INSTABILITY OF AN ACOUSTOMAGNETIC FIELD IN SEMIMETALS AT LOW TEMPERATURES AND ACOUSTOMAGNETIC WAVES

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Submitted April 24, 1968

Zh. Eksp. Teor. Fiz. 55, 1827–1835 (November, 1968)

In semimetals (Bi and Sb), in the presence of a sufficiently strong but feasible sound-beam flux S (0.2–0.5 W/cm^2), weakly damped almost transverse and almost exclusively magnetic waves called acoustomagnetic waves should be possible. This effect can be realized in practice at temperatures below the hydrogen temperature. Instead of being damped, the waves can begin to build up in an isotropic crystal (or when S is parallel to the principal axis of the crystal) and in the presence of an external magnetic field. In crystals of low symmetry (trigonal, rhombic) build-up is also possible in the absence of an external magnetic field, providing S does not have certain "forbidden" directions. The presence of such waves in the core of an induction coil results in oscillations of the real and imaginary parts of the impedance of the circuit. Moreover under build-up conditions of the oscillations detection of oscillations in the external circuit should be possible even in the absence of an external source of current, that is, the system may operate as a generator. The frequency of the oscillations is of the order of kHz. A nonlinear theory of the generation regime for low supercriticality as well as an exact theory of stationary traveling acoustomagnetic waves, are developed.

It is known that in the presence of a thermoelectric field, thermomagnetic waves can exist and can become unstable, i.e., they can change from damped waves into growing waves either in the presence of an external magnetic field,[1] or in anisotropic crystals of sufficiently low symmetry.[2] It is only necessary that the temperature gradient exceed a certain critical value, which incidentally is sufficiently small and can be readily obtained. The situation is exactly the same in the case of an acoustoelectric field. We shall show that it can generate almost transverse waves, which can be called acoustomagnetic, and which can be weakly damped and change into growing waves in the presence of an external magnetic field or in an anisotropic medium. In analogy with the foregoing, this requires that the acoustoelectric field exceed a certain critical value. Both mentioned phenomena are realizable only in semimetals and only in exceptional cases in metals at helium temperatures and somewhat higher, or else in a hot plasma, but the latter will not be considered in this article.

1. ACOUSTOMAGNETIC WAVES IN ISOTROPIC CONDUCTORS

In the presence of an acoustic flux of density **S** and in the presence of a magnetic field **H**, the electric current is

$$\mathbf{j} = \hat{\sigma}(\mathbf{H})\mathbf{E}^* - \hat{\gamma}(\mathbf{H})\mathbf{S},$$

where $\mathbf{E}^* = \mathbf{E} - e^{-1}\nabla\xi$ (ξ—chemical potential of carriers). From this we get the electric field

$$\mathbf{E}^* = \hat{\sigma}^{-1}(\mathbf{H})\mathbf{j} + \hat{\sigma}^{-1}(\mathbf{H})\hat{\gamma}(\mathbf{H})\mathbf{S} = \hat{\eta}(\mathbf{H})\mathbf{j} + \hat{\beta}(\mathbf{H})\mathbf{S}.$$

Here $\hat{\sigma}(\mathbf{H})$ is the electric conductivity tensor, $\hat{\gamma}(\mathbf{H})$ the tensor of the acousto-electric current coefficients, $\hat{\eta}(\mathbf{H})$ the resistivity tensor, and $\hat{\beta}(\mathbf{H})$ the tensor of the acousto electric field coefficients. In the isotropic case we can write in expanded form*

$$\mathbf{E} = \eta\mathbf{j} + \eta_1[\mathbf{jH}] + \eta_2\mathbf{H}(\mathbf{jH}) + \beta\mathbf{S} + \beta_1[\mathbf{SH}] + \beta_2\mathbf{H}(\mathbf{SH}) + e^{-1}\nabla\zeta,$$

where

$$\eta = \frac{\sigma}{\sigma^2 + (\sigma_1 H)^2}, \quad \eta_1 = -\frac{\sigma_1}{\sigma^2 + (\sigma_1 H)^2}, \quad \eta_2 = \frac{\sigma_1^2 - \sigma\sigma_2}{[\sigma^2 + (\sigma_1 H)^2][\sigma + \sigma_2 H^2]}.$$

In the absence of a magnetic field[3] we have

$$\beta \approx \beta_0 = 1/nels,$$

where s and l are the velocity and characteristic absorption length of the sound, and n is the concentration of the sound-absorbing carriers.

