RESONANCE VARIATION OF THE MOBILITY OF HOT ELECTRONS IN A HIGH-FREQUENCY MAGNETIC FIELD

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Resonance variation of the mobility of hot electrons, induced by a high-frequency magnetic field whose frequency is close to the spin resonance frequency, is investigated. The dependence of the amplitude and line shape of the resonance-mobility variation on the spin-lattice relaxation mechanism and conduction-electron energy and momentum relaxation mechanism is determined. Estimates show that for semiconductors of the InSb type the relative variation of the mobility can reach 5–10%. Some manifestations of the effect in two pronouncedly nonequilibrium cases are discussed.

1. INTRODUCTION

SOLOMON and Gueron,[1] in a study of the resonance of conduction electrons in InSb, observed the phenomenon of resonant increase of the mobility of hot electrons situated in a high frequency magnetic field whose frequency was close to the electron spin resonance (ESR) frequency. The gist of the phenomenon consists in the following. It is known that when the frequency of the external magnetic field coincides with the natural frequency of the electron spins, a sharp increase takes place in the absorption of the high-frequency field energy. The average energy of the spin system (or the spin temperature T_S) increases and becomes larger than the average kinetic energy of the electrons. The electron spins relax and transfer part of the absorbed energy to the kinetic degrees of freedom. This leads to an increase (decrease) of the conduction-electron mobility increases (decreases) with increasing average kinetic energy of the electrons. The effect of the change of the mobility is more strongly manifest if the conduction electrons are "heated" by a constant electric field.[1]

Unlike the mobility change connected with the heating of the conduction electrons in a high-frequency electric field, the phenomenon considered in the article is resonant, i.e., it becomes manifest only in a narrow frequency interval near the ESR frequency of the conduction electrons. Naturally, this phenomenon can be used to detect the ESR of conduction electrons. Solomon and Gueron noted that such a method of ESR detection may turn out to be more sensitive than the standard methods, in the case of low conduction-electron concentration or in the case of samples with small dimensions.

In the present paper we propose a theory that explains this phenomenon, and reveals the influence of the spin-lattice relaxation mechanisms and the hot-electron energy and momentum relaxation mechanism on the resonance variation of the mobility.

2. FORMULATION OF PROBLEM. METHOD OF SOLVING THE KINETIC EQUATION

The kinetic equation for the density matrix describing the behavior of the conduction electrons in a constant electric field and a high-frequency magnetic field is given by[2]

$$\frac{\partial f_{\sigma\sigma'}(\mathbf{p})}{dt}+e\mathbf{E}\frac{\partial f_{\sigma\sigma'}(\mathbf{p})}{\partial \mathbf{p}}+\frac{i\beta}{\hbar}[(\mathbf{H}\sigma),f_{\sigma\sigma'}(\mathbf{p})]=-\left(\frac{\partial f_{\sigma\sigma'}(\mathbf{p})}{\partial t}\right)_{\text{coll}}, \quad (1)$$

$$\sum_{\sigma\mathbf{p}} f_{\sigma\sigma}(\mathbf{p})=N. \quad (2)$$

Here $\mathbf{H}$—magnetic field, which is the sum of the dc components H_z and of the hf components H_x and H_y; β—Bohr magneton; $(\partial f_{\sigma\sigma'}(\mathbf{p})/\partial t)_{\text{coll}}$—collision integral, which includes transitions without and with spin flip; N—total number of conduction electrons. Equation (1) is valid if the following condition is satisfied for the constant magnetic field:

$$H_z \ll \min\left(\frac{m^*c\nu}{e}, \frac{\bar{\varepsilon}}{g\beta}\right) \quad (3)$$

where m*—effective electron mass, ν—frequency of collisions between the electrons and the scatterers, $\bar{\varepsilon}$—average kinetic energy of the conduction electrons, and g—their g-factor. For simplicity we consider the quadratic and isotropic spectra of the conduction electrons, i.e.,

$$\varepsilon_{p\sigma}=p^2/2m^*+\sigma g\beta H_z, \quad \sigma=\pm 1/2. \quad (4)$$

We represent the density matrix in the form

$$f_{\sigma\sigma'}(\mathbf{p})=\Phi_0(\mathbf{p})I+\Phi(\mathbf{p})\sigma=\Phi_0(\mathbf{p})\mathrm{I}+\Phi_z(\mathbf{p})\sigma_z+\Phi_+(\mathbf{p})\sigma_-+\Phi_-(\mathbf{p})\sigma_+,$$

where

$$\sigma_\pm=1/2(\sigma_x\pm i\sigma_y), \quad \Phi_\pm(\mathbf{p})=\Phi_x(\mathbf{p})\pm i\Phi_y(\mathbf{p}), \quad (5)$$

and σ_x, σ_y, σ_z, and I are the Pauli matrices and the unit matrix.

