MAGNETIC SURFACE LEVELS

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Stationary states of conductivity electrons skipping along arcs of small curvature ending on the metal surface are induced by a weak magnetic field in the surface layer of a metal. The spectrum of these states is manifest in the oscillatory dependence of the surface impedance of the metal on the magnetic field strength due to resonance absorption of microwave quanta. The experimental results obtained with bismuth at a frequency of 9.4 GHz and sample temperature of 4.2–1.7°K are in complete agreement with the theoretical calculations for field strengths between 0.1 and 0.5 Oe. An analysis of the effect permits one to explain the causes of some familiar phenomena and to predict the existence of some new ones.

An oscillatory dependence of the surface impedance of a metal on a weak magnetic field was observed in[1] in tin, indium, and cadmium at a frequency 9.4 GHz. Detailed experimental investigations of this effect in tin, indium and aluminum, were performed by Koch and Kuo[2] in the range 28--70 GHz. An explanation of the causes of the effect and its theory have been published only very recently.

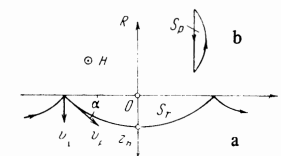
The theory of the effect is based on the idea that it is necessary to take into account the role of the electrons that "jump" over the surface of the metal as they move inside the metal; the trajectories of these electrons are arcs whose ends terminate on the surface of the metal[3,4]. By quantizing the periodic motion of such electrons specularly reflected from the surface of the metal, Nee and Prange constructed a theory of magnetic surface levels[5,6], and this theory agreed with the experiments. It turned out that the experimentally observed complicated dependence of the surface impedance of the metal on the weak magnetic field is the magnetic spectrum of the resonance absorption of microwave quanta by conduction electrons as they pass between the magnetic surface levels.

1. CALCULATION OF THE SPECTRUM OF THE MAGNETIC SURFACE LEVELS

We shall perform a simple calculation of the spectrum of the quantum surface states of the conduction electron in a weak magnetic field. This quasiclassical calculation leads to a result which coincides exactly, at large quantum numbers, with the formula obtained by Nee and Prange[5,6], but the quantum addition, which is essential for the calculation of the energy levels at small quantum numbers, remains undetermined.

We consider an electron moving in a field H in a metal close to its surface, without collisions, and specularly reflected from the surface when impinging on the latter. Part of the trajectory of such an electron jumping over the surface of the metal, consisting of a chain of arcs of large radius of curvature $R = Pc/eH$, is shown in Fig. 1. The orbit of the jumping electron on the Fermi surface is a small arc of curvature radius P subtended by a chord (Fig. 1b), and encloses an area S_p.

FIG. 1. Trajectory of an electron jumping over a reflecting metal surface in a weak magnetic field H: a – in coordinate space, b – in momentum space.



From the Bohr-Sommerfeld quantization rule we obtain a condition for the quantization of the area of the closed orbit of the electron in momentum space:

$$S_p = n_1 ehH / c;$$

$$n_1 = n + \gamma, \quad n = 1, 2, 3, \ldots; \quad 0 < \gamma < 1. \tag{1}$$

As is well known, in medium and strong fields, the quantization of the sections of the Fermi surface at large values of n leads to formation of Landau levels. On the other hand, if the orbit is truncated as a result of reflections of the electron from the surface of the metal, then only part of the section, spanned by the orbit, can be quantized. Such a situation was analyzed by Kosevich and Lifshitz[7] from the point of view of an investigation of the de Haas-Van Alphen effect.

In our case the area S_p can be calculated by transforming in momentum space the area of the small segment S_r, spanned by the trajectory arc in coordinate space:

$$S_p = \left(\frac{eH}{c}\right)^2 S_r = \left(\frac{eH}{c}\right)^2 \frac{4}{3}(2Rz_n^3)^{1/2}. \tag{2}$$

Application of the quantization rule (1) to the area (2) gives the value for the sagitta segment:

$$z_n = \left(\frac{3h}{4\sqrt{2}}\right)^{2/3} \left(\frac{c}{eHP}\right)^{1/3} n_1^{2/3}, \tag{3}$$

which determines the allowed trajectories of the electrons for small values of n.

The energy levels of the allowed states are obtained in the following manner. The electron moving with Fermi velocity v_F is reflected from the surface at an angle α and acquires an initial velocity normal to the surface $v_\perp \approx v_F\alpha$ (Fig. 1a). On the path to the turning point z_n, the electron loses an energy $\Delta E_n = mv_\perp^2/2$. Recognizing that for a small segment $z_n = R\alpha^2/2$, we obtain the quantity

$$\Delta E_n = (e / c) v_F H z_n, \tag{4}$$

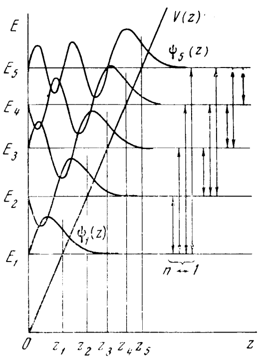


FIG. 2. Level scheme and plots of the wave functions of the magnetic surface states of electrons.

which characterizes the n-th stationary state of the electron.

