \mathcal{H}'$,

$$\mathcal{H}_0 = -\sum_{\mathbf{r}_1\mathbf{r}_2} V(\mathbf{r}_1-\mathbf{r}_2)\sigma_1\sigma_2 - \sum_{\mathbf{r}_2}\mathbf{S}_{\mathbf{r}_2}\Big(\mu_2\mathbf{H} - \sum_{\mathbf{r}_1} V(\mathbf{r}_1-\mathbf{r}_2)\sigma_1\Big) - \sum_{\mathbf{r}_1}\mathbf{S}_{\mathbf{r}_1}\Big(\mu_1\mathbf{H} - \sum_{\mathbf{r}_2} V(\mathbf{r}_1-\mathbf{r}_2)\sigma_2\Big),$$

$$\mathcal{H}' = \sum_{\mathbf{r}_1\mathbf{r}_2} V(\mathbf{r}_1-\mathbf{r}_2)(\mathbf{S}_{\mathbf{r}_1}-\sigma_1)(\mathbf{S}_{\mathbf{r}_2}-\sigma_2).$$

Here H_0 corresponds to the zeroth approximation of the self-consistent field[15,16], and σ_1 and σ_2 denote the mean values of the spin moments of the sublattices.

The free energy F in the zeroth approximation of the self-consistent field is equal to

$$\beta F_0 = -(N_1N_2)^{1/2}\frac{(\mathbf{y}_1-\beta\mu_1\mathbf{H})(\mathbf{y}_2-\beta\mu_2\mathbf{H})}{\beta V_0} - N_1\ln\frac{\operatorname{sh}[(S_1+1/2)(\mathbf{y}_1\alpha_1)]}{\operatorname{sh}[(\mathbf{y}_1\alpha_1)/2]} - N_2\ln\frac{\operatorname{sh}[(S_2+1/2)(\mathbf{y}_2\alpha_2)]}{\operatorname{sh}[(\mathbf{y}_2\alpha_2)/2]};$$

$$\mathbf{y}_1 = \beta(\mu_1\mathbf{H} - V_0^{(12)}\sigma_2),\quad \mathbf{y}_2 = \beta(\mu_2\mathbf{H} - V_0^{(21)}\sigma_1),\quad \alpha_j = \mathbf{y}_j/y_j,\ j=1,2. \quad (2)$$

Here $\beta = 1/T$; N_1, N_2 — number of sites in the sublattices;

$$V_0^2 = V_0^{(12)}V_0^{(21)},\quad V_0^{(12)} = \sum_{\mathbf{r}_2} V(\mathbf{r}_1-\mathbf{r}_2),\quad V_0^{(21)} = \sum_{\mathbf{r}_1} V(\mathbf{r}_1-\mathbf{r}_2),$$

$$N_1V_0^{(12)} = N_2V_0^{(21)} = \sum_{\mathbf{r}_1\mathbf{r}_2} V(\mathbf{r}_1-\mathbf{r}_2).$$

The mean values σ_1 and σ_2 of the spins in the sublattices are obtained from the condition that the free energy F as a function of the variables $\mathbf{y}_j$ and α_j be a minimum:

$$\frac{\partial F}{\partial \mathbf{y}_j} = 0,\quad \partial\Big(F - \frac{1}{2}\sum_{j=1,2}\lambda_jN_j\alpha_j^2\Big)\Big/\partial\alpha_j = 0, \quad (3)$$

where λ_j are undetermined Lagrange multipliers. The system of equations (3) can be rewritten in the form

$$\sigma_1 = (\beta\mu_2\mathbf{H} - \mathbf{y}_2)/\beta V_0^{(21)} = \alpha_1 b_1(\mathbf{y}_1\alpha_1),$$
$$\sigma_2 = (\beta\mu_1\mathbf{H} - \mathbf{y}_1)/\beta V_0^{(12)} = \alpha_2 b_2(\mathbf{y}_2\alpha_2); \quad (4)$$

$$\sigma_j\mathbf{y}_j + \beta\lambda_j\alpha_j = 0,\qquad \alpha_j^2 = 1. \quad (5)$$

Here

$$b_j(y_j) = (S_j+1/2)\operatorname{cth}[(S_j+1/2)y_j] - 1/2\operatorname{cth}(1/2 y_j).$$

The system of equations (5) determines the equilibrium configurations of the spins $\alpha_j = \sigma_j/\sigma_j$[17]. Let the field H be directed along the z axis and let the spontaneous moments of the sublattices be such that $N_1\sigma_1 > N_2\sigma_2$. Then $\alpha_1^z = -\alpha_2^z = 1$ and $\alpha_j^x = \alpha_j^y = 0$, provided the following inequality is satisfied[17]

$$\mu_1\mu_2H \leqslant (N_1\mu_1\sigma_1 - N_2\mu_2\sigma_2)(N_1N_2)^{-1/2}V_0. \quad (6)$$

We confine ourselves to the case of weak fields H, when the inequality (6) is satisfied. Then antiferromagnetic ordering will take place in the system. The magnetization M in the system is in this case equal to

$$M = N_1\mu_1\sigma_1 - N_2\mu_2\sigma_2, \quad (7)$$

where

$$\sigma_j = b_j(y_j),\quad y_1 = \beta(\mu_1H + V_0^{(12)}\sigma_2),\quad y_2 = \beta(-\mu_2H + V_0^{(21)}\sigma_2). \quad (8)$$

In the absence of a magnetic field, $H = 0$, it follows from (8) that a second-order phase transition takes place in the system at a temperature T_c equal

$$T_c = 1/3[S_1S_2(S_1+1)(S_2+1)]^{1/2}V_0.$$

In accordance with (2), (7), and (8), the thermodynamic quantities vary near T_c, in agreement with the phenomenological theory. In particular, when $H = 0$ the magnetization M increases in proportion to $(T_c - T)^{1/2}$.

To calculate the next terms of the expansion of F, we employ the diagram technique described in[6]. Each connected diagram can be represented, in accordance with[16], in the form of single-cell blocks joined by the interaction lines $V(\mathbf{r}_1 - \mathbf{r}_2)$. Each interaction line joins the vertices of blocks belonging to different sublattices, either S_1^z or S_2^z, or $S_1^\pm$ with $S_2^\mp$, where $S^\pm = (S^x \pm iS^y)/\sqrt{2}$. The Fourier components of the single-cell blocks, calculated in[16], must now be labeled with the number of the sublattice. Accordingly, the Green's functions $G(i\omega n)$, used in[16], where $i\omega_n = 2\pi i n T$, have for each of the sublattices the form

$$G_1(i\omega_n) = (y_1 - i\beta\omega_n)^{-1},\qquad G_2(i\omega_n) = -(y_2 + i\beta\omega_n)^{-1}, \quad (9)$$

where the y_j are given by (8).