We consider a stationary process, in which Φ_0 and Φ_z do not depend on the time, and the time dependence of the functions Φ_+ and Φ_- is determined by the external high frequency field $H_\pm = H_x \pm iH_y = H_1 \exp(\pm i\omega t)$, i.e., $\Phi_\pm(t) = \Phi_\pm(t)\exp(\pm i\omega t)$. We can then rewrite (1) in the form

$$e\mathbf{E}\partial\Phi_0/\partial\mathbf{p}=J_0(\Phi_0,\Phi_z), \quad (6a)$$

$$e\mathbf{E}\partial\Phi_z/\partial\mathbf{p}+1/2iw(\Phi_+-\Phi_-)=J_z(\Phi_0,\Phi_z), \quad (6b)$$

$$e\mathbf{E}\partial\Phi_\pm/\partial\mathbf{p}\pm iw\Phi_z\pm i(\omega-\Omega)\Phi_\pm=J_\pm(\Phi_0,\Phi_\pm), \quad (6c)$$

where $w=g\beta H_1/\hbar$, $\Omega=g\beta H_z/\hbar$, and J_0, J_z, J_+, and J_- are the collision integrals, including transitions without

and with spin flip,

$$J_0(\Phi_0,\Phi_z)=-\sum_{\mathbf{p}'}\{W^{\mathbf{pp}'}[\Phi_0(\mathbf{p})(1-\Phi_0(\mathbf{p}'))-\Phi_z(\mathbf{p})\Phi_z(\mathbf{p}')]$$
$$+(W_{+-}^{\mathbf{pp}'}+W_{-+}^{\mathbf{pp}'})[\Phi_0(\mathbf{p})(1-\Phi_0(\mathbf{p}'))+\Phi_z(\mathbf{p})\Phi_z(\mathbf{p}')]$$
$$+(W_{+-}^{\mathbf{pp}'}-W_{-+}^{\mathbf{pp}'})[\Phi_z(\mathbf{p})-\Phi_z(\mathbf{p})\Phi_0(\mathbf{p}')+\Phi_0(\mathbf{p})\Phi_z(\mathbf{p}')]-(\mathbf{p}\rightleftarrows\mathbf{p}')\},\tag{7a}$$

$$J_z(\Phi_0,\Phi_z)=-\sum_{\mathbf{p}'}\{W^{\mathbf{pp}'}[\Phi_z(\mathbf{p})-\Phi_z(\mathbf{p})\Phi_0(\mathbf{p}')-\Phi_0(\mathbf{p})\Phi_z(\mathbf{p}')]$$
$$-(\mathbf{p}\rightleftarrows\mathbf{p}')\}+\{(W_{-+}^{\mathbf{pp}'}-W_{+-}^{\mathbf{pp}'})[\Phi_0(\mathbf{p})(1-\Phi_0(\mathbf{p}'))+\Phi_z(\mathbf{p})\Phi_z(\mathbf{p}')]$$
$$+(W_{+-}^{\mathbf{pp}'}+W_{-+}^{\mathbf{pp}'})[\Phi_z(\mathbf{p})-\Phi_z(\mathbf{p})\Phi_0(\mathbf{p}')-\Phi_0(\mathbf{p})\Phi_z(\mathbf{p}')]+(\mathbf{p}\rightleftarrows\mathbf{p}')\}.\tag{7b}$$

Here and throughout, the symbol $(\mathbf{p}\rightleftarrows\mathbf{p}')$ denotes an expression that differs from that written out explicitly only in that $\mathbf{p}$ under the summation sign is replaced by $\mathbf{p}'$ and vice versa; $W^{\mathbf{pp}'}$ and $W_{+(+-)}^{\mathbf{pp}'}$ are the probabilities of the transitions without and with spin flip, respectively. We shall henceforth need only certain general properties of the collision integrals J_+ and J_-, so that we do not give their explicit form here.[1)]

To solve the system (6), we use a small parameter γ, which equals the ratio of the probabilities of the transitions with and without spin flip. This ratio is small because the transverse and longitudinal spin-relaxation times τ_2 and τ_1, which characterize the time of establishment of the internal equilibrium of the spin system and the equilibrium with the lattice, greatly exceed the characteristic time of establishment of equilibrium of the "kinetic" degrees of freedom. Therefore, that part of the collision integrals which corresponds to transitions with spin flip is of first order in γ.

The terms of Eqs. (6b) and (6c) containing the factors w and $\omega-\Omega$ are also of first order of smallness in γ, for the following reasons. The ESR line width is equal to $1/\tau_2$, and therefore $(\omega-\Omega)\sim 1/\tau_2\sim\gamma$. The investigated change of the mobility occurs upon "saturation" of the ESR line. For "saturation" it is necessary that the amplitude of the high-frequency field satisfy the condition $w^2\tau_1\tau_2\sim 1$,[4] i.e., $w\sim 1/\sqrt{\tau_1\tau_2}\sim\gamma$.

