THEORY OF FLUCTUATIONS OF ACOUSTIC WAVES IN PIEZO-SEMICONDUCTORS

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Submitted October 28, 1967

Zh. Eksp. Teor. Fiz. 55, 1784–1799 (November, 1968)

Fluctuations of acoustic and coupled longitudinal electromagnetic waves in piezo-semiconductors are considered for the case of space and time dispersion of the electron-hole plasma carriers. It is shown that the effective correlation range of the fluctuating elastic displacement components is determined by the reciprocal damping constant per unit crystal length of the corresponding elastic waves in the piezo-semi-conductors. Phonon drag by subsonic carrier drift is considered and a general expression is found for the spectral density of the phonon energy flux. It is shown that for small values of the electron drift the flux is simply proportional to the drift velocity. Drag of longitudinal and transverse sound waves is considered and the angular distribution of the dragged phonons is investigated.

Spontaneous generation of acoustic waves occurs during supersonic electron motion. The spectral intensity of generated acoustic noises is found by solving the boundary problem for this case. It is shown in particular, that irrespective of the crystal orientation the generation threshold is always the same as the velocity of the transverse sound waves. The explicit form of the correlation function for random inductions in a nonequilibrium medium with carrier drift is found in the long-wave approximation and it is shown that it can be expressed in a natural way in terms of the nonequilibrium dielectric tensor of the medium.

In connection with the intensive investigations of the effects of amplification and generation of sound waves in semiconductors, it is of interest to consider the question of the fluctuations of acoustic waves and of the electromagnetic waves coupled to them in piezo-semiconducting crystals. For systems in the stationary and thermodynamic-equilibrium states, a general method of solving such problems was developed by Callen and Welton,[1, 2] who have shown that the fluctuations in the system are determined by the dissipative part of the general response of the system to the corresponding external action (the so-called fluctuation-dissipation theorem). For a non-equilibrium medium in the stationary state, the theory of fluctuations of acoustic waves in piezo-semiconductors can be constructed in closed form, if one knows the correlation function for the random forces entering in the right sides of the corresponding equations. In the presence of a directed electron flux in the semiconductor, only the electronic subsystem is not in equilibrium, so that it suffices only to know the correlator of the random currents (or inductions) for a non-equilibrium plasma medium with drift. For long-wave fluctuations, an explicit form of this correlator is found in Sec. 5 of the present paper, and it turns out that it is expressed in natural fashion in terms of the non-equilibrium dielectric tensor of the medium.

We first construct a theory of thermodynamic-equilibrium fluctuations of acoustic waves and the electromagnetic waves coupled with them in piezo-semiconductors, and then the theory of non-equilibrium stationary fluctuations, when there is a directed carrier drift in the plasma medium. The entire analysis, unlike that of Gurevich,[3] takes into account spatial and temporal dispersions. The correlation function of the random inductions for the electron subsystem, as well as the dielectric constant of the medium itself, are not specified concretely, so that the results of the calculation are applicable to both low and high frequencies. The only assumption used here is connected with the homogeneity of the system (see (2.1) and (3.1)). If the directed velocity of the carriers is smaller than the phase velocity of the waves, then the dielectric constant of the medium and the correlation function of the random inductions, taken at two different points of space, will depend on the difference of the coordinates of these points; on the other hand, if the carrier drift velocity exceeds the wave phase velocity, so that a spatial buildup of the fluctuations takes place in the system, the choice of the difference dependence in the arguments means only neglecting the influence of the growing fluctuations at the ground state of the carrier plasma. Obviously, such an approach is valid within the framework of the linear theory.

In a number of piezo-semiconductors, such as cadmium selenide and cadmium sulfide, and also tellurium, there is a sufficiently strong interaction between the acoustic lattice vibrations and the carrier plasma, and in a number of cases it is precisely the plasma and not the lattice damping which plays the decisive role. It is therefore clear that the correlation between the components of the elastic displacement in such crystals will be determined by the plasma part of the damping of the corresponding sound waves, and consequently the effective correlation radius will simply equal the reciprocal of the damping decrement of the sound waves per unit length of the crystal. This result is obtained in general form in Sec. 2.

We consider also the effect of electron dragging of the phonons by the drifting subsonic electron flux. Owing to the strong interaction between the phonons in the piezo-semiconductor and the carrier plasma, a fraction of the directed electron momentum will be transferred by the phonons and a flux of acoustic energy will appear as a result. Expressions are obtained for the density of the energy flux of such a phonon flux and it is shown that its magnitude is determined primarily by the ratio of the carrier velocity to the velocity of the corresponding sound wave, and by the derivative of the electronic absorption decrement with respect to the dimensionless drift velocity.

In the case of supersonic motion of the carriers, acoustic waves are generated and the fluctuations of the acoustic waves increase in space. By solving the problem for a semi-infinite medium in general form, we obtain the spectral density of the generated acoustic noise, and its angular and spatial distributions. It turns out here that the threshold for the occurrence of spontaneous noise oscillations always corresponds to the velocity of the transverse sound waves, regardless of the crystal orientation.[1)] This result allows us to interpret a number of experiments on current saturation in piezo-semiconductors in electric fields, corresponding to the velocity of the transverse sound waves.

Section 5 is devoted to the question of the correlation function of the random inductions in a non-equilibrium medium with drift. It is shown that this function can be expressed in terms of the non-equilibrium value of the dielectric tensor of the medium in the presence of drifts.

1. GENERAL ANALYSIS OF FLUCTUATIONS IN SEMICONDUCTORS

On the basis of the general fluctuation theory developed by Callen and Welton[1] it is possible to consider fluctuations of acoustic oscillations and of the electric field intensity in piezo-semiconductors. Owing to the presence of the piezoeffect in the medium, there are two sources, i.e., two types of "random forces" leading to fluctuations of the displacement vector: first, spontaneous or, as is customarily said, random oscillations of the elastic stresses in the medium $\sigma_{ik}^{(S)}(\mathbf{r}, t)$, and second, random or spontaneous oscillations of the electric current $\mathbf{j}^{(S)}(\mathbf{r}, t) = (4\pi)^{-1}\partial \mathbf{D}^{(S)}(\mathbf{r}, t)/\partial t$, where $\mathbf{D}(\mathbf{r}, t)$ is the random induction. These quantities play the role of "external forces" in the corresponding equations of elasticity theory for the fluctuating displacement vector $\mathbf{u}(\mathbf{r}, t)$ and in the equations for the fluctuating electric field $\mathbf{E}(\mathbf{r}, t)$:[2,4]

$$\rho\frac{\partial^2 u_i}{\partial t^2} - \lambda_{iklm}\frac{\partial u_{lm}}{\partial r_k} - \mu_{iklm}\frac{\partial^2 u_{lm}}{\partial t\,\partial r_k} - \beta_{l,ik}\frac{\partial E_l}{\partial r_k} = \frac{\partial}{\partial r_k}\sigma_{ik}^{(s)}(\mathbf{r}, t), \quad (1.1)$$

$$\varepsilon_0\frac{\partial E_i}{\partial t} - 4\pi\int_{-\infty}^{t} dt'\int d^3\mathbf{r}\,\sigma_{ij}(\mathbf{r}, \mathbf{r}', t-t')E_j(\mathbf{r}, t) - 4\pi\beta_{i,kl}\frac{\partial u_{kl}}{\partial t} = \frac{\partial}{\partial t}D_i^{(s)}(\mathbf{r}, t). \quad (1.2)$$

Here ρ is the lattice density, λ_{iklm} is the elastic-modulus tensor, μ_{iklm} is the viscosity tensor (its symmetry properties coincide with those of the tensor λ_{iklm}), $\beta_{i,kl}$ is the tensor of the piezo-moduli "in deformation," ε_0 is the dielectric constant of the lattice, and $\sigma_{ij}(\mathbf{r}, \mathbf{r}', t-t')$ is the conductivity tensor of the medium. The electric field accompanying the sound wave in an unbounded piezo-electric medium is potential, and it is therefore necessary to add to (1.2) also the condition $\mathrm{curl}\,\mathbf{E} = 0$. In order to satisfy this condition, we introduce immediately the electric-field potential $\varphi(\mathbf{r}, t)$, so that $\mathbf{E} = -\partial\varphi/\partial\mathbf{r}$.

