CONTRIBUTION TO THE THEORY OF THE CONDUCTIVITY OF SEMICONDUCTORS IN STRONG MAGNETIC AND ELECTRIC FIELDS

V. F. ELESIN

Moscow Engineering-Physics Institute

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We consider the influence of an electric field that is not strong enough to heat the electrons on the conductivity of semiconductors in a quantizing magnetic field. Such an influence is due to the special sensitivity of the density of states to the electric field if the energy of the electrons contributing to the conductivity is close to the Landau level. By generalizing the method of Adams and Holstein[8] a formula is obtained for the current; this formula is valid for strong electric and magnetic fields. The field dependence of the current due to monoenergetic photoelectrons is calculated on the basis of this formula. It is shown that when the electron energy approaches the Landau level, the current reverses sign and depends on the field like $E^{-1/2}$. The current is calculated in the case of a degenerate electron gas. It is shown that when the Fermi energy is close to the Landau level, the current depends on the field like $E^{-1/2}$. This leads to a smoothing of the conductivity oscillations of the Shubnikov-de Haas type, as was apparently observed by Komatzubara[7].

It has been recently established experimentally that it is possible to produce in a semiconductor nonequilibrium electrons concentrated in a narrow energy interval by illuminating it with monochromatic light (see, for example[1]). Such a situation leads to a number of new effects[1,2] and is also useful because it makes it possible to reveal the subtle details of the physical phenomena which sometimes are inaccessible as a result of energy averaging. In particular, interest attaches to the behavior of the monoenergetic electrons in quantizing magnetic fields, since the states of electrons with energy close to the Landau levels turn out to be very sensitive to the electric field.

Indeed, in quantizing magnetic fields ($\hbar\Omega \gg kT$, $\Omega = eB/mc$), the density of states near the Landau levels becomes infinite, leading, for example, to oscillations of the transverse conductivity. Of course, allowance for the attenuation eliminates the divergence. However, a stronger effect can be produced by the decrease of the density of the states as a result of the electric field, since the electron energy depends on the field. We can therefore expect the conductivity to decrease with the field near resonance.

In the present paper we investigate the influence of the electric field on the transverse conductivity near resonance, when the energy of the electrons that contributes to the conductivity approaches the Landau level $\epsilon \to \Omega\hbar/2$. Such a problem arises in the consideration of negative conductivity due to monoenergetic photoelectrons in a quantizing magnetic field (see[3]), when the energy acquired by the electrons from the field becomes comparable with the quantity $\epsilon - \Omega\hbar/2$ (resonance), and also in a consideration of the equilibrium conductivity of degenerate electrons near the peaks of the Shubnikov—de Haas oscillations, if the energy transferred from the electrons from the electric field is comparable with $\xi - \Omega\hbar/2$, where ξ is the Fermi level.

It should be noted that the influence of a strong electric field on galvanomagnetic effects becomes manifest also in the heating of the electrons (the temperature of the electron gas changes close), if the energy acquired from the field exceeds the energy given up to the lattice. These phenomena were investigated by Kazarinov and Skobov[4], who assumed that the influence of the electric field on the density of states can be neglected in the absence of scattering. The equation for the density matrix was expanded in a series in the electric field (see also[5]), the current being expressed in terms of a symmetrical distribution function with an effective temperature. However, in the case of resonance, when $eE\Omega^{-1}\sqrt{\epsilon/m} > \epsilon - \hbar\Omega/2$, such an approximation is not valid. We therefore use in this paper a different approach. Namely, the expression for the current in the quantizing magnetic field and strong electric fields is obtained by generalizing the results of the work of Adams and Holstein[6]. On this basis, we analyze the field dependence of the current for the case of photoelectrons and equilibrium degenerate electrons.

We assume further a quadratic dispersion law for the electrons with effective mass m; the electron spin is neglected; the broadening of the Landau levels is disregarded. The electric field is assumed to be insufficient for heating.