When $H \neq 0$, the corresponding coefficients can be estimated by means of interpolation formulas that are valid when $\mu H/c \ll 1$ and $\mu H/c \gg 1$ (μ—carrier mobility, c—velocity of light)[4]

$$\beta = \frac{\beta_0}{1 + (\mu H/c)^2}, \quad \beta_1 = \beta\frac{\mu}{c}, \quad \beta_2 = \beta\left(\frac{\mu}{c}\right)^2.$$

In the presence of an external magnetic field H_0 the stationary acoustic electric field in an isotropic conductor is of the form (the frequency and the wave vector of the sound are much larger than the frequency and wave vector of the produced oscillations, so that we can confine ourselves to the value of the electric field averaged over the period and over the wavelength of the sound):

$$\mathbf{E}_0 = \beta\mathbf{S} + \beta_1[\mathbf{SH}_0] + \beta_2\mathbf{H}_0(\mathbf{SH}_0). \quad (1.1)$$

In the presence of a weak perturbation of the electric and magnetic fields **E′** and **H′** we have

$$\mathbf{E}' = \eta\mathbf{j}' + \eta_1[\mathbf{j}'\mathbf{H}_0] + \eta_2 H_0(\mathbf{j}'\mathbf{H}_0) + 2\frac{\partial\beta}{\partial H^2}(\mathbf{H}_0\mathbf{H}')\mathbf{S}$$
$$+ \beta_1[\mathbf{SH}'] + 2\frac{\partial\beta_1}{\partial H^2}(\mathbf{H}_0\mathbf{H}')[\mathbf{SH}_0] + \beta_2\mathbf{H}_0(\mathbf{SH}')$$

*$[\mathbf{jH}] \equiv \mathbf{j}\times\mathbf{H}$.

$$+\beta_2\mathbf{H}'(\mathbf{SH}_0)+2\frac{\partial\beta_2}{\partial H^2}(\mathbf{H}_0\mathbf{H}')(\mathbf{SH}_0)\mathbf{H}_0+\frac{1}{e}\nabla\zeta'. \quad (1.2)$$

Using Maxwell's equations

$$\mathrm{rot}\,\mathbf{E}'=-\frac{1}{c}\frac{\partial\mathbf{H}'}{\partial t},\quad \mathrm{rot}\,\mathbf{H}'=\frac{4\pi}{c}\mathbf{j}',\quad \mathrm{div}\,\mathbf{H}'=0, \quad (1.3)$$

in which we neglect the displacement current compared with the conduction current (we shall show that the frequency ω is much smaller than the conductivity σ), we arrive at a dispersion equation (assuming that $\mathbf{E}'$, $\mathbf{H}'$, $\mathbf{j}' \sim \exp\{i(\mathbf{k}\cdot\mathbf{r}-\omega t)\}$). In a weak magnetic field $\mu H_0/c \ll 1$ (μ—carrier mobility) the dispersion equation is

$$\omega=-c\beta_1(\mathbf{kS})-i\nu k^2\pm$$
$$\pm\frac{1}{2}c\beta_2\left\{(\mathbf{k}[\mathbf{H}_0\mathbf{S}])^2-4(\mathbf{H}_0\mathbf{S})(\mathbf{kS})\left[\frac{2(\mathbf{H}_0\mathbf{S})(\mathbf{kS})}{S^2}-(\mathbf{kH}_0)\right]\right\}^{1/2}, \quad (1.4)$$

where $\nu=\eta c^2/4\pi$. We take into account here the fact that when $\mu H_0/c \ll 1$ we have

$$\beta_2 H_0^2 \approx \beta_1 H_0\frac{\mu H_0}{c}\approx\beta_0\left(\frac{\mu H_0}{c}\right)^2,$$

where $\beta + \beta_2 H^2 = \beta_0$; therefore $\partial\beta/\partial H^2 = -\beta_2$.

