INTERACTION OF AN ATOM WITH A STRONG ELECTROMAGNETIC FIELD WITH THE RECOIL EFFECT TAKEN INTO CONSIDERATION

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The problem of the change in the velocity distribution function of atoms interacting with an electromagnetic field and with the recoil effect taken into account is considered by using the method of Wigner quantum distribution functions. The cases of traveling and standing monochromatic waves are discussed. In the resonance approximation the velocity distributions contain characteristic "dips" (in the more populated level) and "peaks" (in the less populated level) which are shifted by an amount equal to twice the recoil shift and having widths determined by the natural line width. As a consequence, the spectral characteristics of a gas laser are changed: The Lamb "dip" and the Lisitsyn-Chebotaev "peak" split into two components whose relative weight is determined by the ratio of the lifetimes of atoms in the combining levels. In addition, it is shown that for a small relative difference in the populations of the levels, the maximum in the unsaturated absorption (or amplification) line is shifted by an amount which appreciably exceeds the recoil shift. Numerical estimates confirm that the indicated effects should be experimentally observable in the optical region of the spectrum and should have a substantial effect on the properties of lasers with a highly-stabilized frequency.

1. INTRODUCTION

THE recoil effect associated with the absorption or emission of a photon consists, as is well-known, in the fact that the corresponding lines turn out to be shifted relative to the Bohr frequency ω_{mn}. The magnitude of the shift is given by the formula

$$\pm\delta = \pm k^2\hbar / 2m = \pm 1.25\cdot 10^6 \lambda^{-2} M^{-1} \text{ [sec}^{-1}\text{]}. \quad (1.1)$$

Here m and M denote the mass and atomic weight of the atom, k is the wave vector, λ is the wavelength expressed in microns. In the x-ray region and for γ rays, the shift δ turns out to be larger than the line widths, and the recoil effect can be easily observed. In the optical and near infrared regions of the spectrum ($\lambda \sim 1\mu$) for atoms with M of the order of a few multiples of ten, we have $\delta \sim 10^5$ sec^{-1}, which is not only much smaller than the Doppler widths ($k\bar{v} = \Delta\omega_D \sim 10^{10}$ sec^{-1}) but in many cases is even smaller than the natural line width. Therefore, it was concluded that the recoil effect should not play any appreciable role in optical spectroscopy.

The situation is altered if the phenomenon of saturation, which arises in strong electromagnetic fields, is taken into consideration. In gaseous systems the Doppler-broadened line acquires a fine structure with a characteristic width of the order of the natural line width Γ. In this situation, manifestation of the recoil effect depends on the ratio δ/Γ. For atomic lines $\Gamma \sim 10^7$ to 10^8 sec^{-1} and hence $\delta \ll \Gamma$. However, in molecular spectra the width Γ may be much smaller (for low pressures in the gas, $\Gamma \sim 10^3$ to 10^5 sec^{-1}), and it is impossible to neglect the recoil effect.

The problem under discussion acquires special significance in connection with the discovery by Lisitsyn and Chebotaev[1,2] of the interesting properties of gas lasers containing an absorption cell. It was shown[1-5] experimentally and theoretically that a graph of the power of such a laser as a function of the frequency has a "peak" whose width is determined by the natural line width of the absorbing gas. This phenomenon has already been utilized in order to stabilize the frequency of a laser,[2] and in the future may acquire great value in this regard. One can apparently select conditions in the absorption cell so that the natural line width amounts to $\Gamma \sim 10^4$ to 10^5 sec^{-1}.[1-3] In accordance with what was said above, here the recoil effect may turn out to be not only quite appreciable, but it may even determine the nature of the phenomenon.

A general theory of the interaction of moving atoms with a strong electromagnetic field, taking the finiteness of the photon momentum into consideration, is developed in Sec. 2 of the present article. The cases of traveling and standing monochromatic waves are considered in Secs. 3 and 4. In Sec. 5 the results which have been obtained are used to analyze the properties of a gas laser.

2. FUNDAMENTAL EQUATIONS

Within the framework of the quasiclassical theory developed below, taking the momentum of the field into account means the necessity of quantization of the translational motion of the center of mass of an atom. Here the method of Wigner quantum distribution functions turns out to be very convenient. However, in contrast to well-known cases of the application of this method (see, for example, [6]), in the problem which we are considering, it is advisable to explicitly single out the states of an optical electron in an atom.