The correlations between the fluctuations of the displacement vector $\mathbf{u}(\mathbf{r}, t)$ at different instants of time and at different points of space are characterized by a correlation function

$$\varphi_{ij}^{uu}(\mathbf{r}, \mathbf{r}', t, t') = {}^1/_2\langle\hat{u}_i(\mathbf{r}, t)\,\hat{u}_j(\mathbf{r}', t') + \hat{u}_j(\mathbf{r}', t')\,\hat{u}_i(\mathbf{r}, t)\rangle, \quad (1.3)$$

where the angle brackets denote averaging with the aid of the exact wave functions of the system, and the quantities $\hat{\mathbf{u}}(\mathbf{r}, t)$ should be regarded, in accordance with the rules of quantum mechanics, as operators. In exactly the same manner, we can introduce correlation functions for the fluctuations of the electric field intensity φ_{ij}^{EE}, and "mixed" correlators φ_{ij}^{uE} and φ_{ij}^{Eu}. We introduce further the Fourier components of the operators $\hat{\mathbf{u}}(\mathbf{r}, t)$ and $\hat{\mathbf{E}}(\mathbf{r}, t)$ in accordance with the relations

$$\hat{u}_i(\mathbf{r}, \omega) = \frac{1}{2\pi}\int dt\, e^{i\omega t}\,\hat{u}_i(\mathbf{r}, t), \qquad \hat{u}_i(\mathbf{r}, t) = \int d\omega\, e^{-i\omega t}\,\hat{u}_i(\mathbf{r}, \omega). \quad (1.4)$$

We shall assume that the system is in the stationary state, and then the correlation function (1.3) will depend only on the difference $\tau = t - t'$. To use immediately the fluctuation–dissipation theorem and to find the correlation functions, we proceed in the following manner. We write the closed system of the equations of elasticity theory and Maxwell's equations for the longitudinal electric field (1.1) and (1.2) in the form[2)]

$$\int \alpha_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega)\,\psi_\beta(\mathbf{r}', \omega)\,d^3\mathbf{r}' = G_\alpha^{(s)}(\mathbf{r}, \omega), \quad (1.5)$$

where $\psi_\alpha(\mathbf{r}, \omega)$ is a four-dimensional row matrix with components $u_i(\mathbf{r}, \omega)$ and $\varphi(\mathbf{r}, \omega)$, i.e., $\psi = (u_i, \varphi)$, and $G_\alpha^{(S)}$ is a column matrix of random forces with components

$$G_i^{(s)} = \frac{\partial}{\partial r_k}\sigma_{ik}^{(s)}(\mathbf{r}, \omega), \qquad G_4^{(s)} = -\frac{1}{4\pi}\frac{\partial}{\partial r_i}D_i^{(s)}(\mathbf{r}, \omega),$$

$\alpha_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega)$ is the matrix of the kinetic coefficients of the system

$$\alpha_{ik}(\mathbf{r}, \mathbf{r}', \omega) = -\left[\rho\omega^2\delta_{ik} + (\lambda_{iplk} - i\omega\mu_{iplk})\frac{\partial^2}{\partial r_p\,\partial r_l}\right]\delta(\mathbf{r}-\mathbf{r}'), \quad (1.6a)$$

$$\alpha_{44}(\mathbf{r}, \mathbf{r}', \omega) = -\frac{1}{4\pi}\frac{\partial^2}{\partial r_i\,\partial r_j'}\varepsilon_{ij}(\mathbf{r}, \mathbf{r}', \omega) \equiv -\frac{1}{4\pi}p^{-1}(\mathbf{r}, \mathbf{r}', \omega), \quad (1.6b)$$

$$\alpha_{i4}(\mathbf{r}, \mathbf{r}', \omega) = \alpha_{4i}(\mathbf{r}', \mathbf{r}, \omega) = \beta_{l,im}\frac{\partial^2}{\partial r_l\,\partial r_m}\delta(\mathbf{r}-\mathbf{r}'), \quad (1.6c)$$

and equality (1.6c) is the consequence of the symmetry of the piezo-electric tensor with respect to the last two indices; $\varepsilon_{ij}(\mathbf{r}, \mathbf{r}', \omega)$ is the dielectric tensor of the medium with allowance for the carrier plasma:

$$\varepsilon_{ij}(\mathbf{r}, \mathbf{r}', \omega) = \varepsilon_0\delta_{ij}\,\delta(\mathbf{r}-\mathbf{r}') - \frac{4\pi}{i\omega}\sigma_{ij}(\mathbf{r}, \mathbf{r}', \omega). \quad (1.7)$$

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1) Of course, without allowance for the viscous or any other non-electronic mechanisms of sound-wave absorption (see section 4).

2) The Greek indices α, β, and γ, unlike the Latin ones, take on the values 1, 2, 3, 4; summation from 1 to 4 is carried out over repeated greek indices.

Thus, the matrix $\alpha_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega)$ is symmetrical, which is of course the direct consequence of the Onsager principle of the symmetry of the kinetic coefficients.

Equations (1.5) play the role of the relations

$$f_{i\omega} = \sum_k \alpha_{ik}^{-1} x_{k\omega}$$

in the general theory of thermodynamic fluctuations of several quantities (see [4], sec. 127), so that we can write down immediately the correlation functions of interest to us:

$$\langle \psi_\alpha(\mathbf{r}', \omega)\psi_\beta(\mathbf{r}, \omega)\rangle = \frac{i\hbar}{4\pi}\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)\{\alpha_{\beta\alpha}^{-1*}(\mathbf{r}, \mathbf{r}', \omega) - \alpha_{\alpha\beta}^{-1}(\mathbf{r}', \mathbf{r}, \omega)\}, \tag{1.8}$$

and consequently the problem reduces simply to a calculation of the inverse matrix (T is the temperature of the system in energy units). Expanding (1.8) separately for the elements $\mathbf{u}(\mathbf{r}, \omega)$ and $\mathbf{E}(\mathbf{r}, \omega)$, we can easily obtain general expressions for the correlation functions φ^{uu}, φ^{uE}, and φ^{EE}. We present only the expression for φ^{uu}:

$$\langle u_i(\mathbf{r}, \omega)u_j(\mathbf{r}', \omega)\rangle = \frac{i\hbar}{4\pi}\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)(L_{ij}^{-1}(\mathbf{r}, \mathbf{r}', \omega) - L_{ji}^{-1*}(\mathbf{r}', \mathbf{r}, \omega)), \tag{1.9}$$

where the tensor of the generalized susceptibility L_{ik} is given by

$$L_{ik}(\mathbf{r}, \mathbf{r}', \omega) = \alpha_{ik}(\mathbf{r}, \mathbf{r}', \omega) + 4\pi\beta_{l,\,ip}\,\beta_{s,\,kq}\frac{\partial^4 p(\mathbf{r}, \mathbf{r}', \omega)}{\partial r_p\,\partial r_l\,\partial r_s\,\partial r_q}. \tag{1.10}$$