In the absence of an external magnetic field we have

$$\omega=-c\beta_1(\mathbf{kS})-i\nu k^2 \quad (1.5)$$

and the damping of waves with frequency (1.5) is small if

$$S \gg S_1 = ck/4\pi\sigma\beta_1, \quad (1.6)$$

i.e., if $k \ll 4\pi\sigma\beta_1 S/c$. In this case waves of a new type can propagate in the crystal, which we shall call acoustomagnetic. Here

$$\mathrm{Re}\,\omega \ll \omega_{max}=4\pi\sigma(\beta_1 S)^2.$$

Substituting β_1, we obtain

$$\omega_{max}=4\pi\mu^3 S^2/nl^2s^2ec^2.$$

An estimate of ω_{max} will be presented below.

The expression under the square root in (1.4) may be negative at certain angles between $\mathbf{k}$, $\mathbf{H}_0$, and $\mathbf{S}$; its absolute magnitude is maximal when $\mathbf{k}\parallel\mathbf{H}_0\parallel\mathbf{S}$. Then

$$\omega=-c\beta_1(\mathbf{kS})-i\nu k^2\pm ic\beta_2 k(\mathbf{H}_0\mathbf{S}). \quad (1.7)$$

Growth of the waves occurs when

$$\beta_2(\mathbf{H}_0\mathbf{S})=\nu k. \quad (1.8)$$

We now consider the case $\mu H_0/c \gg 1$. In the limiting case when the maximal acoustomagnetic term is much smaller than the Hall term (which is much larger than the remaining galvanic terms if the electron and hole densities n_- and n_+ are not equal):

$$\beta_2 H'(\mathbf{SH}_0)+\beta_2 H_0(\mathbf{SH}')+2\frac{\partial\beta_2}{\partial H^2}(\mathbf{H}_0\mathbf{H}')(\mathbf{SH}_0)H_0 \ll \eta_1[\mathbf{j}'\mathbf{H}_0], \quad (1.9)$$

i.e., when

$$H \gg H_1 = 4\pi\beta_0/\eta_1 kc,$$

the dispersion equation takes the form

$$\omega=\pm ck\left\{\frac{c\eta_1}{4\pi}(\mathbf{kH}_0)-i\beta_2\left[\frac{3}{2}(\mathbf{H}_0\mathbf{S})-\frac{1}{2k^2}(\mathbf{kH}_0)(\mathbf{kS})+\frac{1}{k^2}[\mathbf{kH}_0]^2(\mathbf{H}_0\mathbf{S})\frac{\partial}{\partial H^2}(\ln\beta_2)\right]\right\}+\frac{1}{2}c\beta_2(\mathbf{k}[\mathbf{H}_0\mathbf{S}])-i\nu k^2. \quad (1.10)$$

Formula (1.10) again leads to weakly damped or weakly growing waves (the imaginary part of the frequency is much larger than the real one). As before, the increment is maximal when $\mathbf{k}\parallel\mathbf{H}_0\parallel\mathbf{S}$, and growth occurs when

$$\beta_2(\mathbf{H}_0\mathbf{S}) > \eta ck/4\pi. \quad (1.11)$$

Expressions (1.11) and (1.9) are compatible if $n_- \neq n_+$.

Let us proceed to a case when an inequality opposite to (1.9) is satisfied. This occurs always in conductors in which $n_- = n_+$, provided that $S \gg S_1$; on the other hand, if the densities are not equal, then the required condition is $H_0 < H_1$. In this case the dispersion equation

$$\omega={}^1\!/_2 c\beta_2(\mathbf{k}[\mathbf{H}_0\mathbf{S}])-i\nu k^2\pm{}^1\!/_2 c\beta_2\{(\mathbf{k}[\mathbf{H}_0\mathbf{S}])^2-4(\mathbf{H}_0\mathbf{S})(\mathbf{kH}_0)[2H_0^{-2}(\mathbf{kH}_0)(\mathbf{H}_0\mathbf{S})-(\mathbf{kS})]\}^{1/2} \quad (1.12)$$

again leads to a maximum increment at $\mathbf{k}\parallel\mathbf{H}_0\parallel\mathbf{S}$ and to the growth condition (1.11), but, unlike the preceding case, the growth of the magnetic field is aperiodic when $\mathbf{k}\parallel\mathbf{H}_0\parallel\mathbf{S}$.