Let $\psi_m(x)$ denote the wave functions of the electron in time-independent states. The elements $\rho_{mn}(\mathbf{r}, \mathbf{r}')$ of the density matrix depend on the coordinates $\mathbf{r}$, $\mathbf{r}'$ of the atom's center of mass. If relaxation processes are taken into account by introducing an attenuation constant $\Gamma_{mn} = \Gamma^*_{nm}$, then it is not difficult to obtain the following system of equations:

$$i\hbar\left(\frac{\partial}{\partial t}-\frac{i\hbar}{2m}(\nabla^2-\nabla'^2)+\Gamma_{mn}\right)\rho_{mn}(t,\mathbf{r},\mathbf{r}')$$
$$=\sum_l\{V_{ml}(t,\mathbf{r})e^{i\omega_{ml}t}\rho_{ln}(t,\mathbf{r},\mathbf{r}')-\rho_{ml}(t,\mathbf{r},\mathbf{r}')V_{ln}(t,\mathbf{r}')e^{i\omega_{ln}t}\}$$
$$+\delta_{mn}q_m(\mathbf{p}_0)\frac{1}{V}\int e^{-i(\mathbf{r}-\mathbf{r}')\mathbf{p}/\hbar}\Delta(\mathbf{p}-\mathbf{p}_0)\,d\mathbf{p}; \tag{2.1}$$

$$V_{mn}(t,\mathbf{r})=-\frac{e}{\mu c}\mathbf{p}_{mn}\mathbf{A}(t,\mathbf{r}),\quad \mathbf{p}_{mn}=\int\psi_m^*(x)\mathbf{P}\psi_n(x)\,dx, \tag{2.2}$$

where μ is the reduced mass of an electron, **P** and p are the momenta of the electron and of the atom center of mass, $\mathbf{A}(t, \mathbf{r})$ is the vector potential of the field. The last term on the right hand side of Eq. (2.1) describes the excitation of the m-th state of an electron with momentum distribution function $\Delta(\mathbf{p}-\mathbf{p}_0)$. The centers of the packets have a distribution given by $q_m(\mathbf{p}_0)$, which we shall assume to be Maxwellian:

$$q_m(\mathbf{p}_0)=Q_m(\sqrt{\pi}\,\bar{p})^{-3}\exp\{-\mathbf{p}_0^2/\bar{p}^2\},\quad \bar{p}=\sqrt{2kTm}. \tag{2.3}$$

Now let us introduce the Wigner functions

$$f_{mn}(t,\mathbf{p},\mathbf{r})=\frac{1}{(2\pi\hbar)^3}\int e^{-i\mathbf{p}\mathbf{x}/\hbar}\rho_{mn}\left(t,\mathbf{r}+\frac{\mathbf{x}}{2},\mathbf{r}-\frac{\mathbf{x}}{2}\right)d\mathbf{x}, \tag{2.4}$$

for which a system of integro-differential equations follows from (2.1):

$$i\hbar\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma_{mn}\right)f_{mn}(t,\mathbf{p},\mathbf{r})=\delta_{mn}q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)$$
$$+\frac{1}{(2\pi\hbar)^3}\sum\Big\{e^{i\omega_{ml}}\int e^{-i(\mathbf{p}-\mathbf{q})\mathbf{x}/\hbar}V_{ml}\left(t,\mathbf{r}+\frac{\mathbf{x}}{2}\right)f_{ln}(t,\mathbf{q},\mathbf{r})\,d\mathbf{q}\,d\mathbf{x}$$
$$-e^{i\omega_{ln}t}\int e^{-i(\mathbf{p}-\mathbf{q})\mathbf{x}/\hbar}f_{ml}(t,\mathbf{q},\mathbf{r})V_{ln}\left(t,\mathbf{r}-\frac{\mathbf{x}}{2}\right)d\mathbf{q}\,d\mathbf{x}\Big\}. \tag{2.5}$$

If one goes to the limit $\hbar \to 0$ in the arguments of the exponentials, then Eqs. (2.5) give the usual quasiclassical equations when the field and the translational motion of the atoms are described classically. Quantization of the center-of-mass coordinates is most clearly expressed upon expansion of the field in terms of plane waves:

$$V_{mn}(t,\mathbf{r})=\int V_{mn}(t,\mathbf{k})e^{i\mathbf{k}\mathbf{r}}\,d\mathbf{k}. \tag{2.6}$$

Then (2.5) goes over into the following system of equations:

$$i\hbar\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma_{mn}\right)f_{mn}(t,\mathbf{p},\mathbf{r})=\delta_{mn}q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)$$
$$+\sum_l\Big\{e^{i\omega_{ml}t}\int e^{i\mathbf{k}\mathbf{r}}V_{ml}(t,\mathbf{k})f_{ln}\left(t,\mathbf{p}-\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)d\mathbf{k}$$
$$-e^{i\omega_{ln}t}\int e^{i\mathbf{k}\mathbf{r}}f_{ml}\left(t,\mathbf{p}+\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)V_{ln}(t,\mathbf{k})\,d\mathbf{k}\Big\}. \tag{2.7}$$

Thus, the electromagnetic field "connects" states differing in momentum by an amount proportional to the photon momentum $\hbar\mathbf{k}$.

