HIGH FREQUENCY EFFECTS IN SEMICONDUCTORS IN INELASTIC SCATTERING OF CARRIERS BY OPTICAL PHONONS

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We investigate the propagation of a strong circularly-polarized electromagnetic wave along the direction of a static magnetic field in a semiconductor, where the electrons are scattered in a sharply inelastic manner by optical phonons. We show that the reflection coefficient of such a wave from the vacuum–semiconductor interface experiences oscillations following variation of the wave amplitude, the magnitude of the magnetic field, and the electron concentration. The cyclotron-resonance absorption line shape has an almost flat top and sharply dropping edges.

INTRODUCTION

IT is well known that in the case of sharply inelastic carrier scattering the components of the electric conductivity tensor σ_{ik} in crossed fields **E** and **H** have singularities at a certain ratio E/H.[1] The purpose of the present work is to investigate the influence of these singularities on the propagation of an electromagnetic wave.

We consider a circularly polarized wave of amplitude E and frequency ω, propagating in an isotropic medium along the direction of the static field **H**; $\omega > 0$ if the direction of rotation of the vector **E**(t) coincides with the direction of the cyclotron rotation of the carriers (of frequency ω_c). To calculate the current **j**(t) it is possible to change over to a system of coordinates rotating about **H** with velocity ω. In this system **E**(t) and **j**(t) are stationary, and the problem reduces to the static one. However, since the charges rotate relative to this system with frequency $\omega_c - \omega$, the static problem will involve a magnetic field H′ such that the cyclotron frequency corresponding to it is $\omega_c' = \omega_c - \omega$.

The main manifestation of the aforementioned singularities is the vanishing of the dissipative component σ_{xx}. When the field E increases at a fixed value of H′, i.e., at fixed H and ω, this component vanishes abruptly on going through a certain critical field E*, which depends on H and ω. As shown in [2], the jumps of the components σ_{ik}, when considered as functions of the field E, lead to an oscillatory dependence of the surface impedance and of the reflection coefficient on the amplitude of the incident field. The physical reason for this can be readily understood. Assume that a wave is incident on a half-space with amplitude E_0 such that a field $E_0' > E^*$ is produced on the interface. On penetrating into the medium, the field attenuates and becomes equal to E* at a certain distance a. At this point, owing to the jump of σ_{ik}, the properties of the medium change abruptly and a plate of thickness a becomes detached, as it were, from the half-space. It is obvious that a increases monotonically with increasing E_0' and E_0, and since the impedance of the plate is an oscillating function of a, the surface impedance is consequently an oscillating function of E_0.

Another high-frequency effect more directly influenced by the vanishing of the dissipative component σ_{xx} is cyclotron resonance when E and ω are fixed and H varies. We note that in this case H′ is actually the deviation from resonance δH, expressed in units of the magnetic field. In this case there exists a critical value $(\delta H)^*$ at which σ_{xx} drops to zero. Therefore, the cyclotron-absorption line has an unusual form with sharply steepened edges.

1. CONNECTION BETWEEN THE STATIC AND HIGH-HIGH-FREQUENCY CONDUCTIVITIES

Confining ourselves to non-quantizing magnetic fields and neglecting the spatial dispersion, we can seek the connection between the current **j**(t) and the wave field **E**(t) from the kinetic equation without spatial derivatives

$$\frac{\partial}{\partial t} f(\mathbf{p}, t) + \left\{ e\mathbf{E}(t) + \frac{e}{c}[\mathbf{v}(\mathbf{p})\mathbf{H}] \right\} \frac{\partial}{\partial \mathbf{p}} f(\mathbf{p}, t) + S(f|\mathbf{p}) = 0,$$

$$\mathbf{j}(t) = e \int \mathbf{v}(\mathbf{p}) f(\mathbf{p}, t)\, d\mathbf{p}, \quad \mathbf{v}(\mathbf{p}) = \frac{\partial}{\partial \mathbf{p}} \varepsilon(\mathbf{p}). \qquad (1.1)^*$$

Here f—distribution function, **H**—external static field, and S—ordinary Boltzmann collision term expressed in terms of the scattering probability $W(\mathbf{p}_1, \mathbf{p}_2)$, and $\varepsilon(\mathbf{p})$ is the dispersion law. It is also assumed that the wave magnetic field is $H(t) \ll H$.