An intuitive meaning of the growth obtained by us for the acoustomagnetic waves can be understood as follows. (We are considering the case $\mathbf{k}\parallel\mathbf{H}_0\parallel\mathbf{S}$.) In expression (1.2) for the electric field, the term of importance for the growth is $\beta_2\mathbf{H}'(\mathbf{S}\cdot\mathbf{H}_0)$, which must predominate over the term $\eta\mathbf{j}'$, which causes damping, in order for the instability to occur. Discarding the latter, we see that the electric field consists of three terms, of which two govern the real part of the frequency and are therefore of no interest to us at the present. Accurate to these terms, the electric field $\mathbf{E}'$ of the wave thus turns out to be proportional to its magnetic field $\mathbf{H}'$, and curl $\mathbf{E}' \sim \partial\mathbf{H}'/\partial t$ turns out to be proportional to curl $\mathbf{H}'$, by virtue of (1.2). If a circularly polarized fluctuation of the magnetic field is produced, then curl $\mathbf{H}' \sim \mathbf{H}'$ in this fluctuation, and therefore $\partial\mathbf{H}'/\partial t \sim \mathbf{H}'$. Consequently, the magnetic field of the fluctuation will vary exponentially in time, and the sign of this variation will depend on the direction of the circular polarization in such a manner that one of the circularly polarized fluctuations should grow.

2. ACOUSTOMAGNETIC WAVES IN ANISOTROPIC CONDUCTORS

In low-symmetry crystals, the situation is essentially different from the case of isotropic conductors. In these crystals, the state at which there is a sufficiently large acoustic flux is stable even in the absence of an external magnetic field. Such a situation can be realized in crystals of trigonal syngony (Bi, Sb). The sound flux should exceed in magnitude a critical value of the order of S_1. The energy pumping necessary for the occurrence and amplification of these waves is effected as a result of the presence of the sound flux.

If a fluctuation vibrational current $\mathbf{j}'$ and the magnetic field $\mathbf{H}'$ associated with it are produced in the crystal in the presence of the sound flux, then

$$E_i'=\eta_{ik}j_k'+\beta_{ikl}S_k H_l'+\frac{1}{e}\frac{\partial\zeta'}{\partial x_i}. \quad (2.1)$$

Equation (2.1) together with Maxwell's equations (1.3) leads to the dispersion relation (if $S \gg S_1$, when the term $\eta_{ik}j_k'$ can be neglected)

$$\omega = \Omega \pm \sqrt{a_{ij}k_i k_j}, \tag{2.2}$$

where

$$4a_{xx} = [(\beta_{yiz} + \beta_{ziy})S_i]^2 - 4[(\beta_{yiy})S_i][\beta_{zkz}S_k],$$
$$4a_{xy} = -2[(\beta_{yiz} + \beta_{ziy})S_i][(\beta_{xkz} + \beta_{zkx})S_k] + 4[(\beta_{xiy} + \beta_{yix})S_i][\beta_{zkz}S_k],$$
$$2\Omega = k_x[(\beta_{ziy} - \beta_{yiz})S_i] + \ldots, \quad i, k = x, y, z. \tag{2.3}$$

The remaining components a_{ij} and the remaining two terms for Ω are obtained by cyclic permutation of the indices. The instability condition requires that one of the two frequencies (2.2) have a positive imaginary part, i.e., that $a_{ij} k_i k_j < 0$.

