FREQUENCY FLUCTUATIONS OF A GAS LASER AND DETERMINATION OF THE NATURAL WIDTH OF ITS SPECTRAL LINE

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A technique is described for observing and measuring the spectral density of natural frequency fluctuations of laser radiation (of the order of hundredths or tenths of Hz^2/Hz). For this purpose, an auxiliary laser heterodyne is employed whose frequency differs from that of the laser on the average by a constant value. This is attained by using an automatic tuning system. The results of measurement of the spectrum of natural frequency fluctuations are compared with the results of a previous theoretical analysis. Satisfactory quantitative agreement is observed. An experimental investigation of the natural frequency fluctuations of the radiation from a single-frequency He – Ne laser with $\lambda = 0.63\ \mu$ yielded its line width.

As is well known, in any self-oscillating system, and consequently also in lasers, the presence of fluctuations leads to non-monochromaticity of the oscillations, or to a natural spreading (width) of the spectral line of the generator. In the time domain this means that the fluctuations give rise to a random frequency modulation (natural frequency fluctuations) of the oscillation. Besides the fluctuations, the system parameters are subject to random changes (for various technical reasons); these cause, as a rule, large but relatively slow random drifts of the frequency (technical frequency fluctuations), governing the technical (real) width of the spectral lines. This group of questions has been thoroughly investigated, both theoretically and experimentally, mainly as applied to the radio bands (see the monographs[1,2]).

It is important to note that the magnitude of the natural (fluctuation) non-monochromaticity of the oscillation is smaller by several orders of magnitude than the real (technical) one. Nonetheless, a theoretical and experimental determination of the natural width of a spectral line is of undisputed physical interest, and in some cases is also of practical interest. For obvious reasons, a direct measurement (by plotting the oscillation spectrum) of the natural width is practically impossible. A demodulation proposed for the radio band (see[3]) makes it possible to investigate the spectrum of the phase (frequency) fluctuations of the generator and to determine from the latter the natural width of its spectral line. The idea of investigating the spectrum of frequency fluctuations was used to determine the line width of ordinary vacuum tube oscillators, klystrons, magnetrons, and molecular generators; it is natural to apply this idea to the analogous laser problem.

The purpose of the present paper is to describe the development of a procedure for observing and experimentally investigating frequency fluctuations of laser oscillations, and also to compare the experimental data with the results of the theoretical analysis[1)]. An investigation of the emission of a single-frequency He – Ne laser operating with a single axial mode at a wavelength 0.63 μ was carried out by us with the aid of an auxiliary laser, using a method that reduces to partial coherent optical heterodyning with subsequent measurement of the fluctuations of the frequency spectrum of the difference oscillations[2)].

We note that an investigation of the spectrum of laser-frequency fluctuations, as well as of the intensity fluctuations, is of interest in itself in various aspects (questions of short-duration frequency stability, form of the spectral line of the radiation, etc.). In particular, a comparison of the experimental results with the corresponding theoretical analysis makes it possible to clarify the mechanism of the fluctuation processes in the gas laser. An analysis of this type was carried out in[7], where expressions were obtained for the fluctuation spectra of the intensity and frequency of the single-mode laser, due to the spontaneous emission. Some of the results of the experiments on the natural fluctuations of the laser-emission intensity, used in the present paper, are reported in[8].

1. PRELIMINARY REMARKS

To determine the natural width of the spectral line of the laser emission, as noted above, it is necessary to observe the natural fluctuations of the frequency of its oscillations and to perform the appropriate measurement. Observation of the natural fluctuations of the laser frequency is made difficult by the presence of

[1)]A brief report of the work was published earlier [4].

