INTERFERENCE OF NONIDENTICAL PARTICLES

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The traditional viewpoint that there exists a sharp distinction between the behavior of identical and non-identical particles is criticized. The problem of the identity and distinguishability of particles is analyzed with the help of the interference principle. Interference phenomena observed in the registration of nonidentical particles A and B in correlation experiments are considered. The conditions for interference in correlation experiments with wave packets are investigated. It is shown that for $m_A - m_B \to 0$ the probability for recording nonidentical particles in the two counters can in some cases be described by the same formulas as in the case of identical particles. In the extreme relativistic limit the correlations between nonidentical particles have the same character as in the case of identical particles even when the difference between m_A and m_B is large. A general discussion of the concept of the complete wave function of the two particles A and B is presented. A new proof of the symmetry or antisymmetry of the total wave function is given which is based on the superposition principle.

1. INTRODUCTION

In traditional quantum mechanics it is asserted that there is a fundamental difference between the behavior of systems of identical and nonidentical particles, and that the transition from one system to the other is not at all continuous. We believe that this assertion is not, in general, correct. The strong evidence for the special behavior of a system of identical particles does not yet provide a basis for the categorical conclusion that there is no continuous transition from nonidentical particles to identical particles. The traditional argument, that connects the existence of an unsurmountable barrier between the properties of identical and nonidentical particles with certain specific features of quantum mechanics, can in our opinion be regarded as substantial neither in itself nor from a logical point of view. It seems to us that a more detailed analysis is needed of the concepts of indistinguishability, identity, and non-identity, as well as of the concrete phenomena in which the characteristic properties of identical particles show up. The present paper is devoted to such an analysis; we hope to develop the basic assertions of it further in subsequent publications (cf. also[1]).

Our investigation is based on the general quantum-mechanical principle of the interference of the amplitudes of indistinguishable processes. Guided by this principle, R. P. Feynman gave a deep and original treatment of the problem of identity in his lectures on quantum mechanics (cf.[2], Ch. 2). The present paper is an attempt to develop and generalize the Feynman approach.

2. INTERFERENCE AND THE PROBLEM OF IDENTITY

We recapitulate briefly the principle of the interference of the amplitudes of indistinguishable particles. Let us assume that the final state of interest to us (the triggering of a system of counters, the production of some particles, etc.) can be reached in several ways, each with the corresponding amplitude R_i. If the conditions are such that it is impossible to determine, even in principle, which way was chosen in any actual process, then interference occurs, i.e., the total amplitude is

$$R = \sum_i R_i, \tag{1}$$

and the probability for reaching the final state is

$$P = \Big| \sum_i R_i \Big|^2. \tag{2}$$

If the conditions of recording are such that after each measuring act we can in principle distinguish the "chosen path" and therefore, each recording act can be connected with the corresponding concrete path, then there is no interference, and the total counting probability is

$$P = \sum_i |R_i|^2. \tag{3}$$

In resolving the question under what conditions the "chosen paths" are distinguishable or not, the uncertainty principle plays an important role. Indeed, if we fix the coordinate of the particle with an accuracy Δx in the measuring process, then, according to the uncertainty principle, the amplitude for the transition to a state with momenta in the interval $\Delta p \lesssim \hbar/\Delta x$ must interfere. At the same time, states whose momenta differ by $\Delta p \gg \hbar/\Delta x$ are distinguishable under these conditions, and the corresponding transition amplitudes practically do not interfere. Let us now assume that the time for each measuring act is of the order Δt. Then, according to the uncertainty principle for energy and time, transition amplitudes correspond to states with energies differing by $\Delta E \lesssim \hbar/\Delta t$ interfere; for $\Delta E \gg \hbar/\Delta t$ the interference terms are extremely small. An analogous relation holds also in other cases, for example, between the accuracy of the determination of the scattering angle of a particle and the interference of transition amplitudes to states with different angular momenta.

Let us now turn directly to the problem of identity.

We consider the known experiment of the scattering of two "truly" [1)] identical particles. In this case the process as a result of which a given counter counts one of the particles cannot be distinguished from the process as a result of which the same counter counts the other particle.[2] Let the amplitude for the first process be $f(\theta)$, where θ is the scattering angle of one of the particles in the center-of-mass system. Then the amplitude for the second process (exchange amplitude) is $f(\pi-\theta)e^{i\delta}$. Owing to the indistinguishability, the total scattering amplitude has the form

$$A(\theta) \sim f(\theta) + f(\pi - \theta)e^{i\delta}. \tag{4}$$

Experiment shows that the phase δ must be set equal to zero for particles with integer spin and equal to π for particles with half-integer spin. In the framework of quantum mechanics this fact may be taken as a postulate which finds its consistent justification in quantum field theory.