In calculating formula (4), the trajectory has been replaced by a parabolic arc. This is quite an accurate approximation, in view of the smallness of α; it means in essence that the Lorentz force acting on the electron is replaced by a constant force directed normal to the surface of the metal. In such an approximation, the motion of the electron jumping over the surface of the metal is perfectly analogous to the motion, in the gravitational field, of an elastic ball jumping over a horizontal surface.

The frequencies of the absorption of emission lines in transitions between the energy levels (4) are determined by the Einstein formula:

$$\nu_{nk} = \frac{\Delta E_n - \Delta E_k}{h} = \left(\frac{3e}{4\sqrt{2}\,c}\right)^{2/3}\left(\frac{H^2}{Ph}\right)^{1/3} v_F\,(n_1^{2/3} - k_1^{2/3})^{3/2}. \qquad (5)$$

The obtained relation describes a spectrum whose lines form series (in analogy with the series of Lyman, Balmer, Paschen, etc. for the hydrogen-atom spectrum, calculated by the Balmer formula).

The theory of Nee and Prange[5] consists of solving the Schrödinger equation for an electron moving in a potential well

$$V(0) = \infty, \qquad V(z) = (e/c)v_F Hz, \qquad (6)$$

which leads to wave functions of the stationary states in the form of the Airy functions (Fig. 2). The obtained serial formula coincides exactly with formula (5), except that the quantum addition $\gamma = 1/4$ is also determined.

If we regard the frequency ν in (5) as constant, then we obtain the resonance values of the magnetic field:

$$H_{nk} = \frac{4\sqrt{2}h\,c}{3e}\,\nu^{3/2}\left(\frac{P}{v_F{}^3}\right)^{1/2}(n_1^{2/3} - k_1^{2/3})^{-3/2}. \qquad (7)$$

To calculate the positions of the lines of the magnetic surface-state spectrum (in relative units) it is necessary to calculate the values of the quantum factor in (7). The results of the calculation of the first five series of the spectrum are listed in the table, where $\zeta_n = (3\pi n_1/2)^{2/3}$ is the asymptotic expression of the roots of the Airy functions.

When $H = H_{nk}$, resonance absorption of the emission quanta of frequency ν, incident on the sample, should occur. In order to be able to resolve the absorption spectrum into individual lines, it is necessary that the metal contain a large group of electrons differing by the same value of the parameter P/v_F^3. This condition is satisfied by groups of electrons belonging to cylindrical Fermi surfaces. Such a group makes an appreciable contribution to the surface impedance of the metal, and its magnetic absorption spectrum will be manifest in the form of oscillations of the surface impedance Z as a function of the magnetic field H. Indeed, all the metals in which this effect is observed have almost cylindrical Fermi-surface parts, and the orientation of these parts is uniquely related with the anisotropy of the effect[1-4].

Spectrum of magnetic surface levels

n	ζ_n	$(\zeta_n - \zeta_k)^{-3/2}$				
		$k = 1$	2	3	4	5
1	2.3381					
2	4.0879	0.4320				
3	5.5206	0.1761	0,5831			
4	6.7867	0,1066	0,2255	0,7019		
5	7,9442	0,07534	0,1321	0,2650	0,8030	
6	9,021	0,05788	0,09127	0,1527	0,2994	0,8949
7	10,039	0,04679	0,06888	0,1041	0,1705	0,3298
8	11,008	0,03917	0,05493	0,07779	0,1153	0,1865
9	11,935	0,03384	0,04549	0,06156	0,08561	0,1254
10	12,828	0,02943	0,03870	0,05062	0,06734	0,09265

2. EXPERIMENT

To verify the theory of the effect it was necessary to investigate it in a metal whose Fermi surface is well known. In this respect the best substance is bismuth; its Fermi surface has been investigated in detail and many of its quantitative characteristics have been studied with very high accuracy.

The experiment was performed on a single crystal of very pure bismuth (resistance ratio $\rho(300°K)$ $\rho(300°K)/\rho(4.2°K) \gtrsim 100$), in the form of a disc of 17.8 mm diameter and thickness $l \sim 0.2$ mm, grown from the melt in a dismountable mold of fused quartz. The sample surfaces grown on optically polished quartz surfaces were not subjected to any subsequent treatment. The sample was sufficiently thin to eliminate by the cutoff method the higher-order cyclotron resonances that could be observed in a thick sample[8]. The measurements were made by a frequency modulation method[9] at 9.4 GHz and at a sample temperature 1.7°K.