[2)]Javan, Ballik, and Bond [5] attempted to determine the limiting nonmonochromaticity of an infared He-Ne laser, the theoretically-estimated relative value of which is approximately 10^{-17}. Their investigations of the result of the beats between two adjacent modes of one generator (thereby excluding to a certain degree the technical factors) have shown that the nonmonochromaticity of the oscillations does not exceed 1×10^{-14} over several seconds. On the other hand, an investigation of the beats of two independent generators yielded a value 8×10^{-14} over several dozen milliseconds [6]. Obviously, such experiments show only the present-day limit of the possible reduction of the real (technical) nonmonochromaticity of laser emission.

appreciable technical fluctuations, due to different factors. These are, first, the mechanical, acoustic, thermal, etc. disturbances, well known in the radiofrequencies, which lead to variation of the laser-cavity parameters, and also pulsations of the power supply. Among the technical disturbances are factors peculiar to a gas laser, such as the noise of the gas-discharge plasma. By virtue of the entirely different nature of the processes that lead to natural and technical frequency fluctuations, the foregoing fluctuations are statistically independent and differ in character.

Numerous observations of laser-frequency fluctuations (see[9], and also[10]) have shown that the foregoing causes of technical origin lead to relatively slow changes of the parameters of the laser generator, and in the first place to random changes of the resonator length. Under disturbances of this kind, the spectrum of the (technical) frequency fluctuations is concentrated at low observation frequencies, decreasing rapidly in magnitude with increase of the latter. At sufficiently high frequencies one can expect, in the main, results of the action of natural causes. If the intensity and the character of the external disturbances changes, then only that part of the spectrum which is governed by the lack of constancy of the generator parameters should change. It follows therefore that it is necessary to analyze the spectrum of the frequency fluctuations in a rather broad interval of observation frequencies F under variation of the external conditions. On the other hand, the theory makes it possible to determine the spectrum of the natural fluctuations and its dependence on the various generator parameters, such as the radiation power etc. Thus, the aggregate of the measurements of the spectra under controlled variation of both the external conditions and the generator parameters, makes it possible to separate the natural frequency fluctuations, and by the same token to determine the natural line width of the generator.

The laser frequency fluctuations connected with the natural factors were considered in[7]; it is shown there, in particular, that the main cause of the natural fluctuations is a spontaneous emission of the active medium—a process having, as is well known, a very small correlation time. From an analysis carried out in[7] with allowance for the dispersion and nonlinear properties of the active medium, the following expression is obtained for the spectral density of the frequency fluctuations

$$w_\nu(F) = \frac{(\Delta\nu)^2 h\nu\tilde{\beta}\alpha\varkappa_2^0}{P(1+q_2)^2\tilde{\varkappa}_2}\left\{1 + \frac{2p_2}{p^2+(2\pi F)^2}\left[p_2\left(1+\frac{q_1^2}{2(1+q_2)^2}\right) + \frac{q_1 p}{1+q_2} + 2\pi F\sqrt{2}\right]\right\}, \quad (1)$$

where $\Delta\nu$ is the bandwidth of the laser cavity, P the total radiation power given up by the active medium, $a = (n_2^0/g_2)/[(n_2^0/g_2) - (n_1^0/g_1)]$ is a parameter characterizing the degree of population inversion of the optical transition in the absence of the oscillations (initial inversion), $\tilde{\beta} = \tilde{n}_2/n_2^0$ is a parameter characterizing the depletion of the upper level during the generation process, $\kappa_2^0/\tilde{\kappa}_2$ is the ratio of the imaginary components of the dielectric constant of the medium in the absence and in the presence of oscillations, respectively, and q_1, q_2, p, p_2 are parameters describing the dispersion and nonlinear properties of the active medium of the laser. When the generation frequency ν is tuned to the center of the line ν_l of the working transition, the parameters q_1 and p_2 assume zero values, and the factor in the curly brackets of (1) becomes equal to unity. When $\nu \neq \nu_l$, as shown by estimates and by some of our experiments, the value of the indicated factor differs little from unity, owing to the fact that the line width of the working transition is much larger than the bandwidth of the employed resonators.