Thus the differential cross section for the scattering of identical particles (cf., e.g.,[2,3]) is

$$d\sigma(\theta)/d\Omega \sim |f(\theta) \pm f(\pi - \theta)|^2. \tag{5}$$

We note that (5), with a different normalization, also describes the probability for recording two identical particles with two counters in coincidence.

In an analogous fashion we can consider the decay of any system into identical particles. In the most general case, where two or more identical particles are formed as result of one and the same collision or decay process, there is interference for any type of recording these particles. The amplitude for such a process is either symmetric or antisymmetric under the interchange of any pair of identical particles.

Let us now turn to nonidentical particles. If two particles have different internal quantum numbers (charge, spin, baryon number, etc.) and if these quantum numbers are conserved in the measuring process, then the direct and exchange processes are distinguishable and the corresponding amplitudes cannot interfere. In the concrete case of elastic scattering the probability for recording such particles with one counter as well as with two counters in coincidence is proportional to

$$\sigma(\theta) = |f(\theta)|^2 + |f(\pi-\theta)|^2, \tag{6}$$

where $f(\theta)$ is the scattering amplitude. However, numerous examples can be given where the differing quantum numbers of the first and second particles are not conserved in the measuring process. The question arises what one should expect in this case.

As a methodological example we consider the scattering of two electrons with opposite spin projection with respect to the direction of a magnetic field H, assuming that the scattering does not involve spin flip. Clearly, if the electrons after scattering are recorded with two selector counters in coincidence, one of which fixes the spin projection $m_H = +1/2$ and the other the spin projection $m_H = -1/2$, then there is no interference and the electrons behave like nonidentical particles. Assume now that each of the selector counters records a state with spin projection $+1/2$ on the x axis perpendicular to the direction of the magnetic field H. The state $|+x\rangle$ is a superposition of the states $|+H\rangle$ and $|-H\rangle$ (cf., e.g.,[2], ch. 5, sec. 5):

$$|+x\rangle = \frac{1}{\sqrt{2}}|+H\rangle + \frac{1}{\sqrt{2}}|-H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}. \tag{7}$$

Therefore the state $|+x\rangle$ can be registered only if the electron hitting the counter has been, before the act of recording, in the state $|+H\rangle$ as well as in the state $|-H\rangle$. In other words, the direct and exchange processes are indistinguishable and their amplitudes must interfere. The number of delayed coincidences when the first counter acts at the instant t_1 and the second at t_2, is proportional to

$$\begin{aligned} P(t_1, t_2) &= |f(\theta)e^{-i\mu Ht_1/\hbar}e^{i\mu Ht_2/\hbar}\langle +x|+H\rangle\langle +x|-H\rangle \\ &\quad - f(\pi-\theta)e^{-i\mu Ht_2/\hbar}e^{i\mu Ht_1/\hbar}\langle +x|-H\rangle\langle +x|+H\rangle|^2 \\ &= {}^1\!/_4\{|f(\theta)|^2 + |f(\pi-\theta)|^2 - 2\mathrm{Re}\,[f(\theta)f^*(\pi-\theta)e^{2i\mu Ht/\hbar}]\}, \end{aligned} \tag{8}$$

where μ is the magnetic moment of the electron, $t = t_2 - t_1$ is the retardation time, and $2\mu H$ is equal to the difference in the energy of the interaction of the states $|+H\rangle$ and $|-H\rangle$ with the magnetic field (the details of the calculation will be clear from the following).

The minus sign in (8) corresponds to the half-integer value of the electron spin; in a similar experiment with bosons interference with a plus sign would be observed. We note that if the resolution time of the coincidences is $\tau \gg h/\mu H$, then the interference term in (8) vanishes because of time averaging. The absence of a second detector is evidently equivalent to an averaging of (8) over an infinite time t_2. Hence, it is in principle impossible to observe interference when electrons with opposite spins are recorded by a single detector. This result is understandable from the interference principle formulated above. Indeed, by observing the second electron for an arbitrarily long time, we can determine what stationary state it is in, and can thus find out unambiguously which value of the spin projection corresponds to the electron captured in the apparatus.

Thus particles which under certain conditions behave as distinguishable, can interfere with each other under other conditions. It is true that electrons with opposite spins are usually regarded as identical although they are not "truly" identical. In the following sections we consider the conditions for the interference of particles which are manifestly nonidentical from the commonly accepted point of view.