The investigated plane surface of the sample had an orientation that practically coincided with the basal plane of the bismuth crystal. Figure 3 shows the directions of the normal **N** to the surface of the sample, of the microwave currents **J**, and of the field **H** relative to the trigonal, binary, and bisector axes (C_3, C_2, and C_1) of the crystal (accurate to $\sim 1°$). The figure shows also the arrangement of one of the three ellipsoids of the electronic Fermi surface of the bismuth. The mag-

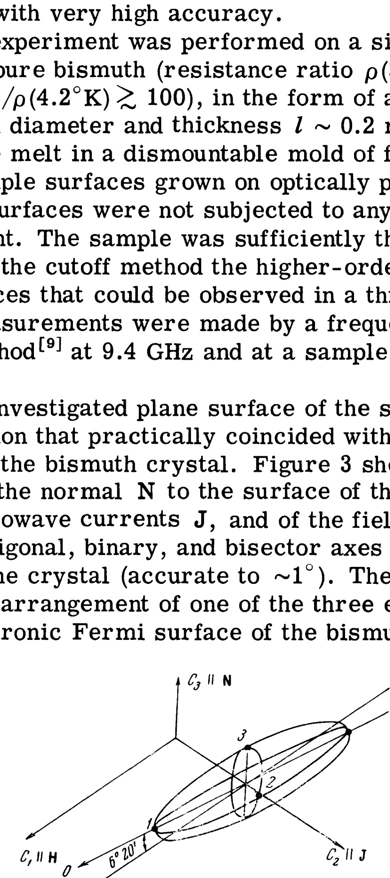


FIG. 3. Arrangement of one of three ellipsoids of the electronic Fermi surface of bismuth.

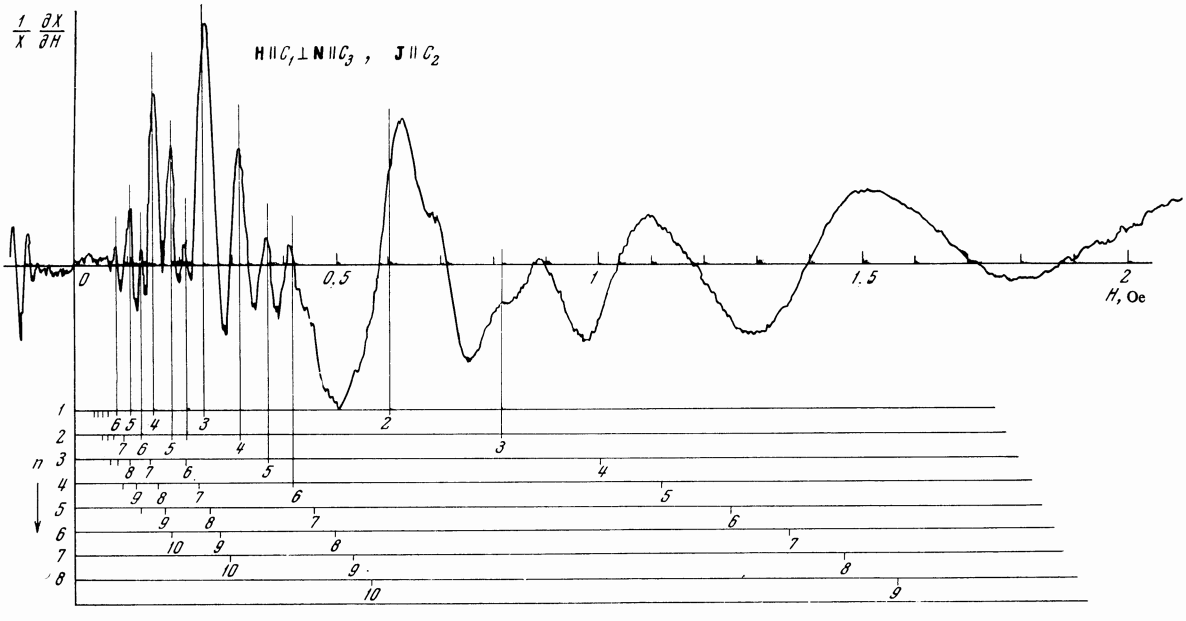


FIG. 4. Plot of oscillations of the surface impedance of bismuth (l = 0.2 mm, T = 1.7°K, f = 9.7 GHz). In the lower part of the figure are shown the calculated positions of several series of resonant lines of the spectrum H_{nk} of the magnetic surface levels. The values of n are given on the left, and the values of k at the markers indicating the positions of the spectral lines.

netic field was produced by a Helmholtz system with compensation for the earth's magnetic field.

3. EXPERIMENTAL RESULTS AND COMPARISON WITH THE CALCULATIONS

3.1. A verification of the quantum theory of the effects should consist of a calculation of the absolute values of the resonance magnetic fields H_{nk} and their comparison with the experimentally obtained spectrum of the oscillations of the surface impedance of the metal. Such a procedure was used in the present investigation with bismuth.