The equations written down for the Wigner functions refer to the simplest model for relaxation. It is assumed, in particular, that during a collision process the momentum of the atom remains unchanged, but excitation of the system takes place incoherently. Generalization of the theory in this direction is quite possible and constitutes the subject of a separate communication. For the purposes of the present article (clarifying the role of the recoil effect), the adopted model is adequate.

3. EFFECT OF SATURATION IN A PLANE, MONOCHROMATIC, TRAVELING WAVE

Let us consider the simplest case of a traveling, monochromatic wave

$$\mathbf{A}(t,\mathbf{r})=\mathbf{A}_0\cos(\omega t-\mathbf{k}\mathbf{r}) \tag{3.1}$$

and let us apply the resonance approximation (the level widths, the energy of interaction with the field, and $\hbar(\omega-\omega_{mn})$ are much smaller than $\hbar\omega_{mn}$). Then in the system of equations (2.7) it is sufficient to retain only the terms with indices $l = m, n$. We shall assume that level m has a larger energy than level n so that $\omega_{mn} > 0$. Omitting the rapidly oscillating terms (with frequencies $\omega+\omega_{mn}$), from (2.7) we obtain

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma_m\right)f_{mm}(t,\mathbf{p},\mathbf{r})=q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)$$
$$-2\,\mathrm{Re}\left[iGe^{-i(\Omega t-\mathbf{k}\mathbf{r})}f_{nm}\left(t,\mathbf{p}-\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)\right]$$
$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma_n\right)f_{nn}(t,\mathbf{p},\mathbf{r})=q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)$$
$$+2\,\mathrm{Re}\left[iGe^{-i(\Omega t-\mathbf{k}\mathbf{r})}f_{nm}\left(t,\mathbf{p}+\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)\right],$$
$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma\right)f_{nm}(t,\mathbf{p},\mathbf{r})$$
$$=-iG^*e^{i(\Omega t-\mathbf{k}\mathbf{r})}\left\{f_{mm}\left(t,\mathbf{p}+\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)-f_{nn}\left(t,\mathbf{p}-\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)\right\}. \tag{3.2}$$

Here the following notation has been introduced:

$$G=-\frac{e}{2\mu c\hbar}\mathbf{p}_{mn}\mathbf{A}_0,\quad \Omega=\omega-\omega_{mn},$$
$$\Gamma\equiv\Gamma_{nm},\quad \Gamma_l\equiv\Gamma_{ll}\quad(l=m,n). \tag{3.3}$$

We shall regard the perturbation as homogeneous and constant in time. Then, obviously, one should seek $f_{nm}(t, \mathbf{p}, \mathbf{r})$ in the form

$$f_{nm}(t,\mathbf{p},\mathbf{r})=f(\mathbf{p})e^{i(\Omega t-\mathbf{k}\mathbf{r})}, \tag{3.4}$$

where the diagonal elements $f_{mm}(t, \mathbf{p}, \mathbf{r})$ and $f_{nn}(t, \mathbf{p}, \mathbf{r})$ turn out to be independent of **r** and t:

$$f_{mm}(t,\mathbf{p},\mathbf{r})=f_m(\mathbf{p}),\quad f_{nn}(t,\mathbf{p},\mathbf{r})=f_n(\mathbf{p}), \tag{3.5}$$

and Eqs. (3.2) reduce to a set of algebraic equations:

$$\Gamma_m f_m(\mathbf{p})=-2\,\mathrm{Re}\left[iGf\left(\mathbf{p}-\frac{\hbar}{2}\mathbf{k}\right)\right]+q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0),$$
$$\Gamma_n f_n(\mathbf{p})=2\,\mathrm{Re}\left[iGf\left(\mathbf{p}+\frac{\hbar}{2}\mathbf{k}\right)\right]+q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0),$$
$$\left[\Gamma+i\left(\Omega-\frac{\mathbf{p}}{m}\mathbf{k}\right)\right]f(\mathbf{p})=-iG^*\left\{f_m\left(\mathbf{p}+\frac{\hbar}{2}\mathbf{k}\right)-f_n\left(\mathbf{p}-\frac{\hbar}{2}\mathbf{k}\right)\right\}. \tag{3.6}$$