It follows from (1) that under the indicated conditions the spectrum of the frequency fluctuations does not depend on the observation frequency, and consequently the shape of the natural spectral line of the laser generation is Lorentzian with width $\delta\nu$. For the line width we get, in accordance with the theory (see, for example,[2]), $\delta\nu = \pi w_\nu(F)$ as $F \to 0$; taking the foregoing into account, we get

$$\delta\nu \approx \frac{\pi(\Delta\nu)^2 h\nu\tilde{\beta}\alpha\varkappa_2^0}{P\tilde{\varkappa}_2}\ (\text{Hz}). \quad (2)$$

Let us make numerical estimates[3)]. From the previous investigations with a single-frequency laser at a wavelength 0.63 μ[8] we can put $\Delta\nu$ = 6 MHz and $\tilde{\beta}\alpha\kappa_2^0/\tilde{\kappa}_2 \approx 2$ at $P \approx 1$ mW; for $w_\nu(F)$ and $\delta\nu$ we have respectively ~0.02 Hz^2/Hz and ~0.07 Hz. The experimental data of[9,10] show that the values of $w_\nu(F)$, up to observation frequencies on the order of 10 kHz, exceed the values calculated from (1) by several orders of magnitude.

As follows from the foregoing, to observe and also to determine quantitatively the spectral component due to the natural fluctuations of the frequency, the measurements must be carried out by a method having a rather high sensitivity in a frequency interval at least up to hundreds of kHz. In the radio and microwave bands, two methods are used for such a problem: the delay-line method and the contour method[2]. Calculations have shown that it is practically impossible to satisfy the foregoing requirements for a single-frequency He – Ne laser operating at 0.63 μ: for a reliable separation (at a signal/noise ratio not less than several units) of the frequency fluctuations from the noise of the measuring apparatus (including the fluctuations of the laser-radiation intensity), it is necessary to have either a very stable optical interferometer with a large path difference, or a stable high-Q optical resonator (in the latter case, a laser with high stability of the mean frequency is necessary). It is possible to obtain satisfactory sensitivity in the measurements by using the procedure of conversion (heterodyning) of the frequency of the investigated laser in the radio-frequency region, as was done in the present work.

2. MEASUREMENT METHOD AND APPARATUS

In the heterodyning of signals of two independent lasers, the studied quantity is the frequency fluctua-

3) In writing down (on the basis of the results of [7]) expressions (1) and (2), a transition was made from circular to ordinary frequencies.

tion of the difference oscillation. Naturally, random deviations of the frequency from its mean value can in this case be quite large, up to several dozen MHz. Since most discriminators for the frequency of the difference oscillation operate effectively in a limited bandwidth, it is necessary to limit the variation of the difference frequency. This is done by using some variant of an automatic frequency control for the laser and for the heterodyne.

In our investigations we used an automatic frequency control system whose main elements are shown in the block diagram of the experimental setup (see Fig. 1). The radiation from the investigated laser 1 interferes with the radiation from the auxiliary laser 2 (heterodyne) and is applied to a photodiode 3, the load of which is a resonant LC circuit 4, turned to the difference frequency ~8.4 MHz; these are followed by an amplifier 5, an amplitude limiter 6, and a frequency discriminator 7. The output voltage of the latter, which is thus determined only by the fluctuations of the difference-oscillation frequency, passes through amplifier 8 and then to the spectrum analyzer 9. In addition, this voltage is fed through an RC network 10 with time constant ~0.3 sec to amplifier 11, and then to the piezoelectric element 12, which controls the heterodyne frequency[4)].

Such a system maintains automatically the mean value of the frequency (8.4 MHz) of the difference oscillation, without exerting any influence on the rather rapid fluctuations of the frequency of the tuned laser, which lie outside the passband of the system; on the other hand, the low frequency fluctuations of the heterodyne and laser frequencies turn out to be correlated as a result of the automatic control. If the parameters of the partially-coherent heterodyne are chosen such that its radiation power and resonator Q are larger than those of the laser, the level of the natural fluctuations of its frequency are much lower than the sought level (see the estimates above), and rapid fluctuations of the frequency of the difference oscillation will be due in practice only to the frequency fluctuations of the investigated laser. To exclude from the observation results the parameters of the photodiode and the radio apparatus, an auxiliary generator 14 was used, which was connected through a weak capacitive coupling circuit ($C \approx 5$ pF) 13 to the photodiode load (the calibration