For simplicity we consider the solution of the system (6) in the case of a low electron concentration, when it is possible to disregard the Pauli principle in the collision integral J_0, J_Z, J_+, and J_-. Then they are linear integral operators acting on the functions Φ_j, and the system of equations is also linear.[2)] The system (6) can be represented in abstract form:

$$\hat{L}_{ij}\Phi_j=0,\qquad \sum_{\mathbf{p}}\Phi_0(\mathbf{p})=N,\tag{8}$$

where i, j = (0, z, +, −). We expand $\hat{L}_{ij}$ and Φ_j in a series in the parameter γ:[3)]

$$\hat{L}_{ij}=\hat{L}_{ij}+\gamma\hat{L}_{ij}^{(1)},\qquad \Phi_j=\Phi_j^{(0)}+\gamma\Phi_j^{(1)}.\tag{9}$$

Then the terms of zeroth and first order yield the following expressions:

$$\hat{L}_{ij}^{(0)}\Phi_j^{(0)}=0,\tag{10}$$

$$\hat{L}_{ij}^{(0)}\Phi_j^{(1)}=-\hat{L}_{ij}^{(1)}\Phi_j^{(0)}.\tag{11}$$

All the $J_i^{(0)}$ (i.e., the collision integrals without spin flip) are equal to each other if $g\beta H_z/\bar{\varepsilon}\ll 1$.[4)] Physically this means that the energy and momentum relaxations of all four components of the density matrix occur in the same manner. Therefore, using the expression (8), we write

$$\hat{L}_{ij}^{(0)}=\hat{L}_0,\tag{12a}$$

$$\hat{L}_0\Phi_j(\mathbf{p})=e\mathbf{E}\frac{\partial\Phi_j(\mathbf{p})}{\partial\mathbf{p}}+\sum_{\mathbf{p}'}\{W^{\mathbf{pp}'}\Phi_j(\mathbf{p})-W^{\mathbf{p}'\mathbf{p}}\Phi_j(\mathbf{p}')\}.\tag{12b}$$

We note that the operator $\hat{L}_0$ is not hermitian.

The solution of the system (10) is the problem of hot electrons. Let us assume that it is solved and that the solution is given by

$$\Phi_j^{(0)}=m_j\Phi_0^{(0)},\tag{13a}$$

$$\sum_{\mathbf{p}}\Phi_0^{(0)}(\mathbf{p})=N,\tag{13b}$$

where $m_0=1$, m_x, m_+, and m_- are undetermined parameters.

Our main task is to solve the system of inhomogeneous linear integral equations (11). The homogeneous equations $\hat{L}_0\Phi_j^{(1)}=0$ have a nontrivial solution, and therefore, generally speaking, Eqs. (11) are contradictory. It is known that the condition for the existence of a solution of the system of Eqs. (11) is orthogonality of the right sides of these equations and of the solution of the homogeneous equation[6]

$$\tilde{L}_0\varphi=0.\tag{14}$$

Here $\tilde{\hat{L}}_0$ is the transpose of $\hat{L}_0$. Thus, if $\varphi_0(\mathbf{p})$ is the solution of (14), then the conditions for the existence of the solution of the system (11) takes the form

$$\sum_{\mathbf{p}}\varphi_0(\mathbf{p})\hat{L}_{ij}^{(1)}\Phi_j^{(0)}(\mathbf{p})=0.\tag{15}$$

It is easy to see that the operator $\tilde{\hat{L}}_0$ is given by

$$\tilde{\hat{L}}_0\varphi=-e\mathbf{E}\frac{\partial\varphi}{\partial\mathbf{p}}+\sum_{\mathbf{p}'}W^{\mathbf{pp}'}(\varphi(\mathbf{p})-\varphi(\mathbf{p}')),\tag{16}$$

and the nontrivial solution of Eq. (14) is $\varphi_0=\text{const}$. Substituting[5)] $\varphi_0=g\beta$ in Eq. (15) and using the concrete form of the collision integrals, we obtain an identity for i = 1 and three Bloch equations for the magnetization of the hot electrons (i = 2, 3, and 4):

$$\frac{iw}{2}(M_+-M_-)=-\frac{M_z-M_0}{\tau_1},\tag{17}$$

$$\pm iwM_z\pm i(\omega-\Omega)M_\pm=-M_\pm/\tau_2;\tag{18}$$

[1)] In [3] they obtained the collision integrals J_Z, J_+, J_- for the case of electron-phonon interaction.

[2)] While allowance for the Pauli principle makes the system (6) nonlinear, it does not introduce any fundamental changes in the solution. This case is considered in the Appendix.

[3)] The methods developed subsequently for solving the system of equations (7) is close in its idea to the Chapman-Enskog method [5] for deriving the hydrodynamics equations from the Boltzmann kinetic equations.