The spin temperature correlation functions are defined as:

$$K_{\nu\mu}^{(lj)}(\mathbf{k}, i\omega_n) = \frac{1}{2\beta}\int_{-\beta}^{\beta} e^{i\omega_n t}\,dt \sum_{\mathbf{r}_j} e^{i\mathbf{k}(\mathbf{r}_j-\mathbf{r}_l)}\langle \hat{T}(S_{\mathbf{r}_l}^{\nu}(t) - \sigma_l^{\nu})(S_{\mathbf{r}_j}^{\mu}(0) - \sigma_j^{\mu})\rangle.$$

Here $\hat{T}$—T-ordering symbol, $S^{\nu}(t) = \exp(\mathcal{H}_0 t) S^{\nu} \exp(-\mathcal{H}_0 t)$; the mean value $\langle \ldots \rangle$ denotes the trace of $\rho_0(\ldots)$ with $\rho_0 = \exp(-\beta\mathcal{H}_0)/\mathrm{Tr}\exp(-\beta\mathcal{H}_0)$; l and j are the sublattice indices.

The function $K_{\nu\mu}^{(lj)}$ is represented by the aggregate of all the singly connected diagrams with two vertices. Denoting by $\Sigma_{\nu\mu}^{(lj)}$ the aggregate of all the irreducible diagrams, we can write the correlation functions in the form

$$K_{\nu\mu}^{(lj)}(\mathbf{k}, i\omega_n) = \Sigma_{\nu\mu}^{(lj)}(\mathbf{k}, i\omega_n) - \Sigma_{\nu\gamma}^{(lp)}(\mathbf{k}, i\omega_n)\beta V_{\mathbf{k}}^{(pm)} K_{\gamma\mu}^{(mj)}(\mathbf{k}, i\omega_n). \quad (10)$$

Here

$$V_{\mathbf{k}}^{(lj)} = \sum_{\mathbf{r}_j} V(\mathbf{r}_l - \mathbf{r}_j) e^{i\mathbf{k}(\mathbf{r}_j - \mathbf{r}_l)}; \quad l, p, m, j = 1, 2;$$

and summation over pairs of identical indices is implied.

Solving the system (10) with respect to $K_{\nu\mu}^{(lj)}$, we obtain

$$K_{-+}^{(11)}(\mathbf{k}, i\omega_n) = \Sigma_{-+}^{(11)}(\mathbf{k}, i\omega_n)/D_{-+}(\mathbf{k}, i\omega_n),$$

$$K_{-+}^{(21)}(\mathbf{k}, i\omega_n) = \{\Sigma_{-+}^{(21)}(\mathbf{k}, i\omega_n) + \beta V_{\mathbf{k}}^{(21)}[\Sigma_{-+}^{(21)}(\mathbf{k}, i\omega_n)\Sigma_{-+}^{(12)}(\mathbf{k}, i\omega_n) - \Sigma_{-+}^{(11)}(\mathbf{k}, i\omega_n)\Sigma_{-+}^{(22)}(\mathbf{k}, i\omega_n)]\}/D_{-+}(\mathbf{k}, i\omega_n), \quad (11)$$

where

$$D_{-+}(\mathbf{k}, i\omega_n) = 1 + \Sigma_{-+}^{(12)}(\mathbf{k}, i\omega_n)\cdot\beta V_{\mathbf{k}}^{(21)} + \Sigma_{-+}^{(21)}(\mathbf{k}, i\omega_n)\cdot\beta V_{\mathbf{k}}^{(12)} + [\Sigma_{-+}^{(21)}(\mathbf{k}, i\omega_n)\Sigma_{-+}^{(12)}(\mathbf{k}, i\omega_n) - \Sigma_{-+}^{(11)}(\mathbf{k}, i\omega_n)\Sigma_{-+}^{(22)}(\mathbf{k}, i\omega_n)]\beta^2 V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)}.$$

The remaining $K_{\nu\mu}^{(lj)}$ are obtained from (11) by permuting the indices of the sublattices and interchanging the spin indices.

It was shown earlier[16] that in the case of a large interaction radius, and also in the case of low or high temperatures, it is sufficient to confine oneself for $\Sigma_{\nu\mu}$ to the simplest diagrams (see, for example,[16], Figs. 1a and b). In this approximation we have

$$\Sigma_{-+}^{(11)}(\mathbf{k}, i\omega_n) = b_1 G_1(i\omega_n), \quad \Sigma_{-+}^{(22)}(\mathbf{k}, i\omega_n) = -b_2 G_2(i\omega_n),$$

$$\Sigma_{zz}^{(11)}(\mathbf{k}, i\omega_n) = \delta_{n0} b_1', \quad \Sigma_{zz}^{(22)}(\mathbf{k}, i\omega_n) = \delta_{n0} b_2',$$

$$\Sigma_{\nu\mu}^{(12)}(\mathbf{k}, i\omega_n) = \Sigma_{\nu\mu}^{(21)}(\mathbf{k}, i\omega_n) = 0, \quad (12)$$

where b_j are given by (8), G_j by (9), $b_j' = \partial b_j(y_j)/\partial y_j$, and δ_{nm} is the Kronecker symbol for the corresponding frequency difference. Substituting (4) in (11) we obtain for the correlation functions

$$K_{-+}^{(11)}(\mathbf{k}, i\omega_n) = \frac{b_1 G_1(i\omega_n)}{1 + \beta^2 b_1 b_2 G_1(i\omega_n) G_2(i\omega_n) V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)}},$$

$$K_{-+}^{(22)}(\mathbf{k}, i\omega_n) = \frac{(-b_2) G_2(i\omega_n)}{1 + \beta^2 b_1 b_2 G_1(i\omega_n) G_2(i\omega_n) V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)}}. \quad (13a)$$

$$K_{zz}^{(11)}(\mathbf{k}, i\omega_n) = \frac{\delta_{n0} b_1'}{1 - \beta^2 b_1' b_2' V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)}},$$

$$K_{zz}^{(22)}(\mathbf{k}, i\omega_n) = \frac{\delta_{n0} b_2'}{1 - \beta^2 b_1' b_2' V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)}}. \quad (13b)$$