Let the wave be circularly polarized, let it have the frequency ω, and let it propagate along $H \parallel z$. We change over now to a coordinate system (p_x', p_y', p_z') which rotates around z with frequency ω together with the field. In this system, the components E(t) do not depend on the time. If the medium is isotropic, then $\varepsilon(\mathbf{p})$ is invariant against the rotation of **p**, and $W(\mathbf{p}_1, \mathbf{p}_2)$ is invariant relative to a simultaneous rotation of $\mathbf{p}_1$ and $\mathbf{p}_2$.

Therefore the transition to a rotating system in the term S and in the term connected with the magnetic field does not lead to the appearance of an "explicit' time." Thus, the distribution f, expressed in terms of

*$[\mathbf{v}(\mathbf{p})\mathbf{H}] \equiv \mathbf{v}(\mathbf{p}) \times \mathbf{H}$.

$\mathbf{p}'$, does not depend explicitly on the time, and we have

$$\frac{\partial f}{\partial t}=\frac{\partial f}{\partial \mathbf{p}'}\frac{\partial \mathbf{p}'}{\partial t}=\omega[\mathbf{e}_z\mathbf{p}']\frac{\partial f}{\partial \mathbf{p}'},\quad \mathbf{e}_z=\frac{\mathbf{H}}{H}. \tag{1.2}$$

We introduce the cyclotron mass and the frequency

$$m=\frac{p}{v},\quad v=\frac{\partial \varepsilon}{\partial p},\quad \omega_c=\frac{eH}{mc}. \tag{1.3}$$

Then

$$\frac{\partial f}{\partial t}=-m\omega[\mathbf{v}'\mathbf{e}_z]\frac{\partial f}{\partial \mathbf{p}'}. \tag{1.4}$$

Substituting (1.4) in (1.1), we obtain in the coordinate system $\mathbf{p}'$ the static problem, in which the magnetic field is

$$\mathbf{H}\to\mathbf{H}'=\mathbf{H}-\frac{mc}{e}\omega\mathbf{e}_z \tag{1.5}$$

or, what is the same,

$$\omega_c\to\omega_c'=\omega_c-\omega. \tag{1.6}$$

Since the expression for $\mathbf{j}$ in (1.1) is also invariant against rotation of the coordinate system, Eq. (1.5) or (1.6) establishes the connection between the static galvanomagnetic tensor $\sigma_{ik}(0, H)$ and the high frequency tensor $\sigma_{ik}(\omega, H)$ for a circularly polarized field, in accordance with

$$\sigma_{ik}(\omega, H)=\sigma_{ik}(0, H'). \tag{1.7}$$

We note that the conditions for the applicability of relation (1.7) are much broader. Actually, all the assumptions made are inessential, with the exception of the assumption that the medium is isotropic in the absence of an external magnetic field. Even axial symmetry with respect to the z axis is sufficient.

2. CONDUCTIVITY TENSOR

We shall henceforth consider an example in which there is inelastic scattering of the electrons of the semiconductor by optical phonons $\hbar\omega_0$ at low temperature $kT \ll h\omega_0$. The static conductivity tensor for this case was calculated in [1], and has the natural form

$$\|\sigma_{ik}\|=\begin{Vmatrix}\sigma_\perp & \sigma_x & 0\\ -\sigma_x & \sigma_\perp & 0\\ 0 & 0 & \sigma_\parallel\end{Vmatrix}; \tag{2.1}$$

Expressions for the dissipative (transverse, $\sigma_\perp$), Hall (crossing, σ_x), and longitudinal ($\sigma_\parallel$) components are given in the table, where

$$\varkappa=\frac{1}{2}\frac{v_0}{c}\frac{H}{E}, \tag{2.2}$$

$$\sigma_0=\frac{1}{2}\frac{eNv_0}{E}=\frac{eNc}{H}\varkappa,$$

$$p_0=mv_0=(2m\hbar\omega_0)^{1/2},$$

N is the carrier density. To obtain the high-frequency conductivity it is necessary to substitute (1.5) in (2.2).