[4)] This is valid if the Pauli principle is not taken into account in the collision integral. When the Pauli principle is taken into account, the situation becomes somewhat more complicated (see the Appendix).

[5)] The constant is chosen here in such a way as to make the solvability conditions coincide with the Bloch equations.

Here $M_{z,+,-} = g\beta m_{z,+,-}N$—components of the density of magnetization of the hot electrons, τ_1 and τ_2—longitudinal and transverse relaxation times, defined by the relations

$$\frac{1}{\tau_1} = \frac{1}{\tau_2} = \frac{1}{N}\sum_{pp'}(W_{+-}^{pp'} + W_{-+}^{pp'})\Phi_0^{(0)}(p), \tag{19}$$

M_0—value of the density of the magnetization of the hot electrons at $H_+ = H_- = 0$. It equals

$$M_0 = g\beta m_z^0 N,$$
$$m_z^0 = \frac{\sum_{pp'}(W_{+-}^{pp'} - W_{-+}^{pp'})\Phi_0^{(0)}(p)}{\sum_{pp'}(W_{+-}^{pp'} + W_{-+}^{pp'})\Phi_0^{(0)}(p)} \tag{20}$$

From (17) and (18) we determine m_Z:

$$m_z = m_z^0\frac{\tau_1^2(\omega-\Omega)^2+1}{\tau_1^2[w^2+(\omega-\Omega)^2]+1}. \tag{21}$$

The change of the mobility is determined by the function $\Phi_0^{(1)}$. To determine this function we must solve the first of the inhomogeneous equations (11), using relations (13a) and (21) for this purpose.

We have disregarded above the collisions between electrons. It can be shown that when they are taken into account the described method of solving the kinetic equation does not change in principle, and the expression obtained for m_Z does not differ from that obtained above.

3. CALCULATION OF THE DISTRIBUTION FUNCTION

In general, the solution of the first of the equations (10) and (11) or of the more general equations (A.1) and (A.2) entails considerable difficulties. We shall solve them in the so-called diffusion approximation,[7] i.e., we shall assume that the collisions of the electrons with the scatterers are almost elastic. The first equation of (A.1) can be written in the form[6)]

$$eE\frac{\partial\Phi_0^{(0)}}{\partial p} + \sum_{p'}\{W^{pp'}\Phi_0^{(0)}(p)[1-\Phi_0^{(0)}(p')] - (p \rightleftarrows p')\} = 0. \tag{22}$$

We represent the function $\Phi_0^{(0)}(p)$ in the form of a sum of parts that are symmetrical and anti-symmetrical in p:

$$\Phi_0^{(0)}(p) = F^{(0)}(\varepsilon) + \psi^{(0)}(p), \tag{23}$$

where $\varepsilon = p^2/2m^*$ is the electron energy. Up to sufficiently high values of the electric field it can be assumed that $\psi^{(0)}(p)$ is much smaller than $F^{(0)}(\varepsilon)$, and we shall therefore disregard terms quadratic in $\psi^{(0)}(p)$ (which appear when the Pauli principle is taken into account).

In the concrete calculations we assume that conduction-electron energy relaxes on acoustic or piezoacoustic phonons. In this case

$$W^{pp'} = \frac{2\pi}{\hbar}\sum_q |V_q|^2[(N_q+1)\delta_{p'+q,p}\,\delta(\varepsilon_p - \varepsilon_{p'} - \hbar\omega_q) + N_q\delta_{p',p+q}\,\delta(\varepsilon_p - \varepsilon_{p'} + \hbar\omega_q)], \tag{24}$$

where V_q—matrix elements of the electron–phonon interaction and N_q—number of phonons with momentum q.[7)]

The method of transforming the integro-differential equation (22) into a system of differential equations, namely expansion of the collision integral in the small ratio $\hbar\omega_q/\varepsilon$, is quite well known,[7-9] so that we present immediately the final results. The system of differential equations for $F^{(0)}(\varepsilon)$ and $\psi^{(0)}(p)$ is of the form

$$\frac{\partial}{\partial\varepsilon}j_\varepsilon = 0, \quad eEv\frac{\partial F^{(0)}}{\partial\varepsilon} = -\frac{\psi^{(0)}}{\tau_{mom}}. \tag{25}$$