1. CURRENT IN STRONG MAGNETIC AND ELECTRIC FIELDS

To find the density of the current in a quantizing magnetic field we shall use the results of[6], which admits of a simple generalization to the case of a strong electric field. The equation of motion for the density matrix

$$-i\partial\rho / \partial t = [\rho, H], \qquad H = H_0 + V \tag{1}$$

is solved with the aid of the Laplace transformation

$$P(s) = s\int_0^\infty e^{-st}\rho(t)\,dt.$$

Here V is the potential of elastic scattering;

$$H_0 = {}^1/_2(k_x^2 + k_z^2) + {}^1/_2(x - X)^2 - XF \qquad (2)$$

is the Hamiltonian of the electron in crossed electric and magnetic fields in dimensionless form (length is measured in units of $L = (\hbar c/eB)^{1/2}$, momentum in units of $\hbar/L$, energy in units of $\hbar\Omega$, and force in units of $\hbar\Omega/L$), where $X = k_y + F$—center of the Larmor orbit and F—force exerted on the electron by an electric field directed along the x axis; **B** is directed along the z axis.

The function P(s) satisfies the equation

$$-isP(s) = [P(s), H] - is\rho(0), \qquad (3)$$

where the initial density matrix is chosen in the form

$$\rho_{\mu\nu}(0) = \delta_{\mu\nu} f(\varepsilon_\mu) \equiv \delta_{\mu\nu} f_\mu. \qquad (4)$$

Here ϵ_μ is the part of the energy that does not depend on the electric field, and the Greek indices denote three quantum numbers $\mu = k_y, k_z, N$, $\epsilon_\mu = N + 1/2 + k_z^2/2$. The total electron energy is $E_\mu = \epsilon_\mu - X_\mu F$.

With the aid of Eq. (3) we can expand the function P(s) in a perturbation-theory series in powers of the scattering potential, and then calculate the current accurate to terms of first order in this parameter. The density matrix obtained in[6] in this manner

$$\rho_{N+1,N}(k_z) = -\pi i \sum_\nu V_{N+1,\nu} V_{\nu,N} \{[f(\varepsilon_{N+1}) - f(\varepsilon_\nu)] \times \delta(\varepsilon_{N+1,\nu} - X_{N+1,\nu}F) + [f(\varepsilon_N) - f(\varepsilon_\nu)]\,\delta(\varepsilon_{N\nu} - X_{N\nu}F)\}, \qquad (5)$$

$$\varepsilon_{\mu\nu} = \varepsilon_\mu - \varepsilon_\nu, \qquad X_{\mu\nu} = X_\mu - X_\nu$$

is expanded in a series in an electric field up to terms of first order, after which the current is calculated. It turns out that it is possible to calculate the current with the aid of (5) without resorting to expansion. Following a procedure similar to that used in[6], we obtain for the dissipative current in dimensionless form

$$j_x = \pi \sum_{\mu\nu} (f_\mu - f_\nu)\,\delta(\varepsilon_{\mu\nu} - X_{\mu\nu}F)\, q_y |V_{q\mu\nu}|^2, \qquad (6)$$

where $\mathbf{q} = \mathbf{k}_\mu - \mathbf{k}_\nu$, $V_{q,\mu\nu}$ is the matrix element of the scattering potential between the initial and final oscillator functions in the electric field.

Substituting $\nu \rightleftarrows \mu$ in the second term of (6) and then changing over from the variable k_z to the energy ϵ, we obtain finally

$$j_x = -\frac{2}{(2\pi)^2} \sum_{N,M,q_x,q_y} \int \frac{f(\varepsilon) q_y |V_{q,N,M}|^2 d\varepsilon}{(\varepsilon - N - {}^1/_2)^{1/2}(\varepsilon - M - {}^1/_2 + q_y F)^{1/2}}. \qquad (7)$$

The expression obtained from the current has a meaningful form that illustrates clearly the mechanism of transverse conductivity in a quantizing magnetic field. It is seen from (7) that the current is proportional to the scattering probability, the densities of the initial and final states, and the distribution function. The electric field enters in the density of the final states and makes transitions with gain and loss of energy not equally probable. This circumstance causes a directional motion along the field, i.e., an electric current. We call attention to the fact that the field enters in the density of states as a first-power term. We note in this connection that the Komatzubara's[7] interpretation of the experimental data on the field dependence of the character of the oscillations of the transverse conductivity, of the Shubnikov—de Haas type, is in error. He assumed that the main contribution to the density of state is made by the quadratic term.