The excitation spectrum is determined by the poles of the analytic continuation of $K(\mathbf{k}, \omega)$ of the correlation function $K(\mathbf{k}, i\omega_n)$. Replacing in (13a) $i\omega_n \to \omega$, $\omega > 0$ and recognizing that $K_{-+}(\mathbf{k}, i\omega_n) = K_{+-}(\mathbf{k}, -i\omega_n)$, we obtain for the spin-wave spectrum in this approximation

$$\beta\omega_{\mathbf{k}} = \frac{y_1 - y_2}{2} + \left[\left(\frac{y_1 + y_2}{2}\right)^2 - \beta^2 b_1 b_2 V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)}\right]^{1/2},$$

$$\beta\tilde{\omega}_{\mathbf{k}} = \frac{y_2 - y_1}{2} + \left[\left(\frac{y_1 + y_2}{2}\right)^2 - \beta^2 b_1 b_2 V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)}\right]^{1/2}. \quad (14)$$

The excitation spectrum (14) coincides with the spectrum predicted by the phenomenological theory. The first branch ω_k is analogous to the spectrum of the ordinary ferromagnets and assumes for small k the form

$$\omega_{\mathbf{k}} = \mu H + bV_0(k^2 R_0^2/3);$$

$$\mu = \frac{N_1\mu_1 b_1 - N_2\mu_2 b_2}{N_1 b_1 - N_2 b_2}, \quad b = \frac{(N_1 N_2)^{1/2} b_1 b_2}{N_1 b_1 - N_2 b_2},$$

$$R_0^2 = r_0^2 (v_{c1} v_{c2})^{1/3} = \sum_{\mathbf{r}} r^2 V(\mathbf{r}) \Big/ \sum_{\mathbf{r}} V(\mathbf{r}), \quad (15)$$

v_{cj} is the volume of the unit cell of sublattice j.

The second branch $\tilde{\omega}_k$ is connected with the presence of two sublattices and is analogous to the optical oscillations in the spin-moment system. At small values of k we have

$$\tilde{\omega}_{\mathbf{k}} = (b_1 b_2/b) V_0 + bV_0(k^2 R_0^2/3). \quad (16)$$

In (15) and (16) it is assumed that $(b_1 b_2/b)V_0 \gg \mu_j H$, as is usually the case in ferrites.

To calculate the next terms of the expansion of $\Sigma_{\nu\mu}^{(lj)}$ it is convenient to introduce in lieu of the initial interaction V effective interactions that take into account the particle correlation in the presence of spin waves:

$$-\beta V_{+-}^{(11)}(\mathbf{k}, i\omega_n) = \beta V_{\mathbf{k}}^{(12)} K_{-+}^{(22)}(\mathbf{k}, i\omega_n)\beta V_{\mathbf{k}}^{(21)},$$

$$\beta V_{+-}^{(12)}(\mathbf{k}, i\omega_n) = \beta V_{\mathbf{k}}^{(12)} - \beta V_{\mathbf{k}}^{(12)} K_{-+}^{(21)}(\mathbf{k}, i\omega_n)\beta V_{\mathbf{k}}^{(12)}. \quad (17)$$

The remaining $V_{\nu\mu}^{(lj)}$ are obtained from (17) by interchanging the indices of the sublattices and the spin indices. The interaction $V_{zz}^{(lj)}$ joins the vertices s^z of the sublattices l and j and is represented by the dotted line. The interaction $V_{+-}^{(lj)}$ joins the vertex s^+ in sublattice l with vertex s^- in sublattice j, and is represented by a solid line.

3. SPIN-WAVE DAMPING

The damping of the spin waves is determined by the imaginary part $\Gamma(\mathbf{k})$ of the denominator of the correlation function K_{-+} (11). The first terms of the expansion of $\Sigma_{\nu\mu}^{(lj)}$ in the parameter r_0^{-3}, shown in Fig. 1, make the following contribution to the spin-wave damping:

$$\Gamma_1(\mathbf{k}) = \pi\sum_{\mathbf{q}} \frac{s_{\mathbf{k}}^2 c_{\mathbf{k}}^2}{1 - \beta^2 b_1' b_2' V_{\mathbf{k}-\mathbf{q}}^2}\left\{\frac{b_1' b_2}{N_1 b_1}(\eta_{\mathbf{k}} V_{\mathbf{k}-\mathbf{q}} - V_{\mathbf{q}})^2 + \frac{b_2' b_1}{N_2 b_2}(\eta_{\mathbf{k}}^{-1} V_{\mathbf{k}-\mathbf{q}} - V_{\mathbf{q}})^2 + \frac{2 b_1' b_2'}{(N_1 N_2)^{1/2}}[V_{\mathbf{q}} V_{\mathbf{k}-\mathbf{q}}(\eta_{\mathbf{k}} - 1)(\eta_{\mathbf{k}}^{-1} - 1) + (V_{\mathbf{k}-\mathbf{q}} - V_{\mathbf{q}})^2]\beta V_{\mathbf{k}-\mathbf{q}}\right\}\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{k}}), \quad (18)$$

where

$$s_{\mathbf{k}} = [(y_1 - \beta\omega_{\mathbf{k}})/\beta(\omega_{\mathbf{k}} + \tilde{\omega}_{\mathbf{k}})]^{1/2}, \quad c_{\mathbf{k}} = [(y_1 + \beta\omega_{\mathbf{k}})/\beta(\omega_{\mathbf{k}} + \omega_{\mathbf{k}})]^{1/2},$$

$$\eta_{\mathbf{k}} = (s_{\mathbf{k}}/c_{\mathbf{k}})(N_1 b_1/N_2 b_2)^{1/2}, \quad V_{\mathbf{k}} = (V_{\mathbf{k}}^{(12)} V_{\mathbf{k}}^{(21)})^{1/2}.$$

The damping determined by formula (18) is connected with the scattering of the spin waves by the fluctua-

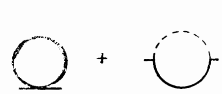

FIG. 1.

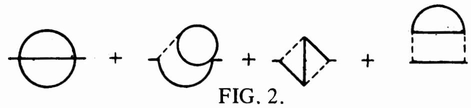

FIG. 2.

tions of the moment S^z. In the absence of a magnetic field, or when $\mu_1 = \mu_2$, it decreases in proportion to k^5. At low temperatures, the damping Γ_1 is exponentially small, and increases on approaching the transition point.