Here

$$j_\varepsilon = eEv\psi^{(0)} - \frac{A(\varepsilon)}{2}\left[\frac{\partial F^{(0)}}{\partial\varepsilon} + \frac{1}{T_0}F^{(0)}(1-F^{(0)})\right], \tag{26}$$

where v—electron velocity, T_0—lattice temperature,

$$A(\varepsilon) = \frac{2\pi}{\hbar}\sum_{p'q}|V_q|^2(2N_q+1)\delta_{p'+q,p}\,\delta(\varepsilon_p - \varepsilon_{p'})(\hbar\omega_q)^2,$$

j_ε has the meaning of the "particle flux density in energy space," and τ_{mom}—electron momentum relaxation time. Using the second equation of (25), we eliminate $\psi^{(0)}$ from the first, integrate it, and obtain, keeping the stationarity condition in mind, the following equation for the determination of $F^{(0)}$:

$$\partial F^{(0)}/\partial\varepsilon + \lambda(\varepsilon)F^{(0)}(1-F^{(0)}) = 0, \tag{27}$$

where

$$\frac{1}{\lambda(\varepsilon)} = T_0\left[1 + \frac{2(eEv)^2\tau_{mom}(\varepsilon)}{3A(\varepsilon)}\right]. \tag{28}$$

As a result of the solution of (27) we get $F^{(0)}$:[8)]

$$F^{(0)}(\varepsilon, N) = \left\{\exp\left[\int_0^\varepsilon \lambda(\varepsilon')d\varepsilon' - \zeta(N)\right] + 1\right\}^{-1}; \tag{29}$$

$\zeta(N)$ is a constant determined from the normalization condition. The concrete form of the dependence of λ on ε is determined by the mechanisms for the dissipation of the energy and momentum of the conduction electrons. In those cases when λ does not depend on ε, the quantity $1/\lambda$ is the temperature of the conduction electrons, and ζ has the meaning of the reduced chemical potential.

We now proceed to calculate $\Phi_0^{(1)}$. From (A.2), (A.4), and (A.6) we obtain the following equation for the determination of this function:

$$eE\frac{\partial\Phi_0^{(1)}}{\partial p} + \sum_{p'}\{W^{pp'}[\Phi_0^{(1)}(p) - \Phi_0^{(1)}(p)\Phi_0^{(0)}(p') - \Phi_0^{(0)}(p)\Phi_0^{(1)}(p')] - (p \rightleftarrows p')\} = -\sum_{p'}\{(W_{+-}^{pp'} + W_{-+}^{pp'})\Phi_0^{(0)}(p) \times (1-\Phi_0^{(0)}(p')) + (W_{-+}^{pp'} - W_{+-}^{pp'})m_z\left[\frac{\partial\Phi_0^{(0)}(p)}{\partial N} - \frac{\partial\Phi_0^{(0)}(p)}{\partial N}\Phi_0^{(0)}(p') + \Phi_0^{(0)}(p)\frac{\partial\Phi_0^{(0)}(p')}{\partial N}\right] - (p \rightleftarrows p')\}. \tag{30}$$

[6)] We have left out here terms equadratic in Φ_Z, inasmuch as m_Z $m_Z \sim g\beta H_Z/\bar{\varepsilon} \ll 1$ and, by virtue of (A.5), $\Phi_Z \ll \Phi_0$.

[7)] The electron momentum can relax also on other scatterers, for example impurities.

[8)] Such a distribution function was obtained first in [9].

The function $\Phi_0^{(1)}(\mathbf{p})$ will also be represented in the form of a sum of symmetrical and antisymmetrical terms:

$$\Phi_0^{(1)}(\mathbf{p}) = F^{(1)}(\varepsilon) + \psi^{(1)}(\mathbf{p}).$$

In semiconductors at low temperatures, the most probable mechanisms for the relaxation of the conduction electron spins are interactions with acoustic or piezoacoustic phonons[10] and with paramagnetic impurities. For phonons the probabilities $W^{\mathbf{pp'}}_{-+(+-)}$ are given by

$$W^{\mathbf{pp'}}_{-+(+-)} = \frac{2\pi}{\hbar}\sum_{\mathbf{q}} |V_{\mathbf{q}}^{s0}|^2 [(2N_{\mathbf{q}}+1)\delta_{\mathbf{p'}+\mathbf{q},\mathbf{p}}\,\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p'}} \mp \hbar\Omega - \hbar\omega_{\mathbf{q}}) + N_{\mathbf{q}}\delta_{\mathbf{p'},\mathbf{p}+\mathbf{q}}\,\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p'}} \mp \hbar\Omega + \hbar\omega_{\mathbf{q}})], \quad (31)$$

where the upper sign on the right is taken for W_{-+}, and the lower one for W_{+-}.

To transform Eq. (30) into a system of two differential equations of the type (25), we employ the same procedure as used in the derivation of the system (25). The right side of (30) is expanded in powers of $\hbar\omega_{\mathbf{q}}/\varepsilon$ and $\hbar\Omega/\varepsilon$, which we assume to be quantities of the same order of smallness. We take into account only those terms of first order in γ, which are connected with the action of the high frequency magnetic field on the hot electrons and which have a resonant character (as functions of the frequency of the external magnetic field). The emitted first-order terms yield a constant addition to the distribution function. This addition is due to allowance for collisions with spin flip, and is of no interest to us.