As seen from formula (7), to calculate H_{nk} it is necessary to know the radius of curvature P of the cylindrical part of the Fermi surface, on which the orbits of the oscillation-producing electrons lie, and the Fermi velocity v_F of these electrons. Figure 3 shows schematically one of three ellipsoids of the electronic Fermi surface of bismuth; its central part is practically indistinguishable from a cylinder[10]. The electrons producing the oscillations in an experiment for which the orientations of the normal to the surface of the sample, of the field, and of the current are indicated in Figs. 3, are represented by points lying along the generatrix, passing through the point 2, of the central cylindrical part of the ellipsoid. Therefore, to compare the results of this experiment with the theory, it is necessary to know the velocity of the electrons at point 2 and the radius of curvature P_{23} of the section 2—3 at the point 2. The Fermi velocity of the electrons has been determined by measuring the Doppler damping of magnetoplasma waves[11]:

$$v_{F2} = (11.0 \pm 0.5) \cdot 10^7 \text{ cm/sec}.$$

The curvature radii can be calculated, assuming that the Fermi surface sections are ellipses, by determining the semiaxes p_2 (parallel to C_2) and p_3 (almost parallel to C_3) by measuring the dimensions of the electron trajectories[8], and by determining the semiaxis p_1 (which is almost parallel to C_1) from the semiaxis ratio 1 : 1.46 : 15 obtained from measurements of the de Haas-van Alphen effect[12]:

$$p_2 = (5.4 \pm 0.15) \cdot 10^{-22} \text{ g-cm/sec}$$
$$p_3 = (7.9 \pm 0.3) \cdot 10^{-22} \text{ g-cm/sec}$$
$$p_1 = (81 \pm 3) \cdot 10^{-22} \text{ g-cm/sec}.$$

Calculation of the curvature radii yields

$$P_{23} = p_3^2 / p_2 = (11.5 \pm 0.7) \cdot 10^{-22} \text{ g-cm/sec}$$
$$P_{21} = p_1^2 / p_2 = (1200 \pm 70) \cdot 10^{-22} \text{ g-cm/sec}.$$

3.2. The surface-impedance oscillation spectrum of bismuth, obtained in an experiment corresponding to Fig. 3, is shown in Fig. 4. In the lower part of the figure are shown the positions of the lines of the magnetic spectrum H_{nk} calculated in accordance with formula (7) with a value of the parameter P/v_F^3 that ensures the best agreement with experiment for several of the sharpest and most intense resonances, designated by thin vertical lines. These are precisely the peaks corresponding to the maxima of absorption of microwave energy by the sample. We see that for the ten or so lines lying below 0.5 Oe it is possible to obtain practically perfect agreement between calculation and experiment; in stronger fields the situation is unclear, and will be discussed somewhat later.

Substituting the Fermi velocity v_{F2} in the obtained value of the parameter P/v_F^3, we obtain the curvature radius

$$P_{23}' = (13.5 \pm 2.5) \cdot 10^{-22} \text{ g-cm/sec},$$

which agrees, within the limits of measurement accuracy, with the value of P_{23} given above. We shall use subsequently the value of the effective mass of the electrons of the limiting point 2 (Fig. 3), measured in[10] by the cyclotron-resonance method:

$$m_2^* = (v_{F2}\sqrt{K_2})^{-1} = (0.137 \pm 0.003)\, m_e,$$

where $K_2 = (P_{23}P_{21})^{-1}$ is the Gaussian curvature of the Fermi surface at the point 2, and m_e is the electron mass. Hence, again using v_{F2} and the obtained value of P'_{23}, we get

$$P_{21}' = (1400 \pm 200) \cdot 10^{-22}\ \text{g-cm/sec},$$

i.e., a quantity practically coinciding with P_{21}.

It should be noted, however, that a somewhat smaller curvature of the Fermi surface, compared with that calculated from the semiaxes of its sections, results from cyclotron-resonance investigations[10], in view of which there are apparently real reasons for P'_{23} and P'_{21} to be somewhat higher than P_{23} and P_{21}.

Thus, our comparison shows that the spectrum of the resonance values of the field H_{nk}, calculated by means of formula (7) with allowance for independently measured characteristics of the bismuth electrons, coincides, within the accuracies of the calculation and of the experiment, with the experimentally observed spectrum of the oscillations of the surface impedance, in the region $H_{nk} < 0.5$ Oe.