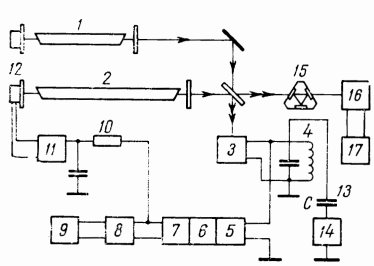

FIG. 1. Diagram of setup for the study of laser-emission frequency fluctuations.

is described in detail later). To observe the optical spectra of the laser and of the heterodyne, and to indicate the tuning of their frequency relative to the center of the optical-transition line, we used a scanning traveling-wave interferometer 15, a photomultiplier 16, and an oscilloscope 17[11].

Let us note the characteristic features of this setup, which make it possible to obtain sufficient sensitivity in the measurement of the frequency-fluctuation spectrum in an interval up to several hundred kHz. These include the use of a photodiode (and not a photomultiplier) as a receiver, the choice of the photoreceiver load in the form of a resonant tuned circuit, and of a corresponding value of approximately 10 MHz for the frequency of the difference oscillation. The significance of these factors will be illustrated by considering the ratio of the useful effect—the spectral density of the laser frequency fluctuations $w_\nu(F)$—to the harmful effect—the spectrum $w_n(F)$ due to the apparatus noise.

The readings of the output instrument of the spectrum analyzer of the described system are governed by the useful effect and by the effect of the parasitic frequency modulation of the difference oscillation, connected with the additive high-frequency noise (adjacent to the 8.4 MHz frequency), essentially across the photoreceiver load[5)]. On the other hand, the amplitude fluctuations of the difference oscillation, with a spectrum extending at least from several dozen to several hundred kHz, are eliminated by the vacuum-tube limiter 6. The noise voltage u across the load impedance consists of the thermal noise of the load and the fluctuations of the photodiode current due to the shot effect and the fluctuations of the laser intensity.

An oscillation of the difference frequency 8.4 MHz, with effective value $U = \sqrt{2\rho I_l I_h z}$, is picked off the impedance z simultaneously with the voltage u; here I_l and I_h are the dc components of the photodiode current due to the laser emission and the emission of the working mode of the heterodyne respectively (a three-mode heterodyne was used), and ρ is a coefficient that takes into account the decrease of the value of U (compared with the maximum possible) as a result of the incomplete spatial overlap and the different divergence of the interfering laser beams. In our experiments we could assume that $\rho \approx 0.5$. Superposition of the noise and regular signals leads to an effect that is equivalent to additional frequency modulation of the difference oscillations; the spectral density of such a noise modulation at the observation frequency F is equal to $w_n(F) = w_u F^2/U^2$, where w_u is the spectral density of the noise voltage u.

It is easy to see that the quantity w_u is determined (with allowance for only the noise noted above) by the expression

$$w_u \approx 4R_c kT + 2eI_{ph}z^2 + 0.5\overline{M_F^2}\, I_l^2 z^2, \qquad (3)$$

where R_c is the active component of the load, k is Boltzmann's constant, T = 290°K, e is the electron charge, I_{ph} is the dc component of the photodiode

4)The authors are grateful to L. A. Khurtin for help in constructing the electronic apparatus.

5)In estimating the sensitivity, the noise of the first and subsequent stages, referred to the input of the radio circutry, could be neglected compared with the thermal noise of the LC circut.

current due to the total radiation of both lasers, and $\overline{M_F^2}$ is the spectral density of the coefficient of laser-intensity modulation at a frequency ~8.4 MHz. The third term is connected with the fluctuations of the laser intensity (we recall that the heterodyne noise has been neglected). An estimate of the magnitude of this term, based on the results of the measurements of [8], has shown that in the described setup, with a laser at wavelength 0.63 μ, the contribution due to the intensity fluctuations (compared with the shot effect of the current I_{ph}) can be neglected.