3. CONDITIONS FOR THE INTERFERENCE IN THE RECORDING OF NONIDENTICAL PARTICLES IN CORRELATION EXPERIMENTS

We consider two generators, one of which emits a beam of particles of type A and the other, particles of type B. At the place where the beams meet, scattering occurs, and the scattered particles are recorded by two

[1)]We call particles "truly" identical if they agree with respect to all internal quantum numbers. Electrons with opposite spin projections are not "truly" identical particles from this point of view.

counters.[2)] It is clear from the preceding section that in this situation a necessary (though not sufficient) condition for observing interference of the nonidentical particles A and B is the nonconservation of the differing intrinsic quantum numbers of these particles in the measuring process. This means that each of the counters must be some sort of filter which separates a certain superposition of the particles A and B according to interaction or decay products. These superpositions may in general be different for the first and second counters. If the masses of the particles A and B are unequal, the states fixed by the counters are nonstationary. In analogy with the case of two electrons in a magnetic field considered above we may expect that there will be a temporal (and also a spatial or space-time) correlation between the counts of the two counters due to the interference of the amplitudes of the direct and exchange processes. We call direct that process as a result of which particle A arrives in counter 1 and particle B in counter 2; in the exchange process particle B arrives in counter 1 and particle A in counter 2.

Let us consider this problem in more detail. Assume that counter 1 singles out a state C by its decay or interaction products, and counter 2, the state D. The transition amplitudes $\langle C|A\rangle$, $\langle C|B\rangle$, $\langle D|A\rangle$, and $\langle D|B\rangle$ will be denoted by A_1, A_2, A_1', and A_2', respectively. Then

$$|C\rangle \sim A_1|A\rangle + A_2|B\rangle, \quad |D\rangle \sim A_1'|A\rangle + A_2'|B\rangle. \tag{9}$$

We assume for simplicity that each of the particles has a well-defined momentum, where the laboratory system coincides with the center-of-mass system of particles A and B. Then

$$\mathbf{p}_A = -\mathbf{p}_B = \mathbf{p}, \quad E_A = c\sqrt{p^2 + m_A^2c^2}, \quad E_B = c\sqrt{p^2 + m_B^2c^2},$$

where $p = |\mathbf{p}_A| = |\mathbf{p}_B|$. We denote the coordinates of the counters by $\mathbf{x}_1$ and $\mathbf{x}_2$ and the moments of registration of the particles in the first and second counters by t_1 and t_2.

The amplitude for the direct process must, according to the laws of quantum mechanics (cf.[2], Ch. 5, Secs. 1 and 2), have the form

$$F_1(\mathbf{p}, t_1, t_2) = A_1A_2'f(1, 2)\, e^{i(\mathbf{p},\, \mathbf{x}_1-\mathbf{x}_2)/\hbar} e^{-iE_At_1/\hbar} e^{-iE_Bt_2/\hbar}, \tag{10}$$

and the amplitude for the exchange process,

$$F_2(\mathbf{p}, t_1, t_2) = A_2A_1'f(2, 1)\, e^{i(\mathbf{p},\, \mathbf{x}_1-\mathbf{x}_2)/\hbar} e^{-iE_Bt_1/\hbar} e^{-iE_At/\hbar}. \tag{11}$$

Here $\mathbf{p}$ is the momentum of the particles hitting the first counter after the scattering, $f(1, 2) = f(\mathbf{p})$ is the amplitude for the formation, in the scattering act, of particle A with momentum $\mathbf{p}$ and particle B with momentum $(-\mathbf{p})$, and $f(2, 1)$ is the amplitude for formation of particle A with momentum $(-\mathbf{p})$ and particle B with momentum $\mathbf{p}$.

According to the interference principle the amplitude for the registration of particles A and B by the two counters in delayed coincidence is, up to a normalization factor,

$$\begin{aligned} F(\mathbf{p}, t_1, t_2) &= F_1(\mathbf{p}, t_1, t_2) + e^{i\delta}F_2(\mathbf{p}, t_1, t_2) \\ &= A_1A_2'f(1,2) \exp\left[-i(E_At_1 + E_Bt_2)/\hbar\right] \\ &\quad + e^{i\delta}A_2A_1'f(2,1) \exp\left[-i(E_At_2 + E_Bt_1)/\hbar\right] \end{aligned} \tag{12}$$

[the common phase factor $\exp(i\mathbf{p}\cdot\mathbf{x}_1 - i\mathbf{p}\cdot\mathbf{x}_2)$, which does not affect observed quantities, is omitted].