At low temperatures, an important role is played by the damping described by the next terms of the expansion in r_0^{-3} and corresponding to the scattering of spin waves by one another. The diagrams of this approximation are shown in Fig. 2. This figure does not show diagrams describing the corrections to the first approximation (18), for in the case of single-sublattice systems[15] these corrections do not change the dependence of the damping on the wave vector **k**.

The corrections of second order in r_0^{-3} in $\Sigma_{\nu\mu}^{(lj)}$ make the following contribution to the spin-wave damping:

$$\Gamma_2(\mathbf{k}) = \frac{\pi}{2N_1N_2}\sum_{\mathbf{pq}} A_{11}^2(\mathbf{k}, \mathbf{p}+\mathbf{q}-\mathbf{k}; \mathbf{p}, \mathbf{q})[n(\omega_{\mathbf{p}+\mathbf{q}-\mathbf{k}}) - n(\omega_{\mathbf{k}} + \omega_{\mathbf{p}+\mathbf{q}-\mathbf{k}})][1 + n(\omega_{\mathbf{p}}) + n(\omega_{\mathbf{q}})]\delta(\omega_{\mathbf{k}} + \omega_{\mathbf{p}+\mathbf{q}-\mathbf{k}} - \omega_{\mathbf{p}} - \omega_{\mathbf{q}})$$
$$+ \frac{\pi}{N_1N_2}\sum_{\mathbf{pq}} A_{12}^2(\mathbf{k}, \mathbf{p}+\mathbf{q}-\mathbf{k}; \mathbf{p}, \mathbf{q})[n(\tilde{\omega}_{\mathbf{p}+\mathbf{q}-\mathbf{k}}) - n(\omega_{\mathbf{k}} + \tilde{\omega}_{\mathbf{p}+\mathbf{q}-\mathbf{k}})][1 + n(\omega_{\mathbf{p}}) + n(\tilde{\omega}_{\mathbf{q}})]\delta(\omega_{\mathbf{k}} + \tilde{\omega}_{\mathbf{p}+\mathbf{q}-\mathbf{k}} - \omega_{\mathbf{p}} - \tilde{\omega}_{\mathbf{q}}) \quad (19)$$

where the amplitudes of the scattering of the spin wave by the first and second branches of the excitations are given respectively by the expressions

$$A_{11}(\mathbf{k}, \mathbf{p}+\mathbf{q}-\mathbf{k}; \mathbf{p}, \mathbf{q}) = s_{\mathbf{k}}c_{\mathbf{p}+\mathbf{q}-\mathbf{k}}s_{\mathbf{p}}c_{\mathbf{q}}(V_{\mathbf{k}-\mathbf{p}} - \eta_{\mathbf{k}}^{-1}V_{\mathbf{p}}) + s_{\mathbf{k}}c_{\mathbf{p}+\mathbf{q}-\mathbf{k}}c_{\mathbf{p}}s_{\mathbf{q}}(V_{\mathbf{k}-\mathbf{q}} - \eta_{\mathbf{k}}^{-1}V_{\mathbf{q}}) + c_{\mathbf{k}}s_{\mathbf{p}+\mathbf{q}-\mathbf{k}}c_{\mathbf{p}}s_{\mathbf{q}}(V_{\mathbf{k}-\mathbf{p}} - \eta_{\mathbf{k}}V_{\mathbf{p}}) + c_{\mathbf{k}}s_{\mathbf{p}+\mathbf{q}-\mathbf{k}}s_{\mathbf{p}}c_{\mathbf{q}}(V_{\mathbf{k}-\mathbf{q}} - \eta_{\mathbf{k}}V_{\mathbf{q}}),$$
$$A_{12}(\mathbf{k}, \mathbf{p}+\mathbf{q}-\mathbf{k}; \mathbf{p}, \mathbf{q}) = s_{\mathbf{k}}s_{\mathbf{p}+\mathbf{q}-\mathbf{k}}s_{\mathbf{p}}s_{\mathbf{q}}(V_{\mathbf{k}-\mathbf{p}} - \eta_{\mathbf{k}}^{-1}V_{\mathbf{p}}) + s_{\mathbf{k}}c_{\mathbf{p}+\mathbf{q}-\mathbf{k}}c_{\mathbf{p}}s_{\mathbf{q}}(V_{\mathbf{q}+\mathbf{p}} - \eta_{\mathbf{k}}^{-1}V_{\mathbf{p}+\mathbf{q}-\mathbf{k}}) + c_{\mathbf{k}}s_{\mathbf{p}+\mathbf{q}-\mathbf{k}}s_{\mathbf{p}}c_{\mathbf{q}}(V_{\mathbf{q}+\mathbf{p}} - \eta_{\mathbf{k}}V_{\mathbf{p}+\mathbf{q}-\mathbf{k}}) + c_{\mathbf{k}}c_{\mathbf{p}+\mathbf{q}-\mathbf{k}}c_{\mathbf{p}}c_{\mathbf{q}}(V_{\mathbf{k}-\mathbf{p}} - \eta_{\mathbf{k}}V_{\mathbf{p}});$$
$$n(\omega) = [\exp(\beta\omega) - 1]^{-1}. \quad (20)$$

We can write down analogously the damping $\tilde{\Gamma}_2(\mathbf{k})$ of the optical branch of the excitations. In this case $A_{11} \to A_{22} = A_{11}$, $A_{12} \to A_{21} = A_{22}$, and $\omega \to \tilde{\omega}$. In the case of small momenta, A_{11} and A_{12} are given by

$$A_{11}(\mathbf{k}, \mathbf{p}+\mathbf{q}-\mathbf{k}; \mathbf{p}, \mathbf{q}) = u_1u_2\left(\frac{V_0R_0^2}{3}\right)\left[2\mathbf{k}(\mathbf{p}+\mathbf{q}-\mathbf{k}) - u\left(\frac{u_1u_2}{b_1b_2}\right)^{1/2}\frac{(\mu_1-\mu_2)H}{V_0}(k^2 + (\mathbf{p}+\mathbf{q}-\mathbf{k})^2)\right],$$
$$A_{12}(\mathbf{k}, \mathbf{p}+\mathbf{q}-\mathbf{k}; \mathbf{p}, \mathbf{q}) = u_1u_2\left(\frac{V_0R_0^2}{3}\right)\left[2\mathbf{k}(\mathbf{k}-\mathbf{p}-\mathbf{q}) + \left(2 - \frac{1}{u_1} + \frac{1}{u_2}\right)(\mathbf{kp})\right] + u\left(\frac{u_1u_2}{b_1b_2}\right)^{1/2}(\mu_1-\mu_2)H;$$
$$u = u_1 + u_2,\ u_1 = \frac{N_1b_1}{N_1b_1 - N_2b_2},\ u_2 = \frac{N_2b_2}{N_1b_1 - N_2b_2},\ u_1u_2 = \frac{b^2}{b_1b_2} \quad (21)$$