2. PHOTOCURRENT IN A QUANTIZING MAGNETIC FIELD

In this section we determine the photocurrent due to nonequilibrium electrons produced in the conduction band under the influence of a monochromatic source $g(\epsilon - \omega)$. The equation for the density matrix has in this case the form

$$-i\partial\rho/\partial t = [\rho, H] - ig(\varepsilon - \omega) + i\rho/\tau_e, \qquad (8)$$

where ρ/τ_e describes the recombination of the electrons and τ_e is the lifetime, which is assumed small compared with the relaxation time of the photoelectrons on the acoustic phonons and with the time of electron-electron interactions[1]. It is assumed that the electron energy ω is smaller than the energy of the optical phonon, and thus the momentum is scattered by point defects $|V_q|^2 = C_1$.

We set the initial density matrix equal to

$$\rho_{\mu\nu}(0) = \delta_{\mu\nu} g(\varepsilon_\mu - \omega)\tau_e \qquad (9)$$

and neglect the broadening connected with the lifetime τ_e. Then we can use for the photocurrent the expression (7), with the substitution

$$f_\mu \to g(\varepsilon_\mu - \omega)\tau_e.$$

If we take the photoelectron distribution in the form of a δ-function[1])

$$g(\varepsilon_\mu - \omega) = \delta(\varepsilon_\mu - \omega),$$

then we get after integration with respect to the energy

$$j_x = \frac{2\tau_e}{(2\pi)^2} \sum_{N,M,q_x,q_y} \frac{q_y |V_{q,NM}|^2}{(\omega - N - {}^1/_2)^{1/2}(\omega - M - {}^1/_2 + q_y F)^{1/2}}. \qquad (10)$$

For simplicity we consider the case $1/2 < \omega < 3/2$. Putting in (10) N = M = 0 and integrating with respect to q_x, we obtain

$$j_x = \frac{\beta}{\sqrt{\omega - {}^1/_2}\sqrt{F}} \int_{-\eta}^{\infty} \frac{q\,dq\,e^{-q^2/2}}{\sqrt{\eta + q}}$$

$$= \frac{\beta}{\sqrt{\omega - {}^1/_2}\sqrt{F}} \exp(-\eta^2/4)[\Gamma({}^3/_2) D_{-{}^3/_2}(-\eta) - \Gamma({}^1/_2) D_{-{}^1/_2}(-\eta)] \qquad (11)$$

where $D_p(z)$ is the parabolic-cylinder function and

$$\eta = (\omega - {}^1/_2)/F, \quad \beta = 2\sqrt{2\pi} C_1 \tau_e (2\pi)^{-4}.$$

Let us investigate the limiting cases. In weak electric fields, when $\eta \gg 1$, simple calculations yield

$$j_x = -\frac{\beta F \sqrt{\pi/8}}{(\omega - {}^1/_2)^2}, \qquad (12)$$

or for the transverse conductivity in dimensional form (see[3])

$$\sigma_{xx} = -\sigma_0 \left(\frac{3\Omega}{32\omega}\right) \frac{\Omega^2}{(\omega - \Omega/2)^2}, \qquad (13)$$

where $\sigma_0 = 4e^2 Ik(\omega)\tau_e[3\Omega^2 \tau_{im} m]^{-1}$ is the "classical" photoconductivity at $\Omega\tau_{im} \gg 1$, τ_{im} the time of relaxa-

1) We disregard here the broadening of $g(\epsilon_\mu - \omega)$ due to the field.

tion on the defect, I the intensity of light, $k(\omega)$ the light absorption coefficient at $B = 0$. The minus sign denotes that the conductivity is negative, i.e., the current flowing in a direction opposite to the applied electric field. When $\omega \to 1/2$, an instant occurs when $\eta \ll 1$; in this case

$$j_x = \frac{\beta}{\sqrt{\omega - 1/2}\sqrt{F}} \frac{\Gamma(3/4)}{2^{1/4}} \tag{14}$$

and the current becomes positive.