It is easy to see that the only nonvanishing components β_{ikl} for hexagonal and tetragonal crystals are those with three different indices, for example β_{xyz} (z—principal axis, the x axis is directed along a twofold axis or is perpendicular to a symmetry plane passing through the z axis), with

$$\beta_{xzy} + \beta_{yzx} = 0. \tag{2.4}$$

It follows therefore that

$$4a_{xx} = [(\beta_{yiz} + \beta_{ziy})S_i]^2, \quad a_{xz} = a_{yz} = 0, \quad a_{ij}k_i k_j \geqslant 0.$$

For trigonal crystals (Bi, Sb), other nonzero components besides those with three different indices are

$$\beta_{xxx} = -\beta_{xyy} = -\beta_{yxy} = -\beta_{yyx}.$$

The quantity $a_{ij} k_i k_j$ is not essentially positive. The growth cannot occur at all directions of S. It is possible only if the flux S is not perpendicular to the twofold axes. In a crystal with rhombic syngony, the growth is possible if the flux is not perpendicular to any of the crystal axes.

The physical mechanism of the growth of the fluctuations can be understood here, too. Applying the curl operation to (2.1), we obtain

$$\partial H_i' / \partial t \sim i\delta_{imn}\beta_{nkl}S_k k_m H_l',$$

which can obviously lead, at definite directions of the vectors **S** and **k**, to an increase of the field **H'** with time (here δ_{ikl} is a unit antisymmetrical tensor).

3. NUMERICAL ESTIMATES

Let us estimate the value of S_1. Since

$$\beta_1 = \mu / cnesl$$

(see [3]), we get in accordance with (1.6)

$$S_1 \approx \left(\frac{c}{\mu}\right)^2 \frac{kls}{4\pi} \tag{3.1}$$

In bismuth, at a temperature T = 1°K and a sound frequency $\omega_S \approx 10^7$ Hz, the absorption length is l = 5 cm.[5] Assuming that $\mu = 2 \times 10^{10}$ absolute units, and the transverse dimensions of the body are L_0 = 0.3 cm ($k \geq \pi/L_0$), we obtain $S_1 \approx 0.2$ W/cm². Under the conditions indicated above and at S = 1 W/cm², we get $\omega_{max} = 10^5$ Hz.

$S_1 \sim \mu^{-2}$ and is therefore realizable only below 8–10°K. In bismuth with residual mobility $\mu = 10^{10}$ absolute units,[6] if the same absorption length l = 5 cm can be realized, then S_1 will be practically the same.

4. IMPEDANCE OF A COIL WITH A CORE IN WHICH THERE ARE ACOUSTOMAGNETIC WAVES

To observe acoustomagnetic waves and their instability, it is most convenient to investigate the impedance of a circuit in which these waves exist. Let us take an inductance coil with a rectangular cross section, the height of which (z axis) is much larger than the width, and the width is much larger than the thickness (x axis). We place inside the coil a core in the form of a plate, in which the height h, the width 2a, and the thickness 2d satisfy similar conditions. We produce a sound flux in the plate. Its presence can greatly change the spectrum of the electromagnetic oscillations in the plate, and consequently the impedance in the coil circuit will have singularities. We assume the plate to be isotropic (this is admissible in the case of Bi and Sb if the sound flux and the external magnetic field are directed along the trigonal axis). Let us consider the case when the sound flux and the external magnetic field are directed along x. Owing to the inequality $h \gg 2a \gg 2d$, the problem can be regarded as one dimensional. If the generator produces in the coil an alternating current of magnitude I, then the boundary conditions x = ± d are given by

$$H_z' = 4\pi NI / c, \quad H_y' = 0$$

(N is the number of turns per cm). In a coil

$$H_z' = \frac{2\pi}{c}NI\,\{[e^{ik_1x}(e^{ik_2d} - e^{-ik_2d}) - e^{ik_2x}(e^{ik_1d} - e^{-ik_1d})] \times [e^{i(k_2-k_1)d} - e^{i(k_2-k_1)d}]^{-1} + [e^{ik_3x}(e^{ik_4d} - e^{-ik_4d}) - e^{ik_4x}(e^{ik_3d} - e^{-ik_3d})][e^{i(k_4-k_3)d} - e^{-i(k_4-k_3)d}]^{-1}\}, \tag{4.1}$$

where k_1 and k_2 are the roots of the dispersion equation

$$\omega + k(u_1 - i\varepsilon u_2) + \nu k^2(i + \varepsilon\gamma) = 0 \tag{4.2}$$

at $\varepsilon = 1$; k_3 and k_4 are the same at $\varepsilon = -1$. Here

$$u_1 = |c\beta_1 S|, \quad u_2 = c\beta_2(\mathbf{H}_0\mathbf{S}), \quad \gamma = \eta_1 H_0 / \eta.$$