The solution of the system (3.6) can be found without difficulty

$$f_m(\mathbf{p})=\frac{q_m(\mathbf{p}_0)}{\Gamma_m}\Delta(\mathbf{p}-\mathbf{p}_0)-\frac{2|G|^2\Gamma}{\Gamma_m}\cdot\frac{\Gamma_m^{-1}q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)-\Gamma_n^{-1}q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\hbar\mathbf{k}-\mathbf{p}_0)}{\Gamma^2(1+\varkappa)+(\Omega+\delta-\eta)^2}$$
$$f_n(\mathbf{p})=\frac{q_n(\mathbf{p}_0)}{\Gamma_n}\Delta(\mathbf{p}-\mathbf{p}_0)+\frac{2|G|^2\Gamma}{\Gamma_n}\cdot\frac{\Gamma_m^{-1}q_m(\mathbf{p}_0)\Delta(\mathbf{p}_0+\hbar\mathbf{k}-\mathbf{p}_0)-\Gamma_n^{-1}q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)}{\Gamma^2(1+\varkappa)+(\Omega-\delta-\eta)^2} \tag{3.7}$$

$$f(\mathbf{p}) = -\frac{iG^*[\Gamma - i(\Omega-\eta)]}{\Gamma^2(1+\varkappa)+(\Omega-\eta)^2}\left\{\frac{q_m(\mathbf{p}_0)}{\Gamma_m}\Delta\left(\mathbf{p}+\frac{\hbar}{2}\mathbf{k}-\mathbf{p}_0\right)\right.$$
$$\left.-\frac{q_n(\mathbf{p}_0)}{\Gamma_n}\Delta\left(\mathbf{p}-\frac{\hbar}{2}\mathbf{k}-\mathbf{p}_0\right)\right\}, \qquad (3.8)$$

where

$$\eta = \mathbf{k}\frac{\mathbf{p}}{m}, \quad \delta = \frac{k^2\hbar}{2m}, \quad \varkappa = \frac{2|G|^2}{\Gamma}\left(\frac{1}{\Gamma_m}+\frac{1}{\Gamma_n}\right). \qquad (3.9)$$

With the aid of (3.7) one can determine the momentum distribution of the atoms in the states m and n. In order to do this it is necessary, according to general rules,[6] to integrate $f_{ll}(\mathbf{p})$ over the volume in which the atom is located and, in addition, to average with respect to $\mathbf{p}_0$:

$$\langle f_{ll}(\mathbf{p})\rangle = \int f_{ll}(t,\mathbf{p},\mathbf{r})\,d\mathbf{p}_0 d\mathbf{r}. \qquad (3.10)$$

We shall assume that integration of $q_l(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)$ with respect to $\mathbf{p}_0$ leads to an equilibrium distribution[1)] with the same temperature T. In the optical spectroscopy case of interest to us, $p = \sqrt{2mkT} \gg k\hbar$. Therefore, from (3.7) we obtain

$$\langle f_{mm}(\mathbf{p})\rangle = (\sqrt{\pi}\bar{p})^{-3}e^{-\mathbf{p}^2/\bar{p}^2}\left\{N_m - \frac{2|G|^2}{\Gamma_m}\Gamma\frac{N_m-N_n}{\Gamma^2(1+\varkappa)+(\Omega+\delta-\eta)^2}\right\}$$
$$\langle f_{nn}(\mathbf{p})\rangle = (\sqrt{\pi}\bar{p})^{-3}e^{-\mathbf{p}^2/\bar{p}^2}\left\{N_n + \frac{2|G|^2}{\Gamma_n}\Gamma\frac{N_m-N_n}{\Gamma^2(1+\varkappa)+(\Omega-\delta-\eta)^2}\right\}. \qquad (3.11)$$

Here the $N_l = Q_l/\Gamma_l$ denote the integral populations of the states $l = m, n$ in the absence of a field. If $N_m > N_n$ (population inversion), then the momentum distribution of the atoms in the state m contains a "dip" of width $\Gamma\sqrt{1+\kappa}$ (see Fig. 1). A "peak" of the same width appears in the state n. The formation of a "dip" is explained by the fact that, owing to the influence of the field, those atoms preferentially "leak out" whose velocities satisfy the energy and momentum conservation laws,

$$E_m + \frac{\mathbf{p}^2}{2m} = E_n + \frac{\mathbf{p}'^2}{2m} + \hbar\omega,$$
$$\mathbf{p}' = \mathbf{p} - \mathbf{k}\hbar,$$

from which we have

$$\eta_m = \mathbf{k}\mathbf{p}_m / m = \Omega + \delta.$$

These atoms supplement the number of atoms in state n with momenta

$$\eta_n = \mathbf{k}\frac{\mathbf{p}_m - \mathbf{k}\hbar}{m} = \Omega - \delta,$$

and a "peak," shifted with respect to the "dip" by an amount 2δ, appears in the distribution $\langle f_{nn}(\mathbf{p})\rangle$.