Formula (10) for the current makes it possible to explain this effect completely, refining the quasiclassical interpretation proposed in[3]. Indeed, it is seen from (10) that the current is proportional to the number of transitions between the stationary states with energies ω and $\omega + q_y F$. In this case, the more probable transition is the one into the state with the lower energy, where the density of state is larger. The δ-like distribution function with $\omega > 1/2 + q_y F$ does not hinder this transition. Consequently, the electron energy in the electric field decreases during collision, i.e., the electron moves along the field, causing a negative current. If the energy ω approaches 1/2, so that $\eta < 1$, motion with a decrease of energy is impossible, and the current becomes positive.

It is easy to see from (10) that when ω increases the resonance situations occur whenever $\omega = \Omega(N + 1/2)$. On the other hand, at $\omega = \text{const}$ the resonance condition is reached by varying the magnetic field with a period

$$\Delta(1/B) = e/mc\omega. \tag{15}$$

The total dependence of the current on the field is given by formula (11).

3. CONDUCTIVITY OF DEGENERATE ELECTRONS

Let us consider degenerate electrons with a distribution function

$$f(\varepsilon) = \begin{cases} 1, & \varepsilon < \xi \\ 0, & \varepsilon > \xi. \end{cases}$$

We assume that the momentum is scattered by point defects, the electric field does not cause heating, and the Fermi level is $1/2 < \xi < 3/2$. We substitute $f(\epsilon)$ in (7) and integrate with respect to the energy ϵ and with respect to q_x with allowance for the assumptions made. As a result we get

$$j_x = \alpha\left\{\int_0^\infty q\,dq\,e^{-q^2/2}\ln\frac{\sqrt{\eta}+\sqrt{\eta+q}}{\sqrt{q}} - \int_0^\eta q\,dq\,e^{-q^2/2}\ln\frac{\sqrt{\eta}+\sqrt{\eta-q}}{\sqrt{q}}\right\}, \tag{16}$$

where

$$\eta = (\xi - 1/2)/F,$$
$$\alpha = 2\sqrt{2\pi}C_1(2\pi)^{-4}.$$

If the field is weak then, letting $\eta \to \infty$ in (16), we obtain the usual expression[6] for the conductivity

$$\sigma_{xx} = \alpha\sqrt{\pi/2}/(\xi - 1/2).$$

In the case of resonance, when $\eta \to 0$, we get from (16)

$$j_x = \alpha\sqrt{\frac{\xi - 1/2}{F}}\frac{\Gamma(3/4)}{2^{1/4}} = \frac{n\alpha}{2\sqrt{F}}\frac{(2\pi)^2}{2^{3/4}}\Gamma(3/4) \tag{17}$$

where n is the electron density

$$n = 2\sqrt{2}\sqrt{\xi - 1/2}/(2\pi)^2.$$

Consequently, the current near resonance depends on the field like $F^{-1/2}$, and decreases with increasing F. Experimentally this effect can appear in the following manner. With increasing electric field (yet remaining too weak for heating), the current in the peaks of the Shubnikov—de Haas oscillations increases more slowly (or even decreases, as in (17)) than far from resonance. Thus, the depth of the oscillations will decrease. It seems to us that this is precisely the phenomenon observed by Komatzubara[7]. We note that for the diffusion coefficients the results will be essentially different, since the electric field has no influence on the diffusion. Therefore, the Einstein relation will be violated near resonance even in the equilibrium state.

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