We introduce the postulate that $e^{i\delta} = +1$ if particles A and B have integer spin and $e^{i\delta} = -1$ if A and B have half-integer spin (the problem of the phase will be considered in more detail in Sec. 5). Assuming the resolution time of the coincidences τ small compared to $\hbar/|E_B - E_A|$, we obtain for the number of delayed coincidences

$$\begin{aligned} P(t_1, t_2) \sim \tau[&|A_1A_2'f(1,2)|^2 + |A_1'A_2f(2,1)|^2 \\ &\pm 2\,\mathrm{Re}\,(A_1A_2'A_2^*A_1'^*f(1,2)f(2,1)\,e^{-i(E_B-E_A)t/\hbar})], \end{aligned} \tag{13}$$

where $t = t_1 - t_2$ is the delay time (we assume that $t \gg \tau$). The plus sign in (13) corresponds to bosons, the minus sign to fermions.[3)]

We see that in our case, where $|\mathbf{p}_A| = |\mathbf{p}_B|$ the counting probability is independent of the coordinates, and hence of the dimensions of the counters. For $\tau \gg h/|E_B - E_A|$ the direct and exchange processes are distinguishable, since a sufficiently accurate measurement of the energy is possible during a long time; accordingly, the interference term in (13) becomes vanishingly small after integration over t. If $\tau < t \ll \hbar/|E_B - E_A|$, particles A and B behave like identical particles. We note that if the difference of the masses of particles A and B tends to zero, the quantity $\hbar/|E_B - E_A|$ and the retardation time t, satisfying the last inequality, can in principle be arbitrarily large.

It is easy to understand that in the registration of monochromatic particles in the general case, when the laboratory system does not coincide with the c.m.s., the absolute values $|\mathbf{p}_A|$ and $|\mathbf{p}_B|$ are not equal when the momenta of the scattered particles A and B have the same direction. The interference term then contains "beats" not only in the retardation time but also in the coordinate difference between the two counters. It is clear that interference is observed if the dimensions of the counters satisfy the condition

$$\Delta R \ll \hbar / \left||\mathbf{p}_A| - |\mathbf{p}_B|\right|.$$

If $\Delta R \gg \hbar/\|\mathbf{p}_A| - |\mathbf{p}_B\|$, the interference term vanishes after integration over $\mathbf{x}_1$ and $\mathbf{x}_2$. For $m_A \to m_B$ we have $E_A \to E_B$ and $|\mathbf{p}_A| \to |\mathbf{p}_B|$, and the exponential in the interference term evidently goes to unity; we thus obtain the same formulas as for identical particles.

4. CORRELATION EXPERIMENTS WITH WAVE PACKETS

We now turn to a less academic case, when particles A and B are represented by quantum-mechanical wave packets with the effective dimensions $\Delta x \sim \hbar/\Delta p$, where Δp is the uncertainty of the momentum. The collision region of the particles under consideration (or the region where they are produced) is evidently also localized

[2)]All that follows applies, with obvious modifications, also to the cases when inelastic scattering occurs in the region where the beams meet or when the particles A and B are produced in one and the same production process, for example, in a decay of the type $C \to A + B$.

[3)]If the first and second counters are completely identical ($A_1 = A_1'$, $A_2 = A_2'$), then (13) has the same structure as (8) for electrons with opposite spins.

with the accuracy Δx. We shall assume that Δx is much smaller than the dimensions of the counters and their distances from the effective collision regions.

It is known that the coordinate wave function of the wave packet has the form[4]

$$\psi(\mathbf{R}-\mathbf{v}t)=e^{i(\mathbf{pR}-Et)/\hbar}f(\mathbf{R}-\mathbf{v}t), \tag{14}$$

where $\mathbf{p}$ is the average momentum, $E = c\sqrt{p^2+(mc)^2}$, $\mathbf{v} = \mathbf{p}c^2/E$ is the group velocity of the wave packet, and $f(\mathbf{R}-\mathbf{v}t)$ is some sufficiently sharp function of its argument.[4)] Neglecting the dimensions of the wave packet, we can assert that the coordinate of the wave packet is related to the time passed from the moment of its creation by the classical formula $\mathbf{R} = \mathbf{v}t$. It is easy to see that for $\mathbf{R} = \mathbf{v}t$ the exponential in (14) has the form

$$\exp\left(-i\frac{Mc^2}{\hbar}\frac{R}{v\gamma}\right),$$

where $R = |\mathbf{R}|$, $v = |\mathbf{v}|$, M is the mass of the particle, and γ its Lorentz factor; the quantity $R/v\gamma$ has the meaning of the proper time of the particle (cf., e.g.,[5]). In the case of unstable particles M must be regarded as complex: $M = m - i\Gamma\hbar/2c^2$, where Γ is the decay constant.