The expressions given in the table do not take into account certain details in the dependence of σ_{ik} on $\varkappa$; these details are connected with the weaker singularities.[3] The interelectron scattering[4] is also neglected. However, these circumstances are not essential

	$\varkappa<1$	$\varkappa>1$
$\sigma_\perp$	$\sigma_0\dfrac{\varkappa}{\arcsin\varkappa}$	0
σ_x	$\sigma_0\left(\dfrac{1}{\varkappa}-\dfrac{\sqrt{1-\varkappa^2}}{\arcsin\varkappa}\right)$	$\sigma_0\dfrac{1}{\varkappa}$
$\sigma_\parallel$	σ_0	σ_0

when it comes to explaining the meaning of the high-frequency effect. These expressions are valid, in addition, in a certain electric-field interval $E^- \ll E \ll E^+$. The characteristic fields $E^\pm$ are determined by the relations $eF^\pm\tau^\pm = p_0$, where τ^+ and τ^- are the electron relaxation times in the regions $\varepsilon > \hbar\omega_0$ and $\varepsilon < \hbar\omega_0$. The average electron energy here is $\bar{\varepsilon}\approx\hbar\omega_0$, so that neglect of quantization signifies $\omega_c \ll \omega_0$. The mean free path l is determined by the time necessary to accelerate the electron from $\varepsilon = 0$ to $\varepsilon = \hbar\omega_0$, and equals $\tau_E = p_0/eE$. Therefore $l \simeq v\tau_E \approx \hbar\omega_0/eE$, and the neglect of spatial dispersion for the wavelength λ means that $eE\lambda \gg \hbar\omega_0$.

3. HIGH-FREQUENCY EFFECTS

In considering the cyclotron resonance, we emphasize that this is a resonance in which the same strong field E "heats" the electrons and resonates with them. Regarding E and ω as fixed, as is usually the case in experiments on cyclotron resonance, we put

$$\varkappa=\frac{\delta H}{(\delta H)^*},\quad (\delta H)^*=2\frac{c}{v_0}E, \tag{3.1}$$

where we have written δH in lieu of H', in order to emphasize that this quantity actually points to a deviation from the resonance of the magnetic field. Using the first of the forms of σ_0 in (2.2), we obtain immediately, from the expressions listed in the table, the form of the absorption line $\sigma_\perp$ as a function of δH (see the figure). The line is symmetrical but deviates greatly from Lorentzian, is almost rectangular, and its edges are very steep. The line width $(\delta H)^*$ is proportional to E; the corresponding relaxation time is τ_E, which indeed corresponds to the collision frequency. Unlike the usual cyclotron resonance, the observation is not limited by the condition $\omega_c\tau_E \gg 1$, or, what is the same, by $\delta H \ll H$, since the resonance is manifest not in the sharp absorption maximum but in abrupt vanishing of the resonance.

Proceeding to the problem of the reflection of a wave from a half-space, when ω and H are fixed, it is convenient to write

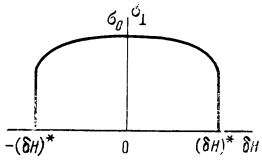
$$\varkappa=\left(\frac{E}{E^*}\right)^{-1},\quad E^*=\frac{p_0|\Delta\omega|}{2e},\quad \Delta\omega=\omega_c-\omega \tag{3.2}$$

and to bear in mind the $\pm$ sign in front of σ_x in accordance with the sign of $\Delta\omega$. Considering a circularly-polarized wave, it is convenient to introduce the complex amplitude $\dot{E}$, defined by the relation