The considerations cited here differ from generally known arguments only in those aspects which are related to the recoil effect. Usually it is implicitly assumed that $\delta = 0$, and the centers of the "dip" and of the "peak" turn out to occur at the same velocity. The possibility of the appearance of a shift due to recoil depends on the value of the ratio $\delta/\Gamma\sqrt{1+\kappa}$, that is, both on the relaxation constants Γ_l, Γ, and on the amplitude of the field.

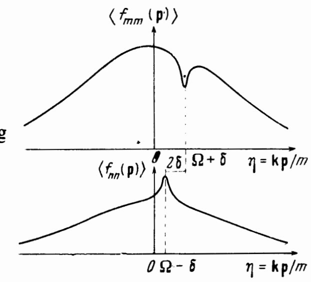

FIG. 1. Momentum distribution of the atoms in the case of a traveling monochromatic wave.

4. EFFECT OF SATURATION IN A STANDING MONOCHROMATIC WAVE

In the majority of optical lasers the adjustable steady-state field is a standing wave

$$\mathbf{A}(t,\mathbf{r}) = \mathbf{A}_0\cos\omega t\cos\mathbf{k}\mathbf{r}. \qquad (4.1)$$

In this case the system (2.7) has the form

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma_m\right)f_{mm}(t,\mathbf{p},\mathbf{r}) = q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)$$
$$\mathrm{Re}\left\{iGe^{-i\Omega t}\left[e^{i\mathbf{k}\mathbf{r}}f_{nm}\left(t,\mathbf{p}-\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)+e^{-i\mathbf{k}\mathbf{r}}f_{nm}\left(t,\mathbf{p}+\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)\right]\right\},$$
$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma_n\right)f_{nn}(t,\mathbf{p},\mathbf{r}) = q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)$$
$$+\,\mathrm{Re}\left\{iGe^{-i\Omega t}\left[e^{i\mathbf{k}\mathbf{r}}f_{nm}\left(t,\mathbf{p}+\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)+e^{-i\mathbf{k}\mathbf{r}}f_{nm}\left(t,\mathbf{p}-\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)\right]\right\},$$
$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\nabla+\Gamma\right)f_{nm}(t,\mathbf{p},\mathbf{r})$$
$$= -\frac{i}{2}Ge^{i\Omega t}\left\{e^{i\mathbf{k}\mathbf{r}}\left[f_{mm}\left(t,\mathbf{p}+\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)-f_{nn}\left(t,\mathbf{p}-\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)\right]\right.$$
$$\left.+\,e^{-i\mathbf{k}\mathbf{r}}\left[f_{mm}\left(t,\mathbf{p}-\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)-f_{nn}\left(t,\mathbf{p}+\frac{\hbar}{2}\mathbf{k},\mathbf{r}\right)\right]\right\}. \qquad (4.2)$$

The system (4.2) no longer has such a simple solution as it had in the case of a traveling wave. One of the reasons for this is that a standing wave creates a periodic spatial inhomogeneity in the medium. If the system of equations is solved by the method of successive approximations and attention is confined to the first-order corrections in $|G|^2$, then one can ignore the inhomogeneity of the medium[7] and seek a solution in the form

$$f_{ll}(t,\mathbf{p},\mathbf{r}) = f_l(\mathbf{p}), \quad l = m, n;$$
$$f_{nm}(t,\mathbf{p},\mathbf{r}) = f^+(\mathbf{p})e^{i(\Omega t-\mathbf{k}\mathbf{r})}+f^-(\mathbf{p})e^{i(\Omega t+\mathbf{k}\mathbf{r})}. \qquad (4.3)$$

In all remaining points we shall adhere to the same assumptions which were made in Sec. 3.

A system of algebraic equations for f_{ll}, f^+, and f^- follows from (4.2) and its approximate solution is given by

$$f_m(\mathbf{p}) = \frac{q_m(\mathbf{p}_0)}{\Gamma_m}\Delta(\mathbf{p}-\mathbf{p}_0) - \frac{|G|^2\Gamma}{2\Gamma_m}\cdot$$
$$\times\left\{\frac{\Gamma_m^{-1}q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)-\Gamma_n^{-1}q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{k}\hbar-\mathbf{p}_0)}{\Gamma^2+(\Omega+\delta-\eta)^2}\right.$$
$$\left.+\frac{\Gamma_m^{-1}q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)-\Gamma_n^{-1}q_n(\mathbf{p}_0)\Delta(\mathbf{p}+\mathbf{k}\hbar-\mathbf{p}_0)}{\Gamma^2+(\Omega+\delta+\eta)^2}\right\}.$$

$$f_n(\mathbf{p}) = \frac{q_n}{\Gamma_n}\Delta(\mathbf{p}-\mathbf{p}_0) + \frac{|G|^2\Gamma}{2\Gamma_n} \qquad (4.4)$$