As a net result, we obtain for the ratio of the useful effect to the harmful one (signal/noise ratio), putting $R_c \approx z$ near resonance,

$$\frac{S}{N} = \frac{w_\nu}{w_n(F)} \approx \frac{w_\nu I_h I_l}{F^2 [4kT/z + 2eI_{ph}]}. \quad (4)$$

From expression (4), we can readily see the role of the quantum efficiency (it is convenient to have a large primary photocurrent) and of the value of z (at larger values of the impedance the relative contribution of the thermal noise of the load decreases).

The difference-oscillation frequency chosen by us has made it possible to obtain an impedance z equal to ~2 kΩ; a higher value of the difference frequency would lead to a lower z, and at lower values it would be difficult to observe and measure the quantity w_ν in the broadband of frequencies F (up to several hundred kHz).

For a quantitative estimate of the S/N ratio in the employed measurement scheme, we put $w_\nu \approx 0.02$ Hz^2/Hz (see above), $z \approx 2 \times 10^3$ ohm, $I_l = 15$ μA, $I_h = 30$ μA, $I_{ph} = 65$ μA (I_h was estimated from the ratio of the intensity of the modes in the optical spectrum of the heterodyne). It follows from (4) that at frequencies F equal to 10, 50, and 300 kHz, S/N assumes the respective values ~3000, 125, and 3.5. This indicates that the limiting sensitivity of the setup, which reaches at F = 10 kHz $w_\nu \approx 5 \times 10^{-6}$ Hz^2/Hz, drops to $w_\nu \approx 10^{-2}$ Hz^2/Hz at frequencies 300–400 kHz. Thus, in the frequency interval up to ~300 kHz, according to the estimates (and confirmed by experiment), there is a real possibility of observing natural frequency fluctuations due to the spontaneous emission of the active medium.

In the observation of such small effects, their measurement involves calibration of the parameters of the photoreceiver and of the radio circuitry. In our experiments we used the calibration method based on a comparison of the investigated frequency fluctuations of the difference oscillation of the frequency $\omega/2\pi = 8.4$ MHz with the effect of frequency modulation of an auxiliary oscillation of frequency ω, produced by superposition of the auxiliary oscillation on a noise signal with a known spectrum. (We note that in considering the sensitivity of the measurement method, the same effect actually plays the role of a parasitic effect.) To this end, the difference oscillation from the laser and heterodyne was imitated by a signal from a type GSS-6 generator (see the calibration circuit in Fig. 1), and the photodiode was illuminated only by "white" light from an incandescent lamp. The auxiliary signal was set equal (using an appropriate instrument at the output of amplifier 5) to the value of U of the difference oscillation from the lasers; the output voltage of the GSS-6 had an effective value E_c. The white-light intensity was chosen such as to make the photocurrent, and consequently also its noise due to the shot effect, equal to the corresponding values when the photodiode is illuminated with the lasers. Obviously (when account is taken of the frequency fluctuations only), the readings of the spectrum analyzer at any observation frequency differ in this case from the readings obtained by measurements with lasers, by an amount governed only by the sought effect[6]. We note that the described procedure automatically takes into account also such factors as the incomplete overlap of the laser beams during interference, the inertia of the photodiode, etc.

The foregoing enables us to indicate the sequence of the procedure for measuring the spectral density of the frequency fluctuations. To determine the latter, it is sufficient to take three readings of the output instrument of the analyzer (with a narrow passband at the frequency F): α_l—when the photodiode is illuminated by the laser and the heterodyne (the automatic frequency control system is in operation), α_c—when an auxiliary signal of magnitude E_c is turned on and the photodiode is illuminated with white light with a photocurrent $I_c = I_{ph}$, and α_d—under conditions analogous to the preceding, but without illuminating the photodiode (the subscripts denote as follows: l—laser, c—calibration, d—dark current). By considering the effects of fluctuations of the frequency $\omega/2\pi$, corresponding to these readings (with allowance for the transfer coefficients of the measuring-setup elements), we can obtain an expression for the sought quantity $w_\nu(F)$; thus, if the amplitude characteristic of the spectral analyzer is linear, we have