It follows from (19) that when $T \ll (b_1b_2/b)V_0$ the damping of the spin waves due to the scattering by the optical branch is exponentially small. In this region of temperatures, at $\mu_1 = \mu_2$, the expression for Γ_2 is analogous to the corresponding expression for the single-sublattice system[15]

$$\text{a) } \omega_{\mathbf{k}} \gg T: \quad \Gamma_2(\mathbf{k}) = \frac{(kR_0)^3}{12r_0^6}\frac{b^{1/2}V_0}{(b_1b_2)^2}\left(\frac{3T}{4\pi V_0}\right)^{5/2} Z_{5/2}(\beta\mu H), \quad (22a)$$

$$\text{b) } \omega_{\mathbf{k}} \ll T: \quad \Gamma_2(\mathbf{k}) = \frac{3(kR_0)^2\omega_{\mathbf{k}}}{32\pi^3r_0^6}\left(\frac{T}{b_1b_2V_0}\right)^2 \cdot \quad (22b)$$

$$\cdot\begin{cases}\ln^2(T/\omega_{\mathbf{k}}) & \text{for } T \ll 2bV_0\\ \ln^2(2bV_0/\omega_{\mathbf{k}}) & \text{for } \omega_{\mathbf{k}} \ll 2bV_0 \ll T,\end{cases}$$

where

$$Z_\alpha(x) = \sum_{n=1}^{\infty} n^{-\alpha}\exp(-nx).$$

If $\mu_1 \neq \mu_2$ the damping of the spin wave does not vanish even when k = 0. From (19)–(21) it follows that in the temperature interval

$$\mu H \ll T \ll 2bV_0$$

the damping of the homogeneous precession is equal to

$$\Gamma_2(0) = \frac{9\mu H}{64\pi r_0^6}\frac{(u_1u_2)^{1/2}u^2}{(b_1b_2)^{1/2}}\left(\frac{T}{V_0}\right)^3\left(\frac{(\mu_1-\mu_2)H}{V_0}\right)^2|\ln(\beta\mu H)|. \quad (23)$$

At temperatures $T \gtrsim (b_1b_2/b)V_0$, it is necessary to take into account the scattering of the spin waves by the optical excitations. It is seen from (21) that this additional damping mechanism does not change the functional dependence $\Gamma_2(\mathbf{k}) \sim k^2\omega_{\mathbf{k}}$ if $\mu_1 = \mu_2$. In the case of inequality of the magnetomechanical ratios, the scattering by the optical excitations also yields a damping proportional to $\omega_{\mathbf{k}}(\mu_1 - \mu_2)^2H^2V_0^{-2}$. Since it follows from experiment that $|\mu_1 - \mu_2|HV_0^{-1} < 10^{-2}$, the relative spin-wave damping $\Gamma_2(\mathbf{k})/\omega_{\mathbf{k}}$ for long spin waves, $kR_0 \ll 1$, is small when $T \ll T_c$. On the basis of (19)–(21) we can draw the same conclusion concerning the damping of the long-wave optical excitations.

When the transition point is approached, the fluctuation damping $\Gamma_1(\mathbf{k})$ and the damping $\Gamma_2(\mathbf{k})$ increase. To calculate $\Gamma_1(\mathbf{k})$ and $\Gamma_2(\mathbf{k})$ when $T \to T_c$, we determine the values of $\sigma_1(T)$ and $\sigma_2(T)$ which enter in expressions (18)–(21). By solving Eqs. (8) at temperatures and fields such that $\tau = (T_c - T)T_c^{-1} \ll 1$ and $\mu HT_c^{-1} \cdot \tau^{-3/2} \ll 1$, we get

$$\sigma_1 = b_1 \approx a_1y_1 \approx \left(\frac{N_2}{N_1}\right)^{1/4}(a_1\theta\tau)^{1/2}, \quad \sigma_2 = b_2 \approx a_2y_2 \approx \left(\frac{N_1}{N_2}\right)^{1/4}(a_2\theta\tau)^{1/2}, \quad (24)$$

where $\theta = 6(N_1N_2)^{1/2}a_1^2a_2^2(N_1c_2a_1^2 + N_2c_1a_2^2)^{-1}$, $a_j = S_j(S_j+1)/3$, $c_j = a_j(6a_j+1)/10$.

Substituting in (18) the expansions (24) and the corresponding expansions for $c_{\mathbf{k}}$, $s_{\mathbf{k}}$, and $\eta_{\mathbf{k}}$ at $kR_0 \ll 1$, we get

$$\Gamma_1(\mathbf{k}) = T_cr_0^{-3}\tau^{-3/2}[\gamma_1(kR_0)^5 + \gamma_2(kR_0)((\mu_1-\mu_2)HT_c^{-1} + \gamma_3(kR_0)^2\tau^{1/2})^2];$$
$$\gamma_1 = \frac{7}{144\pi}\left(\frac{u_1u_2}{\theta}\right)^{1/2}\frac{1}{(a_1a_2)^{1/4}}, \quad \gamma_2 = \frac{3}{4\pi}\left(\frac{u_1u_2}{\theta}\right)^{3/2}(a_1a_2)^{1/4},$$
$$\gamma_3 = \frac{1}{6}\left(\frac{\theta}{u_1u_2}\right)^{1/2}\frac{u}{(a_1a_2)^{1/4}}. \quad (25)$$

It is seen from (25) that the damping increases rapidly near T_c. However, in the region of small k, $(kR_0)^2 < \tau$, the relative damping $\Gamma_1(\mathbf{k})/\omega_{\mathbf{k}}$ is small.