In exactly the same way we can obtain the correlation functions for the "random forces":

$$\left\langle \frac{\partial D_l^{(s)*}(\mathbf{r}', \omega)}{\partial r_l'}\frac{\partial D_k^{(s)}(\mathbf{r}, \omega)}{\partial r_k}\right\rangle$$

$$= i\hbar\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)(p^{-1}(\mathbf{r}, \mathbf{r}', \omega) - p^{-1*}(\mathbf{r}', \mathbf{r}, \omega)), \tag{1.11}$$

$$\left\langle \frac{\partial \sigma_{ik}^{(s)*}(\mathbf{r}', \omega)}{\partial r_k'}\frac{\partial \sigma_{jm}^{(s)}(\mathbf{r}, \omega)}{\partial r_m}\right\rangle = -\frac{i\hbar}{4\pi}\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)$$
$$\times(\alpha_{ij}(\mathbf{r}, \mathbf{r}', \omega) - \alpha_{ji}^*(\mathbf{r}', \mathbf{r}, \omega)),$$

$$\left\langle D_i^{(s)*}(\mathbf{r}, \omega)\frac{\partial \sigma_{ij}^{(s)}(\mathbf{r}, \omega)}{\partial r_j}\right\rangle = 0,$$

where the last relation is a direct consequence of (1.6c).

It is easy to see that the expressions obtained for the Fourier transforms of the correlation functions (1.9)–(1.13) in the absence of an interaction between the elastic waves and the electron-hole carrier plasma, i.e., as $\beta_{i,\,kl} \to 0$, give the known expressions for the correlators. Thus, in an elastically isotropic medium (for example, a liquid), formulas (1.9) and (1.12) for the longitudinal displacement go over into the well known result of Landau and Lifshitz;[5] at the same time, formula (1.11) gives the correlator for the longitudinal electric induction, given for example in the book by Silin and Rukhadze[6] (see Sec. 9).

The correlation functions obtained above make it possible to determine different statistical characteristics of the acoustic and longitudinal electromagnetic waves in piezo-semiconductors under conditions of thermodynamic equilibrium.

2. FLUCTUATIONS IN A HOMOGENEOUS PIEZO-SEMICONDUCTOR

By way of an example of the application of the general formulas (1.9)–(1.13), we consider the fluctuations in a homogeneous piezo-semiconductor, when the tensors of the generalized susceptibility of the system are functions of the coordinate difference. It is then convenient to use the Fourier transformation with respect to the coordinates:

$$f(\mathbf{R}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3}e^{i\mathbf{q}\mathbf{R}}f(\mathbf{q}), \quad f(\mathbf{q}) = \int d^3\mathbf{R}\,e^{-i\mathbf{q}\mathbf{R}}f(\mathbf{R}), \tag{2.1}$$

where $f(\mathbf{R})$ should be taken to mean any of the quantities that depend on the coordinate difference. The normalization volume is set equal to unity throughout. With the aid of (2.1) we can obtain an explicit form for the correlators in a homogeneous medium:

$$\langle u_i(\mathbf{r}, \omega)u_j(\mathbf{r}', \omega)\rangle$$
$$= \frac{i\hbar}{4\pi}\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)\int\frac{d^3\mathbf{q}}{(2\pi)^3}e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')}(L_{ij}^{-1}(\omega, \mathbf{q}) - L_{ji}^{-1*}(\omega, \mathbf{q})), \tag{2.2}$$

where the Fourier component of the tensor $L_{ij}(\omega, \mathbf{q})$ is given by

$$L_{ij}(\omega, \mathbf{q}) = -\rho\omega^2\delta_{ij} + (\lambda_{iklj} - i\omega\mu_{iklj})q_kq_l + \frac{4\pi\beta_{l,\,mi}\beta_{p,\,qj}q_mq_lq_pq_q}{q_rq_s\varepsilon_{rs}(\omega, \mathbf{q})} \tag{2.3}$$

The remaining correlation functions are expressed analogously.

Let us consider further a piezo-semiconductor crystal in which only one piezo modulus differs from 0, say $\beta_{\mathrm{x,\,xx}}$, and the elastic properties are isotropic. Strictly speaking, such a situation is impossible, but in order not to complicate the entire calculation, it is necessary to introduce such a non-fundamental assumption. We assume further that the viscous absorption of the sound waves is small compared with the plasma absorption. Then expression (2.3) is simplified appreciably:

$$L_{ij}(\omega, \mathbf{q}) = -\rho\omega^2\delta_{ij} + \rho v_\perp^2\delta_{ij}q^2 + \rho(v_\parallel^2 - v_\perp^2)q_iq_j + \frac{4\pi\beta_{x,\,xx}q_x^4}{q^2\varepsilon_\parallel(\omega, \mathbf{q})}\delta_{ix}\delta_{jx}, \tag{2.4}$$

where $v_\parallel$ and $v_\perp$ are the velocities of the longitudinal and transverse sound waves, $\varepsilon_\parallel(\omega, \mathbf{q}) \equiv q_iq_j\varepsilon_{ij}/q^2$ is the longitudinal dielectric constant of the medium. In the absence of electric and magnetic fields, the electronic properties of the system are also isotropic, and consequently the dielectric tensor of the medium should not depend on the direction of the wave vector. We therefore obtain for the correlation of the x-components of the displacement vector

$$\langle u_x(\mathbf{r}', \omega)u_x(\mathbf{r}, \omega)\rangle = \frac{\hbar}{4\pi}\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)\frac{\eta^2v_\parallel^2}{\rho\omega^4}\int_0^\pi\frac{d\theta\sin\theta}{(2\pi)^4}$$
$$\times\operatorname{Re}\int_{-\infty}^{+\infty}dq\frac{e^{iqR\cos\theta}q^4\cos^4\theta Z(\omega, \mathbf{q})\operatorname{Im}(\varepsilon_0/\varepsilon_\parallel(\omega, \mathbf{q}))}{|(1 - q^2v_\parallel^2/\omega^2)(1 - q^2v_\perp^2/\omega^2) - \eta^2q^2v_\parallel^2\cos^4\theta Z(\omega, \mathbf{q})/\omega^2\varepsilon_\parallel(\omega, \mathbf{q})|^2} \tag{2.5}$$

where $R = |\mathbf{r} - \mathbf{r}'|$, $\eta^2 = 4\pi\beta_{\mathrm{x,\,xx}}/\rho v_\parallel^2\varepsilon_0$ is the square of the electromechanical-coupling constant for longitudinal waves, and

$$Z(\omega, \mathbf{q}) = 1 - \frac{q^2v_\perp^2}{\omega^2}\cos^2\theta - \frac{q^2v_\parallel^2}{\omega^2}\sin^2\theta.$$