$$w_\nu(F) = \frac{2eI_{ph}F^2}{\omega^2 C^2 E_c^2} \frac{\alpha_l^2 - \alpha_c^2}{\alpha_c^2 - \alpha_d^2} = \frac{2eF^2}{\varepsilon \omega^2 C^2 E_c} [(\alpha_l/\alpha_d)^2 - 1 - \varepsilon I_{ph}], \quad (5)$$

where ϵ denotes the ratio $[(\alpha_c/\alpha_d)^2 - 1]/I_{ph}$. The value of ϵ is determined only by the parameters of the circuit; it is easy to see that ϵ depends on the observation frequency F (in the experiment, the values of ϵ decreased with increasing F by 10–15%).

The reduction of the observation results was based on the last relation of (5), since this increase the accuracy with which $w_\nu(F)$ is determined, owing to the more accurate experimental determination of the values ϵ by means of the foregoing relation[7].

We used in the experiments a single-frequency axial-mode laser with a mixture of helium and natural neon, at an operating wavelength $\lambda = 0.63$ μ. The influence of the strong infrared transition at the 3.39 μ wavelength, which has a common level with the working level, was eliminated by using in the tube Brewster

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[6] Additional experiments have shown that the contribution due to the frequency fluctuations of the GSS-6 generator were small enough to be completely neglected.

[7] The described calibration procedure was verified by additional experiments, which have shown it to be satisfactory at F > 40 – 60 kHz. At smaller values of F, it distorted somewhat the results of the experiments (this being apparently connected with the operation of the limiter). A somewhat different calibration method was used at these frequencies.

windows made of type LK-4 glass. The gas-discharge tube had a diameter 1.5 mm and a working discharge gap approximately 16 cm long; the distance between the flat and spherical mirrors (radius of curvature approximately 250 cm) was ~23 cm and was fixed by means of three invar rods. The use of a piezoelectric element made it possible to vary the length of the resonator and to tune the generation frequency in a range of ~650 MHz. The laser power could be regulated by varying the dc discharge current or by varying the resonator losses. In most experiments, the discharge current was chosen such that the observed intensity fluctuations were due only to the spontaneous emission (see[8]).

The heterodyne employed was a laser with a resonator length approximately 50 cm; as a rule, it operated in the three-mode regime with axial modes. Both lasers, the system of mirrors that ensured interference of their radiation, and the scanning resonator were placed on a heavy steel plate and were maintained under ordinary laboratory conditions.

3. MEASUREMENT RESULTS AND THEIR DISCUSSION

In the investigation of the laser-frequency fluctuations, a cycle of measurements was performed on the fluctuation spectrum in the observation frequency band from 10 to 300 kHz, where a sufficient signal/noise ratio was ensured. The measurements were made principally with the laser generation frequency tuned close to the center of the optical-transition line. To be able to compare more completely the experimental data with the results of the theoretical analysis, the frequency-fluctuation spectra were obtained at different laser power levels and with the heterodyne operating in a fixed regime.

The numerous measurements of the frequency fluctuation spectrum have shown that it is possible to separate dependably the technical component of the spectrum, due primarily to the mechanical and acoustic noise; this section of the spectrum was in the region of frequencies F up to ~40–60 kHz.

The spectral density $w_\nu(F)$, starting with 60--100 kHz, was practically independent of F and remained constant when the noise level was significantly varied (for example in daytime and in the evening, when the indicated noise differed greatly). This section of the spectrum can be attributed to fluctuations due to natural causes.

When the discharge current increased appreciably over its working value ~4.5 mA, an increase of the values of $w_\nu(F)$ by a factor of several times was observed in the spectrum of the frequency fluctuations (up to 60--150 kHz). As shown by parallel measurements, the intensity-fluctuation spectrum reveals at the same time, in the same observation-frequency interval, a contribution from the characteristic noise of the gas plasma.