In transforming (30), we use the inequality that follows from (29) and (A.5)

$$\frac{\partial F^{(0)}}{\partial N} = \frac{\partial \zeta}{\partial N} F^{(0)}(1 - F^{(0)}).$$

We obtain a system of differential equations analogous to (25):

$$\partial j_\varepsilon' / \partial \varepsilon = 0, \qquad e\mathbf{E}v\partial F^{(1)}/\partial\varepsilon = -\psi^{(1)}/\tau_{\text{mom}} \quad (32)$$

where j_ε' —first-order correction to the "electron flux density in energy space," and takes into account the transitions with spin flip.

Using the stationarity condition in exactly the same manner as in the determination of the function $F^{(0)}$, we arrive at the following inhomogeneous differential equation:

$$\partial F^{(1)}/\partial\varepsilon + \lambda(\varepsilon)F^{(1)}(1 - 2F^{(0)}) - \delta\lambda(\varepsilon)F^{(0)}(1 - F^{(0)}) = 0. \quad (33)$$

Here

$$\delta\lambda(\varepsilon) = \frac{\partial\zeta}{\partial N}(m_0 - m_z)\frac{T_0(\hbar\Omega)\nu_{s0}(\varepsilon)}{A(\varepsilon)}\lambda(\varepsilon). \quad (34)$$

The quantity

$$\nu_{s0}(\varepsilon) = \frac{2\pi}{\hbar}\sum_{\mathbf{p'q}} |V_{\mathbf{q}}^{s0}|^2 (2N_{\mathbf{q}}+1)\delta_{\mathbf{p'}+\mathbf{q},\mathbf{p}}\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p'}}) \quad (35)$$

has the meaning of the spin-lattice relaxation frequency. We solve Eq. (33), as usual, by the method of varying the constant. We see that the solution of the homogeneous equation is $C\partial F^{(0)}/\partial\xi$. Varying the constant, we obtain ultimately

$$F^{(1)}(\varepsilon, N) = \left(\int_0^\varepsilon \delta\lambda(\varepsilon')d\varepsilon'\right)\frac{\partial F^{(0)}}{\partial\zeta}. \quad (36)$$

If the spin-lattice relaxation of the conduction electron is determined by the paramagnetic impurities, then the function $F^{(1)}$ has the same form, but

$$\delta\lambda_{\text{imp}}(\varepsilon) = \frac{\partial\zeta}{\partial N}(m_0 - m_z)\frac{T_0\hbar(\Omega - \Omega_{\text{imp}})\nu_{s0}^{\text{imp}}(\varepsilon)}{A(\varepsilon)}\lambda(\varepsilon), \quad (37)$$

where

$$\nu_{s0\,\text{imp}} = \frac{2\pi}{\hbar}\sum_{\mathbf{p'}} |V_{\text{imp}}^{s0}|^2 N_{\text{imp}}\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p'}}) \quad (38)$$

is the frequency of electron-spin relaxation on the paramagnetic impurities, and Ω_{imp} is the resonant frequency of the impurity spins.

4. CALCULATION OF MOBILITY

The expression for the mobility is of the form

$$\mu = -e\int_0^\infty \frac{2\varepsilon}{m}\tau_{\text{mom}}(\varepsilon)\rho(\varepsilon)dF(\varepsilon)\Big/\int_0^\infty \rho(\varepsilon)F(\varepsilon)d\varepsilon, \quad (39)$$

where $\rho(\varepsilon) = \sqrt{2}\,m^{3/2}\varepsilon^{1/2}/\pi^2\hbar^3$ is the density of states. We seek the change of mobility of interest to us in the case when the conduction electrons are nondegenerate, i.e.,

$$F^{(0)}(\varepsilon, N) = \exp\left[-\int_0^\varepsilon \lambda(\varepsilon')d\varepsilon' + \zeta(N)\right], \quad (40)$$

$$F^{(1)}(\varepsilon, N) = \left(\int_0^\varepsilon \delta\lambda(\varepsilon')d\varepsilon'\right)F^{(0)}(\varepsilon, N). \quad (41)$$

We represent the mobility in the form

$$\mu = \mu^{(0)} + \mu^{(1)},$$

and obtain the relative change of the mobility $\mu^{(1)}/\mu^{(0)}$:

$$\frac{\mu^{(1)}}{\mu^{(0)}} = \frac{\nu_{s0}(T_0)}{A_0(T_0)}\frac{(w\tau_1)^2}{1 + \tau_1^2[w^2 + (\omega - \Omega)^2]}m_0T_0(\hbar\Omega)K, \quad (42)$$

where K is a constant whose magnitude depends on the relaxation mechanisms. Its explicit form is given by

$$K = \frac{1}{\langle\varepsilon^{1/2}\rangle\langle(\varepsilon/T_0)^{q+1/2}\rangle}\left\{\left\langle\left[\int_0^\varepsilon \lambda(\varepsilon')\left(\frac{\varepsilon'}{T_0}\right)^{t-r}d\varepsilon'\right]\left(\frac{\varepsilon}{T_0}\right)^{q+1/2}\right\rangle\langle\varepsilon^{1/2}\rangle - \left\langle\left(\frac{\varepsilon}{T_0}\right)^{q+1/2}\right\rangle\left\langle\left[\int_0^\varepsilon \lambda(\varepsilon')\left(\frac{\varepsilon'}{T_0}\right)^{t-r}d\varepsilon'\right]\varepsilon^{1/2}\right\rangle\right\},$$

$$\langle\varphi(\varepsilon)\rangle = \int_0^\infty \varphi(\varepsilon)F^{(0)}(\varepsilon)d\varepsilon. \quad (43)$$