$$E_x(z,t)+iE_y(z,t)=\dot{E}(z)e^{-i\omega t},\quad |\dot{E}|=E. \tag{3.3}$$

Maxwell's equation for this amplitude is given by

$$\frac{d^2}{dz^2}\dot{E}+\varepsilon k^2[1+i\omega f(E)]\dot{E}=0. \tag{3.4}$$



Cyclotron-resonance absorption line shape

Here $k = \omega/c$, ε is the dielectric constant without allowance for the free carriers, $\varepsilon = 1$ in vacuum and $\varepsilon = \varepsilon_L$ in the semiconductor. Further,

$$\alpha = \frac{\omega_p^2}{\omega|\Delta\omega|}, \quad \omega_p^2 = \frac{4\pi Ne^2}{m\varepsilon_L}, \qquad (3.5)$$
$$f(E) = f'(E) + if''(E) = \frac{1}{\sigma_0^*}(\sigma_\perp + i\sigma_x), \quad \sigma_0^* = \frac{1}{2}\frac{eNv_0}{E^*}.$$

The nonlinear equation (3.4) will be solved under the assumption that $\alpha \ll 1$. Methods for the approximate solution have been developed in [2], so that we confine ourselves only to the scheme of the solution.

In accordance with the qualitative picture described in the introduction, the solution is sought separately in three regions:

1) vacuum, $-\infty < z < 0$;

2) "detached plate," $0 < z < a$, where $E > E^*$;

3) remaining part of the semiconductor, $a < z < +\infty$, where $F < E^*$.

When $z > a$, the wave attenuates as a result of the lattice absorption or the small quantity $\sigma_\perp$, unaccounted for in the table, when $E < E^*$. These effects will be disregarded, and we shall therefore assume that when $z > a$ the medium is nondissipative, i.e., $E = E^*$. Nonetheless, we shall neglect reflection from the really existing second interface.

In vacuum we have

$$\dot{E}_1(z) = E_0[\exp\{ikz\} + P\exp\{-ikz\}], \qquad (3.6)$$

where E_0 is the amplitude of the incident wave and P is the sought reflection coefficient. In the nondissipative region 3) we have $f'(E) = 0$, and consequently $E = E^* = \text{const}$, and therefore $f''(E) = f''(E^*) = \text{const}$. As a result we can immediately write out the solution in the form of a wave going to $z = +\infty$

$$\dot{E}_3(z) = E^*\exp\{ikS_3 + ikn(1 - {}^1/_2\alpha f''(E^*))z\}, \qquad (3.7)$$

where we have introduced the refractive index of the lattice $n = \sqrt{\varepsilon_L}$, and S_3 is an undetermined phase. In the dissipative region 2), when $\alpha \ll 1$, there are two waves propagating in both directions, and therefore

$$\dot{E}_2(z) = w_+(z)\exp\{iknz\} + w_-(z)\exp\{-iknz\}, \qquad (3.8)$$
$$w_\pm(z) = u_\pm(z)\exp\{ikS_\pm(z)\}. \qquad (3.9)$$

Substituting (3.8) in (3.4), we can stipulate

$$\frac{d^2}{dz^2}w_\pm \pm 2ikn\frac{d}{dz}w_\pm + ik^2\alpha n^2 f w_\pm = 0. \qquad (3.10)$$

It is also necessary to satisfy the conditions of the continuity of the field E and of the derivative dE/dz at $z = 0$ and $z = a$. From these conditions we should find P, S_2, a, and the integration constants (3.10).