The denominator of the integrand in (2.5) is the dispersion equation for the propagation in a piezo-semiconductor of sound waves that interact with the carrier plasma. It is seen from (2.5) that for the complex variable q there are four poles in the upper half-plane, two of which correspond to longitudinal sound waves and two to transverse waves, propagating in two mutually opposite directions. The fact that the transverse waves correspond to only two rather than four poles is due to the fact that the transverse wave with a polarization perpendicular to the plane containing the wave vector **q** and the x axis does not interact with the carrier plasma if only one piezo modulus $\beta_{x,xx}$ is different from zero. Closing the contour of integration with respect to q from above and assuming the relative plasma damping of the sound waves to be small, i.e., assuming that $|\mathrm{Re}\, q| \gg |\mathrm{Im}\, q|$ at the poles of q, which is always the case, owing to the condition $\eta^2 \ll 1$, we get

$$\langle u_x(\mathbf{r}',\omega)u_x(\mathbf{r},\omega)\rangle = \frac{\hbar\omega}{4\rho}\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)\int_0^\pi \frac{d\theta\sin\theta}{(2\pi)^2} \times\left\{e^{-\gamma_\parallel(\theta)R\cos\theta}\cos\left(\frac{\omega}{v_\parallel}R\cos\theta\right)\frac{\cos^2\theta}{v_\parallel^3} + e^{-\gamma_\perp(\theta)R\cos\theta}\cos\left(\frac{\omega}{v_\perp}R\cos\theta\right)\frac{\sin^2\theta}{v_\perp^3}\right\}, \quad (2.6)$$

where $\gamma_\parallel(\theta)$ and $\gamma_\perp(\theta)$ are the damping decrements of the longitudinal and transverse sound waves, respectively

$$\gamma_\parallel(\theta) = \frac{1}{2}\eta^2\frac{\omega}{v_\parallel}\frac{\varepsilon_0\cos^6\theta}{|\varepsilon_\parallel(\omega,\omega/v_\parallel)|^2}\mathrm{Im}\,\varepsilon_\parallel\left(\omega,\frac{\omega}{v_\parallel}\right)$$

$$\gamma_\perp(\theta) = \frac{1}{2}\eta^2\frac{\omega}{v_\perp}\frac{v_\parallel^2}{v_\perp^2}\frac{\varepsilon_0\cos^4\theta\sin^2\theta}{|\varepsilon_\parallel(\omega,\omega/v_\perp)|^2}\mathrm{Im}\,\varepsilon_\parallel\left(\omega,\frac{\omega}{v_\perp}\right). \quad (2.7)$$

Thus, it follows from (2.7) that the effective "correlation radius" of the elastic displacements is determined simply by the reciprocal damping decrement of the sound waves in the piezo-semiconductor.

Let us examine the limiting transition of formula (2.6) as $R \to 0$, which makes it possible to obtain an expression for the spectral energy density of the radiation of the elastic oscillations. Indeed, it follows from (2.6) (see Sec. 91 of the book[2]), that the thermodynamic-equilibrium value of the energy density of the elastic-oscillation radiations $\langle \mathscr{E}_\omega \rangle$ will be

$$\langle \mathscr{E}_\omega\rangle = 2\rho\omega^2\langle u_x(\mathbf{r}',\omega)u_x(\mathbf{r}',\omega)\rangle = \frac{1}{3}\left(\frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/T}-1}\right)\frac{\omega^2}{2\pi^2}\left(\frac{1}{v_\parallel^3}+\frac{2}{v_\perp^3}\right) \quad (2.8)$$

as $\mathbf{r}' \to \mathbf{r}$.

The first term in the parentheses of (2.8) is due to the zero-point oscillations of the phonon fields; the second term gives the energy of the thermodynamic-equilibrium "phonon radiation" in a non-absorbing medium (in analogy with the black-body radiation of the electromagnetic field). The coefficient $1/3$ in (2.8) is connected with the fact that in place of $(\mathbf{u}\cdot\mathbf{u})_\omega$ we consider only the x-component of the displacement vector. Formula (2.8) could of course be obtained also without considering the fluctuations, simply by a suitable generalization of the Planck formula for the thermal radiation of the phonons in a non-absorbing medium.

3. EFFECT OF ELECTRON DRAGGING OF THE PHONONS

If a subsonic directed stream of electrons flows in a semiconductor or in a semimetal, then the equilibrium distribution of the phonons differs from the Planck distribution and an anisotropy should appear in the phonon distribution function. For example, as shown by Keldysh,[7] when the distribution of the electrons is described by the Fermi distribution function, which is "shifted" by an amount equal to the drift vector in velocity space, then the stationary distribution of the phonons that are in equilibrium with the electrons is determined by a Planck function "shifted" by the same drift vector (without allowance for umklapp processes[8]). Thus, part of the directed momentum from the electrons is transferred to the phonons, giving rise to a directed flux of acoustic energy. We emphasize that this occurs at an electron-drift velocity smaller than the phase velocity of the sound waves, and consequently the Cerenkov mechanism of sound-wave amplification does not come into play here.[9]

To determine the energy flux of the acoustic wave, it is necessary to know the correlation functions of the random inductions and of the random elastic stresses in a system with carrier drift. It is clear that the correlator of the random elastic stresses will be determined as before by formula (1.12), since it contains none of the non-equilibrium properties of the medium. As to the correlator of the random inductions, by virtue of the homogeneity of the medium it is given by

$$\left\langle \frac{\partial D_i^{(s)}(\mathbf{r},t)}{\partial r_i}\frac{\partial D_k^{(s)}(\mathbf{r}',t')}{\partial r_k}\right\rangle = \varphi(\mathbf{r}-\mathbf{r}',t-t'), \quad (3.1)$$

where φ is a certain positive function, the explicit form of which can be established only with the aid of the electron distribution function[10, 11] (see Sec. 5 below, formula (5.12)).

The acoustic energy flux **Q** in the crystals, in the presence of spatial and temporal dispersions, is no longer determined by the simple relation $Q_i = \sigma_{ik}\dot{u}_k$, and should be obtained from the law of energy conservation in the medium.[12] Following the same derivation procedure as in [12, 13], it is easy to show that the flux density of the acoustic energy in a dispersive medium will be $\mathbf{Q} = \mathbf{v}_g \mathscr{E}_\omega$, where $\mathbf{v}_g$ is the group velocity and $\mathscr{E}_\omega$ the energy density of the acoustic waves. For the spectral density of the acoustic-wave energy flux we obtain therefrom

$$\mathbf{Q}(\omega) = 2\rho\omega^2\int\frac{d^3q\,d^3q'}{(2\pi)^6}e^{i(\mathbf{q}-\mathbf{q}')\mathbf{r}}\mathbf{v}_g\langle\mathbf{u}^*(\omega,\mathbf{q})\mathbf{u}(\omega,\mathbf{q}')\rangle. \quad (3.2)$$