Several high-frequency resonances (in the region up to ~80 kHz) of the mechanical system of the laser, leading to narrow "spikes" of the $w_\nu(F)$, were observed. These were mainly resonances of the piezoelectric element, which was constructed in the form of a cylinder to which a mirror was attached. By varying

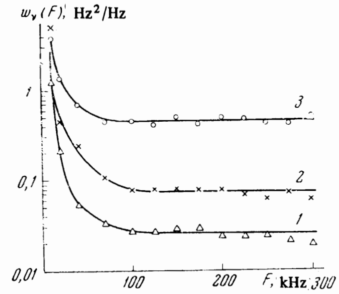

FIG. 2

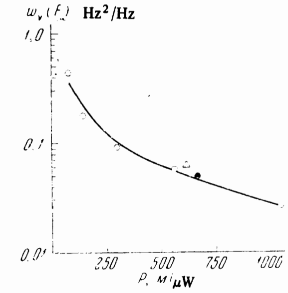

FIG. 3

FIG. 2. Experimental plots of the spectral density of the laser frequency fluctuations against the observation frequency.

FIG. 3. Dependence of the spectral density of the natural laser-frequency fluctuations on the power supplied by the active medium.

the mounting of the mirror and replacing the thin-wall cylinder by a bulky piezoceramic disc, it was possible to eliminate these spikes almost completely.

Out of the available experimental material, we present only those data which were obtained at the maximum reduction of the mechanic and acoustic noise possible under the laboratory condition. The measurements were made in the night time, at various laser-generation power levels P, approximately from 100 to 1000 μW. We note that the output power of the laser was ~20% of P. (The determination of P was described earlier in[8].)

Typical results of the reduction of the experimental data in the determination of $w_\nu(F)$ (see relation (5)) of the 8.4 mHz difference oscillation are shown in Fig. 2, where curves 1, 2, and 3 correspond to the power values ~1000, 300, and 80 μW. It follows from these plots that the values of $w_\nu(F)$ in the frequency interval $F \approx 60$–300 kHz did not depend on F and increased rapidly with decreasing frequency outside this interval. Experiments under other conditions have shown that $w_\nu(F)$ changed only at frequencies up to 60–100 kHz, assuming values larger by 1–2 orders of magnitude than those indicated in Fig. 2. Thus, that part of the spectrum was due to technical factors. We note that technical fluctuations of the laser and heterodyne frequencies could be partly correlated, owing to the common (acoustic, mechanical) sources of such fluctuations.

The spectral component independent of F could be ascribed to the natural fluctuations of the frequency of the investigated laser, since experiments have shown that the heterodyne frequency fluctuations made no appreciable contribution to the observed effect. Thus, when the heterodyne power was decreased by a factor up to 4, the average value of $w_\nu(F)$ in the band from 100 to 300 kHz remained constant, within the limits of experimental error.

To compare the experimental results with the values of w_ν calculated from (1) for the center of the transition line, we plotted (solid line) w_ν against P (see Fig. 3), and marked with light circles the experimental values of $w_\nu(F)$ averaged in the interval of frequencies F from 100 to 300 kHz. In the determination of the values of w_ν by means of (1), we used in the calculation $\tilde{\beta}\alpha\kappa_2^0/\tilde{\kappa}_2 = 2$, since it follows from measurements of the intensity fluctuations of the investigated laser (see[8]) that $\tilde{\beta}\alpha \approx 1.5-1.65$ and $\kappa_2^0/\tilde{\kappa}_2 = 1.05-1.4$ at $P \approx 100-1000\ \mu W$. The resonator bandwidth $\Delta\nu$ was determined with the described setup by observing its resonance curves with the laser tube extinguished; the values of $\Delta\nu$ turned out to be ~ 7 MHz at $P \approx 100\ \mu W$ and ~ 6 MHz at $P \approx 1000\ \mu W$. At the indicated values of the parameters, the discrepancy between the experimental data and the theoretical ones reaches 30%, which does not exceed the measurement errors.