Let us denote the average momenta of particles A and B passing through the first counter by $p_A^{(1)}$ and $p_B^{(1)}$, respectively; and the average momenta of A and B passing through the second counter, by $p_A^{(2)}$ and $p_B^{(2)}$. The following conditions must be satisfied if interference is to occur in the registration of the wave packets A and B by the two counters:

a) particles A and B have at least one common decay or interaction channel and both particles disappear as a result of the measurement;

b) the wave packets of A and B passing through the counters overlap in time and space and hence their group velocities are almost equal [$v_A^{(1)} \approx v_B^{(1)}$, $v_A^{(2)} \approx v_B^{(2)}$ up to terms of order $v\Delta x/R$];

c) the difference of the average momenta

$$|p_A^{(1)}-p_B^{(1)}| \sim |p_A^{(2)}-p_B^{(2)}| \ll \hbar/\Delta x,$$

i.e., is small compared to the uncertainty of each momentum. In other words, the wave packets must also overlap in momentum space. If the last condition is not fulfilled, then the relative phase of the wave packets A and B becomes indefinite because of the too large dimensions of the wave packet. The interference term containing the spatial beats vanishes after averaging over the dimensions of the wave packet, i.e., the interference is absent.

It is easy to see that the above-mentioned conditions can always be satisfied if Δm is sufficiently small (for extreme relativistic particles $\Delta m/\gamma$ plays the role of Δm). If they are fulfilled then the amplitude can be written in the form

$$F = F_1 \pm F_2,$$

where the amplitude for the direct process is

$$F_1 = A_1A_2'f(1,2)\exp\left[-\left(i\frac{m_Ac^2}{\hbar}+\frac{\Gamma_A}{2}\right)\frac{R_1}{v^{(1)}\gamma^{(1)}}\right]$$
$$\cdot\exp\left[-\left(i\frac{m_Bc^2}{\hbar}+\frac{\Gamma_B}{2}\right)\frac{R_2}{v^{(2)}\gamma^{(2)}}\right], \tag{15}$$

and the amplitude for the exchange process is

$$F_2 = A_2A_1'f(2,1)\exp\left[-\left(i\frac{m_Bc^2}{\hbar}+\frac{\Gamma_B}{2}\right)\frac{R_1}{v^{(1)}\gamma^{(1)}}\right]\cdot$$
$$\cdot\exp\left[-\left(i\frac{m_Ac^2}{\hbar}+\frac{\Gamma_A}{2}\right)\frac{R_2}{v^{(2)}\gamma^{(2)}}\right]. \tag{16}$$

Here $f(i, k)$ is the amplitude for the scattering (or production) of particles A and B corresponding to the emission of A in the direction of the i th counter and of B in the direction of the k th counter, the amplitudes A_1, A_1', A_2, and A_2' have the same meaning as above, and R_1 and R_2 are the distances of the counters from the scattering (or production) region of particles A and B.

Using (15) and (16), we obtain the following expression for the probability of retarded coincidences in the registration of particles A and B:

$$P = \int\limits_{\Delta R_1}\int\limits_{\Delta R_2} P(R_1,R_2)\,dR_1\,dR_2,$$

where

$$P(R_1,R_2) = |F|^2 = |A_1A_2'f(1,2)|^2e^{-(\Gamma_A\tau_1+\Gamma_B\tau_2)}$$
$$+|A_2A_1'f(2,1)|^2e^{-(\Gamma_A\tau_2+\Gamma_B\tau_1)} \pm 2\,\mathrm{Re}\left\{A_1A_2'A_1'^*A_2^*f(1,2)f^*(2,1)\right. \tag{17}$$
$$\left.\times\exp\left[-\frac{\Gamma_A+\Gamma_B}{2}(\tau_1+\tau_2)-\frac{i(m_A-m_B)}{\hbar}c^2(\tau_1-\tau_2)\right]\right\},$$

$\tau_1 = R_1/v^{(1)}\gamma^{(1)}$, $\tau_2 = R_2/v^{(2)}\gamma^{(2)}$, and the intervals ΔR_1 and ΔR_2 determine the effective "region of observation" (the dimensions of the counters, etc.).

If the region of observation has the dimensions

$$\Delta R_{(1),(2)} \gg \frac{\hbar v^{(1),(2)}\gamma^{(1),(2)}}{\Delta mc^2},$$

the interference term in (17) vanishes after integration. The quantities

$$l_{1,2} = \frac{\hbar v^{(1),(2)}\gamma^{(1),(2)}}{\Delta mc^2} \tag{18}$$

evidently have the meaning of periods of the spatial beats in the coordinates R_1 and R_2 (actually we are speaking of space-time beats).

If $A_1 = A_1'$ and $A_2 = A_2'$, then we have for $R_1 \ll l_1$, $R_2 \ll l_2$ and $|(\Gamma_A-\Gamma_B)(\tau_1-\tau_2)| \ll 1$

$$P(R_1,R_2) \sim |f(1,2)\pm f(2,1)|^2|A_1A_2|^2e^{-(\Gamma_A\tau_1+\Gamma_B\tau_2)} \sim$$
$$\sim |f(1,2)\pm f(2,1)|^2,$$

which agrees essentially with the corresponding formula for identical particles.