Substituting in (3.2) the values of the correlators from (1.12) and (3.1), we get

$$\mathbf{Q}(\omega) = 2\rho\omega^2\beta_{x,xx}^2\int\frac{d^3q}{(2\pi)^3}\mathbf{v}_g(\omega,\mathbf{q})\frac{q_x^4}{q^4}|L_{ix}^{-1}|^2\varphi(\omega,\mathbf{q})/|\varepsilon_\parallel(\omega,\mathbf{q})|^2 + \frac{\hbar}{\pi}\operatorname{cth}\left(\frac{\hbar\omega}{2T}\right)\rho\omega^3\int\frac{d^3q}{(2\pi)^3}\mathbf{v}_g(\omega,\mathbf{q}) \times\{|L_{ij}^{-1}|^2\mu_\perp q^2 + (\mu_\parallel-\mu_\perp)L_{ij}^{-1}q_jL_{ik}^{-1*}q_k\}, \quad (3.3)$$

where, as before, we consider an elastically-isotropic crystal in which only $\beta_{x,xx}$ differs from 0, and the electron drift is directed along the x axis,

$$\mu_{iklm}q_k q_l \equiv \mu_\perp q^2\delta_{ij} + (\mu_\parallel - \mu_\perp)q_i q_j.$$

In the absence of electron drift, the medium is isotropic, so that the dielectric constant $\varepsilon_\parallel(\omega, \mathbf{q})$ and the correlation function $\varphi(\omega, \mathbf{q})$ depend only on the modulus of the wave vector $\mathbf{q}$, and therefore the flux of the acoustic energy vanishes. The latter is simply the consequence of the condition $\int q_m \, d\Omega = 0$, where $d\Omega$ is the solid-angle element in q-space.

From the expression (3.3) for the flux we see that $\mathbf{Q}$ vanishes when the electron-phonon interaction constant $\beta_{i,kl}$ tends to 0 (when $\mu_\parallel$, $\mu_\perp \neq 0$). On the other hand, if the viscosity is first allowed to tend to 0, then the expression for the flux of the acoustic energy turns out to be independent of the electron-phonon interaction constant. Physically this is connected with the fact that in the hydrodynamic model for the phonons, i.e., in the approximation of the elastic continuum, the viscous absorption of the sound determines the phonon mean free path, which becomes infinite as $\mu \to 0$. Under these conditions, even an infinitesimally small interaction between the electrons and the phonons leads to a finite value of the momentum transfer, and consequently to a finite value of the flux.[3)] This uncertainty indicates that in this case it is necessary to take into account the viscous absorption of the sound waves, and in a more rigorous analysis also the umklapp processes.[8]

If the electron drift v_d is directed along the x axis, then the flux components Q_y and Q_z are equal to 0, and we are left with only Q_x. To find the explicit form of Q_x it is necessary to integrate in (3.3) over the wave vector $\mathbf{q}$, so that it is necessary to refine the form of the longitudinal dielectric constant of the medium $\varepsilon_\parallel(\omega, \mathbf{q})$ and the correlation function $\varphi(\omega, \mathbf{q})$ in the presence of the electron current. The dielectric constant of a plasma medium in which there is a directed current of charged particles (electrons or holes) can always be represented in the form

$$\varepsilon_\parallel(\omega, \mathbf{q}) = f(\omega, q^2, \omega - \mathbf{q}\mathbf{v}_d), \tag{3.4}$$

where f is a certain function, the explicit form of which is already determined by the concrete properties of the plasma medium.[9, 14–16] It is obvious that a relation of the type (3.4) is valid also for the correlation function $\varphi(\omega, \mathbf{q})$. Transforming in (3.3) to spherical coordinates and integrating by taking residues, we obtain for the flux component Q_x

$$Q_x(\omega) = \frac{\omega\eta^2}{4}\int_{-1}^{+1} dt\left\{t^3(\psi_\parallel^+ - \psi_\parallel^-) + \frac{v_\parallel^2}{v_\perp^2}t(1-t^2)(\psi_\perp^+ - \psi_\perp^-)\right\}$$
$$+ \frac{\hbar\omega^3}{32\pi^2 v_\parallel^2}\left(\frac{\omega\mu_\parallel}{\rho v_\parallel^2}\right)\mathrm{cth}\left(\frac{\hbar\omega}{2T}\right)\int_{-1}^{+1} t\,dt\,(\mathcal{G}_\parallel^+ - \mathcal{G}_\parallel^-)$$
$$+ \frac{\hbar\omega^3}{16\pi^2 v_\perp^2}\left(\frac{\omega\mu_\perp}{\rho v_\perp^2}\right)\mathrm{cth}\left(\frac{\hbar\omega}{2T}\right)\int_{-1}^{+1} t\,dt\,(\mathcal{G}_\perp^+ - \mathcal{G}_\perp^-), \tag{3.5}$$

where

$$\mathcal{G}_\parallel^\pm(\omega, t) = \frac{\omega/v_\parallel}{\gamma_\parallel(\omega, \pm\omega/v_\parallel, t) + \omega^2\mu_\parallel/2\rho v_\parallel^2}$$

$$\psi_\parallel^\pm(\omega, t) = \frac{\varepsilon_0\varphi(\omega, \pm\omega/v_\parallel, t)}{|\varepsilon_\parallel(\omega, \pm\omega/v_\parallel, t)|^2}\mathcal{G}_\parallel^\pm(\omega, t)$$

and $t = \cos\theta$. In the derivation of (3.5) it was assumed that the relative change of the phase velocity of the sound waves is small, and in addition the possible appearance of "coupled" acousto-plasma waves was neglected.

The total sound-energy flux will obviously be

$$\mathbf{P} = \int_0^{\omega_m} d\omega \mathbf{Q}(\omega), \tag{3.6}$$

where ω_m is the end-point frequency of the acoustic phonons.

Let us estimate the total flux. In the classical limits its value is

$$P \sim T\frac{\omega_0^2}{v_s^2}\left(\frac{1}{\Gamma}\frac{\partial\Gamma}{\partial\beta}\right)\beta \quad \left(\beta = \frac{v_d}{v_s} \ll 1\right), \tag{3.7}$$

where Γ is the total damping decrement, and ω_0 the characteristic frequency at which the dragging is maximal. At room temperatures and at the frequency $\omega_0 = 3\times10^9\ \mathrm{sec}^{-1}$ we have $v_s = 2\times10^5$ cm/sec, $\beta = 1/3$, $P \sim 10^{-4}\ \mathrm{W/cm}^2$, and the effect apparently can be easily observed experimentally.[4)]

Formula (3.5) is valid at an electron-flux velocity v_d smaller than the velocity v_s of the sound waves; if $v_d > v_s$, then expression (3.5) describes the flux produced by the phonons with wave vectors outside the amplification cone. Integration with respect to the angle θ must then be carried out between the limits θ_0 and π, where θ_0 determines the amplification cone. The latter is determined from the condition $\mathcal{G}_\parallel^{-1}(\theta_0) = 0$, and $\mathcal{G}_\perp^{-1}(\theta_0) = 0$ respectively for the longitudinal and transverse waves.

We note that in a magnetic field, when the conditions for the cyclotron[17] and geometric[18] resonances are satisfied, or for quantum oscillations[19] connected with the Landau levels, the dielectric constant of the medium and the correlation function of the random inductions become oscillating functions of the magnetic field, and consequently the flux of dragged phonons will also oscillate as a function of the magnetic field.

4. GROWTH OF FLUCTUATIONS IN A SYSTEM WITH SUPERSONIC CARRIER DRIFT

If a carrier drift is produced in the crystal, for example by an external electric field, then at a drift velocity exceeding the phase velocity of the wave, amplification and generation of acoustic waves become possible.