3.3. The results of a study of the anisotropy of the oscillations of the surface impedance of single-crystal bismuth in the basal plane are shown in the form of a polar diagram in Fig. 5[4]. The points represent the positions of the maxima on experimental plots similar to Fig. 4, made for different directions of the field vector **H** relative to the C_1 axis. We see that the field of the n-th maximum obeys the law

$$H_n(\varphi) = H_n(0)\sec\varphi,$$

which is represented by straight lines. It should be added that for any direction of the field **H** in space, the ends of the vectors H_n fall on a plane defined by the segment $H_n(0)$, i.e., by the projection of H_n on the direction of the axis of the central almost-cylindrical part of the electronic ellipsoid of the Fermi surface. This is precisely the field component that determines the curvature radius of the electron orbits, and consequently the energy levels of the surface states. The amplitude of the oscillations depends on the polarization of the microwave currents in accordance with the law

$$A(\vartheta) = A(0)\cos\vartheta,$$

where ϑ is the angle between the direction of the current and the C_2 axis, which is parallel to the motion of the electrons jumping over the surface. The corresponding plot (circle) and the experimental points are shown in the left side of Fig. 5.

3.4. In the field region $H > 0.5$ Oe, where the first lines of the spectral series are located, it is difficult to see any agreement between the experimental and the calculated spectrum, owing to the insufficient resolution. In this connection, it should be recalled that the lines of the spectrum of Fig. 4 were calculated for electrons of only one ellipsoid of the Fermi surface of bismuth, located, as indicated in Fig. 3, in the most convenient manner for the observation of the surface-impedance oscillations. Two other ellipsoids, rotated 60° relative to the first, give lines with amplitudes that

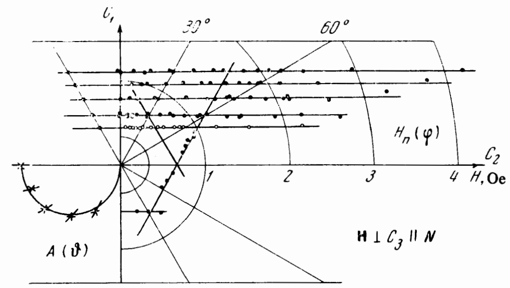


FIG. 5. Anisotropy of the resonance fields $H_n(\varphi)$ and of the oscillation amplitude $A(\vartheta)$ in bismuth (f = 9.4 GHz).

are half as large in fields $H'_{nk} = 2H_{nk}$ that are twice as strong. The presence in the spectrum of these lines, which furthermore should be broadened by splitting as a consequence of possible inaccuracies of the orientation of the crystal and of the field, may greatly reduce the resolution of the spectrum.

Certain relatively broad singularities in the field dependence of the surface impedance Z(H), observed in weak magnetic fields, can be attributed to the existence of the classical effects considered in[2-4]. These effects have that advantage over quantum magnetic surface levels, that no specular reflection of the electrons from the surface of the metal are necessary for their occurrence. In addition, they should be less sensitive to a decrease of the mean free path of the electrons and to an increase of the temperature of the sample. The indicated differences, and also the different dependences of the field of the singularity on the frequency, may serve as attributes for an experimental distinction between classical and quantum effects.

3.5. Using the formulas derived above we can easily obtain expressions for any parameter of the magnetic surface electronic state, particularly for the geometric characteristics of the trajectories of the jumping electrons. Figure 6 shows results of a numerical calculation for the first surface level in bismuth. The trajectories of the electrons in the higher quantum states have correspondingly larger dimensions.

Let us discuss the question of the probability of specular reflection of the electrons from the surface of the metal, necessary for the formation of magnetic surface levels. We can first advance the simple consideration that the smaller the angle α of the encounter between the electron and the rough surface, the larger the probability of its specular reflection. For bismuth, the de Broglie wavelength of the electron is $\sim 10^{-5}$, and the roughnesses of the optically smooth surface of the sample does not exceed the wavelength of light $\sim 10^{-4}$

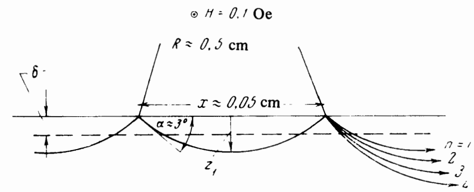


FIG. 6. Numerical characteristics of the trajectories of an electron situated on the first surface level in bismuth ($n = 1$, $\Delta E_1 \approx 0.2$°K, $\delta \approx 10^{-4}$ cm, $z_1 \approx 2 \times 10^{-4}$ cm).

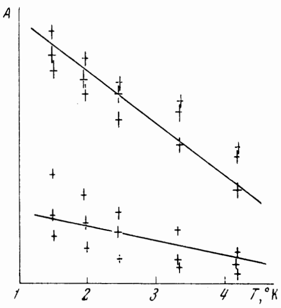


FIG. 7. Dependence of the amplitude of the oscillations A (in relative units) on the sample temperature.

cm, from which it follows that the angle α should be smaller than $\sim 10^{-1}$.