If the mass difference tends to zero, the periods of the spatial beats increase beyond limit. We can therefore assert that the difference of the complex masses is a kind of parameter for the transition from nonidentical to identical particles in correlation experiments with detectors each of which records a certain superposition of the particles A and B. For $\Delta m - i\Delta\Gamma\hbar/2c^2 \to 0$ nonidentical particles interfere in the same way as identical particles.

Another type of transition from nonidentical particles to identical particles occurs in the extreme relativistic limit, since in this case the periods of the beats l_1 and l_2 (and also the decay ranges) become arbitrarily large owing to the relativistic time dilatation effect. This means that in the extreme relativistic limit the corre-

[4)]For simplicity we do not take account of the spreading of the wave packet, which can often be neglected in actual cases (cf. [4]).

lations between nonidentical particles have the same character as in the case of identical particles, even if there is a large difference between the masses m_A and m_B (cf. also[1]).

We note that correlations of the type (17) have been considered earlier in the study of the properties of $K^0\overline{K}^0$ meson pairs.[6] In the production of $K^0\overline{K}^0$ pairs the short-lived K_1 meson plays the role of particle A and the long-lived K_2 meson plays the role of particle B. According to[6] interference occurs when the detectors fix the following linear superpositions of K_1 and K_2: $|K^0\rangle = (|K_1\rangle + |K_2\rangle)/\sqrt{2}$ or $|\overline{K}^0\rangle = (|K_1\rangle - |K_2\rangle)/\sqrt{2}$, in full accordance with the results of the present section.

Besides K_1 and K_2, other elementary particles may also play the role of particles A and B in correlation experiments, for example, π and η, ρ and ω, etc., as well as nuclei, atoms, and molecules with different excited levels. However, usually the mass difference $m_A - m_B$ is so large that interference can occur only at very high, in practice not attainable energies. An exception are (besides the neutral K mesons) atoms and molecules with close-lying energy levels corresponding to different components of the superfluid, Stark, or Zeeman splitting. The interference in the registration of the scattering of atoms or molecules by two detectors is a quite real effect and has been observed in experiment. Here it is essential that one can alter the energy difference between close-lying levels or, in other words, the mass difference $m_A - m_B$ with the help of external electric or magnetic fields. In particular, one can make this mass difference arbitrarily small (level crossing). We shall not here go into the details of the corresponding concrete situations, since this requires a special consideration.

5. SYMMETRY OF THE WAVE FUNCTION OF NONIDENTICAL PARTICLES

The interference effects considered above are intimately connected with the problem of the symmetry of the wave function of a system of nonidentical particles. From the formal point of view, any nonidentical particles A and B can always be treated as two states of the same particle corresponding to different values of some discrete variable. On this basis one can further introduce the concept of a wave function which depends not only on the space coordinates but also on the "internal" coordinates.[5)] The description of this situation is analogous to that in the theory of isobaric spin. One must, however, emphasize the following peculiarity: since particles A and B can differ from each other in any of their properties (charge, mass, strangeness, excitation, etc.) there is no analog of isobaric invariance in the general case.

What is the symmetry of the wave function with respect to the complete interchange of particles, i.e., the usual interchange of coordinates combined with an "isobaric interchange," which takes the state A into B and vice versa? To resolve this question we consider any state of the system of two particles described by a wave function of the type $U(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)}$. This way of writing implies that particles 1 and 2 are distributed in space in a definite manner [this space is not necessarily the coordinate space (it could, e.g., also be momentum space)] in correspondence with the structure of $U(\mathbf{x}_1, \mathbf{x}_2)$, and that particle 1 is in state A and particle 2 in state B.

We note now that in any real production process the formation of the state $U(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)}$ is always accompanied by the formation of the state $U(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(2)}|B\rangle^{(1)}$, which is physically indistinguishable from the former.

Therefore the state of the system is described by the linear combination

$$\psi = aU(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)} + bU(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(2)}|B\rangle^{(1)}, \qquad (19)$$

where a and b are some constant coefficients.