In obtaining (42), all the energy-dependent quantities were represented in the form

$$A(\varepsilon) = A_0(T_0)(\varepsilon/T_0)^r, \quad \tau_{\text{mom}}(\varepsilon) = \tau_0(T_0)(\varepsilon/T_0)^q, \quad \nu_{s0}(\varepsilon) = \nu_{s0}(T_0)(\varepsilon/T_0)^t. \quad (44)$$

In the derivation of (42) we disregarded electron–electron collisions. If the frequency of the electron–electron collisions is much larger than the energy-relaxation frequency, then the distribution function has a Boltzmann (or Fermi) form with effective temperature T_e. But the electron–electron collisions do not influence the energy transfer from the spin system to the

kinetic subsystem.[9] Therefore formula (42) remains valid also for cases when the frequency of the electron–electron collisions is high, i.e., for a large electron concentration.

5. DISCUSSION OF RESULTS

In the case when λ does not depend on ε (i.e., when $r - q = 1$ or in the case when the frequency of the electron–electron collisions is much larger than the energy-relaxation frequency) and the conduction electrons can be described with the aid of the temperature $T_e = 1/\lambda$, expression (43) can be readily integrated. Bearing in mind that $m_0 \approx \hbar\Omega/T_e$,[11, 12] we get

$$\frac{\mu^{(1)}}{\mu^{(0)}} = \frac{(\hbar\Omega)^2}{T_e^2}\frac{\nu_{s0}(T_e)}{\nu_\varepsilon(T_e)}\frac{(w\tau_1)^2}{1+\tau_1^2[w^2+(\omega-\Omega)^2]}K_1, \qquad (45)$$

where $\nu_\varepsilon(T_e) = A_0(T_e)/T_0T_e$ is a quantity that determines the rate of relaxation of the kinetic energy at an electron temperature T_e, and K_1 is a constant coefficient on the order of unity, expressed in explicit form as follows:

$$K_1 = \frac{1}{t-q}\left[\frac{\Gamma(t+3/2)}{\Gamma(q+3/2)} - \frac{\Gamma(t-r+5/2)}{\Gamma(3/2)}\right], \qquad (46)$$

where $\Gamma(x)$ is the gamma function.

In InSb, for example, $\gamma_{s0} \sim \nu_e \sim 10^7$–$10^8$ sec^{-1}, and when $H_z = 500$ Oe ($\Omega\ 10^{11}$ sec^{-1}) the relative change of the mobility for not too strongly heated electrons ($T_e \sim 10°$K) is of the order of 1–10%.

This estimate may turn out to be insufficient if the distribution function of the hot electrons differs from a Boltzmann distribution. One can expect particularly strong deviations from the foregoing results in the case when the electron "runaway" effect appears.[13, 14] Electron "runaway" may come into play in various situations.[14, 15] We present only one example of the possible influence of electron runaway on the change of the mobility, due to the action of a high frequency magnetic field. Electron runaway is manifest in the fact that at electric field intensities much higher than the critical value E_{cr} the moments of the distribution function (all of them, or starting with a certain moment) diverge. There is a possible situation when the moment $\langle\varepsilon^{q+1/2}\rangle$, which determines the mobility, still converges, while the higher-order moments diverge. It is seen from (43) that in the case when the frequency of the spin spin-lattice relaxation $\nu_{s0}(\varepsilon)$ increases more rapidly as a function of the energy than $A(\varepsilon)$ (i.e., $t > r$), the addition to the mobility becomes formally infinite. Physically this means that in the case we can expect a larger mobility resonance amplitude than would follow from the foregoing estimate. Such a situation can be realized, for example, when the momentum relaxes on charged impurities, and the energy and spin relaxation occurs on acoustic phonons, or when all the aforementioned forms of relaxation are due to the interaction between the conduction electrons and piezoacoustic phonons.[15]