In investigations of fluctuations that grow in space it is no longer possible to use, as before, the spatial Fourier transformation, since the sought functions diverge

3) The situation is the same here as in the calculation of the phonon thermal conductivity of crystals, which becomes infinite if no account is taken of the umklapp process or of the scattering of phonons by impurities, dislocations, etc [8].

4) It is expedient to perform the experiment in the following manner: the semiconductor crystal is divided into two regions, and the constant drift field is applied to one of them. The experimentally observed quantity is the potential difference across the second part of the crystal, due to the acousto-electric effect of the acoustic flux from the first part of the crystal.

as $x \to +\infty$. Obviously, the divergence of these functions has an exponential character, so that it is always possible to introduce a certain parameter $s > 0$, such that a function of the type $e^{-sx}\psi(x)$ will already be converging when $x \to \infty$. As is well known, to these functions it is already possible to apply a Fourier transformation (or a Laplace transformation, see [20], Sec. 4.8), but when making the inverse transformation, i.e., when finding $\psi(x)$, the integration contour must be moved to the upper (or lower) half-plane of the corresponding complex variable.

We shall assume that the carrier drift is directed along the x axis. Then it is obvious that the growth of the fluctuations is possible only in the x direction; in directions perpendicular to the drift, i.e., to the x axis, there is no growth of the waves. The latter means that it is sufficient to introduce one parameter $s > 0$, which shifts the integration path in the complex plane q_x; on the other hand, in integration with respect to q_y and q_z, the path of integration can be chosen along the real axis.

In the investigation of fluctuations that grow in space, it is necessary to consider a boundary-value problem, since in an infinite medium the amplitude of the fluctuations is unbounded at any point of space. Therefore Eqs. (1.1) and (1.2) for the fluctuating quantities must be regarded only in the region $x \geq 0$, and the medium will be considered to be unbounded in the y and z directions. Integration with respect to x in (1.2) should be performed from 0 to ∞, but, recognizing that in a medium with a drift in the positive x direction the fluctuation of any quantity at the point x is weakly correlated with the fluctuations of the same quantity at the point x' which is larger than x, the integration in (1.2) can be carried out between 0 and x. Such an assumption is equivalent to neglecting the influence of the region $x' > x$ on the growing fluctuations in the region $0 \leq x < x'$, an assumption which is physically quite obvious since the acoustic waves attenuate rapidly in the direction to the drift. Mathematically this assumption means that instead of using the Wiener–Hopf method to solve (1.2) it is possible to use the Fourier transformation with respect to the coordinate x:

$$\psi(x) = \int_{-\infty+is}^{+\infty+is} \frac{dq_x}{2\pi} e^{-iq_x x}\psi(q_x),$$

$$\psi(q_x) = \int_0^\infty dx e^{iq_x x}\psi(x), \quad \operatorname{Im} q_x = s > 0. \tag{4.1}$$

As the boundary conditions we choose

$$\lambda_{ixxm}\frac{\partial u_m}{\partial x}\Big|_{x=0} - \beta_{x,\,ix}\varphi'(x=0) = 0,$$

$$\frac{\partial u_m}{\partial y} = \frac{\partial u_m}{\partial z} = 0 \quad (\varphi(0) = 0), \tag{4.2}$$

which correspond to a free boundary, on which the surface forces equal 0. Assuming the medium to be quasihomogeneous (i.e., its properties to change little over dimensions on the order of the wavelength of the acoustic fluctuations) and changing over to Fourier components with respect to the coordinate and the time with the aid of (1.4) and (4.1), we obtain from (1.1) and (1.2), with allowance for the boundary conditions (4.2),

$$L_{ij}(\omega, \mathbf{q})u_j(\omega, \mathbf{q}) = -iq_m\sigma_{mi}^{(s)}(\omega, \mathbf{q})$$
$$- \beta_{l,\,ik}q_kq_l\frac{q_pD_p^{(s)}(\omega, \mathbf{q})}{q^2\varepsilon_{\parallel}(\omega, \mathbf{q})} + iP_{im}(\omega, \mathbf{q})u_m(x=0), \tag{4.3}$$

where

$$P_{im}(\omega, \mathbf{q}) = \left(\lambda_{ixxm}q_x + \frac{4\pi\beta_{x,\,xx}^2}{\varepsilon_{xx}(\omega, \mathbf{q})}q_x^3\right)\delta(q_y)\delta(q_z).$$

With the aid of the inverse Fourier transformation we obtain the fluctuating displacement

$$u_i(\mathbf{r}, \omega) = i\int\frac{dq_y\,dq_z}{(2\pi)^3}\int_{-\infty+is}^{+\infty+is} dq_x e^{-i\mathbf{qr}}L_{ij}^{-1}(\omega, \mathbf{q})$$
$$\times\left[P_{jm}u_m(x=0) - q_m\sigma_{jm}^{(s)}(\omega, \mathbf{q}) - \beta_{l,\,jk}\frac{q_kq_lq_pD_p^{(s)}(\omega, \mathbf{q})}{q^2\varepsilon_{\parallel}(\omega, \mathbf{q})}\right] \tag{4.4}$$

at $x \geq 0$.

As before, we consider an elastic-isotropic crystal with only one nonvanishing piezo modulus $\beta_{x,\,xx}$. Furthermore, we set up a bilinear combination $u_i^*(\mathbf{r}, \omega)u_i(\mathbf{r}, \omega)$ and obtain its statistical average, using the values of the correlation functions (1.12) and (3.1). After straightforward but cumbersome transformations, we can obtain a final expression for the spectral density of the radiation energy of the acoustic noise

$$\mathscr{E}_\omega = \frac{\hbar\omega^3}{4\pi v_\perp^3}\int_0^{\pi/2}\frac{d\theta\sin\theta}{2\pi}\left(1 - \exp\left(-\frac{2\Gamma_\perp(\theta)x}{\cos\theta}\right)\right)$$
$$\times\frac{\omega}{\Gamma_\perp(\theta)v_\perp}\left\{\frac{\omega\mu_\perp}{\rho v_\perp^2}\operatorname{cth}\frac{\hbar\omega}{2T} + \eta^2\frac{\varepsilon_0 v_\parallel^4\sin^2\theta\cos^4\theta\varphi(\omega, \omega/v_\perp, \theta)}{2\hbar\omega^2v_\perp^2|\varepsilon_\parallel(\omega, \omega/v_\perp, \theta)|^2}\right\}$$
$$+ \frac{\hbar\omega^3}{4\pi v_\parallel^3}\int_0^{\pi/2}\frac{d\theta\sin\theta}{2\pi}\left(1 - \exp\left(-\frac{2\Gamma_\parallel(\theta)x}{\cos\theta}\right)\right)$$
$$\times\frac{\omega}{\Gamma_\parallel(\theta)v_\parallel}\left\{\frac{\omega\mu_\parallel}{\rho v_\parallel^2}\operatorname{cth}\frac{\hbar\omega}{2T} + \eta^2\frac{\varepsilon_0 v_\parallel^2\cos^6\theta\varphi(\omega, \omega/v_\parallel, \theta)}{2\hbar\omega^2|\varepsilon_\parallel(\omega, \omega/v_\parallel, \theta)|^2}\right\}$$
$$+ 2\rho\omega^2|u_x(0)|^2e^{-2\Gamma_\parallel(0)x}. \tag{4.5}$$

The last term of (4.5) describes the growth of longitudinal waves, the displacement vector of which is specified at the boundary $x = 0$, on going away from the boundary. The shear deformations in the $x = 0$ plane always lead to the appearance of transverse waves with a wave vector along the x direction, but under conditions when only $\beta_{x,\,xx} \neq 0$ the transverse waves "from the boundary" are not amplified in the x direction, and therefore (4.5) will not contain a term that grows in space and corresponds to transverse waves.