The damping $\Gamma_2(\mathbf{k})$ in the considered region of temperatures and fields can be written, in accordance with (19)–(21), in the form

$$\Gamma_2(\mathbf{k}) = \xi_1r_0^{-6}\tau^{-2}\omega_{\mathbf{k}}(kR_0)^2 + \xi_2r_0^{-6}\tau^{-3}\omega_{\mathbf{k}}(\mu_1-\mu_2)^2H^2T_c^{-2}, \quad (26)$$

where ξ_1 and ξ_2 depend on N_1, N_2, S_1, and S_2, and are slowly varying (logarithmic) functions of $(kR_0)^2$ and $\tau^{-1/2}\mu HT_c^{-1}$. For example, the contribution made in (26) by the mutual scattering of the spin waves is described by the second expression of formula (22b):

$$\Gamma_2(\mathbf{k}) = \frac{T_c}{8\pi^3r_0^6}\frac{(u_1u_2)^{1/2}}{(a_1a_2)^{1/4}}\frac{(kR_0)^4}{(\theta\tau)^{3/2}}\ln^2(kR_0) \text{ for } (kR_0)^2 \gg \mu HT_c^{-1}\tau^{-1/2}. \quad (27)$$

It follows from (26) and (27) that, just as in (25), in spite of the growth of the damping near T_c, for the longest spin waves with $k < R_0^{-1}\tau^{1/2}$ the relative damping is small when $T < T_c$.

We have considered above the first two approximations in the parameter $r_0^{-3}\tau^{-1/2}$ at temperatures near T_c such that this parameter is small. It can be assumed, however, that, just as in the case of a single-sublattice system, the dependence on k of the fluctuation damping and of the damping due to the scattering by the spin waves and optical excitations will have the same character also in higher order in the given parameter. In this case, in the absence of a field, the relative damping in the region under consideration is given by[15]

$$\operatorname{Im}\omega/\omega = f_1(r_0^{-3}\tau^{-1/2})(k^2R_0^2/\tau)^{3/2} + f_2(r_0^{-3}\tau^{-1/2})(k^2R_0^2/\tau)\ln^2(kR_0). \quad (28)$$

It is seen from (28) that in the region of applicability of the self-consistent field approximation the damping of the spin waves is small when $kR_0 < \tau^{1/2}$. The quantity $R_0\tau^{-1/2}$ represents in the self-consistent method the spin correlation radius $R_c(\tau)$ near T_c. It can therefore be assumed that the relative damping is small also outside the region of applicability of the self-consistent field method, provided $1/k \ll R_c(\tau)$. If $R_c(\tau) \sim \tau^{-2/3}$, as is indicated by phenomenological considerations, then $\operatorname{Im}\omega/\omega \ll 1$ when $(kR_0)^2 \ll \tau^{4/3}$ [15].

4. WIDTH OF FERROMAGNETIC RESONANCE IN FERRITES

We shall make a few remarks concerning the width of the FMR, i.e., the damping of the spin wave with $k = 0$. The damping of the homogeneous precession is determined by $\Gamma(0)$.

Without taking into account the effects of anisotropy and magnetic interaction, at $\mu_1 = \mu_2$ the damping $\Gamma(0) = 0$, as follows from (22), (23), (25), and (26). The difference of the magnetomechanical ratios of the sublattices leads to a nonzero width of FMR also without account of the anisotropy interactions in the two-sublattice system. In this case, when $T \ll (b_1b_2/b)V_0$, the damping $\Gamma(0)$ is given by (23). Near T_c, in accordance with (26), the FMR width is proportional to $(\mu_1 - \mu_2)^2\tau^{-3}\omega^3T_c^{-2}$, where $\omega = \mu H$. It is possible to estimate more accurately the order of magnitude of $\Gamma(0)$ if $u_1 \gg 1$ and $u_2 \gg 1$ (for yttrium iron garnet $u_1 \approx 5.7$ and $u_2 \approx 4.3$ when $\tau \ll 1$). The main contribution to the integrals of (19) is then made by small $q_2 \lesssim q_0^2$, $(q_0R_0)^2 = 3(u_1, u_2)^{1-}$. When $q^2 \lesssim q_0^2$, the quantities entering in the integrands are of the following order of magnitude: $\omega_q \approx bV_0(qR_0)^2/3$, $\tilde{\omega}_q \approx (b_1b_2/u_1u_2)^{1/2}V_0$, $c_q \approx u_1^{1/2}$, and $s_q \approx u_2^{1/2}$. As a result, in the temperature region $T \gg bV_0 \gg (b_1b_2/b)V_0$ the processes of scattering by spin waves and optical excitations make contributions of equal order to the FMR width:

$$\Gamma_2(0) \sim \frac{1}{10r_0^6}\left(\frac{\mu_1-\mu_2}{\mu}\right)^2\frac{(bT)^2\omega^3}{(b_1b_2V_0)^4}\ln\frac{b_1b_2V_0}{b\omega}, \quad \omega \ll (b_1b_2/b)V_0. \quad (29)$$

Near T_c, $\tau \ll 1$ we get from (29), with allowance for (24),

$$\Gamma_2(0) \sim \frac{(a_1a_2)^{1/2}}{10r_0^6}\left(\frac{\mu_1-\mu_2}{\mu}\right)^2\frac{u_1u_2\omega^3}{(\theta\tau)^3T_c^2}\ln\frac{T_c\theta^{1/2}}{\omega(a_1a_2)^{1/4}(u_1u_2)^{1/2}}. \quad (30)$$

The anisotropy effects can be taken into account by introducing into the Hamiltonian (1) the energy operator of the magneto-crystallographic anisotropy[17]. We confine ourselves for simplicity to the case of a uniaxial ferromagnet, when the anisotropy energy is given by

$$\mathcal{H}_a = \sum_{\mathbf{r}_1\mathbf{r}_2} V^{(a)}(\mathbf{r}_1 - \mathbf{r}_2)S^z_{\mathbf{r}_1}S^z_{\mathbf{r}_2}. \quad (31)$$

Then, according to Sec. 2, it is necessary to replace μ_1H, μ_2H, and V_{zz} throughout by respectively $\mu_1H + b_2V_0^{(a)}(N_2/N_1)^{1/2}$, $\mu_2H - V_1V_0^{(a)}(N_1/N_2)^{1/2}$, and $V_{zz} + V^{(a)}$. The difference $(\mu_1 - \mu_2)H$ in (29) is then replaced by $\delta(\mu H_{eff}) = (\mu H_{eff})_1 - (\mu H_{eff})_2$, which equals

$$\delta(\mu H_{eff}) = (\mu_1-\mu_2)H + (N_1N_2)^{-1/2}(N_1b_1+N_2b_2)V_0^{(a)}. \quad (32)$$