Using the smallness of α, we expand

$$u_\pm(z) = u_\pm^{(0)}(z) + \alpha u_\pm^{(1)}(z) + \ldots,$$
$$S_\pm(z) = S_\pm^{(0)}(z) + \alpha S_\pm^{(1)}(z) + \ldots,$$
$$P = P^{(0)} + \alpha P^{(1)} + \ldots,$$
$$a = \alpha^{-1}a^{(-1)} + \ldots. \qquad (3.11)$$

The expansion of a should contain reciprocal powers of α, since it is obvious from physical considerations that when $\alpha \to 0$ we should have $a \to \infty$; it is easy to verify from the equations that the expansion begins with the term that has been written out. In order to obtain the first correction to P, it is sufficient to retain terms which are explicitly written out in (3.11), solve (3.10) accurate to α^2 inclusive, and satisfy the continuity conditions accurate to α inclusive. As a result we get

$$P^{(0)} = \frac{1-n}{1+n}, \quad P^{(1)} = \bar{P}^{(1)} + \tilde{P}^{(1)}, \quad \bar{P}^{(1)} = -\frac{in}{(1+n)^2}f(E_0'),$$
$$\tilde{P}^{(1)} = \frac{in}{4}\left(\frac{E_0}{E^*}\right)^{-2}f'(E^*)\exp\left\{\frac{i}{\alpha}\Psi(E_0)\right\},$$
$$\Psi(E_0) = 4\int_{E^*}^{E_0'}\frac{dE}{Ef'(E)}, \quad E_0' = \frac{2E_0}{1+n}. \qquad (3.12)$$

Here $\bar{P}^{(1)}$ and $\tilde{P}^{(1)}$ are the monotonic and oscillating corrections to the reflection coefficient, and E_0' is the field on the interface (calculated in the zeroth approximation in α).[1] Inasmuch as $f' > 0$, it follows that $\Psi(E_0)$ is a monotonically increasing function, and $\tilde{P}^{(1)}$ oscillates with increasing E_0. Since, in accordance with (3.5), α depends on N, ω, and H, it follows that $\tilde{P}^{(1)}$ also oscillates when these parameters change.

The obtained solution is meaningful only when $E_0' > E^*$, and in the opposite case the "plate" does not separate and all the oscillatory effects vanish. Since the argument of the exponential of $\tilde{P}^{(1)}$ contains the large factor $1/\alpha$, the periods of the oscillations ΔE_0 and ΔH of E_0 and H contain the small factor α and can be readily determined:

$$\frac{\Delta E}{E_0} = \frac{\pi}{2}\alpha f'(E_0'), \quad \frac{\Delta H}{H} = \frac{\pi}{2}\alpha\left[1 - \frac{E^*}{E_0'f'(E_0')}\right]^{-1}. \qquad (3.13)$$

The oscillations in these parameters are not strictly periodic, since f' depends both on E_0' and on H (via E^*). Since Ψ does not contain the concentration, the oscillations in N^{-1} should be strictly periodic with a period

$$\frac{\Delta N^{-1}}{N^{-1}} = \frac{\pi}{2}\alpha\Psi^{-1}(E_0). \qquad (3.14)$$

We note that the accuracy of the formula (3.13) for ΔH is much worse than the accuracy of the formulas (3.12), owing to the cancellation of the terms in the square brackets. Therefore, at not too small α, it is better to find ΔH directly from the expression for Ψ.

Let us discuss the possibility of experimentally observing the aforementioned effects in p-Ge, where the

[1] We note that in the expression for $\tilde{P}^{(1)}$ the value of a was calculated accurate to the first-order term in the expansion in α. This is valid if $E_0 \gg E^*$ or if n is close to unity. In the general case it is necessary to retain also $a^{(0)}$ in $\tilde{P}^{(1)}$.