[1)]In general a "warming up" of the gas may occur in connection with excitation of the atoms, for example, for excitation of atoms as a consequence of photodissociation of the molecules. However, for excitation by electronic impact the kinetic energy transferred to the atom is not large.

$$\times\left\{\frac{\Gamma_m^{-1}q_m(\mathbf{p}_0)\Delta(\mathbf{p}+\mathbf{k}\hbar-\mathbf{p}_0)-\Gamma_n^{-1}q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)}{\Gamma^2+(\Omega-\delta-\eta)^2}+\frac{\Gamma_m^{-1}q_m(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{k}\hbar-\mathbf{p}_0)-\Gamma_n^{-1}q_n(\mathbf{p}_0)\Delta(\mathbf{p}-\mathbf{p}_0)}{\Gamma^2+(\Omega-\delta+\eta)^2}\right\}$$

Averaging (4.4) with respect to $\mathbf{p}_0$ and $\mathbf{r}$ gives the following momentum distribution for the atoms:

$$\langle f_{mm}(\mathbf{p})\rangle=\frac{e^{-\mathbf{p}^2/\bar{p}^2}}{(\sqrt{\pi}\bar{p})^3}\times\left\{N_m-(N_m-N_n)\frac{2|G|^2}{\Gamma\Gamma_m}\left[\frac{\Gamma^2}{\Gamma^2+(\Omega+\delta-\eta)^2}+\frac{\Gamma^2}{\Gamma^2+(\Omega+\delta+\eta)^2}\right]\right\},$$

$$\langle f_{nn}(\mathbf{p})\rangle=\frac{e^{-\mathbf{p}^2/\bar{p}^2}}{(\sqrt{\pi}\bar{p})^3}\cdot\times\left\{N_n+(N_m-N_n)\frac{2|G|^2}{\Gamma\Gamma_n}\left[\frac{\Gamma^2}{\Gamma^2+(\Omega-\delta-\eta)^2}+\frac{\Gamma^2}{\Gamma^2+(\Omega-\delta+\eta)^2}\right]\right\}. \quad (4.5)$$

It is clear from Eq. (4.5) that a standing wave leads to the formation of two "dips" in $\langle f_{mm}(\mathbf{p})\rangle$ and two "peaks" in $\langle f_{nn}(\mathbf{p})\rangle$ (if $N_m > N_n$), which are shifted in pairs by an amount 2δ (see Fig. 2). An intuitive interpretation of these "dips" and "peaks" follows immediately from the analysis of Sec. 3. To the approximation used here, the two traveling waves, which form a standing wave, independently change the populations of the levels. The wave traveling along the axis $\eta = \mathbf{k}\cdot\mathbf{p}/m$ creates the right-hand "dip" and "peak," and the wave associated with $-\mathbf{k}$ creates the left-hand "dip" and "peak." All of this structure is described by the four resonance terms in Eq. (4.5) with maxima for $\eta = \pm(\Omega \pm \delta)$.

At certain frequencies the "dips" (or "peaks") may merge together. It is essential that such frequencies be different for "dips" and "peaks": The "dips" merge together for $\Omega_1 = -\delta$, and the "peaks" merge together for $\Omega_2 = \delta$. At these frequencies of the field, the change in the population created by one traveling wave turns out to correspond to a resonance for the other wave as well. One would therefore expect that two minima should exist in the energy of the field if it is regarded as a function of the frequency Ω. Let us demonstrate that this is actually so.

The energy radiated by an atom per unit time is given by the general formula

$$W=-\frac{e}{\mu c}\sum_{m,n}\mathbf{p}_{mn}e^{i\omega_{mn}t}\int\frac{\partial\mathbf{A}}{\partial t}f_{mn}(t,\mathbf{p},\mathbf{r})\,d\mathbf{p}\,d\mathbf{p}_0\,d\mathbf{r}. \quad (4.6)$$

In our case, instead of Eq. (4.6) we have

$$W=-\hbar\omega\,\mathrm{Re}\left\{iG\int[f^+(\mathbf{p})+f^-(\mathbf{p})]\,d\mathbf{p}\,d\mathbf{p}_0\right\}. \quad (4.7)$$

Having carried out the calculations, one finds

$$W=\hbar\omega\frac{\sqrt{\pi}}{k\bar{v}}|G|^2\left\{N_m e^{-(\Omega+\delta)^2/(k\bar{v})^2}-N_n e^{-(\Omega-\delta)^2/(k\bar{v})^2}-(N_m-N_n)\frac{|G|^2\tau}{4\Gamma}e^{-\Omega^2/(k\bar{v})^2}\left[1+\frac{\tau_m}{\tau}\frac{\Gamma^2}{\Gamma^2+(\Omega+\delta)^2}+\frac{\tau_n}{\tau}\frac{\Gamma^2}{\Gamma^2+(\Omega-\delta)^2}\right]\right\} \quad (4.8)$$

$$\bar{v}=\frac{\bar{p}}{m}=\sqrt{\frac{2kT}{m}},\quad \tau=\tau_m+\tau_n,\quad \tau_l=\frac{1}{\Gamma_l}\quad (l=m,n). \quad (4.9)$$

