# FALSE POLES OF THE SCATTERING AMPLITUDE IN FIELD MODELS

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Starting from the known expressions for the amplitudes of $V\theta$ scattering in the $V\theta$ sector of the Lee model or the three-particle model, it is shown that at negative energies these amplitudes have not only poles corresponding to discrete states, but also false poles. The properties of these poles and their differences from the false poles of the potential scattering theory are investigated, and a procedure is presented for revealing false poles in simple quantum-field models.

It is known from the theory of potential scattering (see, for example,[1]) that at negative energies the partial scattering amplitudes may have not only poles corresponding to bound states, but also the so-called false poles.

In simple nonrelativistic models of quantum field theory, such as the $N\theta$ sector of the Lee model[2], the model with bilinear interaction[3], or the model of contact interaction[4], discrete energy levels of single-particle and bound states are uniquely determined by the poles of the scattering amplitudes—there are no false poles.

However, the example of the relatively recently solved models, namely the $V\theta$ sector of the Lee model (for a review see[4]) and the three-particle model[5,6] (allowed transitions $W \rightleftarrows V + \theta$, $V \rightleftarrows N + \theta$), shows that it is impossible to conclude on the basis of the simplest models that there are no false poles in field interaction. Indeed, the amplitude of elastic $V\theta$ scattering in the three-particle model contains only the s-wave and is given by[6]

$$T(\omega)=\frac{f^2(\omega)}{2\omega}\frac{1}{1-h(\omega)A(\omega)}\left[-\lambda^2\frac{1+h(\omega)A(\omega)}{h(\omega)}+\lambda_1{}^2\frac{1+M(\omega)}{g(\omega)}\right]. \tag{1}$$

Here $\lambda_1$ and $\lambda_2$ are the renormalized $VN\theta$- and $WV\theta$-interaction coupling constants, $\omega$ is the energy of the $\theta$ particle (the total energy of the in-states is $E = m_V + \omega$); $f(\omega)$ is a real cutoff function, $h^{-1}(\omega)$ is the propagator of the V particle (at $\omega = E - m_N$), whose only pole determines the energy of the V-state if the V particle is stable, as is assumed henceforth, and $g^{-1}(\omega)$ is the W-particle propagator, whose poles determine the energies of the single-particle W states, and whose only zero is determined by the equation

$$1-h(\omega)A(\omega)=0. \tag{2}$$

The function $A(\omega)$ has no poles in the region $\omega < \mu$ ($\mu$—mass of $\theta$ particle). The poles of $M(\omega)$ are found from (2) and from

$$h(\omega)=0. \tag{3}$$

The explicit form of all the foregoing functions is given in[6]. The amplitude of the $V\theta$ scattering in the Lee model is obtained from (1) with $\lambda_1 = 0$.

A direct solution of the Schrödinger equation shows that the energy of the only $V\theta$-bound state of the Lee model is determined by Eq. (2)[7], and that of the single-particle W states by the Eq.[6]

$$g(\omega)=0. \tag{4}$$

From a comparison of (1), (2), and (4) we see that the $V\theta$-scattering amplitudes of both models contain false poles. Their position is determined by the condition (3) for the Lee model (here $\omega = E - m_V$) and by Eqs. (2) and (3) for the three-particle model (the factor $[2\omega]^{-1}$ has a kinematic origin). Without corresponding to any bound states, these poles, if they are close to the threshold energy $\omega = \mu$, are capable, just as are the bound states, of causing an anomalously rapid growth of the scattering cross section near threshold.

It should be noted that the false poles have no relation at all to the "ghost" states[2], the possible appearance of which was excluded beforehand by the conditions $\lambda^2 < \lambda_c^2$ and $\lambda_1^2 < \lambda_{1c}^2$.

The poles considered here differ from the false poles in potential theory. Thus, the analog of the potential in nonrelativistic models is the form factor $f(\omega)$; in any case, the inverse problem of the scattering theory in such models is the reconstruction of the form factor[8]. By cutting off the potential at arbitrarily large distances it is possible to eliminate all the false poles, leaving the positions of the bound states almost unchanged[1]. The false poles (2) and (3) cannot be eliminated by any change of the form factor, so long as the V-particle is stable.

The reason for this non-removability is easy to understand. The false pole of the Lee model (3) is a result of the contribution of the simplest (Born) diagram, in which the energy of the internal N-line is fixed, and the difference $T(\omega) - T_B(\omega)$, where

$$T_B(\omega) = -\lambda^2 f^2(\omega)[2\omega]^{-1}[\omega - m_V + m_N]^{-1}, \tag{5}$$

can now have only a pole corresponding to the $V\theta$-bound state. In the contact and in the bilinear models, there are no elastic-scattering diagrams with internal lines with fixed energy, therefore the poles of the scattering amplitude must correspond to the energy of the bound states. It should be noted that the false pole (3) is a sui generis shadow of the V-particle from the $N\theta$ sector on the $V\theta$ sector. Its energy is shifted relative to the V-state energy by $m_V - m_N$. It always proceeds the $V\theta$-bound state (this follows from Eqs. (2) and (3) and from the properties of the functions $h(\omega)$ and $A(\omega)$[6]), and exists even in the case of weak coupling, when there is no $V\theta$-bound state.

In the three-particle model, the appearance of false poles is due to the summation of diagrams containing no internal lines of W particles corresponding to the first term in formula (1), and diagrams containing internal W-lines, in which at least one of the vertices with a free $\theta$ line is a VN $\theta$ vertex (these diagrams correspond to the term $\lambda_1^2 M(\omega) g^{-1}(\omega)$ in the square brackets of formula (1), having a first-order pole under condition (2), inasmuch as the pole of the factor preceding the bracket cancels out with the zero of $g^{-1}(\omega)$). On the other hand, a correct method of determining the energy of the W states is to find the poles of the W-particle Green's function, i.e., to solve Eq. (4) [5,6]. The Green's functions of the W particle correspond to summation of diagrams with two $WV\theta$ vertices at the free $\theta$ lines—the term $\lambda_1^2 g^{-1}(\omega)$ in the square brackets of formula (1). Since all the diagrams of the $N\theta$ scattering of the Lee model have input and output vertices of the type $VN\theta$, the pole of the $N\theta$-scattering amplitude must coincide with the energy of the single-particle $V\theta$ state, and there are no false single-particle states in this sector.

It is possible to trace the occurrence of false poles in the direct solution of Schrödinger's equation in the continuous-spectrum region [6]. They result from those inhomogeneous terms of the integral equations for the wave functions, which are due to the continuation of the singularity of the type

$$\frac{1}{\omega-\omega_0} = P\frac{1}{\omega-\omega_0} + C\delta(\omega-\omega_0), \quad C \neq 0, \quad \omega_0 > \mu, \tag{6}$$

corresponding to a plane wave, into the unphysical region $\omega_0 < \mu$, where the discrete state is situated.

If the scattering amplitude in models close to those considered is obtained not by solving Schrödinger's equation but by other methods, say by the Lehmann-Symanzik-Zimmerman method, by which the amplitude of the $V\theta$ scattering in the Lee model was obtained for the first time [9], then the number of bound states can be determined with the aid of the Levinson theorem, which was proved for such models in [8], and false poles can be separated only with the aid of additional analysis similar to that presented above.

The considered examples demonstrate that in field theory, just as in potential scattering, there is the danger of the appearance of false poles when the energies of the discrete states are determined from the poles of the scattering amplitude, and these poles can be readily obtained and explicitly separated only in the case of simple models, as was indeed done for the $V\theta$ sector of the Lee model and for the three-particle model.

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