More convincing, however, are, naturally, experiments that confirm directly the existence of specular reflection of the electrons from the surface of the metal. In the experiments of[8], a study was made of the cutoff of cyclotron resonances in a bismuth single crystal ~0.2 mm thick; in a magnetic field weaker than the cutoff field, resonance was observed on electrons experiencing reflections from the surface of the sample. The amplitude of this "resonance with reflection" differ little from the amplitude of the ordinary resonances, from which it follows that under the experimental conditions the reflection probability is of the order of unity. Specular reflection of the electrons from the surface of bismuth was observed also in the study of the dc conductivity of thin single crystals of bismuth[13]. Investigations of the conductivity of thin layers and of whiskers of various metals[14,15] have established that the probability of reflection of the conduction electrons from the surface ranges from ~0.5 to 1.

Thus, the assumption that the electrons are specularly reflected from the surface of the metal as they move on jumping orbits is fully justified.

In analyzing the scattering of electrons moving over jumping trajectories, it is necessary to distinguish between two types of scattering: scattering in the interior of the metal and scattering upon collision with the surface of the metal. In turn, the former must be divided into scattering by phonons and scattering by impurities, while the latter into scattering by surface phonons and scattering by surface roughnesses. Both phonon parts of the scattering, which decrease rapidly with temperature, may cause, owing to the low energy and momentum of the phonons at low temperature, the electron to be scattered only through small angles, on the order of the ratio of the phonon momentum to the electron momentum. But the existence of a spectrum of allowed surface levels can cause the electron, as a result of the small-angle scattering, to go over only from one level to another, or else to be specularly reflected and remain at the previous level, if the energy conditions preclude the possibility of a transition to, say, a higher level. For this reason, collisions of electrons with the surface should not disturb the system of surface levels, and specular reflection of the jumping electrons should be regarded as predominant and as weakly dependent on the temperature.

3.6. Figure 7 shows the experimentally-measured growth of the oscillation amplitude with decreasing temperature. The straight lines drawn on the figure correspond to (T is in degrees K)

$$A(T) = A(0)(1 - T/6).$$

The observed growth of the amplitude is apparently due to an improvement of the resolution of the levels with decreasing temperature, owing to the increase of the level lifetime as a result of the increase of the electron mean free path in the interior of the metal, and also as a result of the change of the level population. The latter should be quite significant, since the excitation energy of the surface levels is close to the temperature of the sample under the experimental conditions. Thus, in the described experiments, a magnetic resonance-absorption spectrum of quanta with energy ~0.5°K was observed at a sample temperature 4.6–1.6°K (in[2], the quantum energy reached ~3.5°K).

As shown by experiment (Fig. 4), the resolution of the resonances and the amplitude of the oscillations decrease with increasing magnetic field above ~0.5 Oe. This results from the fact that when the field increases the trajectories of the jumping electrons are pressed towards the surface of the metal, the frequency of their collisions with the surface of the metal increases, and therefore the probability of electron scattering through a large angle increases, i.e., scattering that knocks out the electron from the system of surface levels. The increase of the angle α with increasing field also contribute to the enhancement of the scattering.

We present below the values of the quantum numbers of the $n \to k$ transitions identified and noted on the magnetic spectrum of Fig. 4 by thin vertical lines:

$1 \to 2$	—	—	—
$1 \to 3$	$2 \to 3$	—	—
$1 \to 4$	$2 \to 4$	—	—
$1 \to 5$	$2 \to 5$	$3 \to 5$	—
$1 \to 6$	$2 \to 6$	$3 \to 6$	$4 \to 6$

The observed line intensities of the absorption spectrum increase in each series with decreasing difference $k - n$, down to $k - n = 2$, at which the most intense line of the series (designated in bold face) takes place; the first line of the series is weaker than all others. The nonmonotonic dependence of the line intensity on the field and its regular variation within the limits of each series indicate that it depends principally on the probability of the transitions between levels, determined by the density of the states and their population.

A quantitative analysis of the data on the width and amplitude of the spectral lines can be postponed until a detailed theory of the effects, in which the singularities considered above are taken into account, is developed.

4. CERTAIN ACCOMPANYING PHENOMENA AND THE POSSIBILITIES OF INVESTIGATING THE EFFECT

The oscillations of the surface impedance of the metal, which result from resonant absorption of microwave-radiation quanta in electron transitions between stationary surface states excited by weak magnetic fields, serve as the most convincing proof of their existence and as a direct method of their quantitative study. There are, however, other phenomena accompanying this effect and observable in experiments of a different nature. It is also possible to indicate a number of still unobserved phenomena, the existence of which follows from the existence of magnetic surface states.