Formula (4.8) shows that the radiation power W has minima at the frequencies $\Omega = \mp\delta$ in accordance with what was said above. The ratio of the depths of the minima is determined by the lifetimes τ_l of the atoms in the

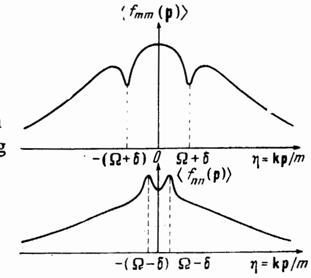

FIG. 2. Momentum distribution of the atoms in the case of a standing monochromatic wave, $\delta = 2\Gamma$.

levels. This is quite natural because the longer an atom existing in a given state can interact with the field, i.e., the longer τ_l is, the larger the change in the population of the level $l = m, n$ will be.

In connection with the derivation of formula (4.8), all Doppler factors in the term proportional to $|G|^2$ have been set equal to $\exp\{-\Omega^2/(k\bar{v})^2\}$ since this entire term is a small correction. In the first two terms, however, the exponents contain $\Omega \pm \delta$ and notwithstanding the smallness of the ratio $\delta/k\bar{v}$, it is in general impossible to discard δ (see the discussion of formula (5.2) given below).

5. SPECTRAL PROPERTIES OF A GAS LASER WITH THE RECOIL EFFECT TAKEN INTO CONSIDERATION

The special features of the saturation effect manifest themselves very strongly in the spectral characteristics of lasers. Therefore the results obtained in Sec. 4 are used below for an analysis of certain characteristic features of a laser.

Under steady-state conditions, the radiation power being emitted from the volume of a laser must be equal to the energy flux across its surface.

$$W=\oint \mathbf{S}\,d\mathbf{F}=\frac{c}{R}U. \quad (5.1)$$

Here $\mathbf{S}$ denotes the average (over a period $2\pi/\omega$) value of the Poynting vector, R is the Q of the resonator, U is the field energy stored in the resonator. From Eq. (5.1) one can determine the steady-state value of the field amplitude and the generation power. Since the latter is proportional to $E^2 \sim |G|^2$, it is sufficient to write down only an expression for $|G|^2$ which corresponds to the solution of Eq. (5.1):

$$|G|^2\frac{\tau}{4\Gamma}=\{\zeta-1-[(\Omega+\Delta)/k\bar{v}]^2\}\left\{1+\frac{\tau_m}{\tau}\frac{\Gamma^2}{\Gamma^2+(\Omega+\delta)^2}+\frac{\tau_n}{\tau}\frac{\Gamma^2}{\Gamma^2+(\Omega-\delta)^2}\right\}^{-1},$$

$$\Delta=\frac{N_m+N_n}{N_m-N_n}\delta. \quad (5.2)$$

Here ζ denotes the excess excitation above the threshold value (for the frequency $\Omega = 0$). It is clear from formula (5.2) that as a function of Ω, $|G|^2$ has two minima near those points where the resonance terms of the denominator are maximal, that is, near $\Omega = \mp\delta$. In this connection it is, of course, necessary that the width of the generation region $\Omega \equiv k\bar{v}/\sqrt{\zeta-1}$ be much greater than Γ and δ, which is usually easy to fulfill and, in addition, the shift δ due to recoil must exceed the width Γ (see Fig. 3a). If $\delta < \Gamma$, then both "dips" (Fig. 3a) merge to-

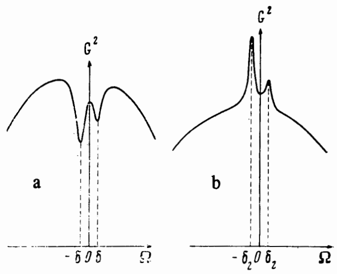

FIG. 3. Graph showing the dependence of the generation power on frequency: Fig. 3a refers to a homogeneous system with $\Delta = \delta = 2\Gamma$, $\tau_m = 2\tau_n$; Fig. 3b refers to a system containing an absorption element with $\Delta' = \delta_1 = \delta_2 = 2\Gamma_2 \ll \Gamma_1$, $\tau_{m2} = 2\tau_{n2}$

gether, and in the limiting case $\delta/\Gamma \to 0$ we arrive at the usual picture with a single dip. If $\tau_m \neq \tau_n$, then the dips have different depths, and the graph $|G|^2(\Omega)$ is asymmetric.