It follows from (4.2) that the sound flux greatly influences the spectrum of the oscillations in the plate if

$$\omega \ll \frac{|u_1 - i\varepsilon u_2|^2}{\nu\sqrt{1+\gamma^2}} \sim S^2. \tag{4.3}$$

In the case of a weak external magnetic field, for low frequencies satisfying (4.3) and for a sufficiently large sample thickness, $\exp\{-u_1 d/\nu\} \ll 1$, we have

$$Z(\omega) = \frac{8\pi}{c^2}N^2 ahu_1\left[1 - \exp\left(-\frac{2\omega^2}{u_1^3}\nu d\right)\mathrm{ch}\left(\frac{2\omega u_2}{u_1^2}d\right) \times \cos\frac{2\omega d}{u_1} - i\exp\left(-\frac{2\omega^2}{u_1^3}\nu d\right)\mathrm{ch}\left(\frac{2\omega u_2}{u_1^2}d\right)\sin\frac{2\omega d}{u_1}\right]. \tag{4.4}$$

When $H_0 = 0$ we have

$$Z(\omega) = \frac{8\pi}{c^2}N^2 hau_1\left[1 - \exp\left(-\frac{2\omega^2}{u_1^3}\nu d\right)\cos\frac{2\omega d}{u_1} - i\exp\left(-\frac{2\omega^2}{u_1^3}\nu d\right)\sin\frac{2\omega d}{u_1}\right]. \tag{4.5}$$

It is seen from (4.5) that the reactance oscillates as a function of the frequency, reversing sign at

$$\omega = \frac{m\pi}{2}\left|\frac{c\beta_1 S}{d}\right| \quad (m = 1, 2 \ldots).$$

The active resistance is positive at all frequencies, but oscillates as a function of the frequency, with the same period as the reactance.

When $H_0 \neq 0$, the active resistance is positive if $\omega > |u_1 u_2/\nu|$. The amplitude of the oscillations exceeds the mean value, i.e., the active resistance reverses sign if $\omega < |u_1 u_2/\nu|$. It is seen from (4.4) that the amplitude of the impedance oscillations has a maximum at $\omega = |u_1 u_2/2\nu|$.

5. GENERATOR OF OSCILLATIONS EXCITED BY ACOUSTOMAGNETIC WAVES

The electric current produced by the acoustomagnetic waves can be delivered to an external circuit. To realize such a generator we construct a circuit with a coil having a core that possesses negative active resistance, and a load impedance $Z_0(\omega)$ which includes the active resistance of the coil and that part of its reactance which is connected with the magnetic flux not passing through the plate. Assume that at the instant of time $t = 0$ fluctuations of the magnetic field $\mathbf{H}'$ are produced in the core. The component H_z' of this fluctuation produces a magnetic flux which induces a current in the circuit of the coil; this current is determined from the condition

$$IZ_0 = -\frac{N}{c}\frac{d}{dt}\int H_z'\,dx\,dy\,dz. \tag{5.1}$$

The field $\mathbf{H}'$ satisfies the equation

$$\left[\frac{\partial}{\partial t} - u_1\frac{\partial}{\partial x} - \nu\frac{\partial^2}{\partial x^2} + i\varepsilon\left(u_2\frac{\partial}{\partial x} + \gamma\nu\frac{\partial^2}{\partial x^2}\right)\right][H_y'(x,t) + i\varepsilon H_z'(x,t)] = 0. \tag{5.2}$$

with boundary conditions $H_z' = 4\pi NI/c$, $H_y' = 0$ at $x = \pm d$.