4.1. In a superconductor situated in a weak magnetic field weaker than the critical value, a stationary surface state of the normal electrons is produced[16,17]. It is revealed by the peak of the resonance absorption, observed when the dependence of the surface impedance of the superconductor on the magnetic field is measured. The surface state exists for electrons jumping over the surface of the superconductor without emerging from a layer of the order of magnitude of the penetration depth λ of the magnetic field. Their motion takes place in the potential well

$$V(0) = \infty, \qquad V_s(z) = (e/c)v_F H\lambda e^{-z/\lambda},$$

and an analysis similar to that described in Sec. 1 establishes the existence of a single level with energy $\Delta E_1 \approx (e/c)v_F H\lambda$, smaller than the gap in the energy spectrum of the superconductor.

4.2. In a normal metal, the wave functions $\psi_n(z)$ of the magnetic surface states are of the form shown in Fig. 2. The distribution of the probability density for electrons situated in the n-th state is given by the function $|\psi_n(z)|^2$, which oscillates with increasing depth z at distances on the order of $10^{-5}-10^{-3}$ cm, with a period that depends on the metal, on the magnetic field intensity, and on the number of the state. Thus, application of a magnetic field leads to an inhomogeneity in the distribution of the electrons in the surface layer of the metal, and influences the conditions of their motion, particularly their interaction with the surface, thereby increasing the probability of specular reflection.

An inevitable consequence of this should be a dependence of the surface impedance of the metal on the weak magnetic field. This dependence, which is not connected with the conditions of resonant absorption of quanta should be observed at any frequency at which the depth of the skin layer δ is comparable in order of magnitude with the period of the oscillating part of the wave function $\psi_n(z)$, i.e., $\delta \sim z_n \sim 10^{-3}-10^{-5}$ cm. In the limiting case of zero frequency, this phenomenon should be observed in thin single crystals, for example in whiskers and in epitaxial films with thickness on the order of z_n. It should be noted here that the spectrum of the surface states in thin samples should differ appreciably from the spectrum in thick samples of the same metal. Thus, in a thin plate there should take place interference of the ψ functions of the states connected with opposite surfaces of the sample; it is also possible that the higher states are cut off by the thickness of the sample.

This phenomenon has a well known analog, namely the Schubnikov—de Haas effect, wherein the volume impedance of the metal depends on a strong magnetic field that causes splitting of the Fermi surface into Landau levels, a splitting which is appreciable near the extremal sections of the Fermi surface. In analogy to this effect, the surface impedance of the metal should depend on a weak magnetic field exciting electronic surface levels localized near the effectiveness strip of the Fermi surface. There is no doubt that this is precisely the cause of the hitherto unexplained nonlinear dependence of the surface impedance of tin, bismuth, and gallium on a weak magnetic field at frequencies on the order of several MHz, observed in[18-20].

The result of the investigations of gallium is particularly interesting, because oscillations of its surface impedance at microwave frequencies have so far not been observed, and the phenomenon observed in[20] is so far the only evidence that magnetic surface levels are excited in gallium. An analogous situation takes place also in potassium, in which a nonlinear dependence of the surface impedance on a weak magnetic field was also recently noted at a frequency of several MHz (private communication from Tsoĭ).

The anomaly in the behavior of the surface impedance of copper in a weak field, observed in[21] at microwave frequencies, obviously also points to the existence of surface levels. To be sure, the cause of this anomaly may be the quantum resonance absorption, which cannot be resolved on the spectral lines.

Returning to the conclusion that the impedance of thin single crystals depends on a weak magnetic field at zero frequency, we note that an experimental investigation of this phenomenon has, strictly speaking, not been performed to date. In the experiments of[13-15] the investigated thin samples were subjected to the earth's magnetic field and to the field of measuring current, both of the order of 1 Oe; the dependence of the impedance on a field of low intensity was not investigated. The only direct conclusion that can be drawn from their results is that the observed unexpectedly large probability of specular reflection of the electrons by the surface of the sample is possibly due to the occurrence of surface levels excited by the uncontrolled magnetic field applied to the sample.

4.3. The magnetic susceptibility of a conductor should inevitably change when surface levels are excited by a weak magnetic field, in analogy with the manner in which a strong magnetic field, which gives rise to Landau levels, produces the de Haas—van Alphen effect, which consists in a dependence of the volume magnetic susceptibility of the metal on the magnetic field. The gist of the phenomenon lies in the fact that excitation of the surface levels leads to the appearance of infinite trajectories of the electrons jumping over the surface of the metal and forming a layered surface current whose magnetic moment is directed parallel to the external magnetic field (Fig. 6). Thus, paramagnetism should appear in the surface layer of the conductor and should be manifest in the occurrence of an unusual dependence of the magnetic susceptibility of the metal on the field. We note that this paramagnetism should have an anisotropy connected with the shape of the Fermi surface, and particularly strong for its cylindrical parts; the latter circumstance is of great significance for the organization of experiments.

The magnetic properties of metals were investigated theoretically in a number of papers by Dingle; in particular, he considered the influence of the surface of the metal on the Landau diamagnetism and on the de Haas—van Alphen effect[22]. However, the singularities of the magnetic susceptibility of the metal, which should result from the excitation of surface states of the electrons by a weak magnetic field, have not yet been investigated.