The foregoing results pertained to the case of laser generation near the center of the optical-transition line. In addition, a series of measurements was also made on the fluctuation spectrum with a detuning off the center of the transition up to $\sim +200$ and -250 MHz (in this case the heterodyne signal constituted the oscillations of the corresponding modes of the auxiliary laser, separated in frequency by ~ 300 MHz.) In these experiments, the emission of the investigated laser was decreased at the indicated detunings by 20--30% from the value $P \approx 700\ \mu W$ at the center of the line. The measurement results have shown that the spectra of the frequency fluctuations, in the case of detuning, are similar to those shown in Fig. 2; the values of $w_\nu(F)$ in the interval from 60 to 300 kHz differed from the corresponding values for the center of the line by 20—40%. For one of the series of the experiments, the results are given in Fig. 3, where the black point corresponds to the mean value of $w_\nu(F)$ near the center of the transition line at $P \approx 670\ \mu W$, and the triangle corresponds to detunings $\sim \pm 200$ MHz and $P \approx 620\ \mu W$. The observed small difference between the indicated spectra indicates that, for the optical transition at wavelength $\lambda = 0.63\ \mu$, the influence of the dispersion properties of the transition on the natural laser frequency fluctuations during the course of the detuning within the limits of the Doppler width of the line, as noted above, is small, and this influence can be disregarded in practice.

The foregoing results have enabled us to compare the experimental data with the appropriate calculations. The observed agreement between the experimental dependence of the frequency-fluctuation spectrum (in the interval from ~60 to 300 kHz) on the observation frequency F and on the generation power P with the results of the theoretical analysis allows us to state that in the indicated band of the frequencies F the frequency fluctuations of the laser oscillations are due only to natural causes (spontaneous emission of the active medium). In this case, in order to determine the total width $\delta\nu$ of the natural spectral line of the investigated laser generator, due to the spontaneous emission, we can extrapolate, in agreement with the theory, the experimental $w_\nu(F)$ plot from the frequency interval of around several tens--several hundreds of kHz to the lower frequencies. Thus, setting $w_\nu(0)$ equal to the mean value of w_ν in the interval of F from 100 to 300 kHz, we obtain for the line width of the investigated laser at $\lambda = 0.63\ \mu$ the experimental value $\delta\nu_l = \pi w_\nu(0) = 0.08$ Hz (the relative width $\delta\nu_l/\nu_l \approx 1.7 \times 10^{-16}$) at a generation power $P \approx 1$ mW.

In conclusion we note that such small random deviations of the frequency of the optical oscillations can be observed and measured experimentally by using the heterodyne method with a photoreceiver having a large quantum efficiency (to decrease the relative role of the photocurrent noise due to the shot effect) and by using a sufficiently high load impedance (to reduce the relative role of the thermal noise of the load).

The possibility of determining the width of the spectral line of a laser by analyzing the frequency fluctuations of the result of beating the laser signals was discussed also in[10]. The authors of that paper, in our opinion, certainly did not pay enough attention to the circumstances noted above. As a result, it was impossible to observe the natural frequency fluctuations with the apparatus constructed by them.

The authors take the opportunity to express once more their gratitude to I. L. Bezshteĭn, under whose guidance this work was performed.

Note added in proof (5 October 1968). In a new paper [12], Siegman and Arrathoon report determination, with the aid of the apparatus described in [10] of the natural line width of an He-Ne laser at $\lambda = 0.63\mu$, operating, unlike the laser used in [10], at very low power levels ($\sim 0.1 - 1\ \mu W$). To obtain stable generation of the investigated laser at such power levels, an addition system of automatic laser-emission intensity stabilization was used. Because of these modifications of the setup, it was possible to determine the width of the natural spectral line of a laser in the interval approximately from 50 to 5000 Hz.

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