It is indicated in [1] that in the case of a small number of electron spins, the method of detecting carrier ESR by measuring the change of the mobility may turn out to be more sensitive than the standard electromagnetic methods. It can therefore be used to study effects that occur only at low concentration of the conduction electrons, when the standard methods are not applicable. For example, it is possible to investigate the InSb instability that leads to the current pinching observed at low carrier density ($n \sim 10^{13}$ cm^{-3}).[16] Such a possibility is connected with the fact that the g-factor of the conduction electrons depends on their kinetic energy (temperature), i.e., it is different for electrons located in the pinch and outside the pinch. For this reason, when the electric field becomes sufficient for the pinch to occur, the resonance mobility line splits into two if the boundaries of the pinch are sharp, or broadens if these boundaries are smooth. Estimates show that the magnitude of this splitting is comparable with the ESR line width when the difference between the temperatures of the hot and cold regions is of the order of 5–10°K. In practice, in [17], the ESR method was used to measure the temperature of the hot electrons in InSb with even higher accuracy ($\Delta T_e \approx 1°$K). A study of this phenomenon can yield information on the dimensions of the pinch and on the difference between the temperatures of the hot and cold regions.

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APPENDIX

When account is taken of the Pauli principle in the collision integrals, the system (6) becomes nonlinear. Expanding the collision integrals and the functions Φ_j in terms of the parameter γ, we obtain in the zeroth approximation

$$\hat{L}_1^{(0)}(\Phi_0^{(0)}) = 0, \qquad (A.1)$$

$$\hat{L}_{ij}(\Phi_0, \Phi_j) = 0, \quad i,j = (z, +, -) \qquad (A.2)$$

and in the first approximation

$$\hat{L}_1^{(0)}\Phi_0^{(1)} = -\hat{L}_1^{(1)}(\Phi_0^{(0)}, \Phi_z^{(0)}), \qquad (A.3)$$

$$\hat{L}_i^{(0)}(\Phi_0^{(1)}, \Phi_j^{(1)}) = -\hat{L}_i^{(1)}(\Phi_0^{(0)}, \Phi_j^{(0)}). \qquad (A.4)$$

We have neglected here the terms $\Phi_j\Phi_{j'}$, which are of the order of $(g\beta H_z/\bar{\varepsilon})^2$.

Let $\Phi_0^{(0)}(\mathbf{p}, N)$ be the solution of Eq. (A.1), where N is the electron density; then the solutions of (A.2) can be represented in the form

$$\Phi_j^{(0)} = m_j\partial\Phi_0^{(0)}(\mathbf{p}, N)/\partial N, \qquad (A.5)$$

where m_j are arbitrary constants. Indeed, substituting $\Phi_0^{(0)}(\mathbf{p}, N)$ and (A.5) in $\hat{L}_0^{(0)}(\Phi_0^{(0)}, \Phi_j^{(0)})$, we get

$$\hat{L}_i^{(0)}(\Phi_0^{(0)}, \Phi_j^{(0)}) = m_j\frac{\partial}{\partial N}[\hat{L}_i^{(0)}\Phi_0^{(0)}(\mathbf{p}, N)]. \qquad (A.6)$$

Inasmuch as $\hat{L}_i^{(0)}(\Phi_0^{(0)}(\mathbf{p}, N)) = 0$ for arbitrary N, the function (A.5) satisfies Eq. (A.2).

In first order in γ, we obtain inhomogeneous equations. It is obvious that they are linear in $\Phi_0^{(1)}$ and $\Phi_j^{(1)}$.

[9] More accurately, they influence only indirectly, via the distribution function, and this influence leads only to a small change of the constant K in formula (42).

These equations have solutions if the right sides of (A.3) and (A.4) and the solutions of the equations $\tilde{L}_1^{(0)}\varphi = 0$ and $\tilde{L}_1^{(0)}\varphi = 0$ are orthogonal. It is easy to verify that when the Pauli principle is taken into account the operators $\tilde{L}_1^{(0)}$ and $\tilde{L}_1^{(0)}$ have a form similar to (16), and these equations are satisfied by the function $\varphi_0 = \text{const}$. Putting $\varphi_0 = g\beta$, we obtain as the conditions for the solvability of Eqs. (A.3) and (A.4) the Bloch equations (17) and (18) for $M_j = g\beta m_j N$. Unlike (19) and (20), we have here

$$\frac{1}{\tau_1} = \frac{1}{\tau_2} = \sum_{pp'} (W_{+-}^{pp'} + W_{-+}^{pp'}) \frac{\partial}{\partial N} [\Phi_0^{(0)}(p)(1 - \Phi_0^{(0)}(p'))], \quad \text{(A.7)}$$

$$m_0 = \left[\sum_{pp'} (W_{+-}^{pp'} - W_{-+}^{pp'}) \Phi_0^{(0)}(p)(1 - \Phi_0^{(0)}(p)) \right] \times \left\{ \sum_{pp'} (W_{+-}^{pp'} + W_{-+}^{pp'}) \frac{\partial}{\partial N} [\Phi_0^{(0)}(p)(1 - \Phi_0^{(0)}(p))] \right\}^{-1} \quad \text{(A.8)}$$

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