In spite of the fact that formula (4.5) contains the Fourier transform of the correlation function (3.1), nonetheless the main physical conclusions can be drawn without using the explicit form of the correlation function.

First of all, it is seen from (4.5) that "oblique" transverse waves, with a directivity pattern comprising two lobes located inside the Cerenkov amplification cone will be initially generated in the medium. It is known that a number of experiments[21–24] have revealed saturation of the current on the current-voltage characteristics practically in all cases, regardless of the crystal orientation, when the carrier drift velocity exceeded somewhat the velocity of the transverse sound waves, but was considerably smaller than the velocity of the longitudinal sound waves. Physically this phenomenon can be attributed to the generation of noise, whose threshold, as shown above, always corresponds to the velocity of the transverse sound waves. For a quantitative analysis of this effect it is necessary, following Hutson,[25] to calculate the acousto-electric current produced as the result of generation of the acoustic noise.[5)] This problem will be considered separately.

At a drift velocity exceeding the velocity of the longitudinal sound waves, i.e., when $v_d > v_{\parallel}$, one more mechanism that leads to the growth of acoustic fluctuations comes into play, and is connected with generation of longitudinal sound waves.

It is seen from (4.5) that the spatial growth of the acoustic waves is determined by the corresponding total increments $\Gamma_{\parallel}(\theta)$ and $\Gamma_{\perp}(\theta)$, whereas the initial fluctuation level is determined by the correlation functions. Under conditions of thermodynamic equilibrium, when the correlation function of the random inductions is determined by (1.11), we get from (4.5) Planck's formula if we go away from the boundary $x = 0$ a distance much larger than the reciprocal damping decrement of the acoustic waves. It is important, that the Planck distribution follows from (4.5) at any thermodynamic-equilibrium value of the dielectric tensor of the medium.

5. CORRELATION FUNCTION OF THE RANDOM INDUCTIONS IN A NONEQUILIBRIUM MEDIUM WITH DRIFT

The expressions obtained above for the spectral energy density and for the noise energy flux contain the correlation function of the random inductions in a nonequilibrium medium with drift. To determine the correlation function in explicit form, we consider, following Angeleĭko and Akhiezer,[27] the kinetic equation for the electrons with a "random force"

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+\frac{e\mathbf{E}}{m}\frac{\partial f}{\partial \mathbf{v}}=\hat{\mathrm{St}}\,\{f-y\}, \qquad (5.1)$$

where f is the electron distribution function, $\widehat{\mathrm{St}}\{\ldots\}$ is the collision integral with a random force chosen such as not to violate the conservation law for the number of particles, and $\mathbf{E}(\mathbf{r}, t)$ is the total value of the electric field.

Using (5.1), we can represent the time derivative of the entropy of the system in the form[27]

$$\dot{S}(t)=-\int d^3\mathbf{r}d^3\mathbf{v}\,\frac{\partial f'(\mathbf{r},\mathbf{v},t)}{dt}\,\frac{f'(\mathbf{r},\mathbf{v},t)}{f_0(v)}=\int d^3\mathbf{r}d^3\mathbf{v}\dot{x}(\mathbf{r},\mathbf{v},t)X(\mathbf{r},\mathbf{v},t), \qquad (5.2)$$

where $f'(\mathbf{r}, \mathbf{v}, t)$ is the deviation, due to the action of the random forces, from the equilibrium function $f_0(v)$, and $X(\mathbf{r}, \mathbf{v}, t) = -f'(\mathbf{r}, \mathbf{v}, t)/f_0(v)$ and $\dot{x}(\mathbf{r}, \mathbf{v}, t) \equiv \widehat{\mathrm{St}}\{f' - y\}$. In the absence of random forces, the change of the entropy of the system per unit time, $\dot{S}$ is a bilinear function with respect to the deviations of the distribution function from the equilibrium value, so that we can use the general method (see [4], Ch. XII) for determining the correlation function for the random forces. According to this method, the quantities $\dot{x}(\mathbf{r}, \mathbf{v}, t)$ should be represented in the form

$$\dot{x}(\mathbf{r},\mathbf{v},t)=\int d^3\mathbf{v}\gamma(\mathbf{v},\mathbf{v}')X(\mathbf{r},\mathbf{v}',t)+y(\mathbf{r},\mathbf{v},t). \qquad (5.3)$$

The kinetic coefficients $\gamma(\mathbf{v}, \mathbf{v}')$ will determine the correlation of the random forces (see [4], Secs. 121–127):

$$\langle y(\mathbf{r},\mathbf{v},t)y(\mathbf{r}',\mathbf{v}',t')\rangle=\{\gamma(\mathbf{v},\mathbf{v}')+\gamma(\mathbf{v}',\mathbf{v})\}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \qquad (5.4)$$

To determine the coefficients $\gamma(\mathbf{v}, \mathbf{v}')$ it is necessary to use the explicit form of the collision integral $\widehat{\mathrm{St}}\{\ldots\}$. At low frequencies, when $\omega \ll \delta\nu$, where ν is the effective frequency of the collisions between the carriers and the scattering centers and $\delta \approx 2m/M$ is a parameter characterizing the fraction of the energy transferred in each elastic collision between the electron and the center with mass M (see [28], Sec. 38), we can use the Davydov diffusion collision integral[28, 29]

$$\hat{\mathrm{St}}_0\{f_0\}=-\frac{1}{2mv^2}\frac{\partial}{\partial v}\left(v\delta\nu\left(\frac{\partial f_0}{\partial v}\frac{T}{m}+vf_0\right)\right), \quad \hat{\mathrm{St}}_1\{\mathbf{f}_1\}=\nu \mathbf{f}_1, \qquad (5.5)$$

where f_0 and f_1 are the zeroth and first harmonics of the expansion of the distribution function in spherical functions. Substituting further (5.5) in (5.2) and noting that $\dot{x}(\mathbf{r}, \mathbf{v}, t) = \delta\dot{S}/\delta X(\mathbf{r}, \mathbf{v}, t)$, we can easily establish the explicit form of $\gamma(\mathbf{v}, \mathbf{v}')$ and by the same token determine the correlator (5.4) (for details see [27]). We shall need in what follows only the correlator of the first harmonics of the random forces, which is given by:[27]

$$\langle y_{1i}(\mathbf{r},v,t)y_{1j}(\mathbf{r},v,t)\rangle=\frac{3}{2\pi}\frac{f_0(v)}{v^2\nu}\delta_{ij}\delta(v-v')\delta(t-t')\delta(\mathbf{r}-\mathbf{r}'). \qquad (5.6)$$

Here $f_0(v)$ is the zeroth harmonic of the distribution function with allowance for the constant electric field E_d in the system:[28]

$$f_0(v)=\left(\frac{n_0}{2\pi T_e}\right)^{3/2}\exp\left(-\frac{mv^2}{2T_e}\right), \quad T_e=T\left(1+\frac{e^2E_d^2}{3m\delta\nu^2T}\right). \qquad (5.7)$$