The broadening connected with the inhomogeneity of the effective field (32) has, in accordance with (29), an order of magnitude

$$\Gamma_2^{(a1)}(0) \sim \frac{1}{r_0^6}\left(\frac{V_0}{V_0}\right)^2\left(\frac{T}{V_0}\right)^2\frac{b^2\omega}{(b_1b_2)^3}\ln\frac{b_1b_2V_0}{b\omega} \quad (33)$$
$$|\mu_1-\mu_2|H \ll (N_1N_2)^{-1/2}(N_1b_1+N_2b_2)V_0^{(a)}.$$

The line width due to the decay of the spin wave with $k = 0$ in scattering processes in which three other spin waves take part equals, as follows from (19), where $A_{11} \approx 4s_0^2c_0^2V_0^{(a)}$

$$\Gamma_2^{(a2)}(0) \approx \frac{27}{48\pi r_0^6}\left(\frac{V_0^{(a)}}{V_0}\right)^2\left(\frac{T}{V_0}\right)^2\frac{bV_0}{(b_1b_2)}, \quad T \gg bV_0. \quad (34)$$

The damping $\Gamma_2(0)$, obtained above for a large interaction radius, when the parameter $r_0^{-3}\tau^{-1/2}$ is small, is appreciable if the fluctuation damping $\Gamma_1(0)$ is small, for example, when $T \ll T_c$. However, at temperatures on the order of the transition temperature, in the case of degeneracy of the energy of the homogeneous precession with the energy of the inhomogeneous spin waves ($\omega \equiv \omega_0 = \omega_p$), the damping $\Gamma_1(0)$ can become larger than or of the order of the damping $\Gamma_2(0)$. If the interaction radius is not large, $r_0^{-3} \sim 1$, then an expression of the type (28) where $k(R_0)^2$ must be replaced by a quantity on the order of the ratio of the dipole interactions the exchange interaction[14], is valid for the FMR line width, when $\tau \lesssim 1$. The terms from (28) can be separated experimentally, since the fluctuation damping depends essentially on the demagnetizing factors, which impose limitations on the possibility of energy transfer from the homogeneous precession to the inhomogeneous spin waves[11].

The contribution of the fluctuation damping to the FMR line width can be estimated in the first approximation in the parameter $r_0^{-3}\tau^{-1/2}$ with the aid of (18) and (25). It follows from (18) and (25) that the fluctuation width $(\Delta H)_1$, with allowance for the dipole interactions λ in the considered region of temperatures and fields ($\tau \ll 1$, $h \equiv \mu HT_c^{-1} \ll \tau^{3/2}$), can be written in the form

$$\mu(\Delta H)_1 \equiv \Gamma_1(0) = \rho_1\frac{\lambda}{r_0^3\tau^{3/2}}\left(\frac{\lambda}{V_0}\right)^{3/2}I_1(n_s) + \rho_2\frac{\lambda}{r_0^3\tau^{1/2}}\left(\frac{\lambda}{V_0}\right)^{3/2}I_2(n_s) + \rho_3\frac{(\mu_1-\mu_2)^2H^2}{r_0^3\tau^{3/2}T_c}\left(\frac{\lambda}{V_0}\right)^{1/2}I_3(n_s), \quad (35)$$

where $\rho_1 = (a_1a_2)^{1/2}\gamma_1$, $\rho_2 = (a_1a_2)^{1/2}\gamma_2\gamma_3^2$, $\rho_3 = \gamma_2$, and $I(n_s)$ is a certain function of the demagnetizing factors n_s, which determines the dependence of $(\Delta H)_1$ on the shape of the sample[11]. It is assumed in (35) that the condition $\mu H \gg b_j\lambda \sim \sqrt{\tau\lambda}$ is satisfied, so that the sample can be regarded as single-domain.

Let us estimate $(\Delta H)_1$ in a spherical sample of yttrium iron garnet. In this case $N_1/N_2 = 3/2$, $S_1 = S_2 = 5/2$, $T_c = 560°K$, $\lambda/\mu \sim 10^3$ Oe, $\lambda/V_0 \sim 10^{-3}$, $I(n_s) \sim 1$, and $r_0^{-3} \sim 1$. The inequality of the magneto-mechanical ratios in the case of yttrium iron garnet may be connected with the fact that the Fe^{3+} ions occupy non-equivalent positions in the crystal lattice. The difference between μ_1 or μ_2 and $2\mu_B$, where μ_B is the Bohr magneton, is due to spin-orbit interactions in the lattice. However, the ratio $|\mu_1 - \mu_2|/\mu$ is small, since the sublattices are made up of identical ions in the S-state. If we assume that the difference (3–6%) between the experimental and theoretical values of the magnetic moment[18] per formula unit of $Y_3Fe_5O_{12}$ is due to effects of spin-orbit coupling, then we can conclude that the ratio does not exceed several percent. As a result, the fluctuation width is of the order of

$$(\Delta H)_1 \sim (10^{-3}\tau^{-3/2} + 10^{-1}\tau^{-1/2} + 10^{-4}\tau^{-3/2}\mu H^2 T_c^{-1})\,\omega \tag{36}$$

and can reach several Oe when $\tau \sim 10^{-2}$ and $\omega \sim 10^4$ MHz, in agreement with the experimental data[5,19-21]. It is seen from (36) that when $\tau \gg 10^{-2}$ the value of $(\Delta H)_1$ depends little on H and is proportional to $\tau^{-1/2}$. When $\tau \sim 10^{-2}$ the width is $(\Delta H)_1 \sim \tau^{-3/2}$ and depends noticeably on H. In strong fields, however, the dependence of $(\Delta H)_1$ on H will not be quadratic, since the average spin moments σ_1 and σ_2 have a more complicated dependence on τ and h in the region of fields $h > \tau^{3/2}$[15]. The functions $y_1(\tau, h)$ and $y_2(\tau, h)$ can be obtained from the system (18), which takes near the transition the form

$$a_1\left(\frac{N_1}{N_2}\right)^{1/2} y_1^3 - \theta\tau y_1 - \frac{\theta h}{2u_1} = 0, \quad a_2\left(\frac{N_2}{N_1}\right)^{1/2} y_2^3 - \theta\tau y_2 - \frac{\theta h}{2u_2} = 0. \tag{37}$$

In the limiting case $1 \gg h \gg \tau^{3/2}$ we get from (37) that $y_1 \sim y_2 \sim h^{1/3}$. Therefore the fluctuation width at $h \gg \tau^{3/2}$ will be of the order of $(\Delta H)_1 \sim 10^{-3}$ H Oe.