Let us apply the unilateral Fourier transformation in time

$$H'(x,t) = \int_{-\infty+i\sigma}^{\infty+i\sigma} H'(x,\omega)e^{-i\omega t}\,d\omega \qquad (\sigma \geqslant 0),$$

$$\left[i\omega + (u_1 - i\varepsilon u_2)\frac{\partial}{\partial x} + \nu(1 - i\varepsilon\gamma)\frac{\partial^2}{\partial x^2}\right][H_y'(x,\omega) + i\varepsilon H_z'(x\ \omega)] = -[H_y'(x,0) + i\varepsilon H_z'(x,0)]. \tag{5.3}$$

The solution of (5.3) is the sum of the free solution, which satisfies equation (5.3) with zero right-hand side and with boundary conditions, and the forced solution, for which we require that it vanish on the boundary. The forced solution can be expanded in the eigenfunctions of the systems ψ_{l+} and ψ_{l-}, which correspond to $\varepsilon = \pm 1$ and satisfy the equations

$$\left[i\omega_{l\pm} + (u_1 \mp iu_2)\frac{\partial}{\partial x} + \nu(i + \gamma\varepsilon)\frac{\partial^2}{\partial x^2}\right]\psi_{l\pm}(x,\omega_{l\pm}) = 0 \tag{5.4}$$

with zero boundary conditions $\psi_l(\pm d) = 0$.

Let us expand $H_y'(x, 0) \pm iH_z'(x, 0)$ in the eigenfunctions of the system

$$H_y'(x,0) \pm iH_z'(x,0) = -\sum_l f_{l\pm}(\omega_{l\pm})\psi_{l\pm}. \tag{5.5}$$

We can then readily see that

$$I(t) = -\frac{i}{2\pi c}Nah\sum_l \int_{-\infty+i\sigma}^{\infty+i\sigma} d\omega\frac{\omega e^{-i\omega t}}{Z(\omega) + Z_0(\omega)}\left[\frac{f_{l+}}{\omega - \omega_{l+}}\int_{-d}^{d}\psi_{l+}\,dx - \frac{f_{l-}}{\omega - \omega_{l-}}\int_{-d}^{d}\psi_{l-}\,dx\right]. \tag{5.6}$$

The contour of integration with respect to ω can be closed by means of a semicircle of infinitely large radius in the lower half-plane. The integral then reduces to residues at the points $\omega = \omega_{l\pm}$ and at the zeros of total impedance $Z(\omega) + Z_0(\omega)$.

If Re $Z > 0$, then the roots of $Z(\omega) + Z_0(\omega) = 0$ lie in the lower half plane, corresponding to an exponential damping of the fluctuations. If Re $Z < 0$ in some frequency region, then it is always possible to choose the load in such a way that the root of $Z_0 + Z = 0$ lies in the upper half plane. The contribution of the similar poles in (5.6) corresponds to growing fluctuations. These frequencies are characterized by a dependence on the load impedance. When R_{cr} + Re $Z \geq 0$, generation vanishes, and when Re $Z_0 > R_{cr}$ the generation will no longer be observed.

We now consider the contribution made to the current by the frequencies $\omega = \omega_{l\pm}$. The eigenfunctions are of the form

$$\psi_{l\pm} = \text{const}\,(e^{ik_{1l\pm}x} + e^{ik_{2l\pm}x}),$$

where

$$k_{l\pm} = \frac{\pi l}{2d} - \frac{u_1 \mp iu_2}{2\nu(i \pm \gamma)},$$

$$k_{2l\pm} = -\frac{\pi l}{2d} - \frac{u_1 \mp iu_2}{2\nu(i \pm \gamma)},$$

$$\omega_{l\pm} = -\nu\left(\frac{\pi l}{2d}\right)^2(i \pm \gamma) - \frac{1}{4\nu(1+\gamma^2)}[i\alpha u_1^2 + \varepsilon(2u_1u_2 + \gamma u_2^2 - \gamma u_1^2)],$$

with (5.7)

$$\alpha = 1 - \left(\frac{\beta_2 H_0}{\beta_1}\right)^2 + 2\frac{\beta_2 H_0}{\beta_1}\frac{\eta_1 H_0}{\eta}.$$