The shift Δ in the maximum of the numerator in (5.2) is associated with the fact that the lines of amplification and absorption without saturation are displaced by an amount 2δ [see Eq. (4.8)]. From the expression for Δ it is evident that for $N_m - N_n \ll N_m + N_n$ the magnitude of Δ may be much larger than δ. In the case $N_m - N_n \cong 10^{-2}(N_m + N_n)$, which is comparatively easy to realize in a gas laser, we obtain $\Delta \sim 10^2\delta \sim 10^7\ \text{sec}^{-1}$. Such line shifts are experimentally observed.[8]

In existing gas systems which are being used as active media, the widths Γ are appreciable ($\Gamma \sim 10^7$ to $10^9\ \text{sec}^{-1} \gg \delta$). In this connection, lasers into whose resonators an absorption cell has been introduced turn out to be of interest. One can make the gas pressure inside the cell much smaller than in the active region of the laser, which brings about a decrease of Γ. To this end, a gas different from the amplifying gas may serve as the absorbing medium.[2,3] Extremely small relaxation constants will correspond, apparently, to times of flight by atoms through that region where the field is concentrated, i.e., $\tau^{-1} \sim 10^4\ \text{sec}^{-1}$.

This direction in the development of lasers with super-stabilized frequencies is now being pursued very intensively. Therefore it is of interest to clarify the role of the recoil effect in this case.

Let us assume the same relaxation model for the absorbing gas as for the amplifying gas. We shall furnish the quantities pertaining to active and passive parts with subscripts 1 and 2, respectively. In analogy to condition (5.1) we now have the equation

$$W_1 - W_2 = \frac{c}{R} U, \tag{5.3}$$

where W_2 is the power absorbed in the cell. From (5.3) instead of (5.2) we obtain

$$\frac{|G|^2\tau_1}{4\Gamma_1} = \left[\zeta - 1 - \left(\frac{\Omega_1 + \Delta'}{k\bar{v}_1}\right)^2\right]\left\{1 + \frac{\Gamma_1{}^2\tau_{m1}/\tau_1}{\Gamma_1{}^2 + (\Omega_1 + \delta_1)^2} + \frac{\Gamma_1{}^2\tau_{n1}/\tau_1}{\Gamma_1{}^2 + (\Omega_1 - \delta_1)^2} - \frac{p_{mn2}^2}{p_{mn1}^2}\frac{N_2}{N_1}\frac{\Gamma_1}{\Gamma_2}\frac{\tau_2}{\tau_1}\left[1 + \frac{\Gamma_2{}^2\tau_{m2}/\tau_2}{\Gamma_2{}^2 + (\Omega_2 + \delta_2)^2} + \frac{\Gamma_2{}^2\tau_{n2}/\tau_2}{\Gamma_2{}^2 + (\Omega_2 - \delta_2)^2}\right]\right\}^{-1},$$

$$\Delta' = [\bar{v}_1{}^2 N_2(\omega_{mn2} - \omega_{mn1}) + \delta_1\bar{v}_2{}^2 n_1 - \delta_2\bar{v}_1{}^2 n_2]/[\bar{v}_2{}^2 N_1 - \bar{v}_1{}^2 N_2],$$

$$N_i = (N_{mi} - N_{ni})l_i, \quad n_i = (N_{mi} + N_{ni})l_i \quad (i = 1, 2), \tag{5.4}$$

l_1 and l_2 are the lengths of the active and passive parts of the laser. The resonance terms, which are determined by the active part, lead just as in (5.2) to the appearance of "dips" in the graph $|G|^2(\Omega)$. The last line in (5.4) gives maxima in the power separated by a distance 2δ and possessing widths Γ_2. In practice the case $\Gamma_2 \ll \Gamma_1$, when the "peaks" are most clearly expressed (Fig. 3b), is of interest.

If $\Gamma_1 \gg \Gamma_2 > \delta_2$, then one "peak" will exist which, generally speaking, is asymmetric (if $\tau_{m2} \neq \tau_{n2}$). The latter property is very important because stabilization of the frequency is carried out at the maximum of the "peak."[2] It is easy to show that the position of an extremum of the power in "combined peaks" is given by the formula

$$\Omega_{max} \cong \delta_2 \frac{\Gamma_{m2} - \Gamma_{n2}}{\Gamma_{m2} + \Gamma_{n2}}. \tag{5.5}$$

Thus, if the ratio Γ_{m2}/Γ_{n2} is unknown, then the position of the "peak" and at the same time the frequency of generation is determined to within the magnitude of δ_2. This fact will be important in systems with relative reproducibility of frequency of order 10^{10} to 10^{11}.

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