In estimating the fluctuation width of the FMR in the yttrium iron garnet we did not take into account the crystallographic isotropy energy, which is very small: $V^{(a)}/\lambda \sim 10^{-2}$–$10^{-1}$. In the general case, introduction of the anisotropy energy, for example with the aid of (31), leads to an additional fluctuation width in the form

$$\mu(\Delta H)_{1a} = r_0^{-3}(V_0^{(a)})^2\lambda^{1/2}V_0^{-3/2}(\rho_4\tau^{-3/2}I_4(n_s) + \rho_5\tau^{-1/2}I_5(n_s)), \tag{38}$$

$$|\mu_1 - \mu_2|H \ll \tau^{1/2}V_0^{(a)}, \quad h \ll \tau^{3/2};$$

$$\rho_4 = \frac{3(a_1a_2)^{1/4}}{4\pi}\left(\frac{u_1u_2}{\theta}\right)^{1/2}, \quad \rho_5 = \rho_4 u^2.$$

It is seen from (38) that the second term, which is connected with the inhomogeneity (32) of the effective field, is larger than the first when $\tau > u^{-2}$ and the fluctuation width in this region of temperatures is proportional to $\tau^{-1/2}$.

The contribution to the width $(\Delta H)_2$ (in yttrium iron garnet) made by the scattering of the homogeneous precession by the spin waves and by the optical excitations, with allowance for dipole interaction and the inequality of the magnetomechanical ratios of the sublattices, does not depend, in accordance with (28) and (30), on the shape of the sample and its order of magnitude near the transition is

$$(\Delta H)_2 = \mu^{-1}\Gamma_2(0) \sim (10^{-3}\tau^{-1/2} + 10^{-4}\tau^{-3}\mu^2H^3T_c^{-2})\ \text{Oe} \tag{39}$$

When $\tau \sim 10^{-2}$ and $\omega \sim 10^4$ MHz, the width $(\Delta H)_2 \sim (\Delta H)_1 \sim 1$ Oe. With increasing τ, $(\Delta H)_2$ decreases rapidly, and the frequency dependence of the width $\Delta H = (\Delta H)_1 + (\Delta H)_2$ is determined by the fluctuation width $(\Delta H)_1$, inasmuch as $(\Delta H)_{2\omega}/(\Delta H)_{1\omega} \sim h\tau^{-3/2} \ll 1$. We can therefore expect in a sample in the form of a disc magnetized perpendicular to its surface ($I(n_s) = 0$) that the FMR width $\Delta H = (\Delta H)_2$ and that in the region of temperatures $\tau > 10^{-2}$ and fields $h < \tau^{3/2}$ it is much smaller than the corresponding width in a spherical sample or a disc magnetized parallel to its surface ($I(n_s) \sim 1$), and depends much less on the frequency. The experimental data[20] apparently confirm this fact.

In the field region $h \gg \tau^{3/2}$ the width is $(\Delta H)_2 \sim 10^{-2}$H and we can expect the FMR width in this temperature and field region to be proportional to the frequency and to be independent of the shape of the sample. The latter calls for experimental verification.

CONCLUSION

From the results obtained in the present investigation we can draw the following conclusions:

1. When the magnetomechanical ratios of the sublattices are equal, the spin waves, just as in the case of a single-sublattice system[15], exist near the transition for sufficiently small momenta.

With the aid of the diagram technique described in Sec. 1, it is easy to investigate a compensated antiferromagnet ($\mu_1 = \mu_2$, $S_1 = S_2$, $N_1 = N_2$), in which $\sigma_1 = \sigma_2 = \sigma$ in the absence of a magnetic field. In this case $\omega_k = \tilde{\omega}_k = \sigma V_0 k R_0/\sqrt{3}$, as seen from (19), and the damping of the spin waves is proportional to $r_0^{-3}\sigma^{-1}V_0(kR_0)^2$ or $r_0^{-6}\sigma^{-3}V_0(kR_0)^2$, as follows directly from (18) and (19). However, a compensated antiferromagnet calls for a special analysis, in view of the strong dependence of its properties on the anisotropic interactions.

2. The difference between the magnetomechanical ratios of the sublattices causes the damping of the homogeneous precession to be not equal to zero even without allowance for the anisotropic interactions: $\Gamma(0) \sim (\mu_1 - \mu_2)^2$. In ferrites that have no compensation points, $|\mu_1 - \mu_2|/\mu \lesssim 0.1$ and in weak fields, the inequality does not affect the existence of spin waves with $k^2 > \mu H/bV_0R_0^2$.

The behavior of the spin waves and the resonant properties of the ferrites in the vicinity of the compensation points $N_1\mu_1\sigma_1(T_k) = N_2\mu_2\sigma_2(T_k)$ can be investigated by the method described above.

3. The $\Delta H(\tau)$ dependence agrees qualitatively and in order of magnitude with the results of experiments[5,19-21] performed on the most perfect single crystals of yttrium iron garnet, the samples of which were thoroughly polished and contained insignificant amounts of impurities.

The difference between the magnetomechanical ratios of the sublattices can cause a frequency dependence of FMR width. The experimental data[9-21] agree with the obtained results. However, the order of magnitude of the ratio $|\mu_1 - \mu_2|/\mu$ used in the estimate of $\Delta H(\omega)$ can be regarded only as the upper limit of the values of μ_1 and μ_2. On the other hand, if $|\mu_1 - \mu_2|/\mu$ is smaller by one order of magnitude, as indicated by certain indirect determinations of the values of μ_1 and μ_2 in yttrium iron garnet[22-24], then the frequency dependence of the

FMR width becomes appreciable, owing to the foregoing effect, in fields $H \gtrsim 10^4$ Oe.

In conclusion, I am grateful to V. G. Vaks, A. I. Larkin, and A. G. Gurevich for interest in the work and useful discussions. I am sincerely grateful to B. E. Rubinshteĭn for calling my attention to[22-24].

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Translated by J. G. Adashko
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