inelasticity of the scattering is appreciable,[5,6] and where the conditions for the applicability of the expressions listed in the table are well satisfied.[1] The parameters of the material are as follows: $m = 0.3\,m_0$ (heavy holes), $\omega_0 = 6\times10^{13}$ sec^{-1}, whence $v_0 = 2\times10^7$ cm/sec; $\tau^+ = 10^{-12}$ sec (spontaneous emission of optical phonons), whence $E^+ = 3000$ V/cm. In order to ensure a break between E^+ and E^- it is necessary to use hydrogen or helium temperatures. Then in pure samples $\tau^- = 10^{-10}$ sec[7] (scattering by acoustic phonons) and $E^- = 30$ V/cm. We choose $\omega = 2.4\times10^{11}$ sec^{-1} ($\lambda_0 = 8$ mm) and $H = 5.6$ kOe, which yields $\omega_c = 3.3\times10^{11}$ sec^{-1}. We then get $\Delta\omega = 0.9\times10^{11}$ sec^{-1}, and the critical field $E^* = 150$ V/cm falls in the permissible interval between E^- and E^+. Choosing $E_0 = 500$ V/cm (incident power $W = 400$ W/cm^2) and using $\varepsilon_L = 16$, we get $E_0' = 200$ V/cm, which is larger than E^* and also lies in the permissible field interval. In such fields, the shallow impurities are completely ionized, so that for pure samples we have $N = 10^{12}$ cm^{-3}, i.e., $\omega_p = 10^{11}$ sec^{-1}. This corresponds to $\alpha = 0.3$. The period of the oscillations here is $\Delta E_0 = 150$ V/cm. The value of E_0 can be increased until E_0' becomes comparable with E^+. It is obvious that this should give rise to a large number of oscillations (on the order of 10). In principle it is possible to observe also oscillations in H and N, the latter with the aid of additional elimination. The periods are then estimated at $\Delta H \approx 2$ kOe and $\Delta N \approx 3\times10^{12}$ cm^{-3}. However, changes of H and N soon cause α to be no longer small, so that only a qualitative retention of the picture can be expected.

It is useful to estimate the value of a at which the first oscillation is observed. Its order of magnitude is half the wavelength in the material, $a \approx \lambda/2$, $\lambda = \lambda_0/n$, i.e., $a \approx 1$ mm.

Cyclotron resonance may be observed under the same conditions. Assuming $E_0 = 500$ V/cm and $E_0' = 200$ V/cm, we get $(\delta H)^* = 2$ kOe. Thus, with the center of the absorption line at $H = 4.1$ kOe, the vanishing should take place in fields 2.1 and 6.1 kOe.

We point out that there exists one necessary condition for the observation of the reflection oscillations. Owing to the finite time τ^+, the vanishing of the dissipative current is actually smeared out near E^* in a certain field interval ΔE^*. Using the concepts developed in [1], we obtain the order-of-magnitude estimate

$$\frac{\Delta E^*}{E^*} \approx \left(\frac{E^*}{E^+}\right)^{1/2} \equiv \xi^{1/2}. \tag{3.15}$$

This smearing brings about a smearing of the boundary of the "detached plate" by an amount

$$\Delta a \approx \left(\frac{dE}{dz}\right)^{-1}\Delta E^* \approx \frac{\Delta E^*}{E^*}\frac{\lambda}{\alpha} \approx \frac{\xi^{1/2}}{\alpha}\lambda. \tag{3.16}$$

The reflection oscillations will not be smeared out if the condition $\Delta a \lesssim \lambda$ is satisfied, i.e., if $\xi \lesssim \alpha^2$. For the figures given above, $\xi = 0.05$ and $\alpha^2 = 0.1$.

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[1] I. I. Vosilyus and I. B. Levinson, Zh. Eksp. Teor. Fiz. **50**, 1660 (1966) [Sov. Phys.-JETP **23**, 1104 (1966)].

[2] F. G. Bass and Yu. G. Gurevich, ibid., **53**, 1058 (1967) [**26**, 630 (1968)].

[3] I. I. Vosilyus and I. B. Levinson, ibid., **52**, 1013 (1967) [**25**, 672 (1967)].

[4] F. G. Bass, I. B. Levinson, and O. N. Chavchanidze, ibid. **52**, 1263 (1967) [**25**, 839 (1967)].

[5] R. Bray and W. E. Pinson, Phys. Rev. **136A**, 1449 (1964).

[6] E. G. S. Paige, J. Phys. Soc. Japan **21**, Suppl. 397 (1966).

[7] D. M. Brown and R. Bray, Phys. Rev. **127**, 1593 (1962).

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