From the kinetic equation (5.1) it is easy to obtain equations for the zeroth and first harmonics of the expansion of the distribution function. Using subsequently the customary procedure for taking moments, we can obtain the hydrodynamic equations with account taken also of the random forces. If we confine ourselves to the low-frequency region $\omega \ll \delta\nu$, then the hydrodynamic system is

$$\frac{\partial n}{\partial t}+\operatorname{div} n\mathbf{v}=0, \quad \frac{e\mathbf{E}}{m}-\frac{1}{n}\frac{\partial}{\partial \mathbf{r}}\frac{P}{m}-\nu\mathbf{v}+\frac{1}{n}\mathbf{Y}=0. \qquad (5.8)$$

The first equation is the ordinary continuity equation and the second is the Navier–Stokes equation in the region of low frequencies $\omega \ll \delta\nu$ with allowance for the random force

$$\mathbf{Y}(\mathbf{r},t)=\frac{\nu}{3}\int_0^\infty v^3y_1(\mathbf{r},v,t)\,dv \qquad (5.9)$$

and the electron-gas pressure

$$P\equiv\frac{1}{3}\int_0^\infty f'(\mathbf{r},v,t)\,v^4dv\approx nT_e,$$

[5)] Expression (4.5), unlike the analogous formula in the paper by V. L. Gurevich [3], contains the contribution due to the generation of transverse waves, which play the decisive role in many effects. In particular analysis of the current-voltage characteristic of a piezo- semi-conductor, carried out by V. L. Gurevich and V. D. Kagan [26] and based on the results of [3], is in error since it does not take into account the generation of "oblique" transverse waves.

where the electron temperature T_e can be regarded as constant and independent of the coordinates and the time in the region of low frequencies.

With the aid of (5.6) it is now easy to establish the explicit form of the correlator for the "hydrodynamic" forces $\mathbf{Y}(\mathbf{r}, t)$, and by the same token determine the correlation function of the random inductions of interest to us. The solution is obtained in the following manner. The system (5.8) is used to determine the small deviations from the stationary values of the electric field, the concentration, and the electron velocity, due to the action of the random forces $\mathbf{Y}(\mathbf{r}, t)$. The self-consistent electric field is eliminated with the aid of the Poisson equation. Then

$$\langle \mathbf{q}\mathbf{D}^{(s)}(\omega, \mathbf{q})\mathbf{q}'\mathbf{D}^{(s)}(\omega, \mathbf{q}')\rangle = -\frac{16\pi^2 e^2}{\varepsilon_0^2}\varepsilon_{\|}(\omega, \mathbf{q})\varepsilon_{\|}(\omega, \mathbf{q}')\langle n_{\sim}(\omega, \mathbf{q}) n_{\sim}(\omega, \mathbf{q}')\rangle, \quad (5.10)$$

where

$$n_{\sim}(\omega, \mathbf{q}) = \mathbf{q}\mathbf{Y}(\omega, \mathbf{q}) \Big/ \left[\nu(\omega - \mathbf{q}\mathbf{v}_d) - \frac{4\pi e^2 n_0}{m\varepsilon_0} i(1 + q^2 r_0^2)\right],$$

r_0 is the Debye radius of the electrons, and the random-force correlator is

$$\langle Y_i(\omega, \mathbf{q}) Y_j(\omega', \mathbf{q}')\rangle = n_0 \frac{\nu T_e}{2m}(2\pi)^2 \delta(\omega + \omega')\delta(\mathbf{q} + \mathbf{q}')\delta_{ij}. \quad (5.11)$$

Substituting (5.11) in (5.10) we obtain for the function $\varphi(\omega, \mathbf{q})$, which determines the correlation of the random inductions,

$$\varphi(\omega, \mathbf{q}) = \frac{4T_e q^2}{\omega}\frac{4\pi\sigma_0/\omega}{(1 - \mathbf{q}\mathbf{v}_d/\omega)^2 + q^4 v_T^4/\omega^2\nu^2} = \frac{4T_e q^2}{\omega - \mathbf{q}\mathbf{v}_d}\operatorname{Im}\varepsilon_{\|}(\omega, \mathbf{q}). \quad (5.12)$$

Comparison of (5.12) and (1.11) shows that the correlation function (5.12) for a nonequilibrium medium with drift can be obtained in the classical limits from the equilibrium function with the aid of the formal substitutions $\omega \to \omega - \mathbf{q}\mathbf{v}_d$, $\varepsilon_{\|}^{eq}(\omega, \mathbf{q}) \to \varepsilon^{noneq}(\omega, \mathbf{q})$.[6] When the carrier drift velocity exceeds the phase velocity of the wave, the imaginary part of the dielectric tensor of the medium, i.e., the conductivity, becomes negative,[16] so that the correlation function as a whole remains a positive quantity, as it should.[7]

A formula of the type (5.12) can also be obtained for the high-frequency region, when $\omega \gg \nu$, if it is assumed that the distribution function in the presence of the electron drift is Maxwellian but shifted by an amount equal to the drift vector in velocity space. The correlation function then has the form (5.12) (the last equation). In this equation, $\varepsilon_{\|}(\omega, \mathbf{q})$ should be taken to mean the high-frequency dielectric constant of the medium in the presence of drift.

Substituting now (5.12) in (4.5) and (3.5) we obtain the final expressions for the spectral energy density of the radiation of the acoustic noise and the energy flux of the dragged phonons.

We have investigated here only the linear problem, and assumed that the energy of the acoustic noise has not increased enough to change the ground state of the system. It is clear that the change of the ground state of the system as a result of the growing noise can be accounted for within the framework of the linear theory, but then the carrier density, the drift velocity, and the drift electric field are already slow functions of the coordinates. This possibility of making the problem self-consistent is brought about by the fact that the intensity of the growing fluctuations does not change noticeably over the characteristic wavelengths, and this in turn makes it possible to use the well-known method of geometrical optics.[28] However, when the electron energy in the field of the sound waves becomes comparable with its average energy, the problem already calls for a nonlinear approach.

In conclusion, I am deeply grateful to V. L. Ginzburg and the members of the seminar under his direction for a useful discussion and valuable remarks. I am particularly grateful to L. V. Keldysh for advice and useful remarks, which have contributed greatly to the performance of this work.

[6] We note that the choice of the correlation function for the random currents (or inductions), made by V. L. Gurevich [3] in the form $\langle jj\rangle \sim \sigma_0$, where σ_0 is the dc conductivity, is incorrect since it does not take into account the spatial and temporal dispersions in the carrier plasma which, on the one hand, are very important for the analysis of the effects of amplification and generation of acoustic waves. The method itself of obtaining the kinetic equation for the mean square of the fluctuating displacement is based on the fact that in expressions (4.6) and (5.14) of [3] it is possible to omit a number of terms proportional to the random current; in general, there is no justification whatever for such a procedure.

[7] It is seen in particular from expression (5.12) that a nonequilibrium medium with a drift, for waves whose phase velocity is smaller than the drift velocity, is formally equilavent to a medium with negative temperature. It is then easy to see that the transition from "positive" to "negative" temperatures can be effected in a continuous fashion only through the region $T = \pm\infty$, as in the case of the magnetic moments [4].

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Translated by J. G. Adashko
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