A spatial interchange transforms ψ into

$$\tilde{\psi} = aU(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(1)}|B\rangle^{(2)} + bU(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(2)}|B\rangle^{(1)},$$

and a subsequent "isobaric interchange" transforms $\tilde{\psi}$ into

$$\tilde{\tilde{\psi}} = aU(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(2)}|B\rangle^{(1)} + bU(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)}.$$

It is clear that $\tilde{\tilde{\psi}}$ and ψ describe the same physical state; hence we must have $\tilde{\tilde{\psi}} = e^{i\epsilon}\psi$, i.e.,

$$\begin{aligned} &aU(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(2)}|B\rangle^{(1)} + bU(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)} = \\ &= e^{i\epsilon}\{aU(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)} + bU(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(2)}|B\rangle^{(1)}\}. \end{aligned} \qquad (20)$$

It follows from (20) that $a = be^{i\epsilon}$ and $b = ae^{i\epsilon}$, i.e., $a^2 = b^2$ or $a = \pm b$. In other words, the wave function (19) has the form

$$\psi = U(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)} \pm U(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(2)}|B\rangle^{(1)}, \qquad (21)$$

i.e., it is either symmetric or antisymmetric under a complete interchange.[6)]

The invariance of all interactions with respect to a complete interchange of particles further leads to the result that the plus or minus sign is rigorously conserved in the time evolution of the system. Therefore all pairs of particles are divided into two classes; in the framework of quantum mechanics the principle of this division can only be established with the help of a special postulate. In the following we shall assume that the complete wave function is symmetric with respect to an interchange in the case of bosons and antisymmetric in the case of fermions.[7)]

The result obtained above plays an essential role in the analysis of any interference phenomena in a two- or

5)Among the internal coordinates one may also include the projection of the spin of the particle on some direction.

6)It is clear that this argument applies also to the "truly" identical particles.

7)This choice is in full accord with the corresponding postulate for "truly" identical particles. We note also that in the language of quantum field theory the complete wave function of particles A and B is symmetric when the field operators A and B commute and antisymmetric when the field operators A and B anticommute at points separated by space-like distances. By the general theorems of local field theory [7,8] one can show that under some additional conditions (in particular, if the superposition of particles A and B is, in principle, observable with the help of some apparatus) the field operators A and B anticommute for half-integer spins s_A and s_B, and commute for integer s_A and s_B and also if one of the spins is integer and the other, half-integer. Thus our postulate concerning the choice of sign can be justified in the framework of quantum field theory.

more-particle system. In particular, the symmetry character of the complete wave function (21) determines the choice of the sign of the interference term between the direct and exchange processes in the correlation experiments described above. Indeed, in the most general case the probability for recording particles A and B with two counters at the positions $\mathbf{x}_1$ and $\mathbf{x}_2$ is determined by

$$P = |U(\mathbf{x}_1, \mathbf{x}_2)A_1A_2' \pm U(\mathbf{x}_2, \mathbf{x}_1)A_2A_1'|^2, \qquad (22)$$

where A_1, A_2, A_1', and A_2' are the amplitudes introduced in Secs. 3 and 4, and the signs agree with the corresponding signs in (21).

For a second example, one may mention the resonance interaction via the radiation field of the ground and excited states of identical nuclei in a diatomic molecule.[9] The dependence of the lifetime of the excited state on the quantum numbers of the molecular levels considered in[9] is in the last instance connected with the symmetry of the coordinate wave function of the nuclei, which, owing to the definite symmetry character of the complete wave function, determines uniquely the symmetry of the "isobaric" part of the wave function (and vice versa).

This correlation occurs also in the general case of arbitrary particles A and B. Indeed, if the coordinate function $U(\mathbf{x}_1, \mathbf{x}_2)$ in (21) has a definite symmetry, i.e.,

$$U(\mathbf{x}_1, \mathbf{x}_2) = \pm U(\mathbf{x}_2, \mathbf{x}_1), \qquad (23)$$

then the complete wave function (21) can be written in the form

$$\psi = U(\mathbf{x}_1, \mathbf{x}_2)\{|A\rangle^{(1)}|B\rangle^{(2)} \pm |A\rangle^{(2)}|B\rangle^{(1)}\}. \qquad (24)$$

It follows from (24) that in this case the normalized internal wave function has the form

$$\psi_{\text{int}} = \frac{1}{\sqrt{2}}\{|A\rangle^{(1)}|B\rangle^{(2)} \pm |A\rangle^{(2)}|B\rangle^{(1)}\}, \qquad (25)$$

i.e., also has a definite symmetry with respect to interchanges (the plus sign corresponds to "isobaric spin 1" and the minus sign to "isobaric spin 0"). Equations (24) and (25) also imply that the symmetries of the coordinate and internal wave functions coincide for integer spins s_A and s_B, and are opposite to one another for half-integers s_A and s_B!

It should be especially emphasized that this result is in no way connected with some "isobaric invariance" or mass equality of particles A and B. In particular, it can easily be shown in connection with the above discussion that the isobaric spin of the deuteron is strictly equal to zero (if we neglect nuclear interactions which do not conserve parity) even when isobaric invariance is violated (for example, on account of electromagnetic interactions).