Im $\omega_l < 0$ when $\alpha > 0$ (it is easy to show that this corresponds to convective instability). On the other hand, in the region of absolute instability ($\alpha < 0$) we have Im $\omega_l > 0$. By increasing the magnetic field to a value such that the condition $\alpha < 0$ is satisfied, we go over into the region of absolute instability, where oscillations whose frequency depends on S and H_0, but not on the load, arise in addition to the frequencies that depend on the load.

6. NONLINEAR THEORY

The nonlinear theory of acoustomagnetic waves can be constructed in analogy with [7]. Let us formulate the main results.

1. At low supercriticality $|(S - S_{cr})/S| \ll 1$, a nonlinear theory can be constructed for an acoustomagnetic generator in which random fluctuations of the magnetic field, occurring in the presence of an external stationary sound flux and an external magnetic field, become amplified and produce standing acoustomagnetic waves in the core. These waves generate in the coils a current that becomes amplified and emerges to the outside. The excitation regime is soft is $\partial \ln \beta_1/\partial H^2 > 0$. The amplitude of the oscillations of the alternating magnetic field is

$$H_{\text{stat}} = H_0\left|\frac{S - S_{\text{cr}}}{S}\right|^{1/2}. \tag{6.1}$$

2. Stationary traveling acoustomagnetic waves can be produced in an inductance-coil core through which a sound flux S is produced parallel to the coil axis

($\mathbf{S} \parallel \mathbf{H}_0$). These waves can be produced with the aid of mutually perpendicular transverse coils on one end of the core, in which the oscillating fields are shifted 90° in phase. This induces a circularly polarized traveling wave, which can be observed by placing an analogous system on the other end of the core. The excitation regime is soft if

$$\frac{\partial \ln \beta_2}{\partial H^2} + \frac{H^2}{\eta} \frac{\partial \eta_2}{\partial H^2} < 0. \tag{6.2}$$

To realize the stationary regime it is necessary to have

$$S > S_{\text{cr}} = \frac{ck\eta(H_0)}{4\pi\beta_2(H_0)H_0}, \tag{6.3}$$

where $\mathbf{k}$ is the wave vector, whose value is fixed by the boundary conditions.

The frequency of the waves

$$\omega = -c(kS)\left(\beta_1 + \frac{\eta_1 ck}{4\pi S}\right) \tag{6.4}$$

is of the order of several kHz. The magnetic field H in the stationary state is determined by the equation

$$(\mathbf{H}_0\mathbf{S}) = \frac{ck[\eta(H) + \eta_2(H)H_\perp^2]}{4\pi\beta_2(H)} \tag{6.5}$$

where $H_\perp$ is the amplitude of the alternating magnetic field in the core.

[1] L. É. Gurevich and B. L. Gel'mont, Zh. Eksp. Teor. Fiz. 47, 1806 (1964) [Sov. Phys.-JETP 20, 1217 (1965)].

[2] L. É. Gurevich and E. F. Shender, Fiz. Tverd. Tela 10, 625 (1968) [Sov. Phys.-Solid State 10, 491 (1968)].

[3] G. Weinreich, T. M. Sanders, and H. G. White, Phys. Rev. 114, 303 (1959).

[4] Yu. M. Gal'perin and V. D. Kogan, Fiz. Tverd. Tela 10, 2037 (1968) [Sov. Phys.-Solid State 10, 1600 (1969)].

[5] D. H. Reneker, Phys. Rev. 115, 303 (1959).

[6] J. Long, C. Greiner, and J. Reynolds, Phys. Rev. 140, 1A, 187 (1965).

[7] L. É. Gurevich and B. L. Gel'mont, Zh. Eksp. Teor. Fiz. 51, 183 (1966) [Sov. Phys.-JETP 24, 124 (1967)].

Translated by J. G. Adashko
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