4.4. In experiments at microwave frequencies, well resolved lines of the resonance-absorption spectra were observed for transitions of electrons belonging to cylindrical parts of the Fermi surface. The spectra of the surface levels of electrons belonging to other non-cylindrical parts of the Fermi surfaces were not resolved in these experiments, and could be manifest only as a broad region, compared with the resolved lines, in which the surface impedance of the metal has an irregular dependence on the magnetic field. Such a phenomenon was apparently observed in copper[21].

The resolution of the surface-level spectrum of electrons belonging to non-cylindrical Fermi surfaces can be effected provided the resonance lines are sufficiently narrowed by lowering the temperature and improving the quality of the samples. In this case one should observe surface-level spectra of the electrons from the extremal sections of the Fermi surface, in analogy with the situation obtained in cyclotron resonance.

4.5. An entire group of new phenomena is caused by the use of energy pumping at the resonant frequency of any one of the transitions between the surface magnetic levels. The following can be proposed by way of the simplest experiment: the pumping is effected by bombarding the sample with quanta whose energy equals the excitation energy of one of the upper levels, and the radiation emitted by the sample at lower-frequencies, corresponding to transitions between lower levels, is investigated (the pump and radiation frequencies will not have a simple ratio). In such an experiment it is possible to obtain stimulated emission. A sufficiently strong pump can change the distribution of the electrons over the surface levels, and consequently can influence the admittance and the magnetic susceptibility, and with it the surface impedance of the metal. Finally, energy pumping can lead to creation of a potential difference between different points of the sample, as is the situation in plasma phenomena.

4.6. Another group of new phenomena are the geometrical and size effects. Notice should be taken first of the dependence of the surface-level spectrum on the surface curvature of the metal. From the classical point of view, owing to the large length of the electron jumps over the surface of the metal, the spectrum of a sample with a small-curvature surface should differ from the spectrum of a flat sample. In turn, the spectrum of a thin plate, whose thickness is of the order of the spatial period of the ψ functions of the magnetic surface states, should differ from the spectrum of a thick sample as a result of interference between the states connected with the opposite surfaces of the plate and satisfying two boundary conditions. In addition, in thin samples the higher quantum numbers should be cut off by the dimensions of the sample, in complete analogy with the classical size effect, wherein the electron trajectories are cut off by the sample thickness[23].

4.7. The fundamental connection between the considered effect and the surface of the conductor gives rise to new possibilities of investigating the properties of the surface or, as a more general program, of an interface from which carriers can be reflected. The distinguishing feature of this approach lies in the possibility of investigating the surface from the inside of the conductor and investigating an internal interface that is located at a macroscopic depth.

The presently available experimental data concerning the physical properties of conductor surfaces, particularly the character of the interaction of the carriers with the surface, are quite scanty. The surface states have been the subject of only a few theoretical articles (for example[24]), starting with a paper by Tamm[25]; the role of the magnetic field was not taken into account in these papers.

4.8. The principles of the theory of magnetic surface states, developed in Sec. 1, are of very general significance and can be applied to the analysis of the behavior of any charged quasiparticle in any conductor placed in a magnetic field. The conductor must be cooled to a sufficiently low temperature, have a sufficient mean free path for the charged quasiparticles, and a smooth surface. At the same time, the occurrence of surface states is not connected in principle with any limitations involving the energy spectrum of the quasiparticles, the shape of the Fermi surface, or even its very existence—the quasiparticle distribution may also be Maxwellian. There is no need for a single-crystal sample, since this does not reduce the quasiparticle mean free path to inadmissible limits. On the basis of these considerations we can expect the excitation of magnetic surface levels to be a highly widespread phenomenon.

In conclusion we note that the presented considerations do not exhaust at all the group of phenomena that should result from the excitation of magnetic surface levels. Any property of a surface or thin layer of a conductor, connected with the state of the energy spectrum of the carriers in the conductor, should exhibit a dependence on a weak magnetic field, inasmuch as for the surface layer of a conductor or for a thin conducting sample the magnetic surface levels play the same role as the Landau level for the volume of a bulky metal.

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Note added in proof (27 September 1968). Article [26] contains an exposition of the theory of magnetic surface levels, a calculation of the surface impedance of a sample, and a discussion of the width of the spectral line. In [27] are calculated the spectra of the magnetic surface levels for three geometric modifications of the effect: the spectrum of a thin sample, the spectrum of a sample with a cylindrical surface, and the spectrum of a sample whose surface has a periodic structure; the existence of the first two effects is indicated in Sec. 4 (Item 4.6) of this article. Observation of the microwave spectrum of magnetic surface levels in gallium, the existence of which was noted on the basis of indirect evidence in Sec. 4.2, is reported in [28].

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