In the general case the symmetry or antisymmetry of the complete wave function with respect to a complete interchange does not imply the symmetry or antisymmetry of the coordinate part of the wave function with respect to an interchange of the space coordinates of the particles alone. In this connection the logical analysis of a specific case is of interest, when some process leads to the simultaneous formation of particles A and B which agree in all internal quantum numbers except the mass.[8)] Assume that immediately after the production of such a system its wave function has the form (21). Then the formation of a system with interchanged particles, A ⇄ B, is described by the wave function

$$\psi' = U(\mathbf{x}_1, \mathbf{x}_2)|B\rangle^{(1)}|A\rangle^{(2)} \pm U(\mathbf{x}_2, \mathbf{x}_1)|B\rangle^{(2)}|A\rangle^{(1)}. \qquad (21')$$

On the other hand, if the mass difference between particles A and B is very small, their interactions with any other particles must be almost identical because of the agreement of all quantum numbers. It follows from this that the wave functions (21) and (21′) must be almost the same, i.e.,

$$U(\mathbf{x}_1, \mathbf{x}_2)|A\rangle^{(1)}|B\rangle^{(2)} \pm U(\mathbf{x}_2, \mathbf{x}_1)|A\rangle^{(2)}|B\rangle^{(1)} \approx$$
$$\approx U(\mathbf{x}_1, \mathbf{x}_2)|B\rangle^{(1)}|A\rangle^{(2)} \pm U(\mathbf{x}_2, \mathbf{x}_1)|B\rangle^{(2)}|A\rangle^{(1)}.$$

The last equality can hold only if

$$U(\mathbf{x}_1, \mathbf{x}_2) = \pm U(\mathbf{x}_2, \mathbf{x}_1), \qquad (26)$$

i.e., when the coordinate wave function is almost symmetric (for bosons) or almost antisymmetric (for fermions). The internal wave function is in both cases almost symmetric.

The equation (26) is satisfied the more precisely the smaller the quantity Δm.[9)] For $\Delta m \to 0$ the probability for formation of a system of such particles in states which are absolutely forbidden for identical particles, becomes vanishingly small. We see that also from this point of view, the parameter Δm plays a fundamental role in the analysis of the transition from a system of similar particles to a system of identical particles.[10)]

6. CONCLUSION

The differences in the behavior of systems of identical and nonidentical particles are absolute only when this problem is considered without an analysis of the specific peculiarities of the given concrete situation in each separate case. It was shown above that such an analysis exhibits the existence of various interference phenomena and correspondingly, of a continuous transition from distinguishable particles to identical particles. The criterion determining this transition is the relation between the mass difference Δm and the characteristic duration of the process under consideration: the smaller

[8)]Particles A and B are here introduced by postulate; the question of their existence in reality is not considered. However, the following results hold also when certain quantum numbers of particles A and B do not coincide, if only they do not play a role in the interactions under consideration.

[9)]For the particles under consideration (26) is valid not only in the act of production but also at subsequent time instants. The smaller Δm the more exactly is the invariance of the interactions fulfilled, and the larger is the length of the time intervals during which (26) remains in force. At the same time, for any finite value of Δm the character of the spatial symmetry of the wave function may change radically sooner or later.

[10)]If the quantum numbers of the particles A and B are different, this correspondence between the wave function of these particles in the limit $\Delta m \to 0$ and the wave function of identical particles does not in general exist. For example, the pair of particles K_1 and K_2 having different CP parity can be in states with odd orbital angular momenta. [6].

the quantity Δm, the larger the time interval during which the behavior of the distinguishable particles is similar to the behavior of identical particles.

In this connection some additional questions arise which require special consideration. One of these refers to the characteristics of the quantum statistics of systems of particles with almost equal masses. The properties of the equilibrium state of such systems are in many cases the same as for systems of nonidentical particles. However, one might think that the equilibrium is reached in two stages: first an intermediate quasi-equilibrium state is attained, the same as for identical particles, which then gradually goes over into the final equilibrium—the more slowly the smaller the quantity Δm.

Another range of questions is connected with the analysis of specific phenomena occurring in cases when particles of type A as well as of type B can be formed in each of the two generators considered above. Interesting effects also arise in the analysis of systems of unstable particles, in particular, in the registration of such particles by a single counter, not two, as in correlation experiments (cf. also[1]).

In connection with the existence of a continuous transition from the properties of systems of similar particles to the properties of systems of identical particles, there arises also the following question of principle: to what extent can one regard as proven the assertion that particles which are traditionally considered identical are indeed so?

We expect to discuss these problems in subsequent publications.

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